Non-Gaussianities in String Inflation

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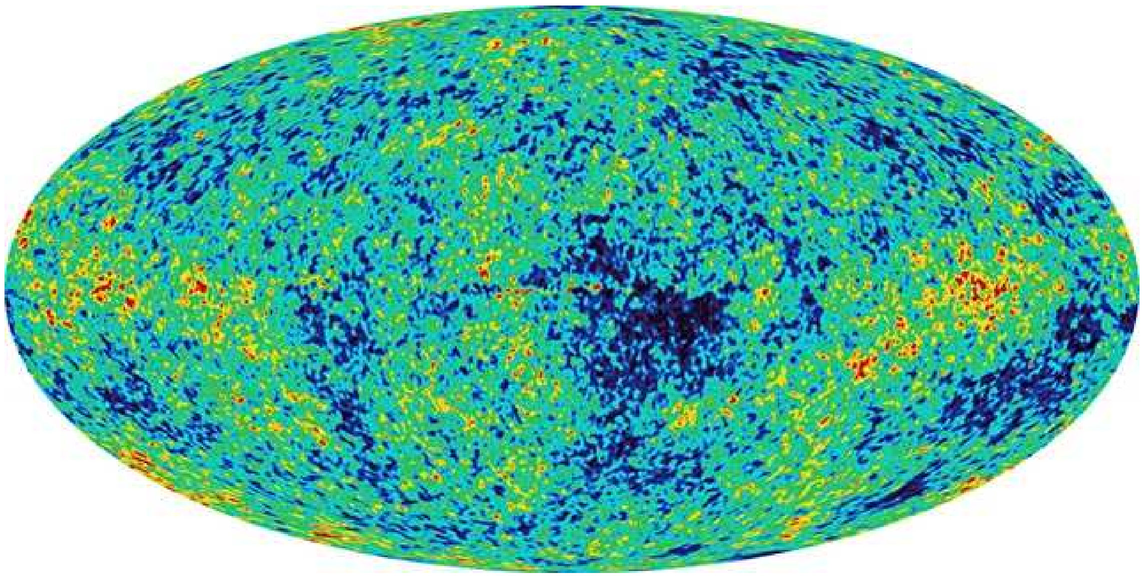
Cosmic Microwave Background



from WMAP

- An almost **scale invariant, adiabatic, Gaussian** primordial fluctuation predicted by **inflation** is in good agreement with CMB data.

- A tantalizing upper bound on the energy density during inflation:

$$V \sim M_{GUT}^4 \sim (10^{16} GeV)^4 \qquad \text{i.e.,} \qquad H \sim 10^{14} GeV.$$

  The relevant energy scale is close to the scale where **stringy physics** becomes important.

Construct inflationary models from string theory

- String theory contains higher dimensional extended objects such as D-branes.
  A simple scenario: $D - \bar{D}$ annihilation. Inflaton is the separation between D-brane and anti-D-brane. D-brane and anti-D-brane have an attractive potential. Inflation is ended by $D\bar{D}$ annihilation, providing also a mechanism for reheating.

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- Inflaton potential is not sufficiently flat:

- Need to be embedded in a setting where all other moduli are fixed.

- Flux are the most well known mechanism for fixing close string moduli in string theory. Flux generate a superpotential for the dilaton and the complex structure moduli of the compact space. Kahler moduli can be also fixed by some non-perturbative effects.

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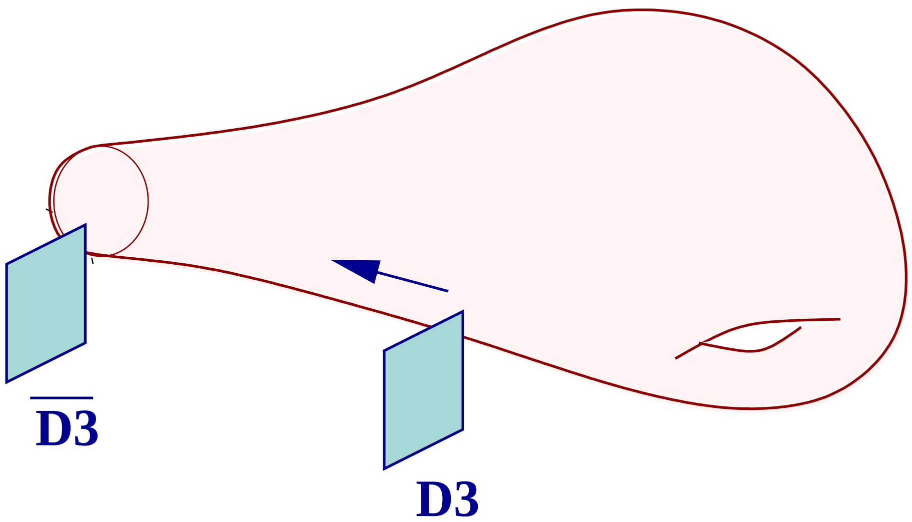

- **DBI inflation**: Warped inflation avoids the fine tuning problem, due to the scalar speed limit imposed by the Dirac-Born-Infeld action of the D-brane. (M. Alishahiha, E. Silverstein and D. Tong)

$$
\begin{aligned}
S &= \frac{M_{pl}}{2}\int d^4x\sqrt{-g}R \\
&- \int d^4x\sqrt{-q}[f(\phi)^{-1}\sqrt{1-f(\phi)(\dot{\phi}^2-(\nabla\phi)^2)}-f(\phi)^{-1}+V(\phi)]
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- Speed limit for inflaton:

$$\dot{\phi}\simeq\frac{1}{\sqrt{f(\phi)}}=\frac{\phi^2}{\sqrt{\lambda}}$$

In a highly warped throat, the inflaton can move very slowly even though the potential is not so flat, generating enough e-folding to solve the cosmological problems.

- Distinctive phenomenological features of warped inflation (DBI inflation) (X. Chen, M. Alishahiha, E. Silverstein and D. Tong):
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  - The models predict large enough primordial non-Gaussianities that can be observed in future experiments.

Our Results

- **General analysis** for an arbitrary action of the form (c.f. Garriga-Mukhaov):

$$S = \frac{1}{2}\int d^4x\sqrt{-g}[M_{pl}^2 R + 2P(X,\phi)] \qquad X = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

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- Laboratory for testing the dS/CFT proposal Strominger
  [Maldacena]; [Larsen, McNees]; [van der Schaar]

$$\langle f_{\mathbf{k}_1} f_{\mathbf{k}_2} f_{\mathbf{k}_3}\rangle' = \frac{2Re\langle\mathcal{O}_{\mathbf{k}_1}\mathcal{O}_{\mathbf{k}_2}\mathcal{O}_{\mathbf{k}_3}\rangle'}{\prod_i(-2Re\langle\mathcal{O}_{\mathbf{k}_i}\mathcal{O}_{\mathbf{k}_i}\rangle')}$$

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- Cosmic Microwave Background contains a lot of information about the primordial density perturbation $\zeta_{\mathbf{k}}$.

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- WMAP ansatz for the primordial non-Gaussianities

$$\zeta(x) = \zeta_g(x) - \frac{3}{5} f_{NL} (\zeta_g(x)^2 - \langle \zeta_g^2(x) \rangle$$

here $\zeta_g(x)$ is a purely Gaussian perturbation with vanishing three point functions.

- The size of the non-Gaussianities is measured by the parameter f_{NL} in the above ansatz. The experimental bound is

$$-58 < f_{NL} < 134 \quad \text{at } 95\% \text{ C.L.}$$

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- However, the primordial non-Gaussianities contain much more information than the power spectrum. It has a shape.

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) F(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$

The delta function constrains the three momentum to form a triangle. The shape of non-Gaussianities $F(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ is the symmetric, homogeneous function of degree -6.

- The WMAP experimental bound analyze the non-Gaussianities at the equilateral triangle limit $k_1 = k_2 = k_3$. The experimental bounds are slightly different for different shapes of non-Gaussianities. P. Creminelli, A. Nicolis, L. Senatore, M. Tegmark and M. Zaldarriaga, arXiv:astro-ph/0509029.

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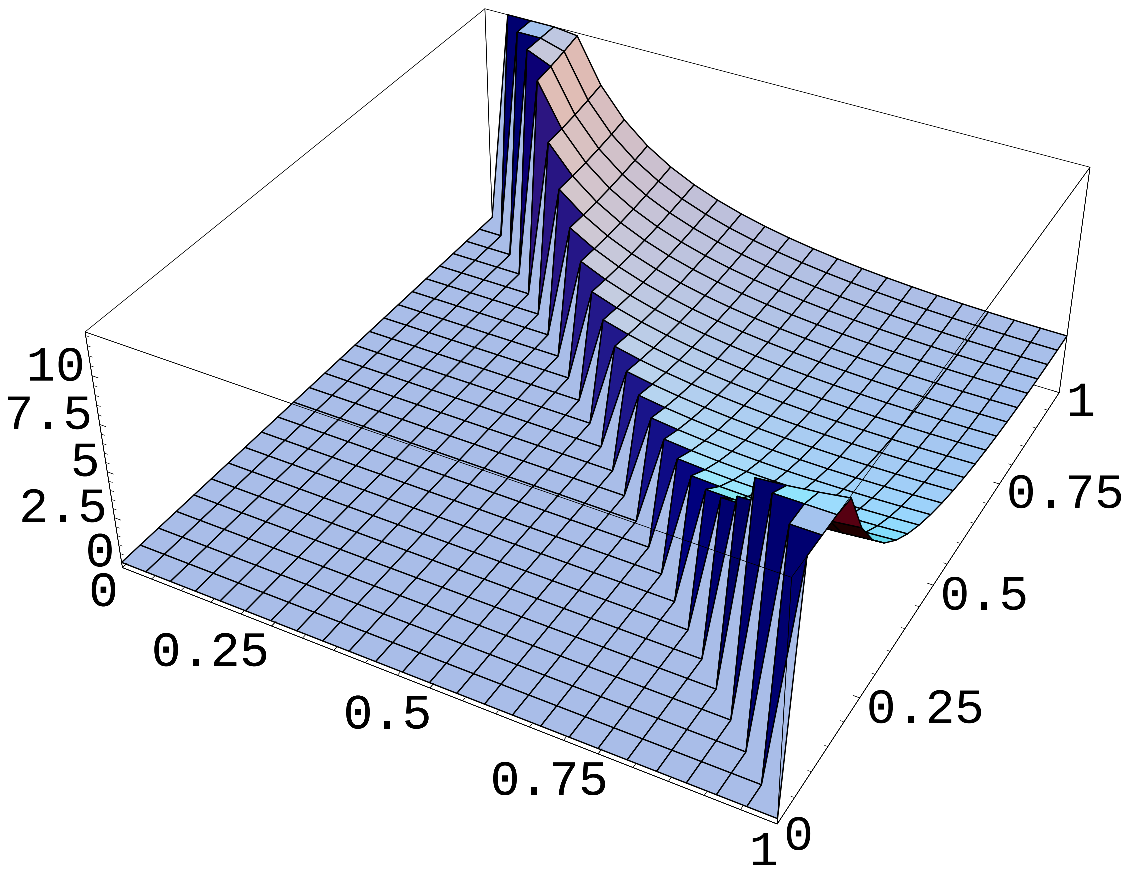
- Due to the symmetry and scaling property of the shape function, all information about the shape can be viewed by plotting

$$F(1, k_2, k_3)k_2^2 k_3^2$$

D. Babich, P. Creminelli and M. Zaldarriaga, astro-ph/0405356.

- The shape of non-Gaussianities for the WMAP ansatz

$$F(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \sim f_{NL}(P_k^{\zeta})^2 \frac{k_1^3 + k_2^3 + k_3^3}{k_1^3 k_2^3 k_3^3}$$



Computing Non-Gaussianities in an inflationary model

Computing Non-Gaussianities in an inflationary model

- It is useful to work in ADM metric formalism

$$ds^2 = -N^2dt^2 + a^2e^{2\zeta}\delta_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

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- The primordial non-Gaussianities are the three point functions of the density perturbation ζ. It is encoded in the cubic terms of the Lagrangian.

Shape of Non-Gaussianities

$$F(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = (2\pi)^4 (P_k^\zeta)^2 \frac{1}{\prod_i k_i^3} \times (\mathcal{A}_\lambda + \mathcal{A}_c + \mathcal{A}_\epsilon + \mathcal{A}_\eta + \mathcal{A}_s)$$

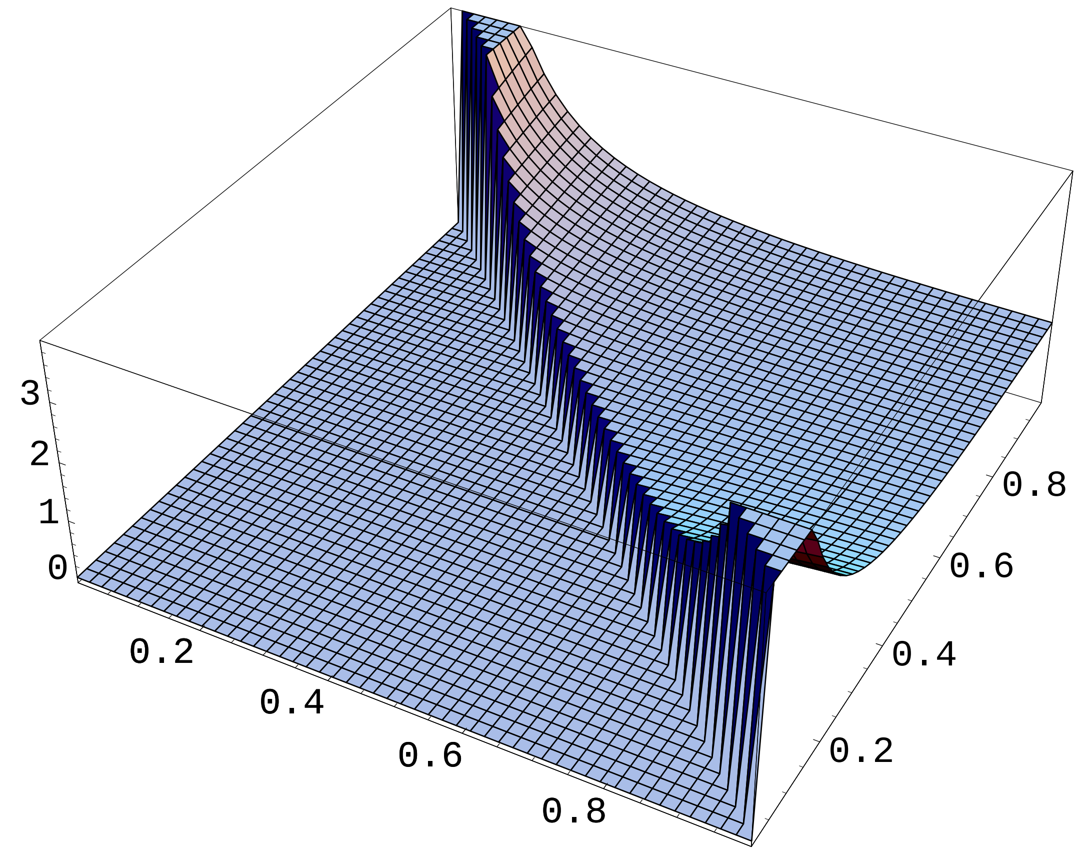
where
$$\begin{aligned}
\mathcal{A}_\lambda &= \left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma}\right) \frac{3k_1^2k_2^2k_3^2}{2K^3} \ , \\
\mathcal{A}_c &= \left(\frac{1}{c_s^2} - 1\right) \left(-\frac{1}{K}\sum_{i>j} k_i^2 k_j^2 + \frac{1}{2K^2}\sum_{i\neq j} k_i^2 k_j^3 + \frac{1}{8}\sum_i k_i^3\right) \ , \\
\mathcal{A}_\epsilon &= \frac{\epsilon}{c_s^2} \left(-\frac{1}{8}\sum_i k_i^3 + \frac{1}{8}\sum_{i\neq j} k_i k_j^2 + \frac{1}{K}\sum_{i>j} k_i^2 k_j^2\right) \ , \\
\mathcal{A}_\eta &= \frac{\eta}{c_s^2} \left(\frac{1}{8}\sum_i k_i^3\right) \ , \\
\mathcal{A}_s &= \frac{s}{c_s^2} \left(-\frac{1}{4}\sum_i k_i^3 - \frac{1}{K}\sum_{i>j} k_i^2 k_j^2 + \frac{1}{2K^2}\sum_{i\neq j} k_i^2 k_j^3\right) \ .
\end{aligned}$$

and $K = k_1 + k_2 + k_3$, $\Sigma = XP_{,X} + 2X^2 P_{,XX}$, $\lambda = X^2 P_{,XX} + \frac{2}{3} X^3 P_{,XXX}$.

- These five types of shapes are potentially observable in small sound speed limit. Two of shapes A_λ, A_c look similar to DBI inflation, three others A_ϵ, A_η, A_s look similar to the shape of slow roll inflation. We reproduce various known results above by taking different limit of the parameters. The sizes of the five types of shapes in equilateral triangle limit are

$$\begin{aligned}
f_{NL}^{\lambda} &= -\frac{5}{81}\left(\frac{1}{c_s^2}-1-\frac{2\lambda}{\Sigma}\right) , \\
f_{NL}^{c} &= \frac{35}{108}\left(\frac{1}{c_s^2}-1\right) , \\
f_{NL}^{\epsilon} &= -\frac{55}{36}\frac{\epsilon}{c_s^2} , \\
f_{NL}^{\eta} &= -\frac{5}{12}\frac{\eta}{c_s^2} , \\
f_{NL}^{s} &= \frac{85}{54}\frac{s}{c_s^2} .
\end{aligned}$$

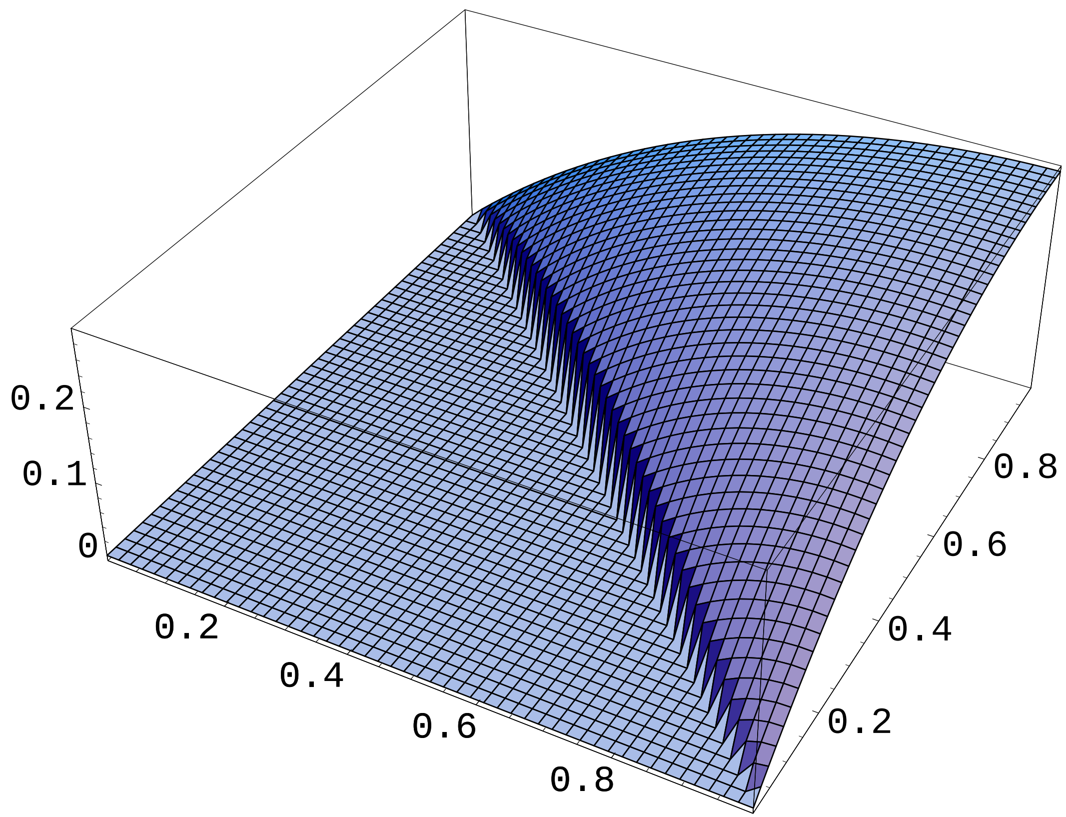
- The shape of A_ϵ is



- The shape of A_η is similar. The shapes of both slow roll inflation look similar to the shape WMAP ansatz we plot earlier.

Non-Gaussianities in DBI inflation

- The shape of non-Gaussianities vanishes in the squeeze triangle limit $k_3 \ll k_1, k_2$.



- A simple realization of all the shapes of non-Gaussianities: k-Inflation models.

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- Comparing the sizes of the two shapes

$$\begin{aligned} f_{NL}^{\lambda} &= -\frac{5}{81}\frac{16(2-\gamma)^2}{\gamma(8-3\gamma)} \\ f_{NL}^{c} &= \frac{35}{108}(\frac{8}{\gamma} - 4) \end{aligned}$$

When $\gamma \to 0$, the two shapes are both large, and have opposite signs. The DBI non-Gaussianities are 5.25 times bigger in size.

- We also compute small corrections to the first two leading contributions to non-Gaussianities. They are the same of order as the three sub-leading slow-roll type shapes. There are two possible sources of corrections

  1. Time variations of nearly constant parameters H, ϵ, η, c_s etc. Expand around the point of horizon crossing $\tau_0 = -\frac{1}{c_s k}$,

  $$f(\tau) = f(\tau_0) - \frac{1}{H_0}\frac{\partial f}{\partial t}\log(\frac{\tau}{\tau_0}) + \mathcal{O}(\epsilon^2 f(\tau_0))$$

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  2. Slow roll corrections to the density fluctuation $u(\tau, \mathbf{k})$.

- If the sound speed is small enough

  $$c_s^2 \ll \epsilon, \eta$$

  it is possible to observe the slow roll shapes A_ϵ, A_η and these correction terms. This can be realized in DBI inflation, consistent with current experimental bound on the sound speed.

Probing the inflationary vacuum with primordial non-Gaussianities

- There have been some interest in the question whether we can observe trans-Planckian physics in the CMB radiation. The answer depends on whether we can allow a slight deviation from the standard Bunch-Davies vacuum of de-Sitter space during inflation. A general phenomenological feature of these deviations is a small modulation of the power spectrum in the log scale of the fluctuation mode.

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- A different perspective: We investigate whether there are pronounced effects of non-standard vacua to be observed in non-Gaussianities.

- There are two potentially observable contributions $\tilde{A}_\lambda$ and $\tilde{A}_c$ due to deviation from Bunch-Davies vacuum. The size of the non-Gaussianities are

$$\begin{aligned} \tilde{f}^{\lambda}_{NL} &= -5 Re(C_-)(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma}) \\ \tilde{f}^{c}_{NL} &= \frac{25}{4} Re(C_-)(\frac{1}{c_s^2} - 1) \end{aligned}$$

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- In certain optimistic scenarios, the deviation from Bunch-Davies vacuum might be of the order

$$\begin{aligned}C_- &\sim \frac{H}{c_s M_{\text{cut-off}}} \\ f_{NL} &\sim \frac{1}{c_s^3}\frac{H}{M_{\text{cut-off}}}\end{aligned}$$

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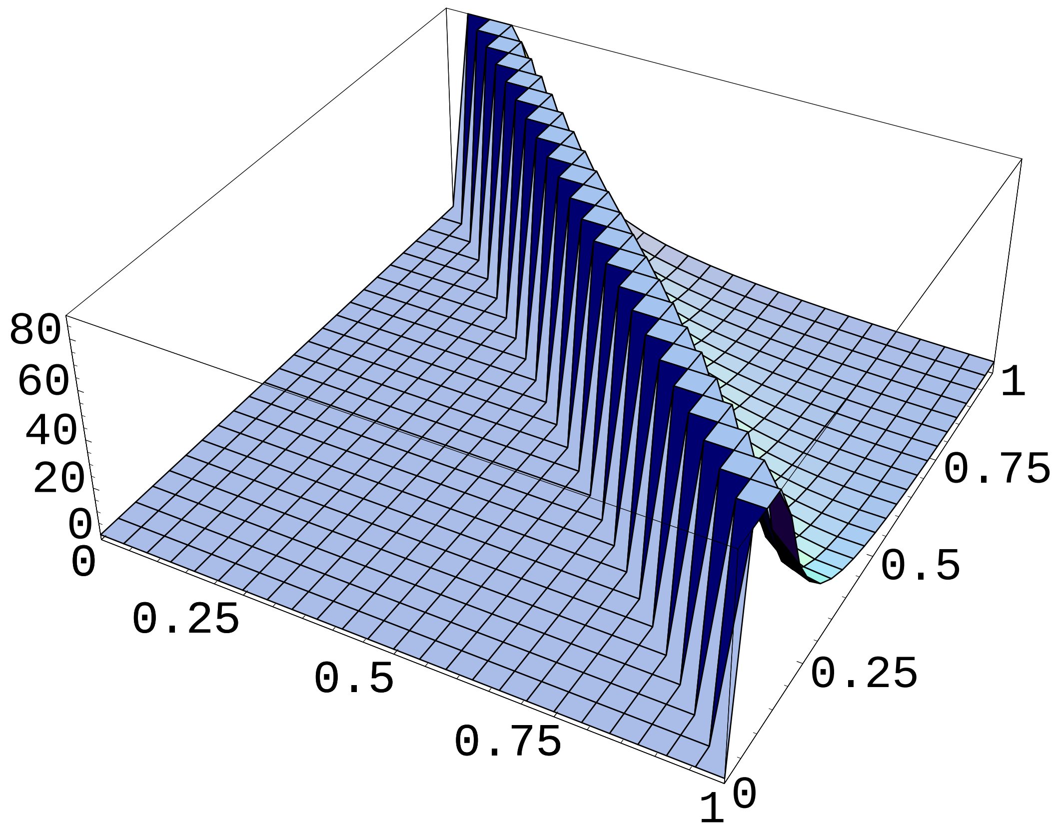
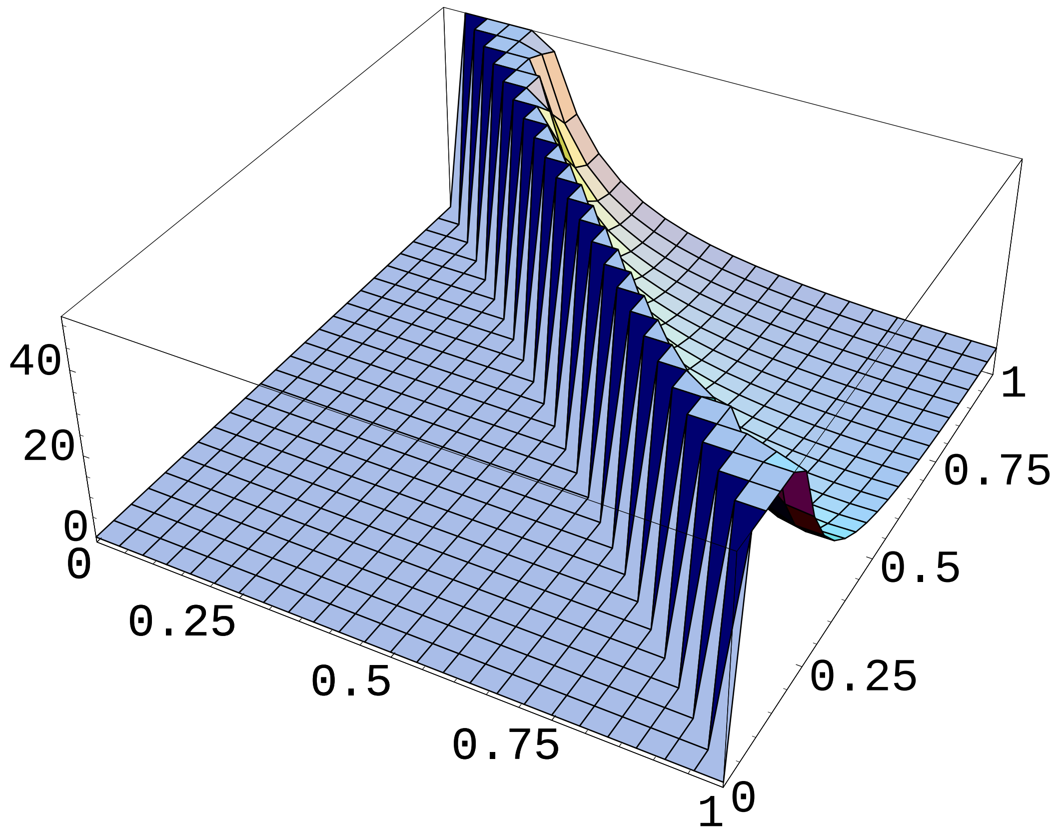
- More importantly, the shapes are very distinctive...

- The shapes are highly peaked at the "folded triangle" limit $k_1 = k_2 + k_3$ for arbitrary values of k_2, k_3. This feature is not shared by any known inflationary models. The effects are potentially more pronounced than power spectrum where the modulation is running in the log scale of the wave mode.





40
20
0
0
0.25
0.5
0.75
1
0
0.25
0.5
0.75
1

Conclusion and Outlook

- Inflation models in string theory such as DBI inflation capture many interesting features of a general inflation Lagragian with small sound speed. Some general features of these models include:

  – Modification of the "consistency relation", the tensor-scalar ratio can be much smaller tensor index.

  – Trans-Planckian effects are enhanced compared to the slow roll inflations.

  – Generically predict large non-Gaussianities.

  It is very interesting to continue this inflation model building business in fundamental theory, and see how natural and general these features are. It would be interesting to construct string models for k-inflations.

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- It will interesting to see whether the dS/CFT proposal can shed some light on the universality of the shape of non-Gausianities. This will be very useful especially for multi-field inflation.

Thank You