Dynamics of supertubes

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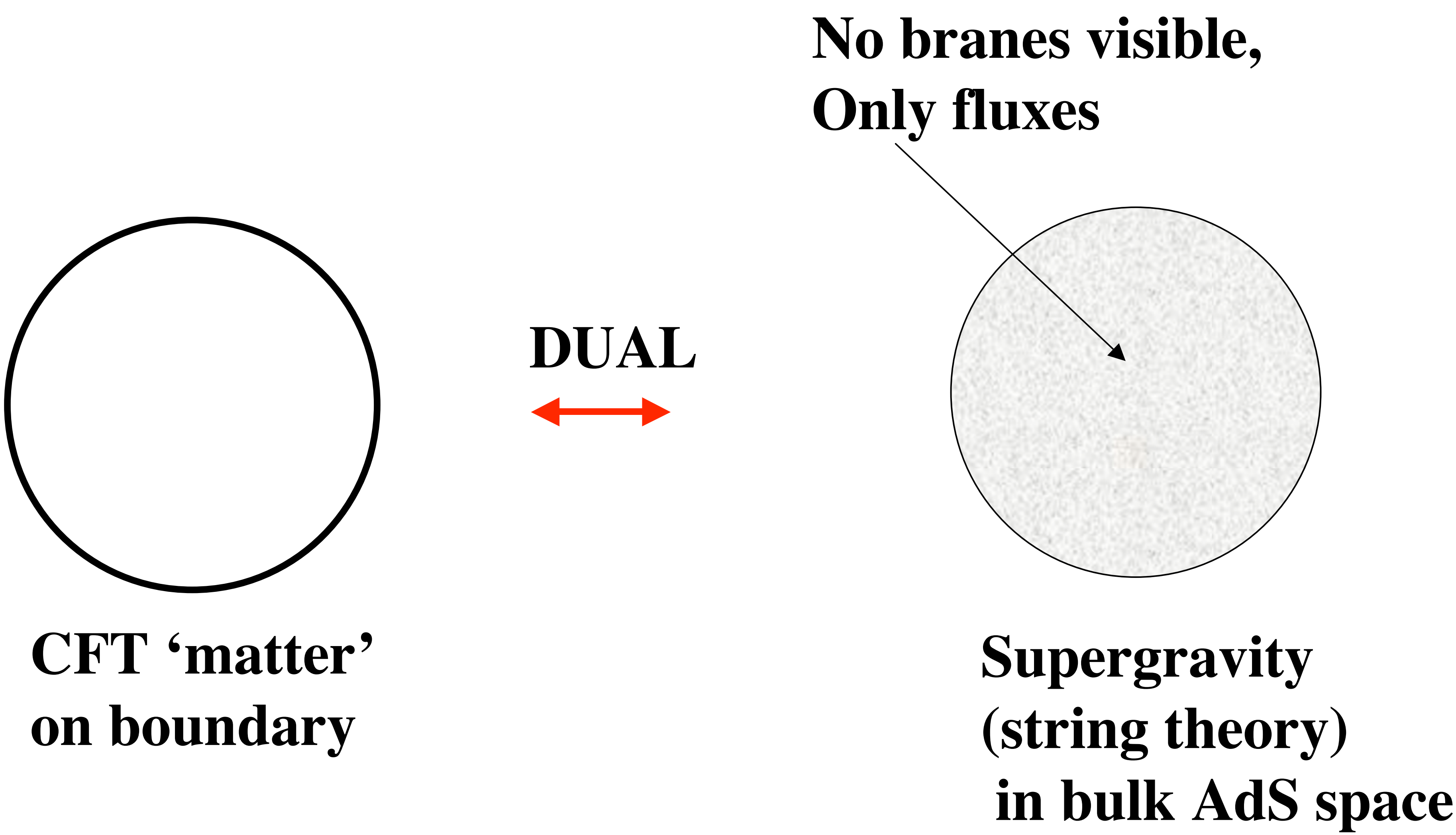
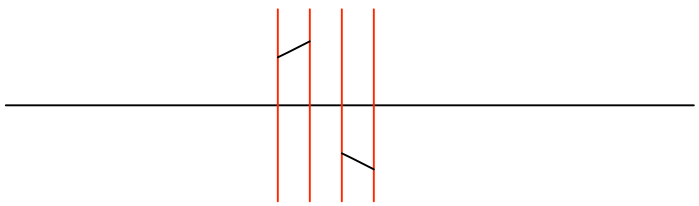
(Work with S. Giusto, O. Lunin, Y. Srivastava)

AdS/CFT duality

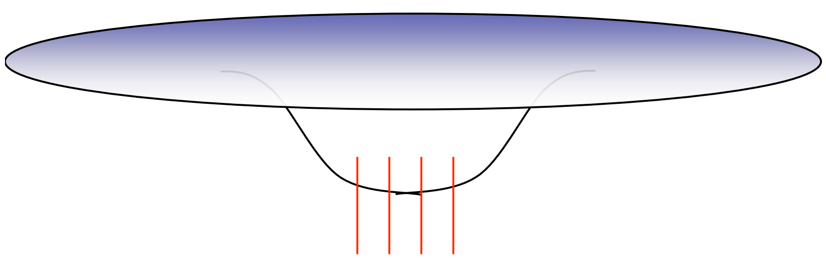


But for the black hole problem, we start a bit differently …

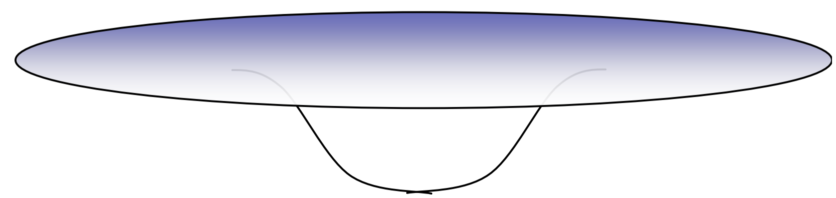


$g \to 0$ D branes in flat space, give CFT

At larger g :



OR ??



At what g can we replace the branes by supergravity?

Notation:

Type IIA string theory: gravitons, NS1, NS5

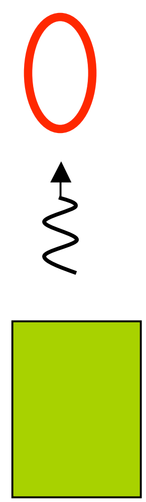
Compactify: $M_{9,1} \to M_{4,1} \times S^1 \times T^4$

Radius of S^1 **is** R_y , **Volume of** T^4 **is** $(2\pi)^4 V$

NS1 wrapped on S^1

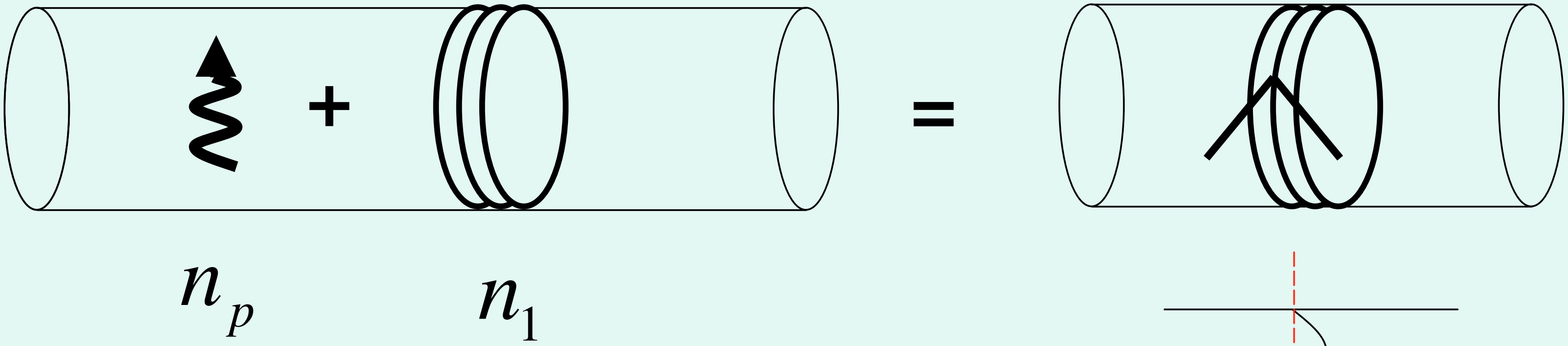
Momentum modes P along S^1

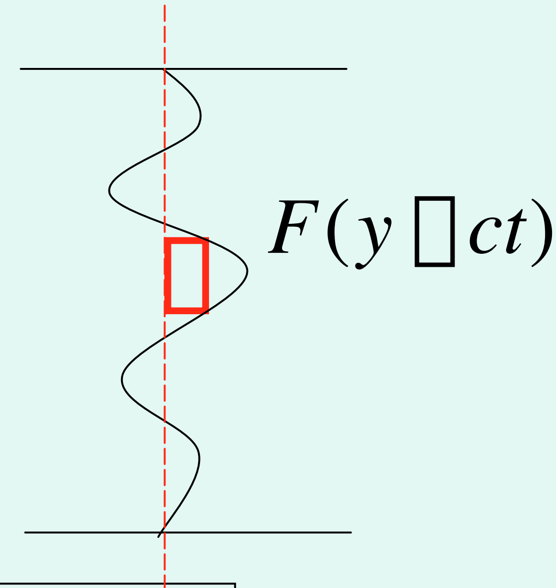
NS5 branes wrapped on $S^1 \times T^4$



Dualities permute NS1, P, NS5 in all possible ways

2-charge systems



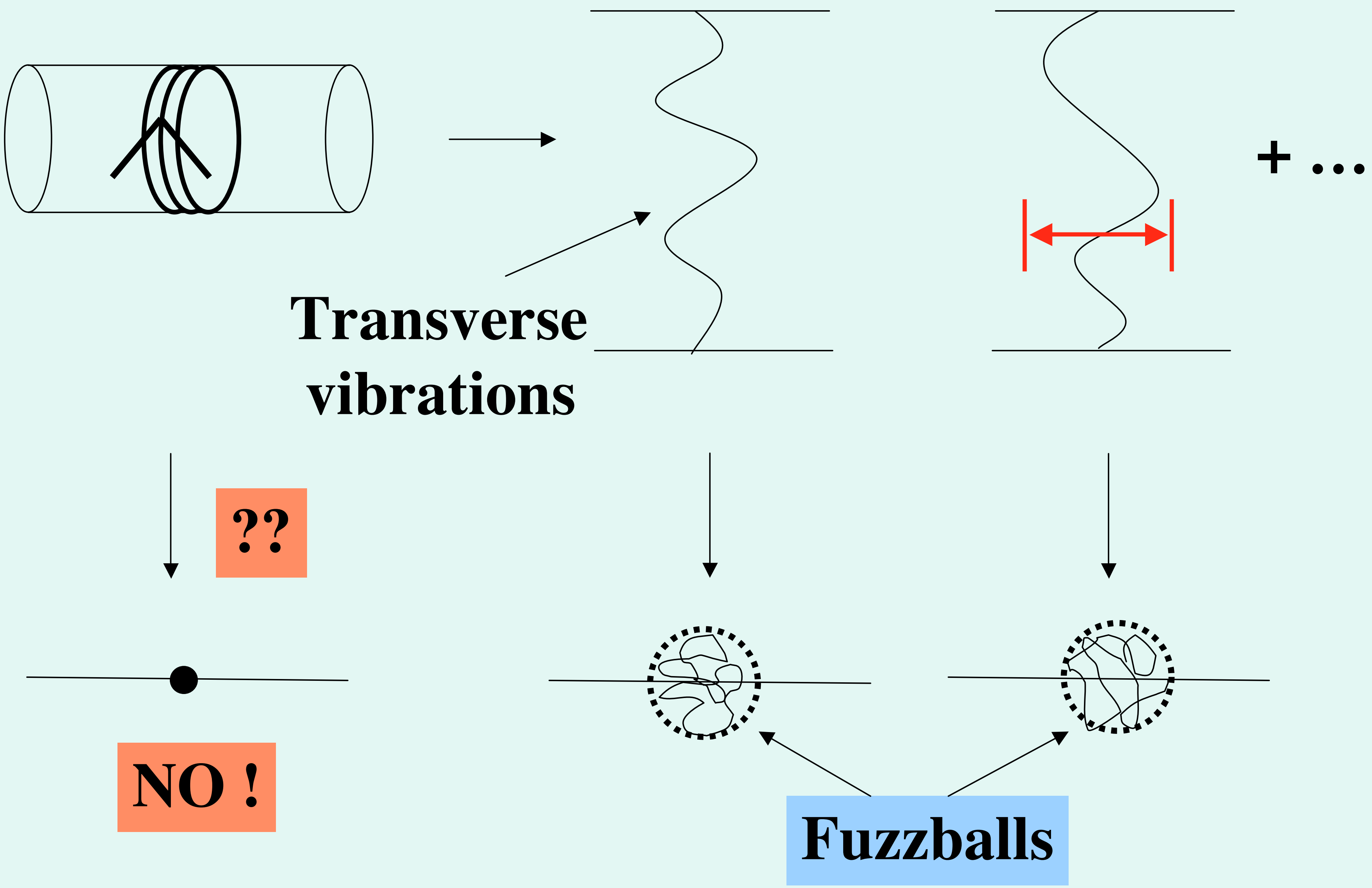


Traveling waves

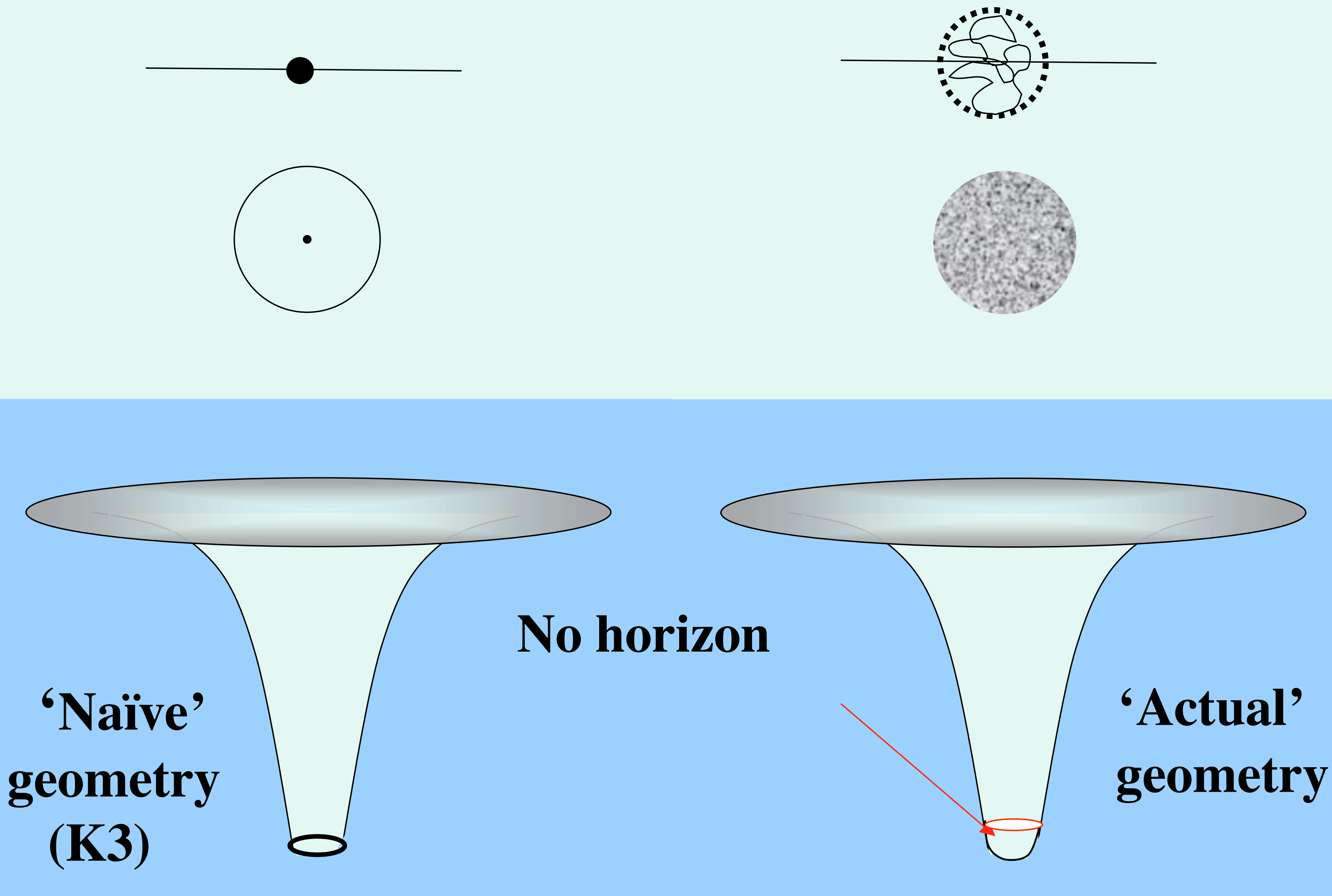
Many ways to partition the momentum among different harmonics

→ $e^{2\pi\sqrt{2}\sqrt{n_w n_p}}$ **states**

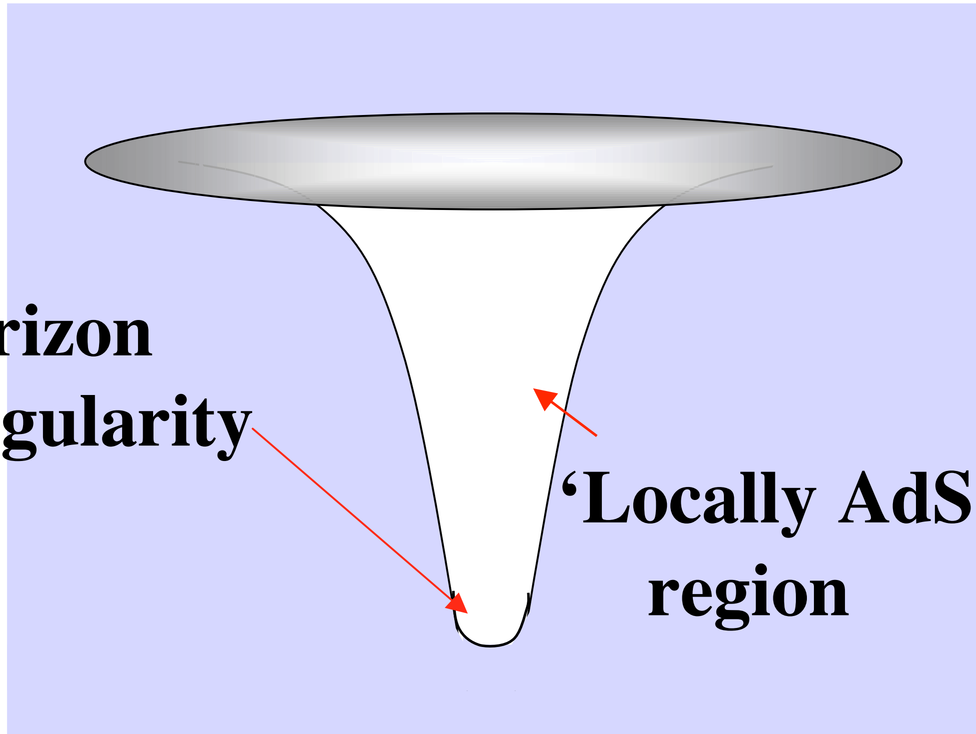
'Size' of the bound state



Geometry created by the fuzzball



Dualize NS1-P ⟶ NS1-NS5

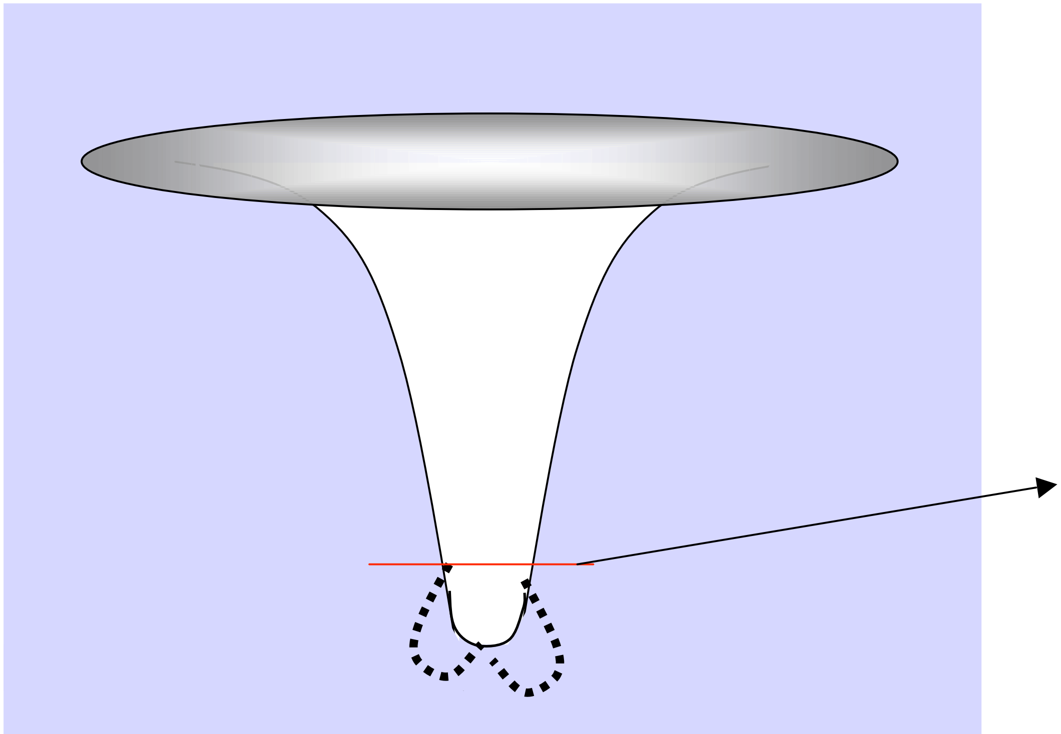


First such metric:
Balasubramanian+
De Boer+ Keski-Vakkuri
+ Ross; Maldacena+Maoz

General metrics: Lunin+SDM

Also,
`Supergravity supertubes'
Emparan+Mateos+Townsend

We have used coupling g such that the throat is much 'deeper' than its 'width'. In this case we see that there are no branes at the end of the throat.



$$\frac{A}{4G} \sim S$$

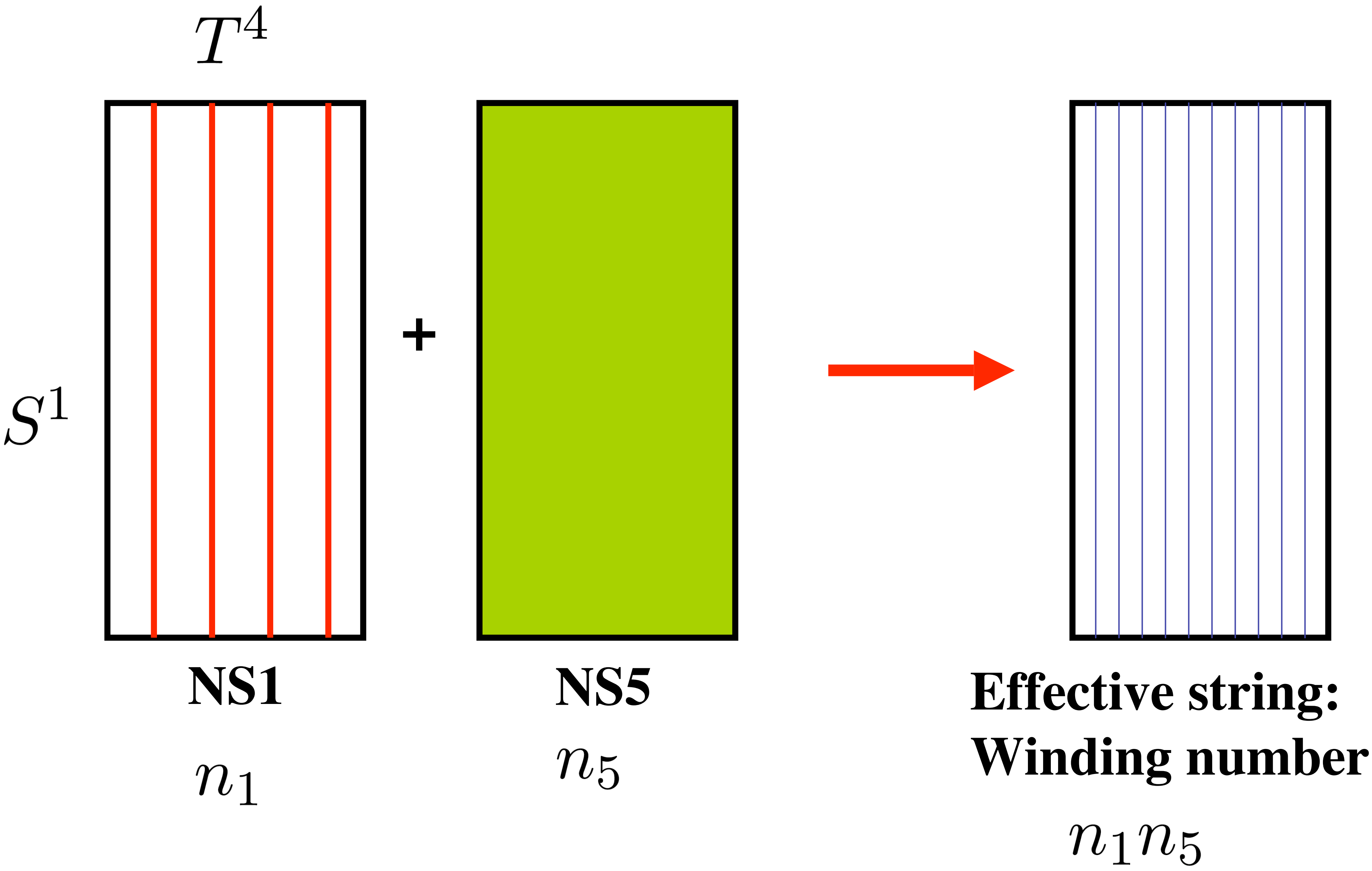
The ‘size’ of the typical fuzzball is such that the area of its surface yields a Bekenstein type relation

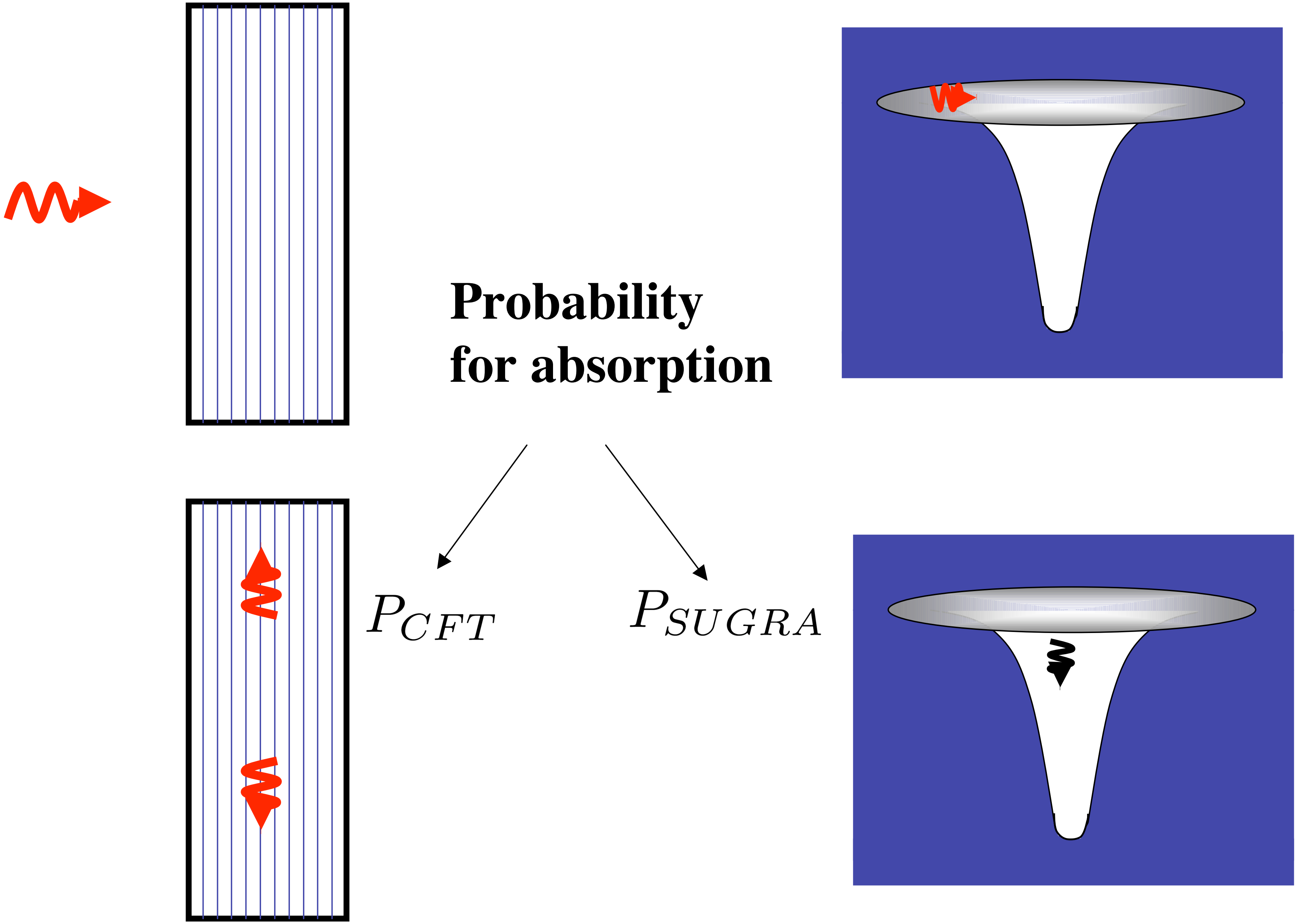
And in this domain of g (where the throat is much deeper than its width) we also get a good illustration of AdS/CFT duality …

To see this, consider the NS1-NS5 system (which has an AdS type region)

First we must understand a little bit about the NS1-NS5 CFT …

Bound state of NS1 and NS5



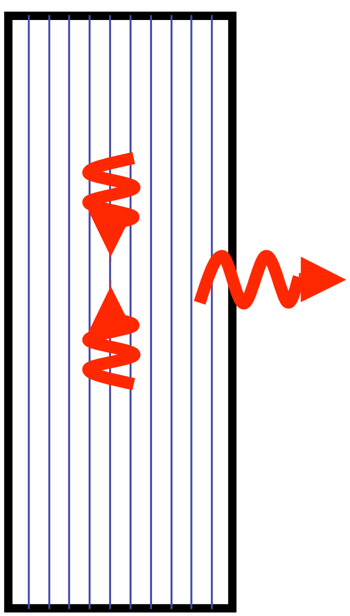

Probability
for absorption
P_{CFT}
P_{SUGRA}

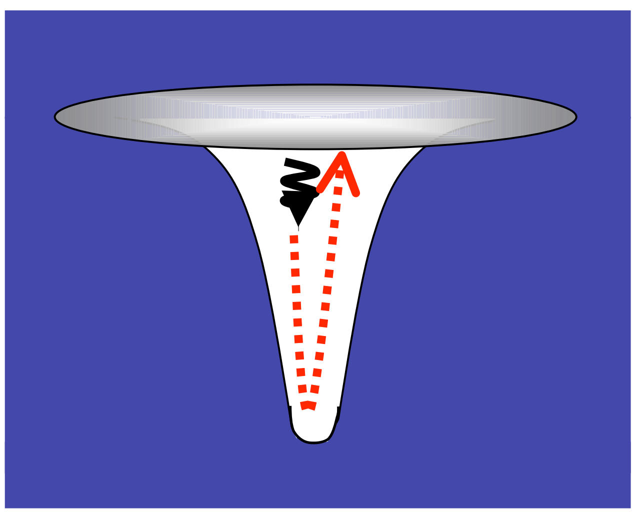
We find

$$P_{CFT} = P_{SUGRA}$$

$$P_l = 4\pi^2 \left(\frac{\bar{Q}'_1 \bar{Q}'_5 \omega'^4}{16}\right)^{l+1} \left[\frac{1}{(l+1)!l!}\right]^2$$

(Das + SDM 1996, Maldacena+Strominger 1996)





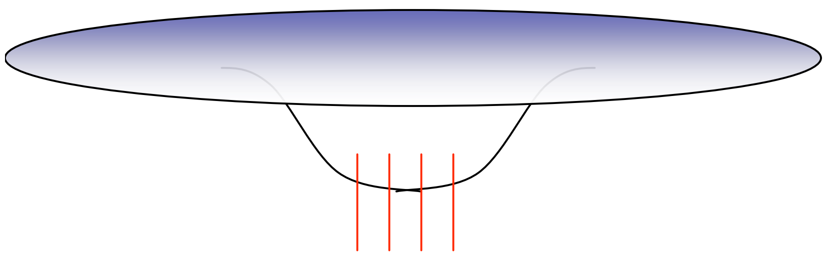
Re-emission possible after some time:

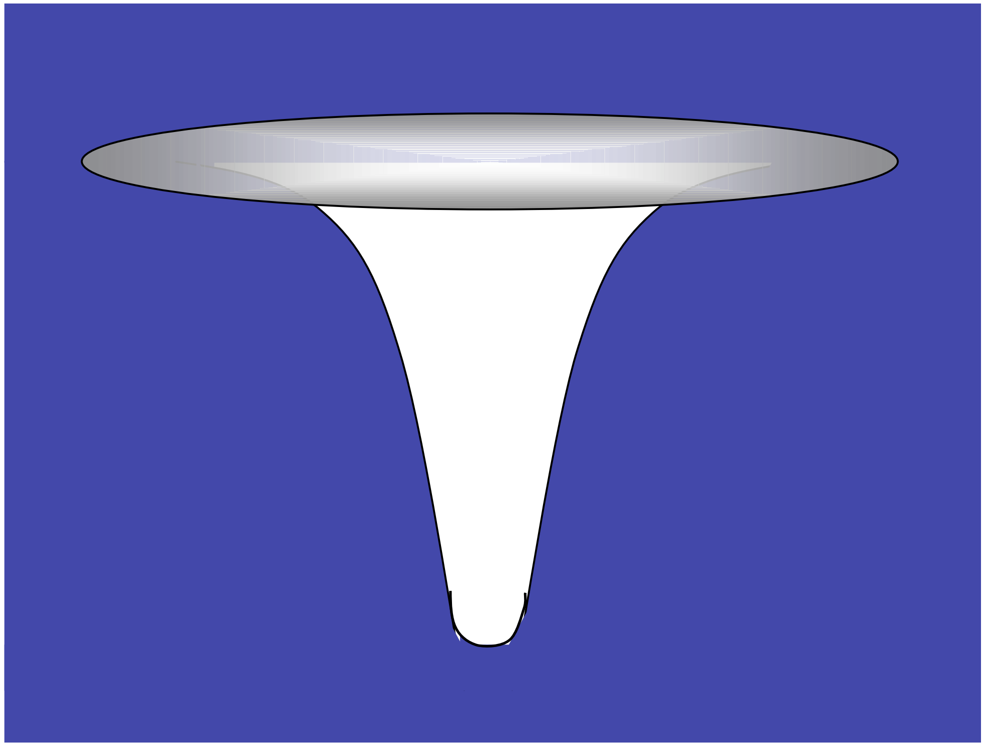
$$\Delta T_{CFT} = \Delta T_{SUGRA}$$

(Lunin + SDM 2001)

Two Questions

(A):



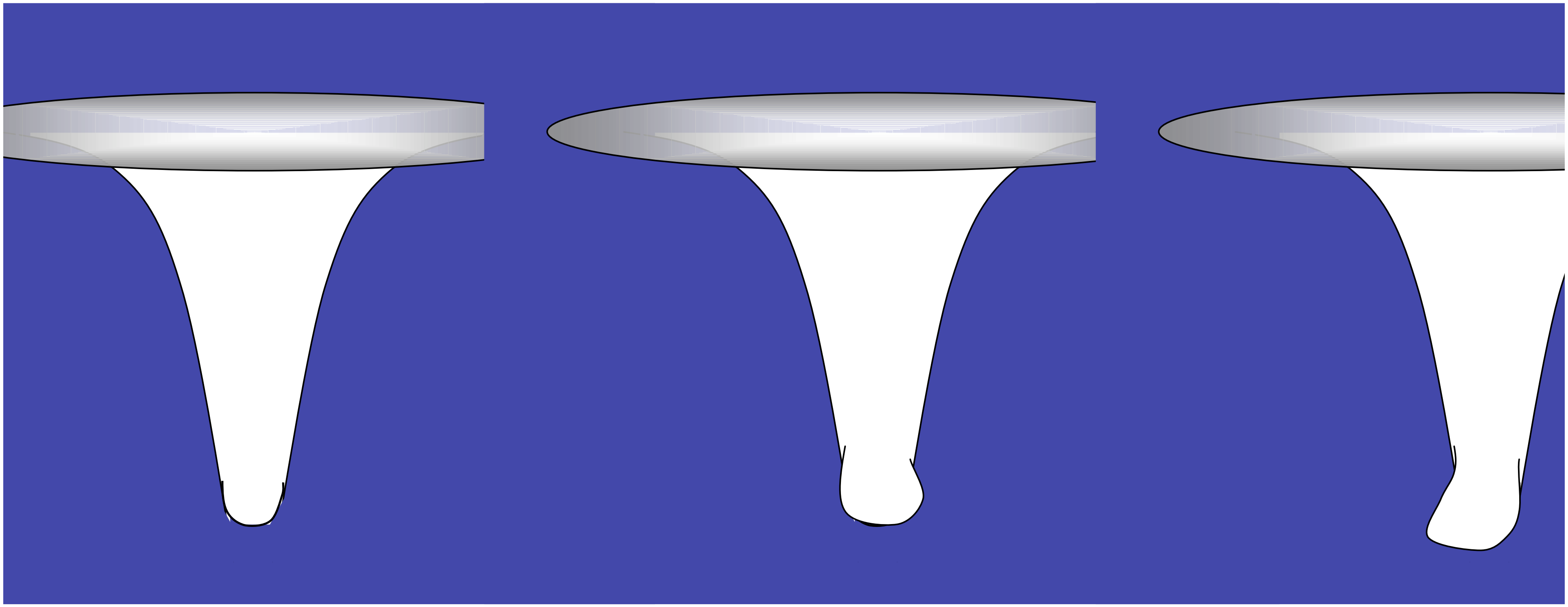


Small g, ‘see’ the branes,
Expect brane dynamics ..

Larger g, branes ‘disappear’,
Can be replaced by
Supergravity fields ….

Does the dynamical behavior change with g?
At what g does the change occur?

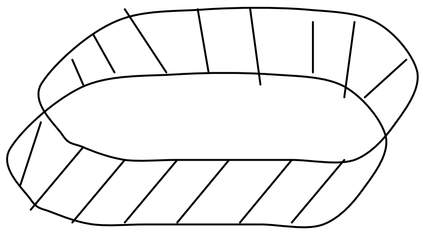
(B):



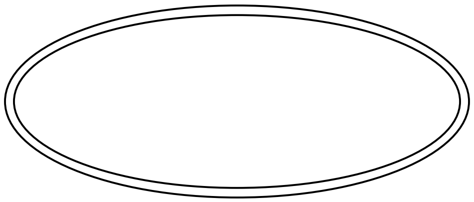
We have a continuous family of smooth supergravity solutions, all with the same mass, charge

Is the low energy dynamics a ‘drift’ over a moduli space?

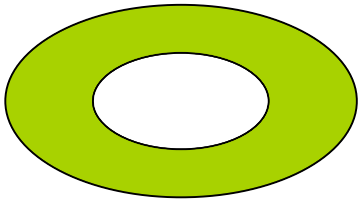
Summary of result for (A)



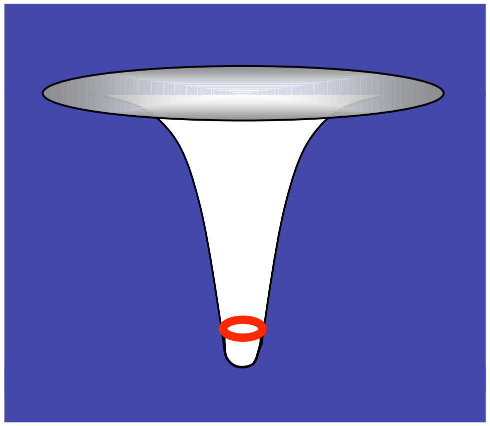
$g = 0$



$g \ll g_c$

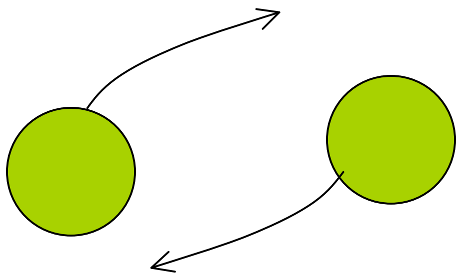


$g \sim g_c$

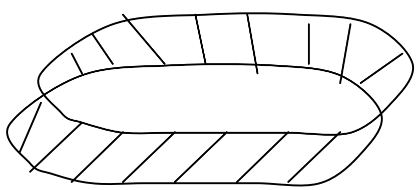


$g \gg g_c$

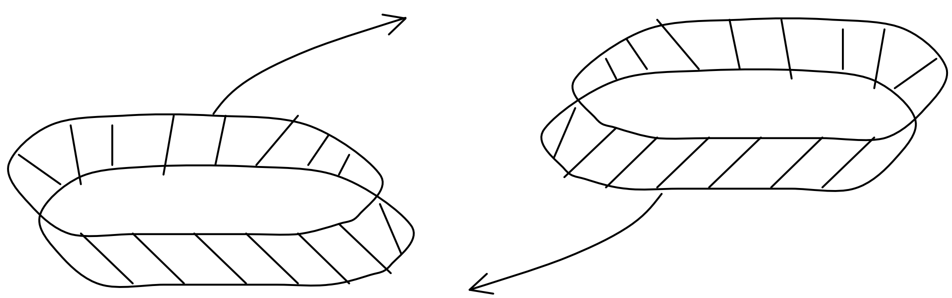
Summary of result for (B):



Unbound state, drift on moduli space



Bound state: ‘quasi-oscillations’,
NO drift on moduli space

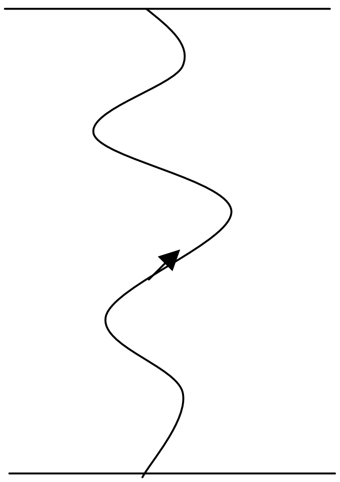


Unbound states:
Expect drift mode for center of mass degree of freedom ….

Can use to distinguish bound from unbound states …

2-charge systems: 'Phase transition' in the 'Matter picture'

A question:

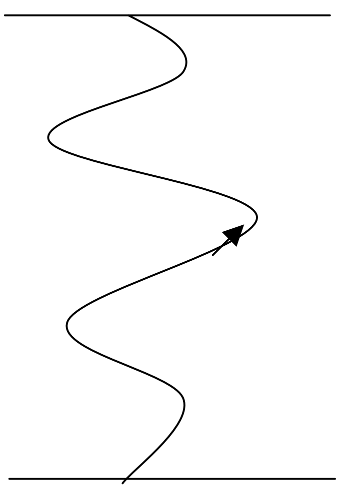


NS1-P extremal state

Add energy ΔE

What happens?

g=0, free string, we just get more excitations



Since total charges don't change, we can call this excitation

$$P\bar{P} \qquad NS1\overline{NS1}$$

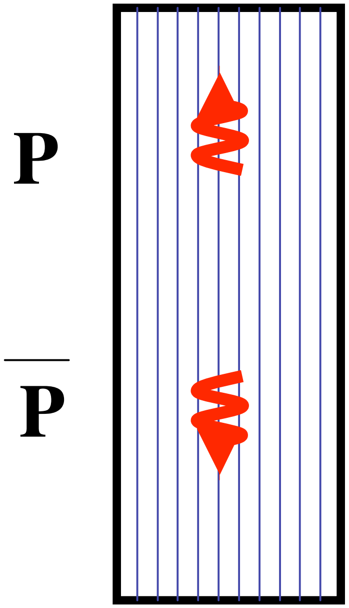
Minimum energy of excitation

$$M^2 = (\frac{R_y n_1}{\alpha'} + \frac{n_p}{R_y})^2 + \frac{4}{\alpha'} N_L = (\frac{R_y n_1}{\alpha'} - \frac{n_p}{R_y})^2 + \frac{4}{\alpha'} N_R$$

$$\delta N_L = \delta N_R = 1 \ , \qquad 2M \Delta M = \frac{4}{\alpha'}$$

$$\Delta E = \Delta M = \frac{2}{\alpha' M}$$

This is for g=0 … but it cannot be true for all g



NS1-NS5

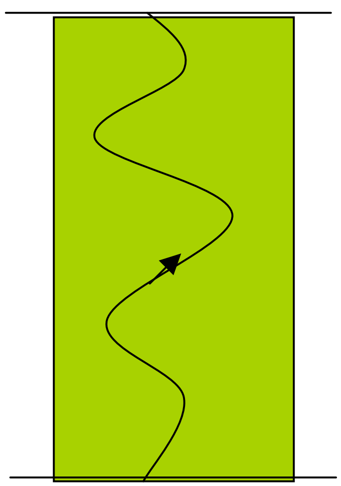
$$NS1\ NS5 + \Delta E \rightarrow P\ \bar{P}$$

By duality we can permute NS1, NS5, P

$$P\ NS1 + \Delta E \rightarrow NS5\ \overline{NS5}$$

Minimum energy of excitation:

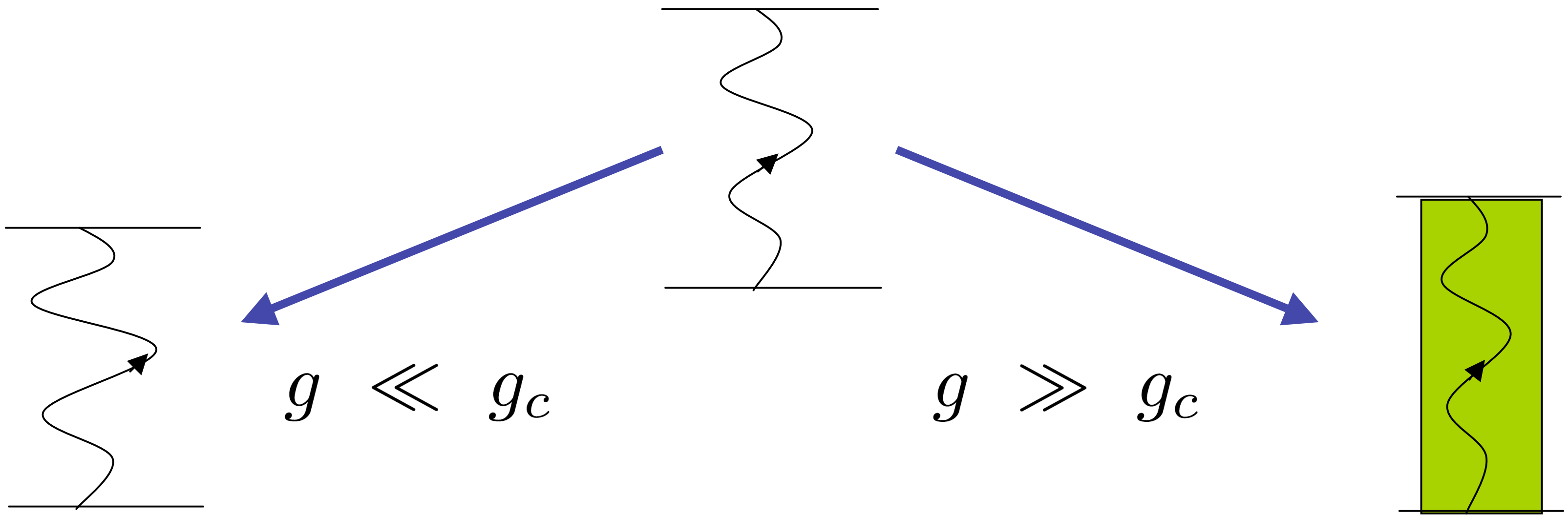
$$\Delta E = 2m_5 = 2\frac{V R_y}{g^2 \alpha'^3}$$



NS1-P

Heavy at small g, but light at large g

Phase transition - microscopic (matter) picture



$g \ll g_c$ $\qquad$ $g \gg g_c$

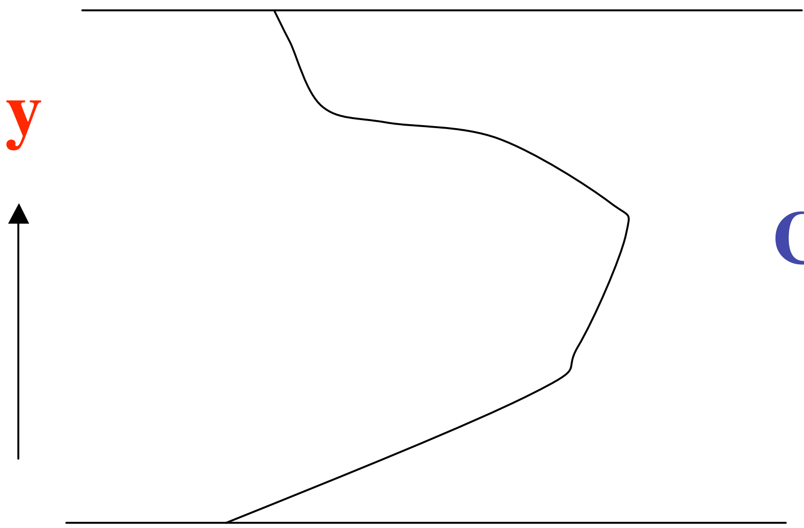
$$\Delta E = \frac{2}{\alpha' M}$$

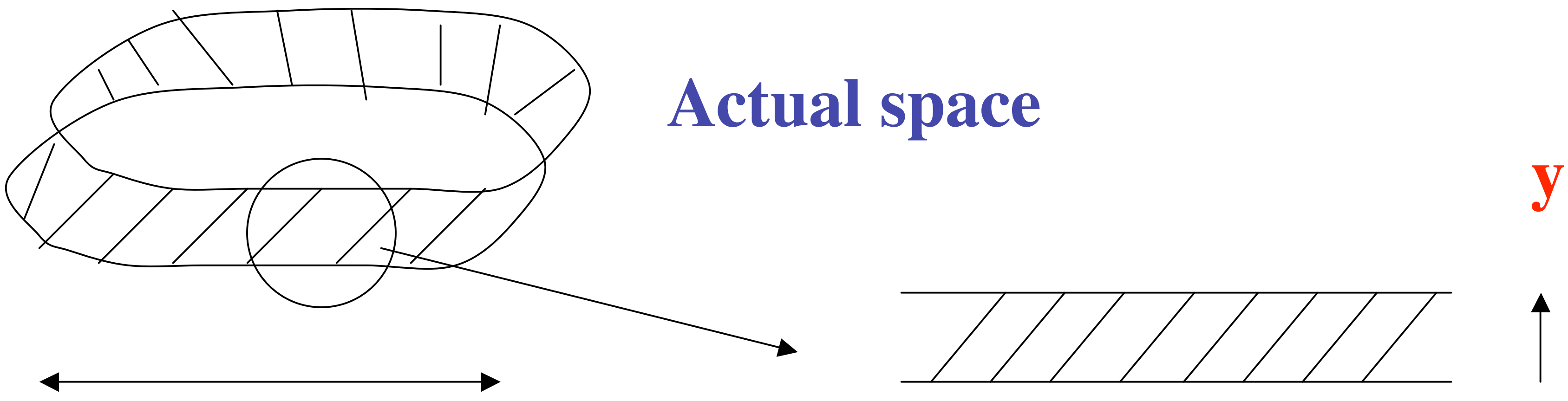
$$\Delta E = 2\frac{V R_y}{g^2 \alpha'^3}$$

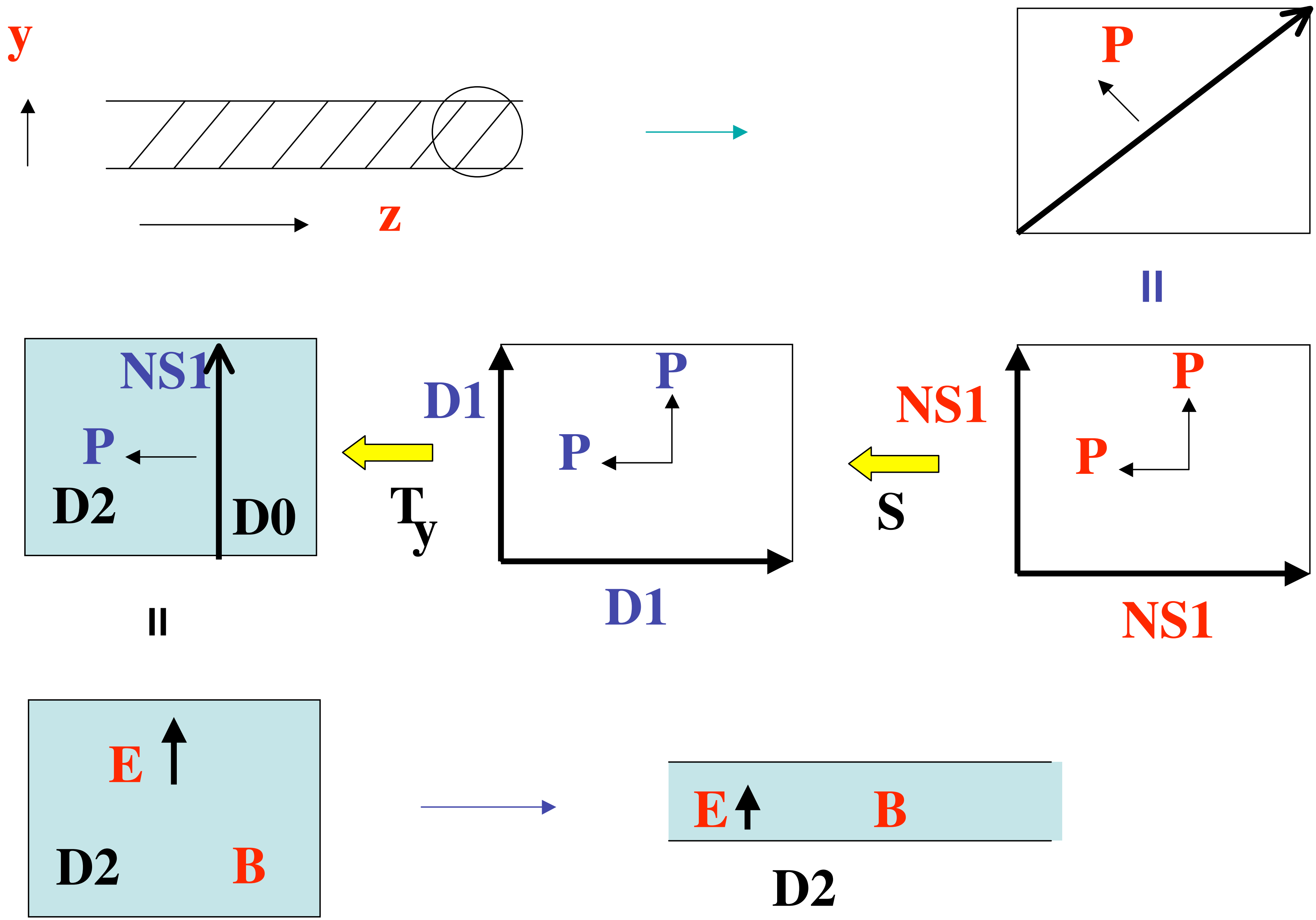
$$g_c = \sqrt{\frac{V R_y M}{\alpha'^2}}$$

2-charge systems:
'Phase transition' in the 'Gravity picture'

NS1-P






y
z
P
||
NS1
P
D2
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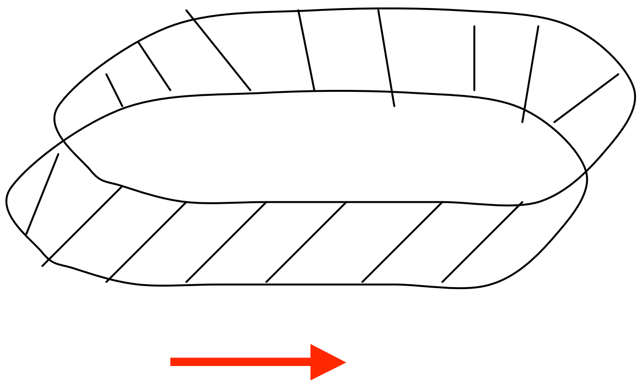
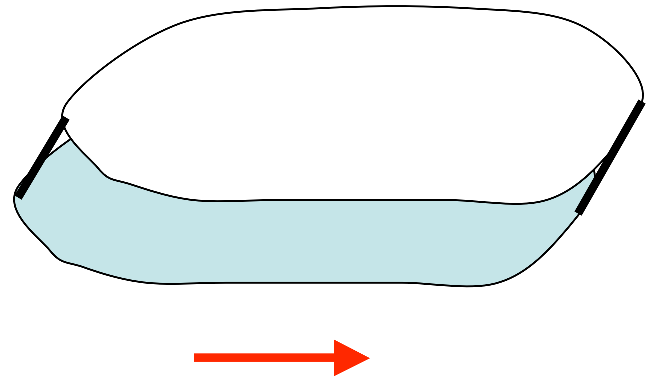
B
D2





Dipole charge NS1
True charges, NS1, P

Dipole charge D2
True charges NS1, D0

Arbitrary shape, arbitrary slope of lines at any point

E=1, arbitrary shape, arbitrary B at any point

Polyakov action for A free string

DBI action for D2

$$\mathcal{L} = -T_2 \sqrt{-\det(g+F)}$$

We wish to solve the DBI equations for general motion of the supertube.

Marolf + Palmer 2004 studied small perturbations around supertube with maximal angular momentum J

The perturbations exhibited *oscillatory* behavior

There is only one supertube with this J, so we could not have found any 'drift over moduli space' in this case

We need to look at generic supertubes.

The DBI equations look hard to solve. But we can solve the NSI-P system (free string), and then dualize …

Polyakov action:

Coordinates on world sheet $\chi^0 \equiv \hat{\tau},\ \chi^1 \equiv \hat{\sigma}$

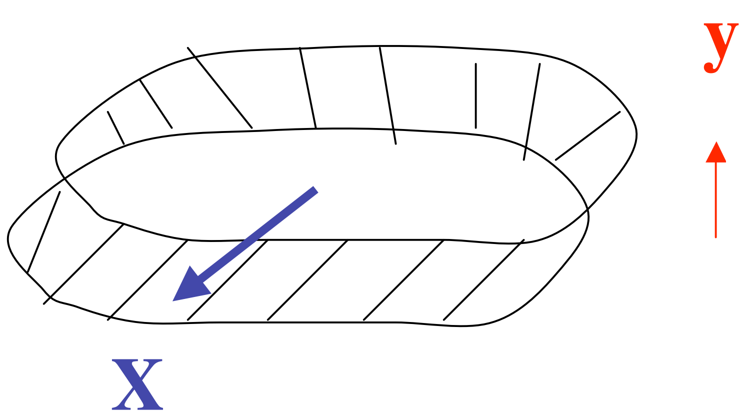
Let $\chi^+ = \chi^0 + \chi^1, \quad \chi^- = \chi^0 - \chi^1$

The solution separates into left and right movers:

$$X^\mu = X^\mu_+(\chi^+) + X^\mu_-(\chi^-)$$

With the constraints

$$\frac{\partial X^\mu}{\partial \chi^+}\frac{\partial X_\mu}{\partial \chi^+} = 0, \qquad \frac{\partial X^\mu}{\partial \chi^-}\frac{\partial X_\mu}{\partial \chi^-} = 0$$



Choose a gauge:

$$X^0 = \hat{a} + \hat{b}\hat{\tau} = \hat{a} + \hat{b}\frac{1}{2}(\chi^+ + \chi^-)$$

Solve the constraints for y

$$\partial_+ y_+ = S_+, \qquad \partial_- y_- = -S_-$$

where

$$S_+ = \sqrt{\frac{\hat{b}^2}{4} - \partial_+ X_+^i \partial_+ X_+^i}\,, \qquad S_- = \sqrt{\frac{\hat{b}^2}{4} - \partial_- X_-^i \partial_- X_-^i}$$

We can now dualize this to get the solution for the D2 Supertube …

Solution of DBI action for D2:

$$X^i = X^i_+(\chi^+) + X^i_-(\chi^-)\,, \qquad A_y = y_+(\chi^+) + y_-(\chi^-)$$

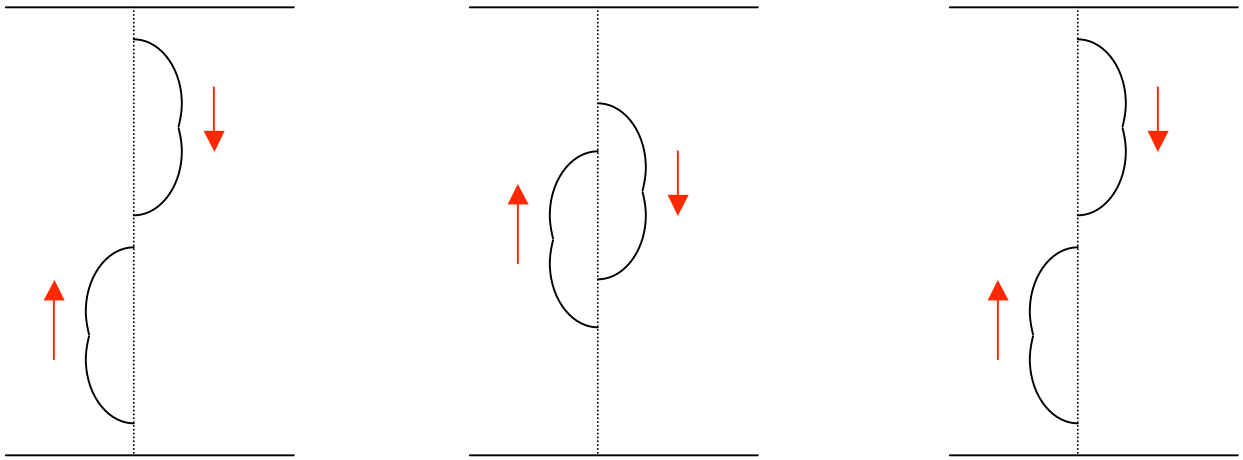
$$E = \partial_{\hat{\tau}} A_y = \partial_+ y_+ + \partial_- y_- = S_+ - S_-$$

$$B = -\partial_{\hat{\sigma}} A_y = -\partial_+ y_+ + \partial_- y_- = -(S_+ + S_-)$$

We can also look at the small oscillations around the equilibrium configuration, to get a better picture of this dynamics …

But it can already be seen that all motion is ‘periodic’, not a ‘drift over moduli space’ …

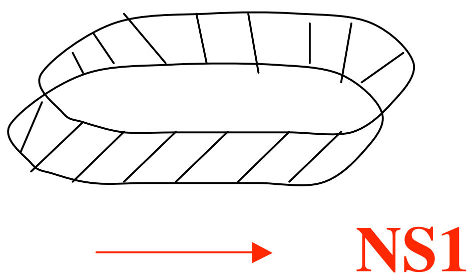
NS1-P: Waves travel around the string and come back



Time period

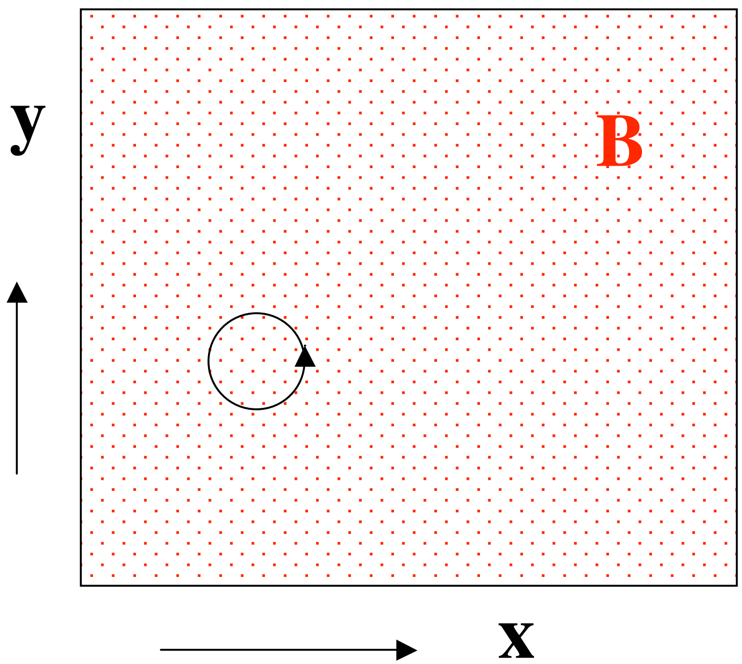
$$\Delta t = \alpha' \pi E = \frac{1}{2}\frac{E}{m_d} = \frac{\text{Total energy of supertube}}{\text{2 X dipole mass per unit length}}$$

$$m_d = \frac{1}{2\pi\alpha'}$$



We had a family of degenerate configurations, but the system did not 'drift' along this family …

An example: particle in a magnetic field



$$\ddot{x} = \frac{e}{m}\dot{y}, \qquad \ddot{y} = -\frac{e}{m}\dot{x}$$

'Quasi-oscillations'

$$v \sim \epsilon, \quad \Delta t \sim 1, \quad \Delta x \sim \epsilon$$

Compare: 'drift'

$$v \sim \epsilon, \quad \Delta t \sim \frac{1}{\epsilon}, \quad \Delta x \sim 1$$

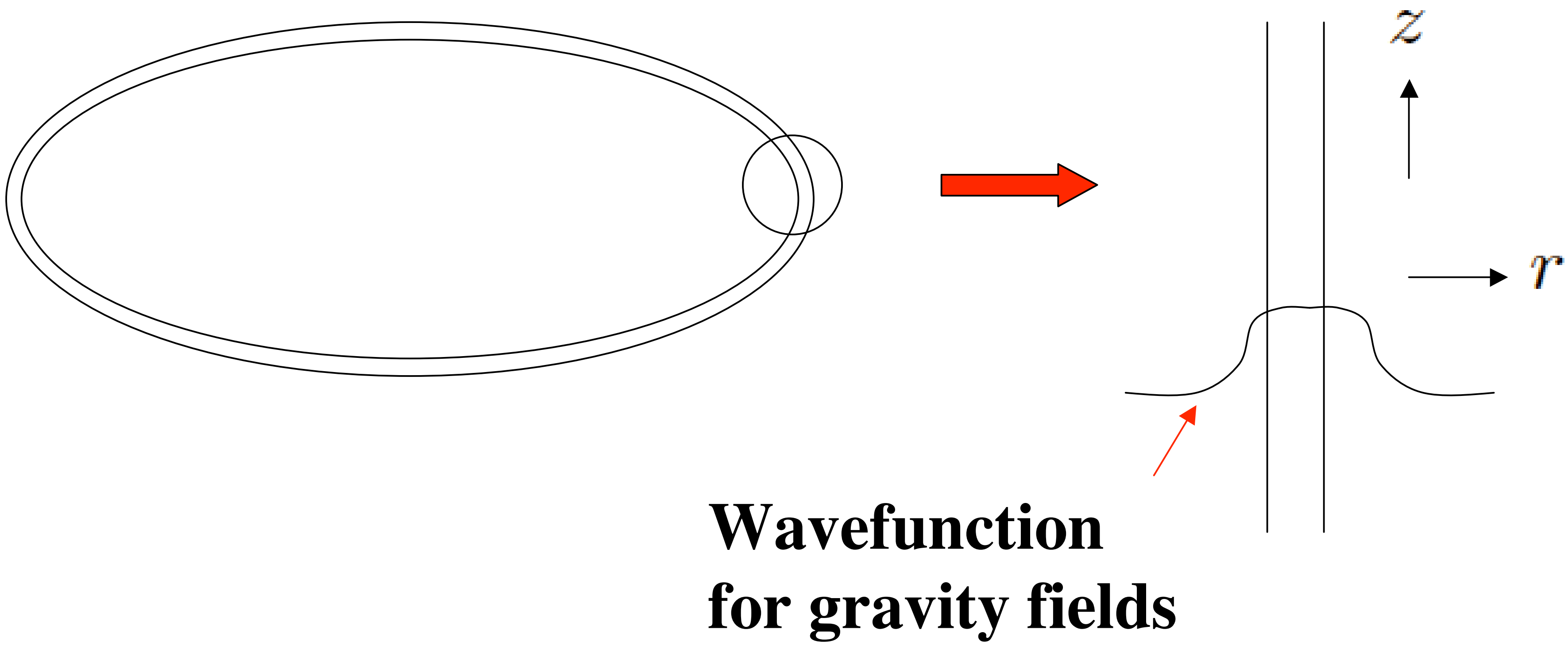
A little work shows that the supertube equations of motion are exactly like the equations for the particle in a magnetic field

Thus the supertube has 'quasi-oscillations' instead of usual oscillation zero modes which would give 'drift along moduli space'

But this was all at g=0 …. What happens as we increase g ?

Small but nonzero coupling g: Supertube is a 'thin ring'

Ring thickness shows range of gravitational field



Metric around the 'thin tube'

$$
\begin{aligned}
ds^2_{string} &= H^{-1}\,[-2dt\,dv + \tilde{K}\,dv^2 + 2A\,dv\,dz] + dz^2 + dx_i dx_i + dz_a dz_a \\
B &= (H^{-1} - 1)\,dt \wedge dv + H^{-1}\,A\,dv \wedge dz \\
e^{2\Phi} &= H^{-1}
\end{aligned}
$$

$$
H = 1 + \frac{Q_1}{r}\,, \quad \tilde{K} = 1 + K = 1 + \frac{Q_p}{r}\,, \quad A = \frac{\sqrt{Q_1\,Q_p}}{r}
$$

Perturbation: String oscillation in a torus direction

$$
ds^2_{string} \to ds^2_{string} + 2\,\mathcal{A}^{(1)}\,dz_{\bar{a}}\,, \qquad B \to B + \mathcal{A}^{(2)} \wedge dz_{\bar{a}}
$$

Let $\mathcal{A}^{\pm} = \mathcal{A}^{(1)} \pm \mathcal{A}^{(2)}$

$$\mathcal{A}_v^- = (\tilde{\alpha} + \tilde{\beta})\, H^{-1}\, (Q_1 - Q_p)\, e^{ik\, z - i\omega\, t}\, \frac{e^{-|\tilde{k}|\, r}}{r}$$

$$\mathcal{A}_t^- = -2(\tilde{\alpha} + \tilde{\beta})\, H^{-1}\, Q_1\, e^{ik\, z - i\omega\, t}\, \frac{e^{-|\tilde{k}|\, r}}{r}$$

$$\mathcal{A}_z^- = -2(\tilde{\alpha} + \tilde{\beta})\, H^{-1} \sqrt{Q_1 Q_p}\, e^{ik\, z - i\omega\, t}\, \frac{e^{-|\tilde{k}|\, r}}{r}$$

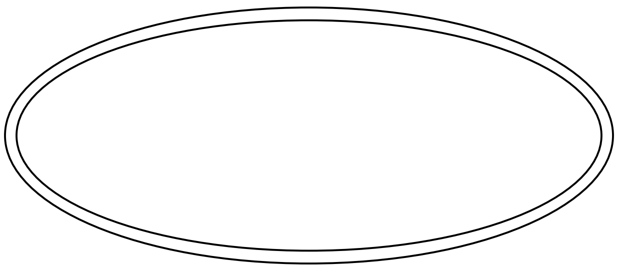
$$\omega = -k\, \frac{2\sqrt{Q_1 Q_p}}{Q_1 + Q_p}$$

$$\tilde{k}^2 = k^2 - \omega^2 = k^2 \left(\frac{Q_1 - Q_p}{Q_1 + Q_p}\right)^2$$

Period of oscillations

Speed of wave along tube

$$v = \frac{\omega}{|k|} = 2\frac{\sqrt{Q_1 Q_p}}{Q_1 + Q_p}$$



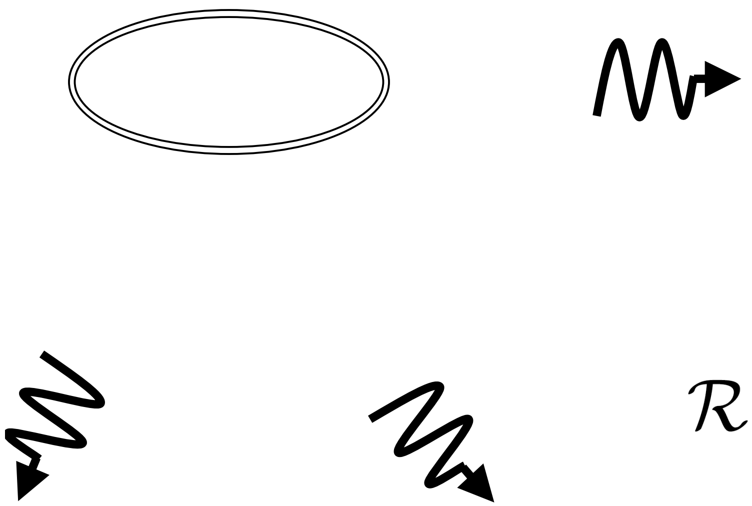
$$\Delta t = \int_0^{L_z} \frac{dz}{v} = \int_0^{L_z} dz \frac{Q_1 + Q_p}{2\sqrt{Q_1 Q_p}} = \frac{1}{2}\int_0^{L_z} dz [\sqrt{\frac{Q_1}{Q_p}} + \sqrt{\frac{Q_p}{Q_1}}]$$

$$= \quad \frac{1}{2T}(M_{NS1} + M_P)$$

This agrees with the period found for the g=0 supertube

But far away from the tube …

$$\Box\Psi = 0$$



$$\Psi = e^{-i\omega\, t}\,\mathcal{R}(\bar{r})\,Y^{(l)}(\theta,\phi,\psi)$$

$$\mathcal{R} = \frac{r_+\, e^{i\omega\,\bar{r}} + r_-\, e^{-i\omega\,\bar{r}}}{\bar{r}^{3/2}}(1 + O(\bar{r}^{-1}))$$

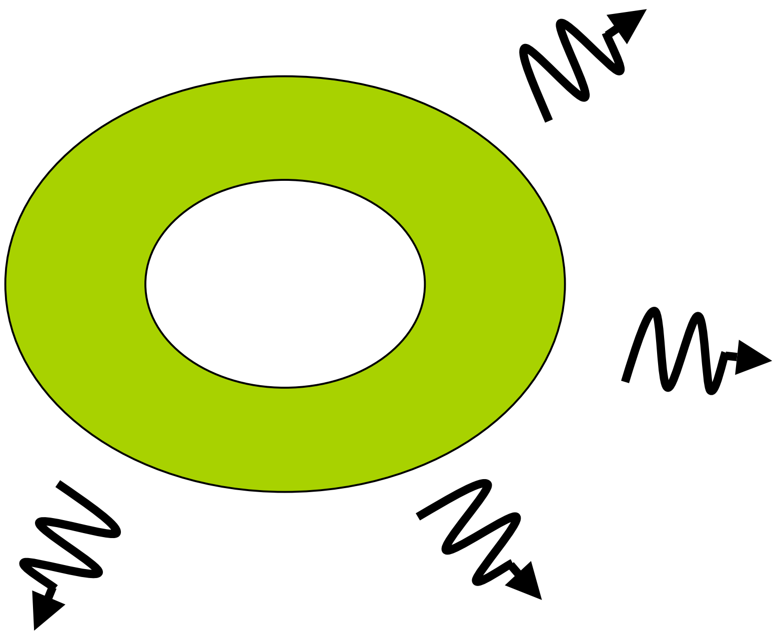
$$e^{-i\omega\, t}, \quad \omega^2 > 0$$

Oscillation mode embedded in the continuum

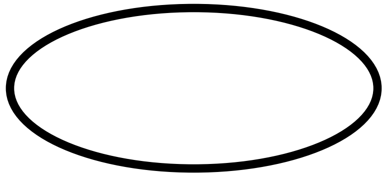
Mixes with the radiation modes of the gravitational field

Energy of oscillation leaks off to infinity, but only slowly since amplitude is small in radiation zone

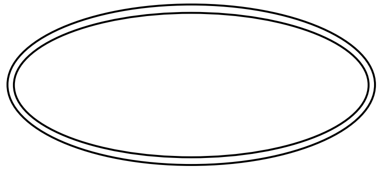
At still larger values of the coupling g …



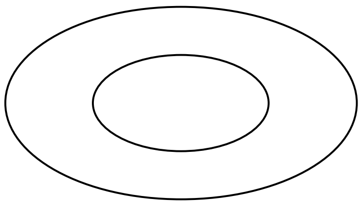
**No oscillation mode at all,
Energy radiates away on same timescale as period of oscillation**



g=0, periodic oscillations

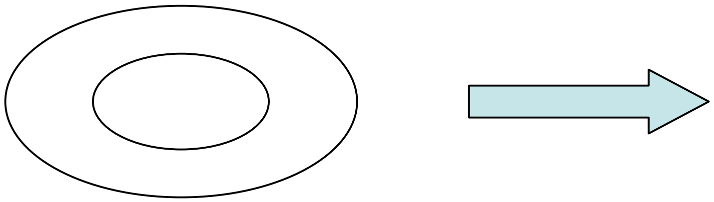


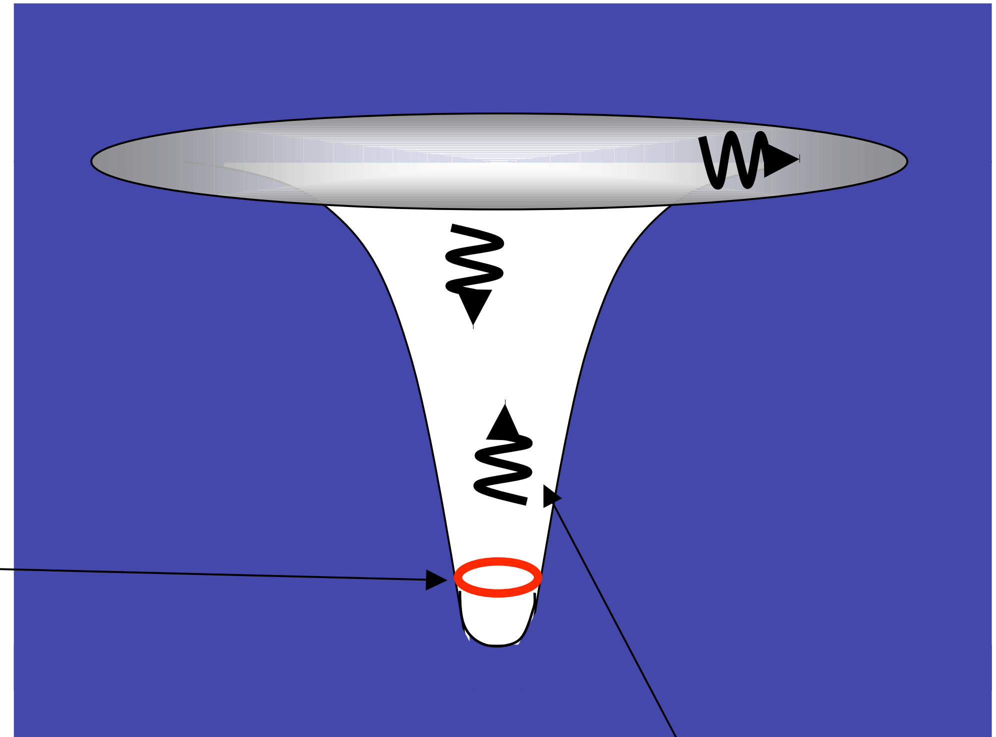
small g, long lived oscillations



larger g, no oscillatory behavior

Let us increase g still further …





Location of original supertube

Supergravity quanta bounce up and down the throat for long times, Slow leakage to infinity

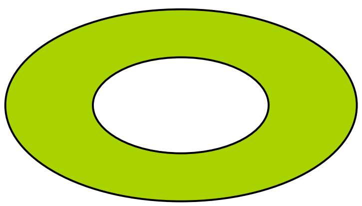
So we again find long lived excitations …

$$\beta = \frac{(\bar{Q}_1 \bar{Q}_p)^{\frac{1}{4}}}{a}$$

a = radius of ring

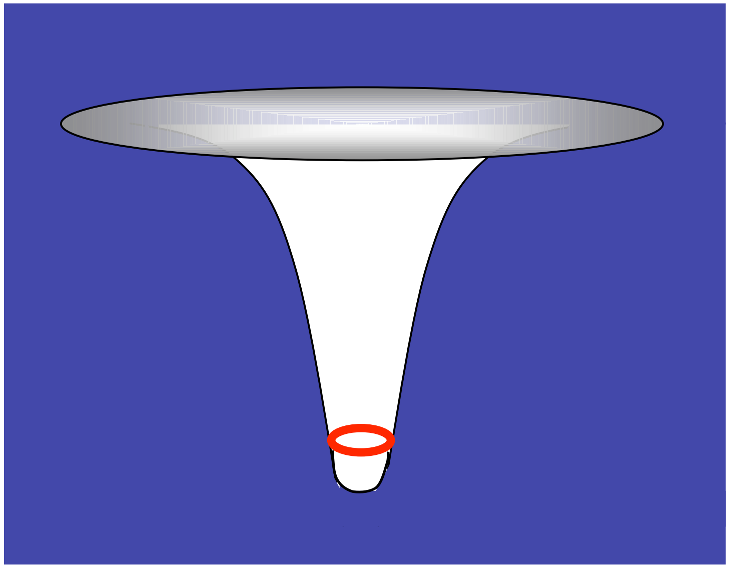
$\bar{Q}_1, \ \bar{Q}_p$ **= charge radii**

$$\alpha \equiv \frac{\Delta t_{escape}}{\Delta t_{osc}} \sim \beta^4$$



Ring thickness comparable to ring radius

$$\beta \sim 1, \quad \alpha \sim 1$$



$$\beta \gg 1, \quad \alpha \gg 1$$

So we get long lived excitations

Putting in the numbers …

$$\beta = \frac{(\bar{Q}_1 \bar{Q}_p)^{\frac{1}{4}}}{a}$$

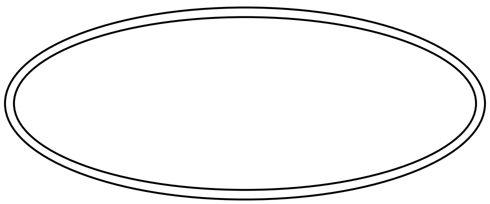
$$\bar{Q}_1 = \frac{g^2 \alpha'^3 n_1}{V}, \quad \bar{Q}_p = \frac{g^2 \alpha'^4 n_p}{V R_y^2}, \quad a = \sqrt{n_1 n_p \alpha'}$$

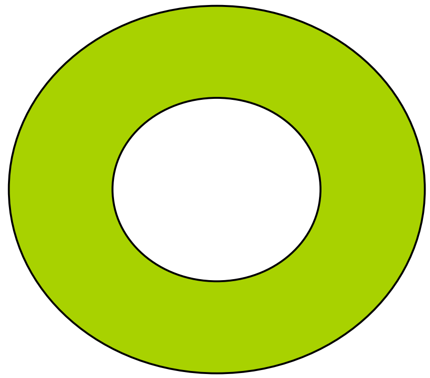
We find

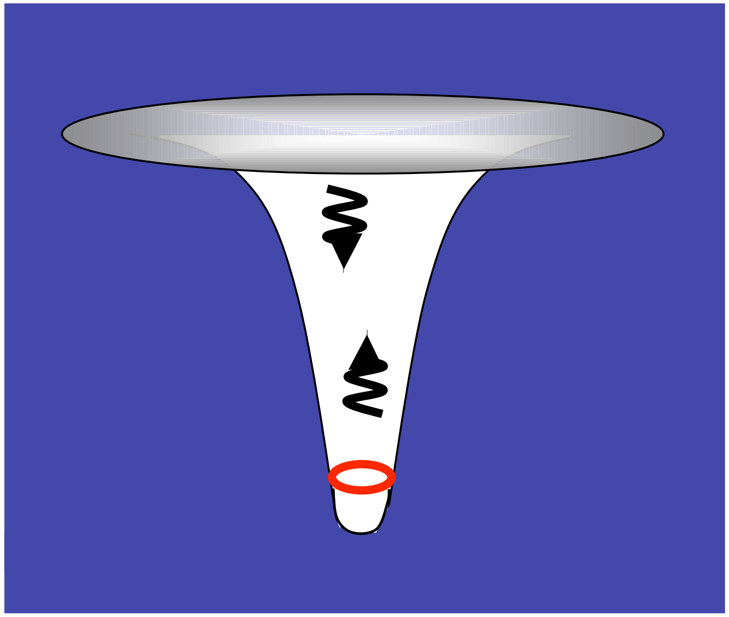
$\beta \sim 1$ **for** $$g \sim g_c = \sqrt{\frac{M V R_y}{\alpha'^2}}$$

(M is the total mass of the NS1-P state)

Supergravity ‘phase transition’







$g \ll g_c$

Long lived oscillations of supertube ‘matter’, gravity not involved

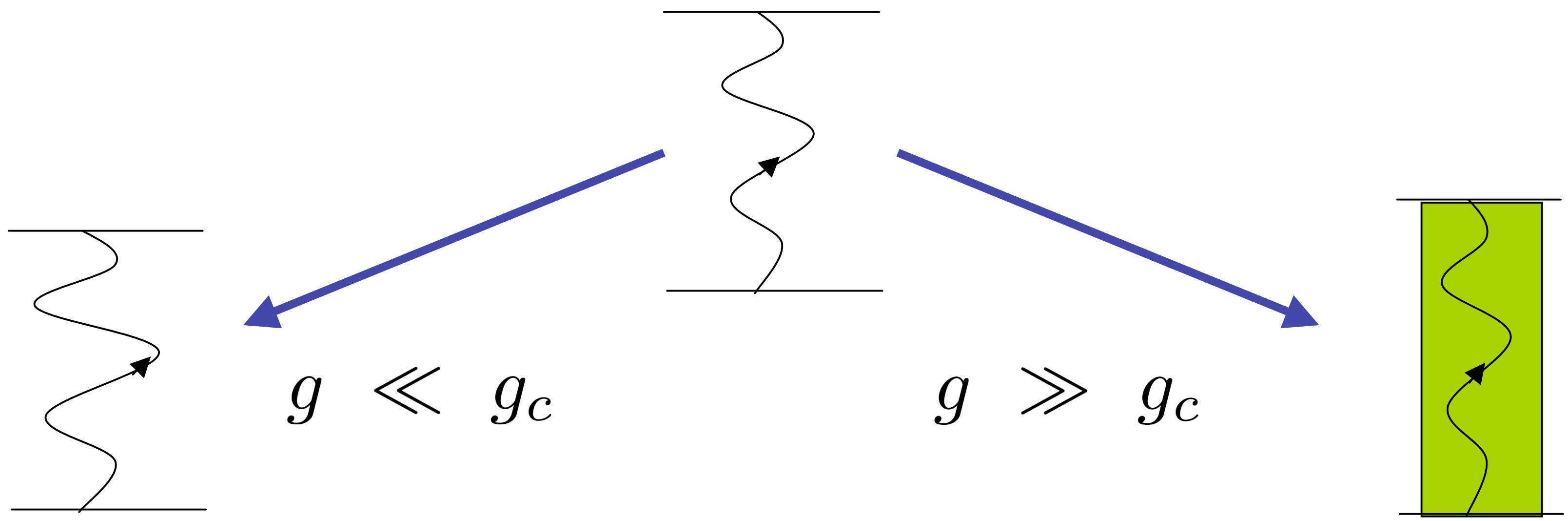
$g \sim g_c$

Oscillations disappear, merge with continuum of Supergravity modes

$g \gg g_c$

Long lived excitations, ‘captured from Supergravity modes of the neighborhood

Phase transition - microscopic (matter) picture



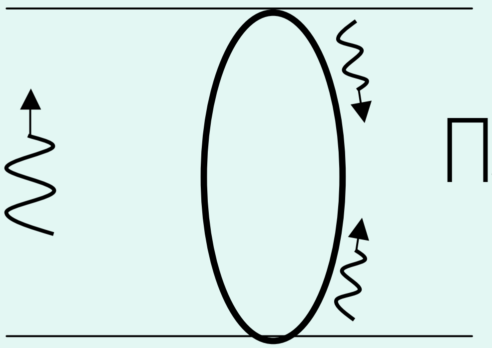
$$g \ll g_c \qquad\qquad g \gg g_c$$

$$\Delta E = \frac{2}{\alpha' M} \qquad\qquad \Delta E = 2\frac{V R_y}{g^2 \alpha'^3}$$

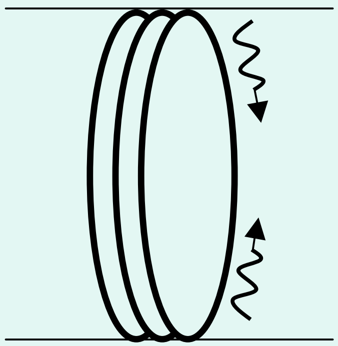
$$g_c = \sqrt{\frac{V R_y M}{\alpha'^2}}$$

Conclusions

Significance of 5 brane pairs: Fractionation



$\prod E \sim \dfrac{1}{R}$

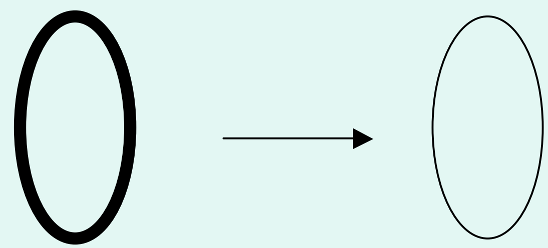


$\prod E \sim \dfrac{1}{nR}$

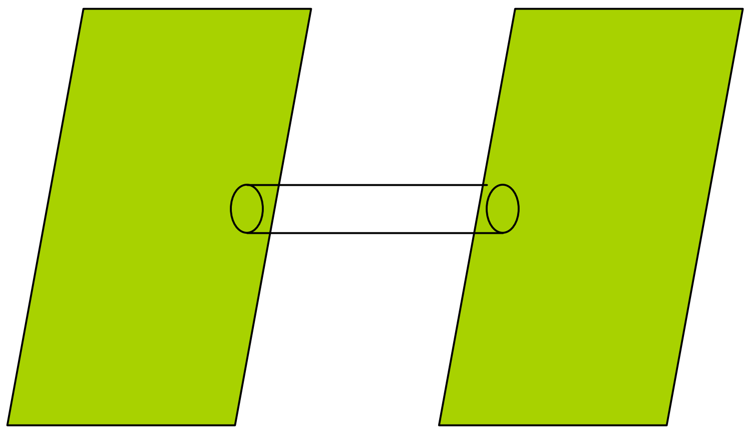
Gravitons appear in FRACTIONAL units 1/n when bound to n strings

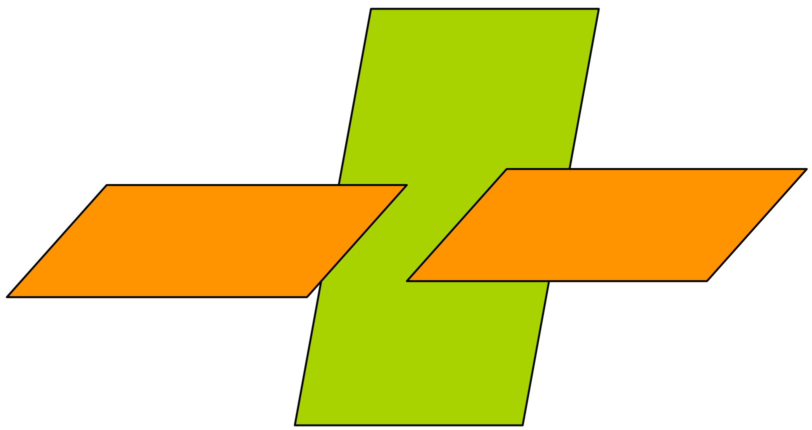
Dualities: graviton □ strings, branes

□ fractional objects:



Very low tension
`floppy' objects, stretch far



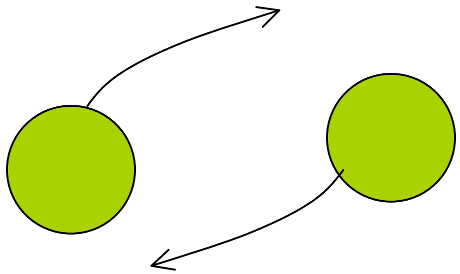


Open string loop, closed string in other channel: GRAVITY

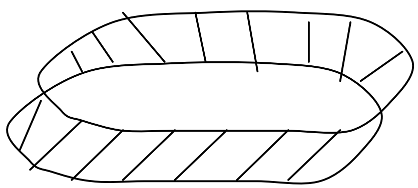
Fractional brane Pairs: reach upto horizon distance:

This effect needs to be understood separately in low energy physics

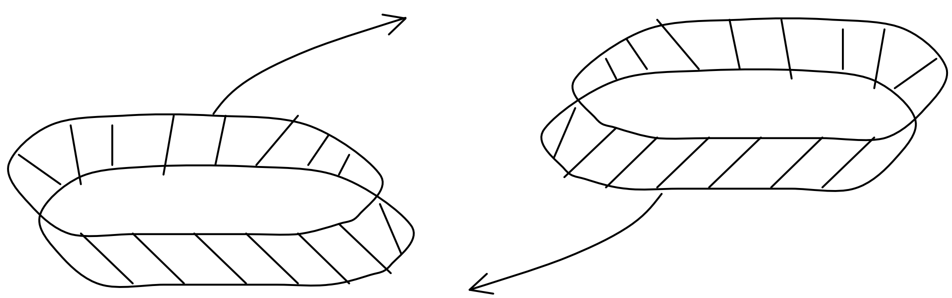
What about the results on black hole moduli space?



Unbound state, drift on moduli space



Bound state: quasi-oscillations



Expect drift mode for center of mass degree of freedom ….

Conjecture: Bound states have no 'drift' modes, while unbound states do

Use of the conjecture:

We have made all BPS 2-charge bound states.

We do not know how to make all BPS 3-charge bound states, but if these could be constructed then we would essentially solve all black hole paradoxes.

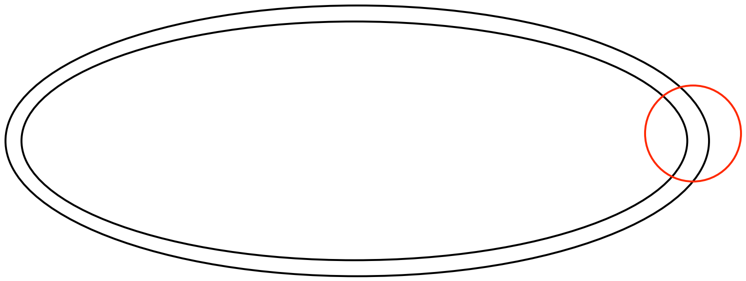
There is a way to write down all 3-charge BPS states, but this includes bound and unbound states

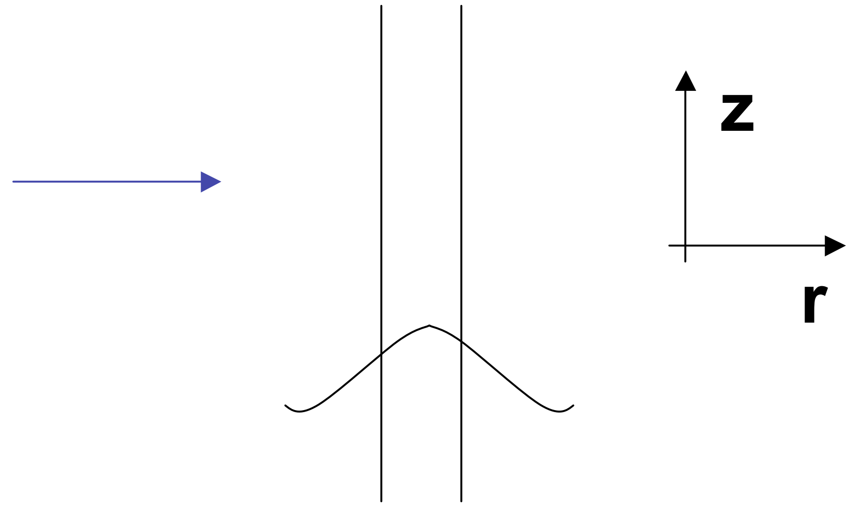
(Gauntlett,+Gutowski+Hull+ Pakis+ Reall 2002, Gutowski+ Martelli+ Reall 2003, Bena+Warner 2004) Bena+Warner 2005, Berglund+Gimon+Levi 2005

Conjecture: Look at this set of BPS states, select those that have no drift modes …
these should be the microstates of the 3-charge black hole

If these all look like 'fuzzballs', and the degeneracy is correct, then we would resolve the information paradox

A microstate for the 3-charge black ring





Smooth D1-D5 geometry

Add p units of P

CFT state $|\psi\rangle = J^{-}_{-1}|\psi\rangle_R$

Wavefunction

$$w = e^{-ip(t+y)-ikz}\,\tilde{w}(r,\theta,\phi)$$

$$B^{(2)}_{MN} = e^{-ip(t+y)-ikz}\,\tilde{B}^{(2)}_{MN}(r,\theta,\phi)$$

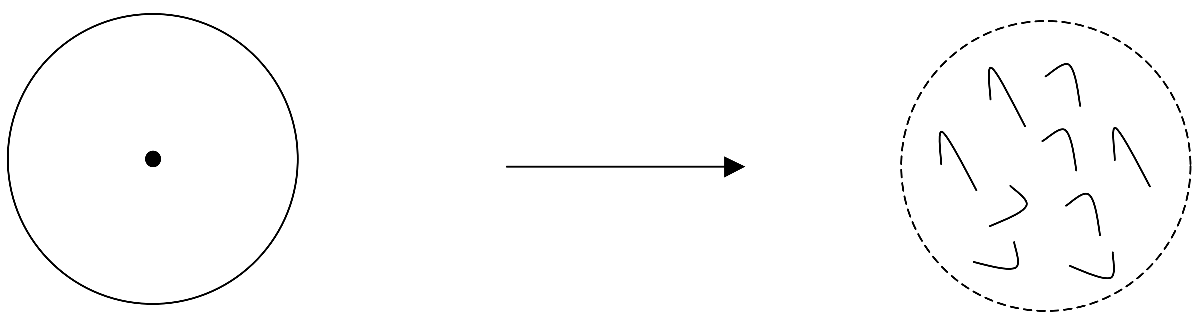
$$w \;=\; e^{-\frac{1}{2Q}(t+y)}\, e^{i(\phi-kz)}\, \cos\frac{\theta}{2}\, e^{-kr}\, \frac{r^{1/2}}{Q+r}$$

$$\begin{aligned}B^{(2)} \;=\;& e^{-\frac{1}{2Q}(t+y)}\, e^{i(\phi-kz)}\, e^{-kr}\, r^{1/2} \Big\{ -\frac{1}{2Q}\, \cos\frac{\theta}{2}\, dt\wedge dz \\ &+ \frac{r}{2(Q+r)^2}\, \cos\frac{\theta}{2}\, [dy - Q(1+\cos\theta)d\phi]\wedge\Big[dt - \frac{2Q+r}{Q}\, dz\Big] \\ &+ ik\, \cos\frac{\theta}{2}\, dr\wedge dz + \frac{1}{2}\sin\frac{\theta}{2}\, dr\wedge[d\theta - i\sin\theta d\phi] \\ &\qquad - \frac{i}{2}\, r\, \cos\frac{\theta}{2}\sin\theta\, d\theta\wedge d\phi \Big\}\end{aligned}$$

Planck length $l_p = \left(\frac{G\hbar}{c^3}\right)^{1/2}$ **= 1.6 x 10^{-33} cm**

When we bind N particles together

$$l_p \longrightarrow N^{\square}\, l_p$$ **??**



'Fuzzball'