Black Hole Vacua

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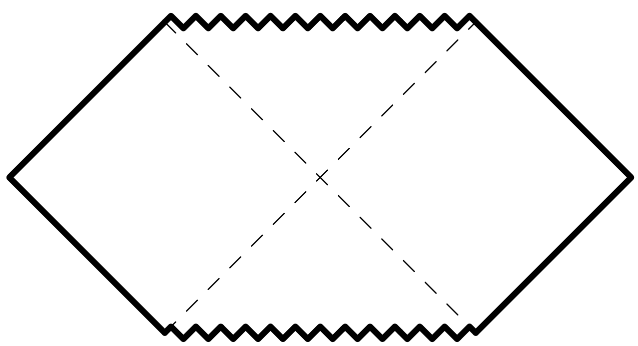
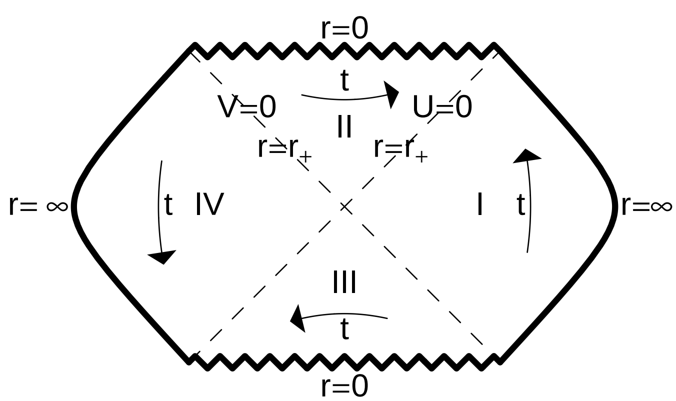
based on work with Andrew Chamblin

Motivation

- String Theory is supposed to solve quantum gravity problems
  - e.g. Black hole singularity?



- Fidkowski, Hubeny, Kleban and Shenker suggested using AdS/CFT



  - Consider geodesics that bounce off singularity
  - AdS/CFT relates this to correlators

Generalities

Fidkowski, Hubeny, Kleban, Shenker

Consider

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-1}^2$$

Where $f(r)$

- monotonically increasing function
- singular at $r = 0$
- zero at $r = r_+ > 0$ [horizon]

Examples:

- Schwarzschild: $f(r) = 1 - \frac{\omega_d M}{r^{d-2}}$
- AdS-Schwarzschild: $f(r) = \frac{r^2}{\ell^2} + 1 - \frac{\omega_d M}{r^{d-2}}$

Technicalities:

Define tortoise coordinate: $r^* = \int_0^r \frac{dr'}{f(r')} + \frac{\pi i}{f'(r_+)}$

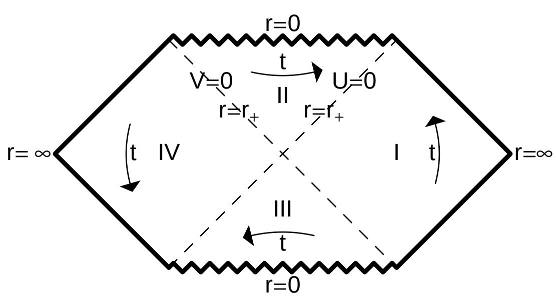
Double null coordinates:

$$u = t - r^* \qquad\qquad v = t + r^*$$

Kruskal coordinates:

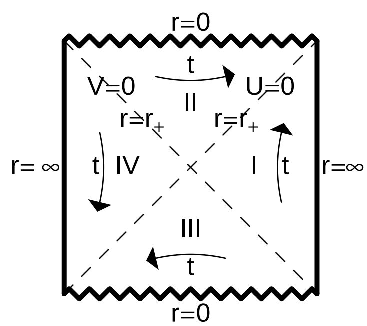
$$U = e^{-\frac{f'(r_+)}{2}u} \qquad\qquad V = e^{\frac{f'(r_+)}{2}v}$$

Kruskal coordinates $\Leftrightarrow$ Penrose diagram:



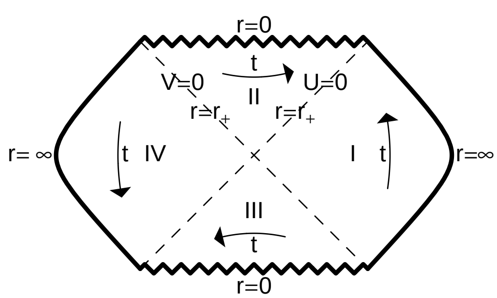
Schwarzschild

$(r^*(r=\infty)=\infty)$



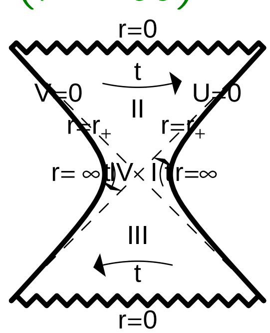
BTZ black hole

$(r^*(r=\infty)=0)$



AdS_5-Schwarzschild

$(0 < r^*(\infty) < \infty)$



$(r^*(\infty) < 0)$

- Which Penrose $\Leftarrow$ asymptotics of r^*

Symmetries:

- $\frac{\partial}{\partial t}$ is timelike (in asymptotic regions) Killing vector
- S^{d-1} symmetries
- $\overset{\frac{1}{2}(U+V)}{\overset{\downarrow}{T}} \to -T$ (vertical reflection of Penrose)
  - inverts time (t), preserves S^{d-1}
  - fixed points at $T = 0$
- $\overset{\frac{1}{2}(-U+V)}{\overset{\downarrow}{Z}} \to -Z$ (horizontal reflection of Penrose)
  - inverts time, preserves S^{d-1}
  - fixed points at $Z = 0$
- antipodal map of S^{d-1} (not on the Penrose diagram)
  - does not affect time; symmetry of S^{d-1}
  - no fixed points

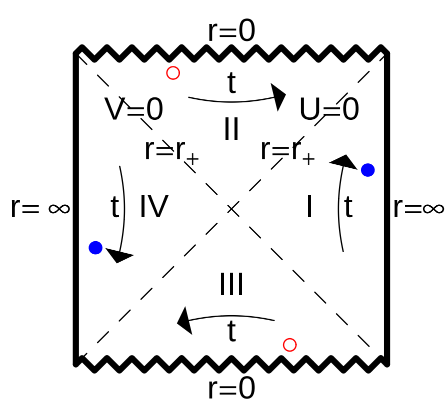
Antipodal Map

Consider combination

$T \to -T, Z \to -Z,$ antipodal map on S^{d-1}

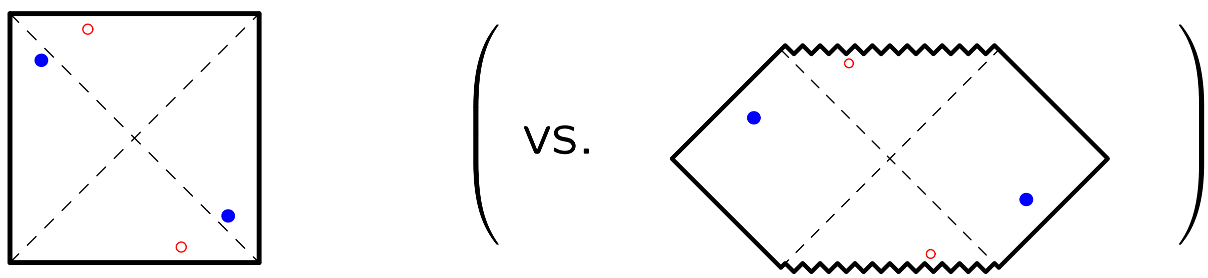
- acts freely
- preserves direction of time
- preserves (obvious) symmetries of the space-time
  - commutes with $\frac{\partial}{\partial t}$
  - commutes with S^{d-1} symmetries

Call this the **antipodal** map



Black Hole Vacua

- $J = R_T R_Z P \Rightarrow \mathbb{Z}_2$ “antipodal” map
  - Commutes with all symmetries
- Cf. de Sitter space:



  - $\exists\ \mathbb{Z}_2$ antipodal map $X \to -X$ on covering space
  - Commutes with all symmetries
  - Can partially correlate point and antipodal point
    - $\Rightarrow$ Mottola-Allen transformation
    - Defines one complex-parameter family of vacua:
      - α-Vacua

Consider a scalar field on this spacetime.

Can choose modes ϕ_n so that

$$\phi_n(\overset{\substack{\text{antipodal point}\\ \downarrow}}{x_A}) \to \phi_n(x)^*$$

antipodal map:

positive frequencies $\leftrightarrow$ negative frequencies

(Antipodal map includes time reversal)

Can Bogoliubov standard vacuum:

$$b_n = \cosh \overset{\substack{\text{real}\\ \downarrow}}{\alpha}\, a_n - e^{-i\overset{\substack{\text{real}\\ \downarrow}}{\gamma}} \sinh \alpha\, \overset{\substack{\text{Hartle-Hawking}\\ \downarrow}}{a_n^\dagger}$$
$$b_n^\dagger = \cosh \alpha\, a_n^\dagger - e^{i\gamma} \sinh \alpha\, a_n$$

- One **complex** parameter family of vacua
- Preserve all (obvious) symmetries

No reason to choose one over another ...

α Vacua

These are *exactly* like dS α-vacua

- including construction

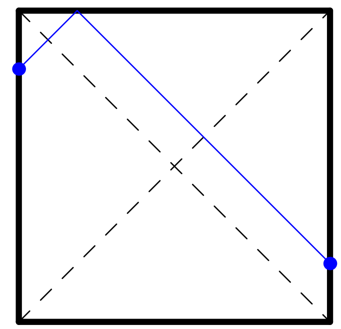
But dS α-vacua are frowned upon:

Einhorn, Larson
Goldstein, Lowe
Kaloper, Kleban, Lawrence, Shenker, Susskind

- Causality problems
- Unphysical Poles
- Pinch Singularities
- Not Thermal

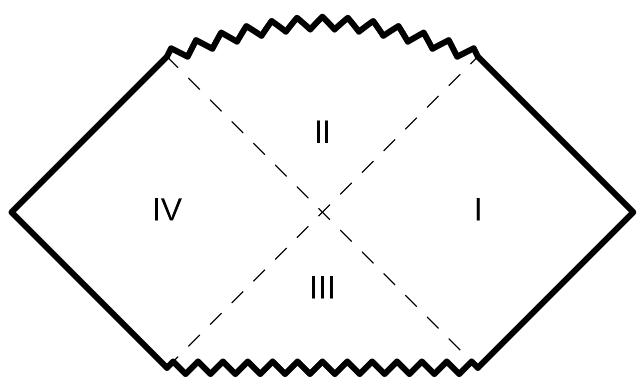
We need not have these problems!

Causality



α-vacua

- send in signal from south pole
- correlation propogates from north pole
  $\Rightarrow$ Causality problems?
- Yes! Naïvely, lightcones only intersect on horizon, **but**
- dS gets taller (gravitational backreaction) Leblond, Marolf, Myers
- $\Rightarrow$ lightcones intersect!



But for black holes,

- gravitational backreaction increases horizon size
- lightcones only intersect inside horizon

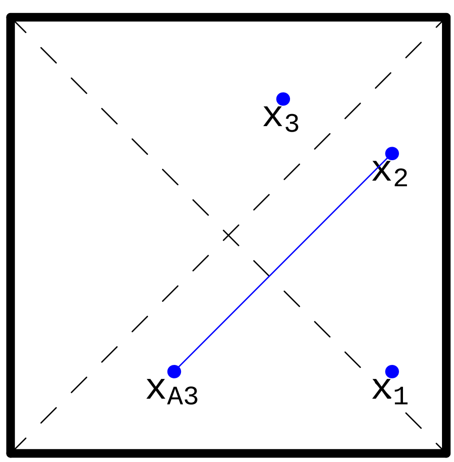
Unphysical Poles

Consider

$$\langle\phi(x_1)\phi(x_2)\phi(x_3)\rangle \sim \int dy G^F(x_1,y)G^F(x_2,y)G^F(x_3,y)$$

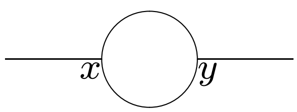
For α vacua, poles when y is on lightcone of point *or antipodal point*

- Divergences if, say, also x_2 on lightcone of x_{3A}
  - coincident poles
- Surprising if x_1, x_2, x_3 causally connected



But for black holes this requires one of x_1, x_2, x_3 to be inside horizon.

Pinch Singularities



$$\sim \int dx \int dy\, G^F_{\alpha\gamma}(x,y) G^F_{\alpha\gamma}(y,x),$$

$$\sim \ldots + \int dx \int dy\, \sinh^2 2\alpha\, G^F_0(x,y) G^F_0(y,x)^* + \ldots$$

Has both $i\epsilon$ prescriptions

$\Rightarrow$ can't evade singularity!

Change time-ordering prescription?

No! (In principle) calculate string loops (*unlike* dS!)

$\Rightarrow$ No pinch singularities

Thermality

In an α-vacuum

$$\frac{P_{\alpha\gamma}(E_i \to E_j)}{P_{\alpha\gamma}(E_j \to E_i)} = \left| \frac{\cosh\alpha + \sinh\alpha e^{i\gamma} e^{\frac{\beta}{2}\Delta E}}{\cosh\alpha + \sinh\alpha e^{i\gamma} e^{-\frac{\beta}{2}\Delta E}} \right|^2 e^{-\beta\Delta E}.$$

- Only thermal (temperature β) if $\alpha = 0$
- Contradicts detailed balance?
  - i.e. $\rho(E_i)P(E_i \to E_j) \neq \rho(E_j)P(E_j \to E_i)$

NO! Just means *nonequilibrium, steady state*.

Holography?

- For AdS-Schwarzschild, have AdS/CFT.
- Two boundaries $\Rightarrow$ two CFTs
- Ordinary vacuum $\Leftrightarrow$ Pure state of (doubled) CFT
  - Trace over CFT_1 $\Rightarrow$ thermal state of CFT

For α-vacuum, Bogoliubov CFT:

$$b_1^\dagger = \cosh\alpha\, a_1^\dagger - e^{i\gamma}\sinh\alpha\, a_2,$$
$$b_2^\dagger = \cosh\alpha\, a_2^\dagger - e^{i\gamma}\sinh\alpha\, a_1$$

Note "1" and "2" no longer bdy_1 and bdy_2.

$\mathrm{Tr}_2 \Rightarrow$ *nonthermal* density matrix

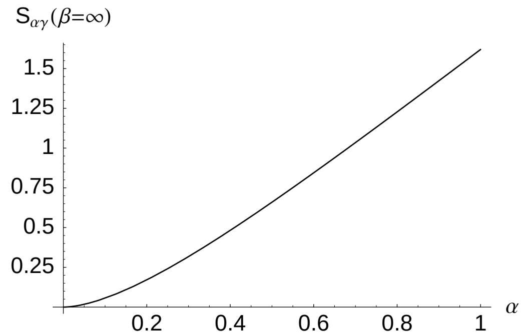
Agrees with nonthermal formula on AdS side!

Sim. propagators $\leftrightarrow$ correlation functions

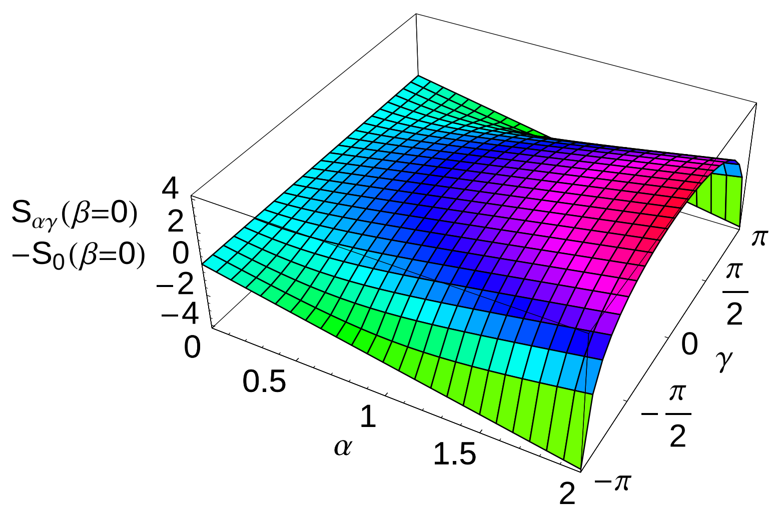
Entropy

Using AdS/CFT, compute $S = -\operatorname{Tr}\rho\ln\rho$:

- Entropy at low temperature:



- Entropy at high temperature:



Conclusions

- Black holes have α-Vacua
  - Very general
  - For AdS-Schwarzschild, can use CFT as well
- Can avoid problems of dS α-Vacua
- Compute α-dependent entropy from CFT
  - What does it mean???