Tachyon Condensation and "Non-BPS" D-Branes in a Ramond-Ramond Plane Wave Background

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Plan

- Review of Tachyon Condensation in Flat Background

- Existence of Tachyon Potential in PP-Wave

- Consequence: D-Brane Descent Relations

- Non-BPS D-branes of Flat Space in Green-Schwarz Formalism

- Open String Theory (light-cone gauge) in PP-Wave

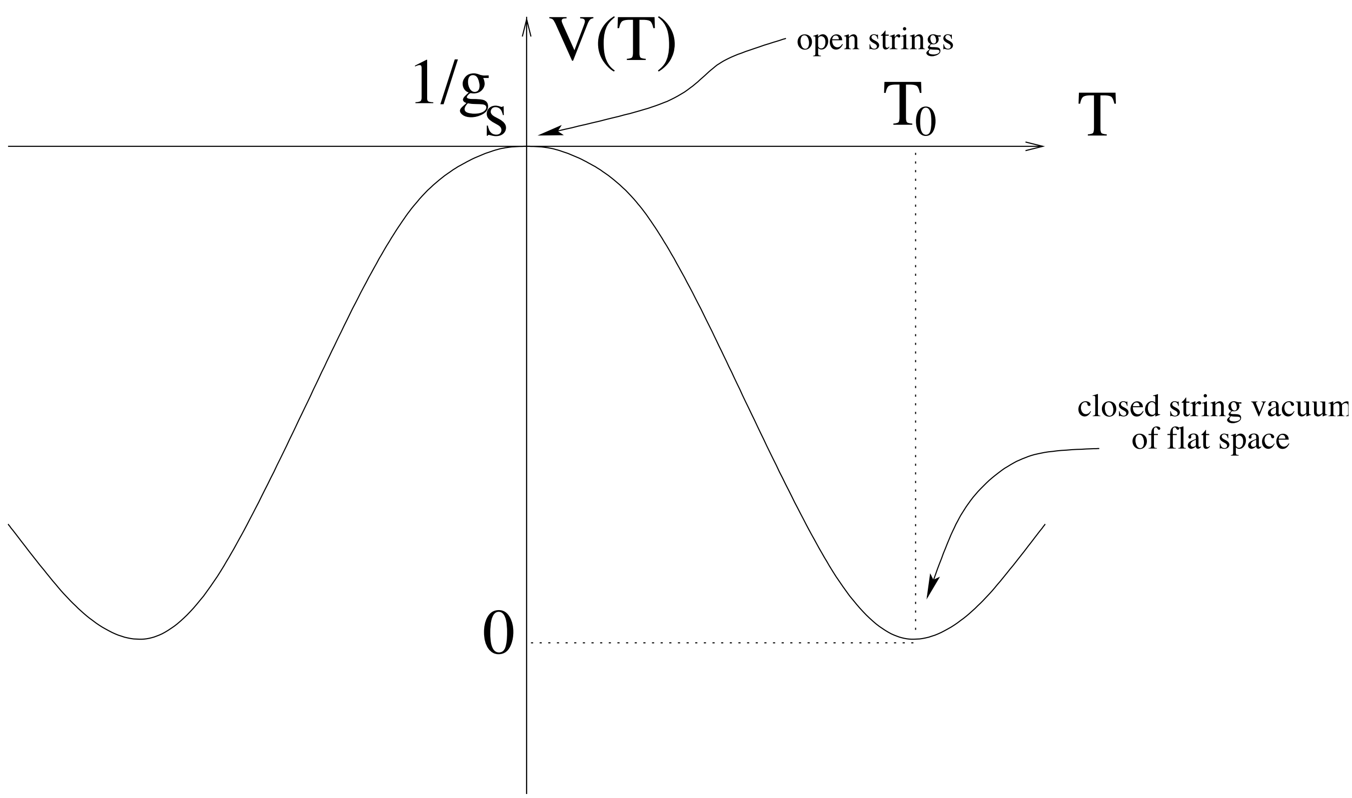
- Future Directions

Review of Tachyon Condensation in Flat Background

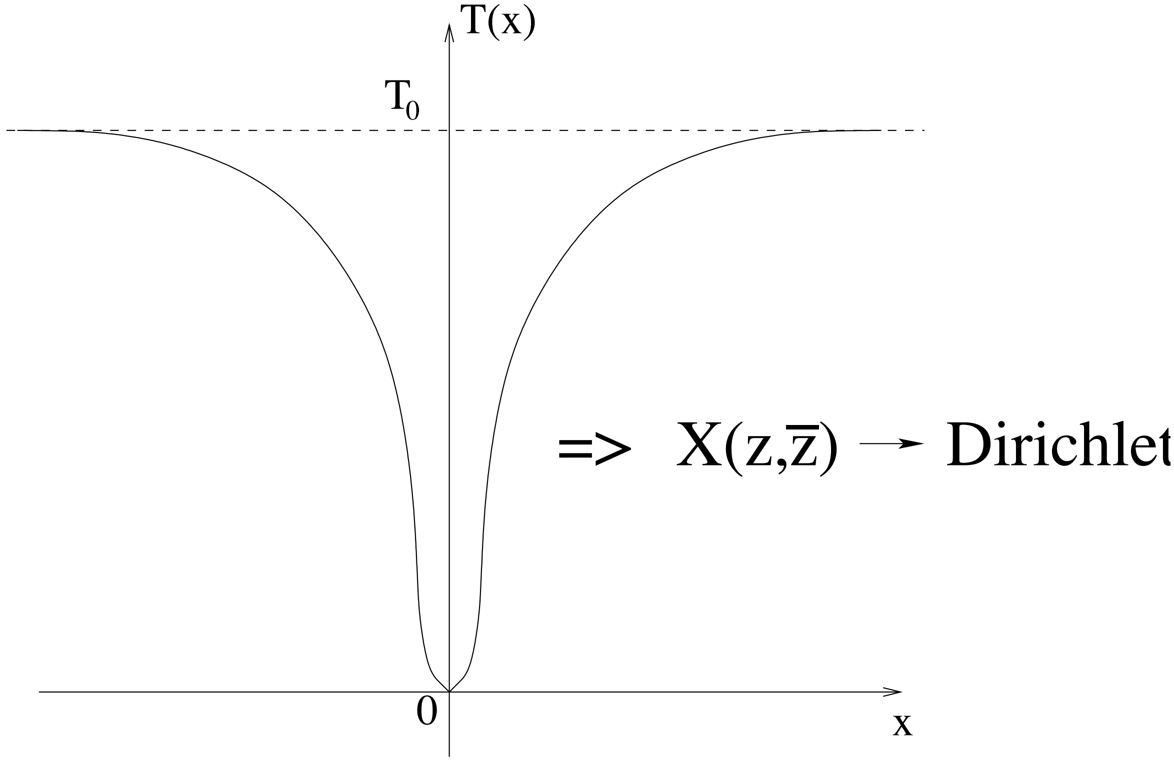
[Sen]

Consider a coincident brane-anti-brane system in type IIB string theory in flat space.

<u>Tachyon potential</u>



Lower dimensional D-branes as lump solutions



D-brane descent relation

D-branes (Dp) of all dimensions are allowed.

BPS $\rightarrow$ $p+1 =$ even.

Non-BPS $\rightarrow$ $p+1 =$ odd.

Type IIB pp-wave

[Blau, Figueroa-O'Farrill,
Hull, Papadopoulos]

We would like to address similar issues in the particular case of type IIB pp-wave background with R-R flux:

$$\begin{aligned} ds^2 &= 2dx^+dx^- - \mu^2 x^I x^I dx^+ dx^+ + dx^I dx^I \ , \\ F_{+1234} &= F_{+5678} \propto \mu \ , \end{aligned}$$

$x^\pm = (x^9 \pm x^0)/\sqrt{2}, \quad I = 1, 2, \cdots 8,$

Flat space limit: $\mu \to 0$.

String theory can be solved in light-cone gauge (Green-Schwarz-Metsaev-Tseytlin covariant action):

$$\begin{aligned}\mathcal{L}_B &= \frac{1}{2}(\partial_+ X^I \partial_- X^I - m^2 X^I X^I) \ , \\ \mathcal{L}_F &\sim S^1 \partial_+ S^1 + S^2 \partial_- S^2 - 2m S^1 \Pi S^2 \ .\end{aligned}$$

$\partial_\pm = \partial_\tau \pm \partial_\sigma$,

$$m \propto \mu p^+ \ , \qquad \Pi_{ab} = \sigma^{1234}_{ab} = \sigma^{5678}_{ab} \ .$$

$$\Gamma^\mu = \begin{pmatrix} 0 & \gamma^\mu \\ \bar{\gamma}^\mu & 0 \end{pmatrix} \ ,$$

$$\gamma^0 = \bar{\gamma}^0 = I_{16} \ , \quad \gamma^9 = \bar{\gamma}^9 = \begin{pmatrix} I_8 & 0 \\ 0 & -I_8 \end{pmatrix} \ ,$$

$$\gamma^I = \bar{\gamma}^I = \begin{pmatrix} 0 & \sigma^I_{a\dot{b}} \\ \bar{\sigma}^I_{\dot{a}b} & 0 \end{pmatrix} \ .$$

A claim

Given a world-volume field $\Phi(x^+, x^I)$ on a coincident brane-anti-brane pair which satisfies $\partial_- \Phi = 0$, the following is true for the world-volume action:

$$S_{pp}[\Phi] = S_{flat}[\Phi] \ .$$

An argument

Let $\tilde{\Phi}(p^+, p_-, p^I)$ be the Fourier transformed field.

Any given term in $S_{pp}[\Phi(x^+, x^-, x^I)]$ has the form

$$T_{pp} = \int \delta(\sum p_i) f(\mu, p_i^+, p_i^-, p_i^I) \prod_i \tilde{\Phi}_i(p_i^+, p_i^-, p_i^I) \ ,$$

Corresponding term in flat background,

$$T_{flat} = \int \delta(\sum p_i) f(0, p_i^+, p_i^-, p_i^I) \prod_i \tilde{\Phi}_i(p_i^+, p_i^-, p_i^I) \ ,$$

We need to argue,

$$f(\mu, p_i^+ = 0, p_i^-, p_i^I) = f(0, p_i^+ = 0, p_i^-, p_i^I) \ .$$

or

$$f = f(m_i, p_i^+, p_i^-, p_i^I) \ .$$

This will be true if all μ dependence can be absorbed into the m dependence and symmetries of theories match for the field configurations considered.

Symmetry currents:

Kinematical generators

PP-wave	Flat
$P^{\pm}, P^{I}, J^{+I}, J^{ij}, J^{i'j'}, Q^{+Aa}$	$P^{\pm}, P^{I}, J^{+-}, J^{\pm I}, J^{IJ}, Q^{+Aa}$

Dynamical generators

PP-wave	Flat
P^{-}, Q^{-A}	P^{-}, Q^{-A}

Missing generators:

$$J^{+-}, J^{-I}, J^{ii'}$$

We need to

1. account for the missing generators,

2. show that in expressions for generators and symmetry algebras

   (a) Π dependence go away in $p^+ \to 0$ limit.

   (b) μ only appears as m,

   (Since $\mu \to 0$ is the flat space limit where Π should not appear, Π must always come with a factor of μ. Therefore establishing 2(b) only is sufficient)

- Kinematical generators evaluated in light-cone gauge do not depend on μ (hence Π). They formally pretend as if the full $SO(8)$ symmetry were present.

- Dynamical generators do depend on μ, but the dependence is only through m.

- Above is not true of symmetry algebra. μ explicitly appears in commutation relations. Schematically,

$$[P^-, P^I] \sim \mu^2 J^{+I} \ , \qquad [P^I, Q^-] \sim \mu Q^+ \ ,$$

$$[P^-, Q^+] \sim \mu Q^+ \ , \qquad \{Q^+, Q^-\} \sim \mu J^{+I} \ ,$$

$$\{Q^-, Q^-\} \ \sim \ \mu J^{IJ} \ .$$

But generators have additional p^+ dependence:

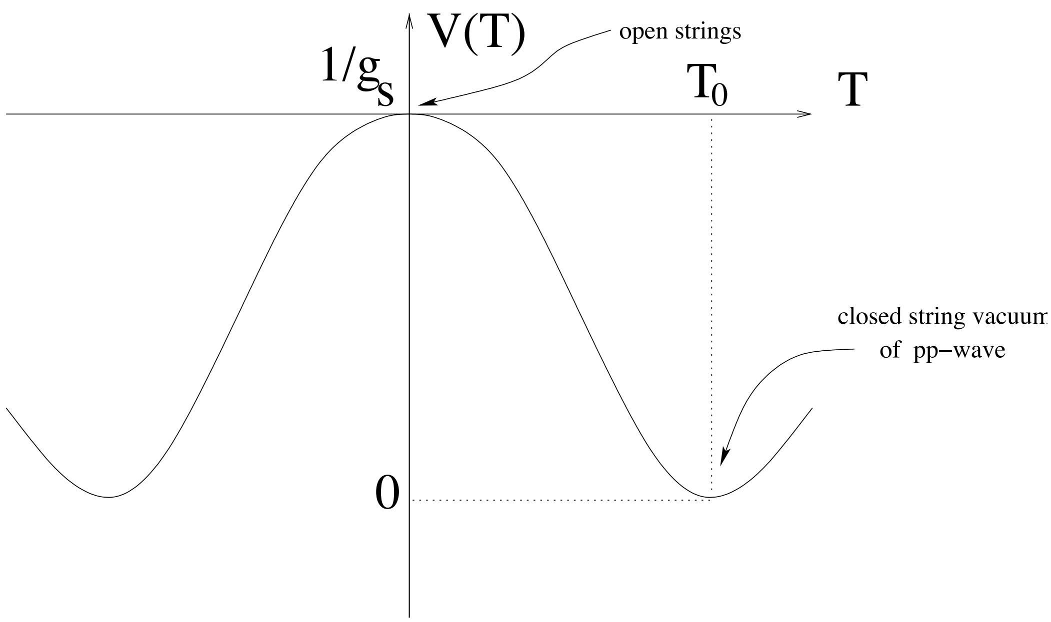
$$P^+ = p^+ \ , P^- \sim \frac{1}{p^+} \ , J^{+I} \sim p^+ \ ,$$

$$Q^{+} \sim \sqrt{p^{+}}\ , Q^{-} \sim \frac{1}{\sqrt{p^{+}}}\ .$$

This dependence is precisely the one that allows one to write the commutation relations in terms of m.

Consequences:

1. world-volume theory on a brane-anti-brane pair in pp-wave admits the same tachyon potential as in flat space.



2. All D-branes, in particular non-BPS D-barnes, in flat space should have pp-wave analogues!!

   Detailed properties are, in general, expected to be different.

Non-BPS D-branes in flat space

[PM]

Bosonic part is as simple as for BPS D-branes. Complication arises in writing down fermion boundary condition in Green-Schwarz formalism.

In light-cone gauge: $S^a(z)$, $\tilde{S}^a(\bar{z})$.

Variation of action gives,

$$S^a(z)\delta S^a(z) = \tilde{S}^a(\bar{z})\delta\tilde{S}^a(\bar{z}) \ , \quad \text{at } z = \bar{z} \ .$$

BPS boundary condition:

$$\begin{aligned} S^a(z) &= M_{ab}\tilde{S}^a(\bar{z}) \ , \qquad \text{at } z = \bar{z} \ , \\ M_{ab} &= \gamma_{ab}^{I_1 \cdots I_{p-1}} \ , \quad \text{(Recall } p \text{ odd)} \ . \end{aligned}$$

For non-BPS D-branes p is even. It turns out that boundary condition is bi-local.

$$S^a(z)S^b(w) \;=\; \mathcal{M}^{ab}_{cd}\tilde{S}^c(\bar{z})\tilde{S}^d(\bar{w}) \text{ ,at } z=\bar{z}$$

$$\begin{aligned}
\mathcal{M}^{ab}_{cd} \;&=\; \frac{1}{8}\delta_{ab}\delta_{cd} + \frac{1}{16}\sum_{I,J}\lambda_{(IJ)}\sigma^{IJ}_{ab}\sigma^{IJ}_{cd} \\
&\quad + \frac{2}{384}\sum_{\{I,J,K,L\}\in\mathcal{K}}\lambda_{(IJKL)}\sigma^{IJKL}_{ab}\sigma^{IJKL}_{cd} \;.
\end{aligned}$$

$$\lambda_{(IJ\cdots)} \;=\; \lambda_I\lambda_J\cdots \;,$$

$$\lambda_I \;=\; \begin{cases} -1 & \text{if } X^I \text{ Neumann ,} \\ 1 & \text{if } X^I \text{ Dirichlet .} \end{cases}$$

$$\{\{I,J,K,L\}\} = \mathcal{K} + \mathcal{K}_D \;,$$

such that, for every $\{I,J,K,L\}\in\mathcal{K}$ there exists a dual element $\{M,N,O,P\}\in\mathcal{K}_D$ satisfying,

$$\epsilon^{IJKLMNOP} \neq 0 \;.$$

Q. How to deal with this boundary condition?

[PM]

Generalise the usual "doubling trick".

$$\mathcal{S}^a(u)\cdots\mathcal{S}^b(v) = \begin{cases} S^a(z)\cdots S^b(w)|_{z=u,w=v}\ , \\ (\Im u, \Im v \geq 0) \\ \\ \mathcal{M}^{ab}_{cd}\ \tilde{S}^c(\bar{z})\cdots\tilde{S}^d(\bar{w})|_{\bar{z}=u,\bar{w}=v}\ . \\ (\Im u, \Im v \leq 0) \end{cases}$$

One consequence:

$$\begin{aligned} \mathcal{S}^a(\tau,2\pi)\mathcal{S}^b(\tau',2\pi) &= \mathcal{S}^a(\tau,0)\mathcal{S}^b(\tau',0)\ , \\ \text{i.e. } \mathcal{S}^a(\tau,2\pi) &= \pm\mathcal{S}^a(\tau,0). \end{aligned}$$

Open-string spectrum: R + NS.

R: $\quad S^{a_1}_{-n_1}S^{a_2}_{n_2}\cdots\begin{pmatrix}|I\rangle \\ |\dot{a}\rangle\end{pmatrix}\ ,$

NS: $\quad S^{a_1}_{-r_1}S^{a_2}_{r_2}\cdots|0\rangle\ .$

Non-BPS D-Branes in PP-Wave

- $SO(8)$ fermions in light-cone gauge:

$$S^{1a}(\tau,\sigma)\ , \quad S^{2a}(\tau,\sigma)\ .$$

- EOM:

$$\partial_+ S^1 - m\Pi S^2 = 0\ , \quad \partial_- S^2 + m\Pi S^1 = 0\ .$$

- Boundary condition (at $\sigma = 0, \pi$):

$$S^{1a}(\tau,\sigma)S^{1b}(\tau',\sigma) = \mathcal{M}^{ab}_{cd} S^{2c}(\tau,\sigma) S^{2d}(\tau',\sigma)\ .$$

- Spectrum: NS + R (without zero modes) (one can argue that R-sector zero modes are inconsistent. This feature is different from flat space. One recovers zero modes in the limit $m \to 0$.)

Hamiltonian

$$p^+ H_R = m\vec{a}^\dagger_{||}.\vec{a}_{||} + \frac{1}{4} m \vec{x}^2_\perp \frac{\sinh(2m\pi)}{\sinh^2(m\pi)} + \frac{m}{2}(p-1) \sum_{n=1}^{\infty} w_n \left[\vec{a}^\dagger_n.\vec{a}_n + \mathcal{S}^a_{-n}\mathcal{S}^a_n\right] \ .$$

$$p^+ H_{NS} = m\vec{a}^\dagger_{||}.\vec{a}_{||} + \frac{1}{4} m \vec{x}^2_\perp \frac{\sinh(2m\pi)}{\sinh^2(m\pi)} + \frac{m}{2}(p-1) \sum_{n=1}^{\infty} w_n \vec{a}^\dagger_n.\vec{a}_n + \sum_{r=1/2}^{\infty} w_r \mathcal{S}^a_{-r}\mathcal{S}^a_r + E_0$$

$$\frac{1}{2} E_0(m) = \sum_{n=1}^{\infty} w_n - \sum_{r=1/2}^{\infty} w_r \ ,$$

$$w_n = \sqrt{n^2 + m^2} \ , \quad w_r = \sqrt{r^2 + m^2} \ .$$

Regularisation of $E_0(m)$

$$\frac{1}{2}E_0(m) = 2\sum_{n=1}^{\infty}\left(\sqrt{n^2+m^2} - \sqrt{\left(\frac{n}{2}\right)^2+m^2}\right)$$

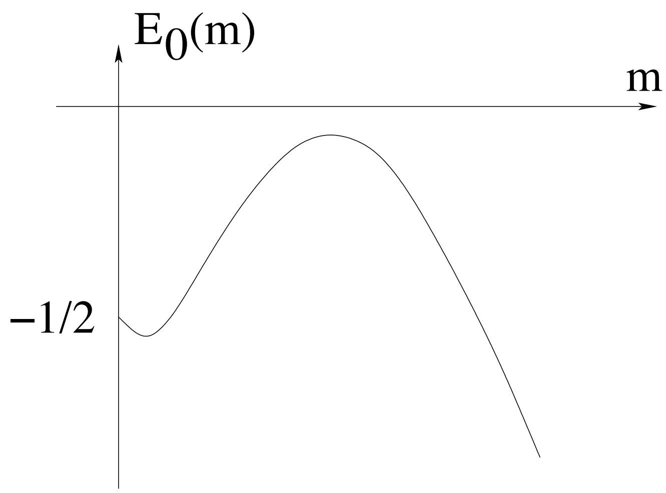
Use

$$\sum_{n=1}^{\infty}\sqrt{\left(\frac{n}{a}\right)^2+m^2} \;=\; -\frac{m}{2} - \frac{am^2}{2\pi}\Gamma(-1) - \frac{2m}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}K_1(2\pi nm) \; .$$

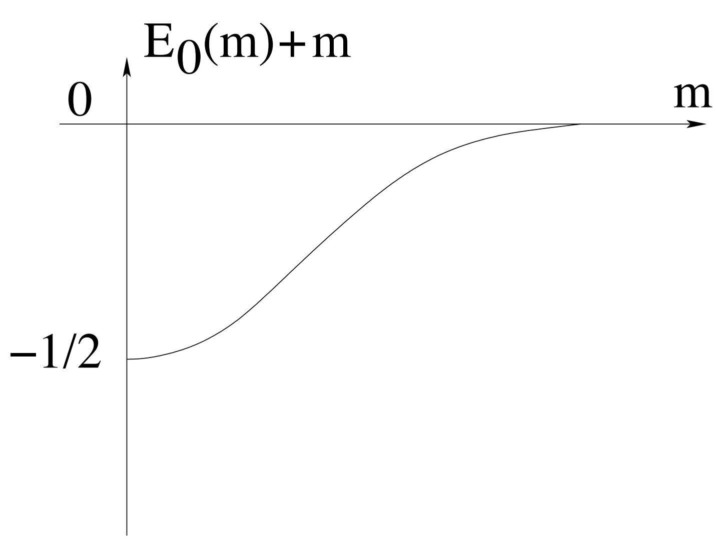
$$E_0(m) = -m + \frac{4}{\pi}m\sum_{n=1}^{\infty}\frac{1}{n}\left[K_1(4\pi nm) - 2K_1(2\pi nm)\right] \; .$$

$$\lim_{m\to 0} E_0 \to -\frac{1}{2}$$

Correct flat space limit, but no lower bound!!



On the other hand:



Future Directions

1. Boundary state construction.

    - Demonstrate open-closed duality.

    - Show zero RR-part.

    - Clarify zero-point energy in NS-sector hamiltonian.

2. Use covariant formulation to study non-BPS D-branes that can not be seen in light-cone gauge.
   Example: D-particle

   An application $\rightarrow$ study decay process.

3. Study defect CFT representing coincident brane-anti-brane system in dual gauge theory.

   - How to see $V(T)$ in gauge theory?

   - Can we understand

     $$\lim_{m\to\infty} E_0(m)\ , \qquad \text{(recall } \lambda' \propto \frac{1}{m} \to 0\text{)}$$

     in dual gauge theory?

   - Open-closed duality in gauge theory?