Recent AdS/CFT results for near-equilibrium strongly coupled thermal gauge theories

Andrei Starinets

Perimeter Institute for Theoretical Physics

Great Lakes Strings Conference
Ann Arbor

March 31, 2006

- Viscosity-entropy ratio at non-zero chemical potential
- Critical exponents for the shear viscosity from AdS/CFT
- Thermal conductivity of N=4 SYM

  (Dam Son, A.S., hep-th/0601157)

- Thermal spectral functions of N=4 SYM

  (Pavel Kovtun, A.S., hep-th/0602059)

- Photon and dilepton production in strongly coupled plasma

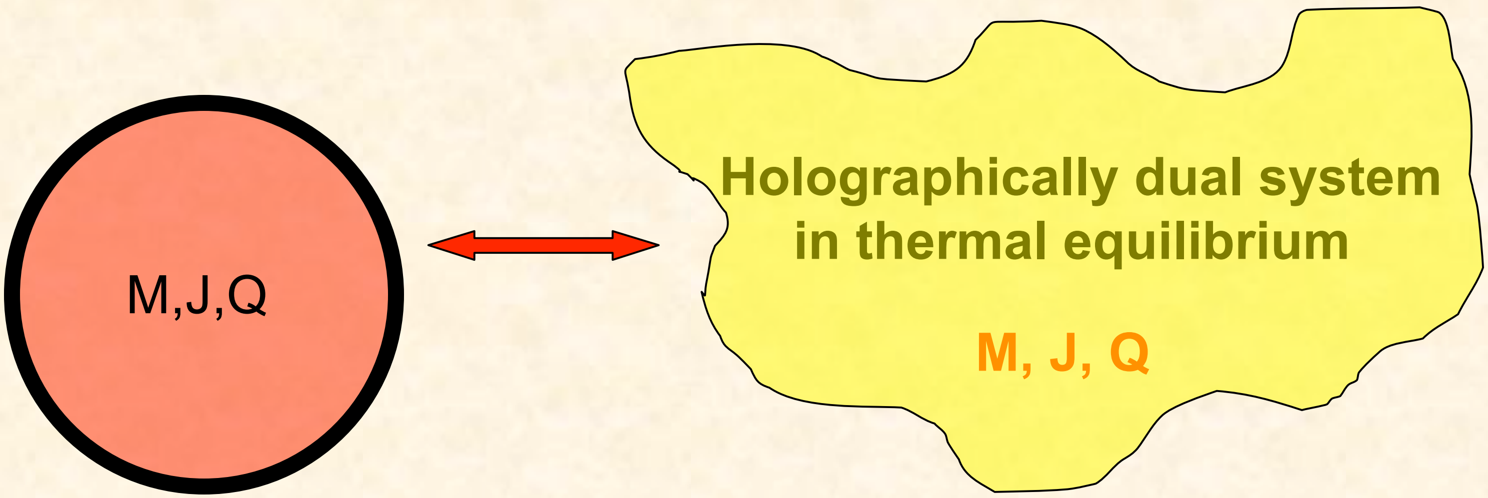
  (Pavel Kovtun, A.S., to appear)

- AdS/CFT correspondence can be used for studies of the near-equilibrium regime of strongly coupled gauge theories

- This is interesting, since this regime remains inaccessible for other non-perturbative methods such as the (direct) lattice simulations

- The Lorentzian version of the AdS/CFT computes thermal correlation functions of a dual theory directly from gravity. This is all we need since the near-equilibrium properties then follow from the fluctuation-dissipation theorems.

- In particular, transport coefficients of strongly coupled thermal gauge theories can be extracted from quasinormal spectrum of the dual gravity background



T_{Hawking} $\quad S_{\text{Bekenstein-Hawking}}$ $\quad \Longleftrightarrow \quad$ T $\quad$ S

Gravitational fluctuations $\quad \Longleftrightarrow \quad$ Deviations from equilibrium

$g_{\mu\nu}^{(0)} + h_{\mu\nu}$ $\quad \Longleftrightarrow \quad$????

$$j_i = -D\partial_i j^0 + \cdots$$

$''\Box'' \; h_{\mu\nu} = 0$ and B.C. $\quad \Longleftrightarrow$

$$\partial_t j^0 + \partial_i j^i = 0$$

$$\partial_t j^0 = -D\nabla^2 j^0$$

Quasinormal spectrum $\quad \Longleftrightarrow$

$$\omega = -iDq^2 + \cdots$$

Transport (kinetic) coefficients

- Shear viscosity η
- Bulk viscosity ζ
- Charge diffusion constant D_Q
- Thermal conductivity κ_T
- Electrical conductivity σ

What is known?

✓ **Shear viscosity/entropy ratio:** $\frac{\eta}{s} = \frac{1}{4\pi}$

- in the limit $g^2 N = \infty$ $N = \infty$
- universally for a large class of theories

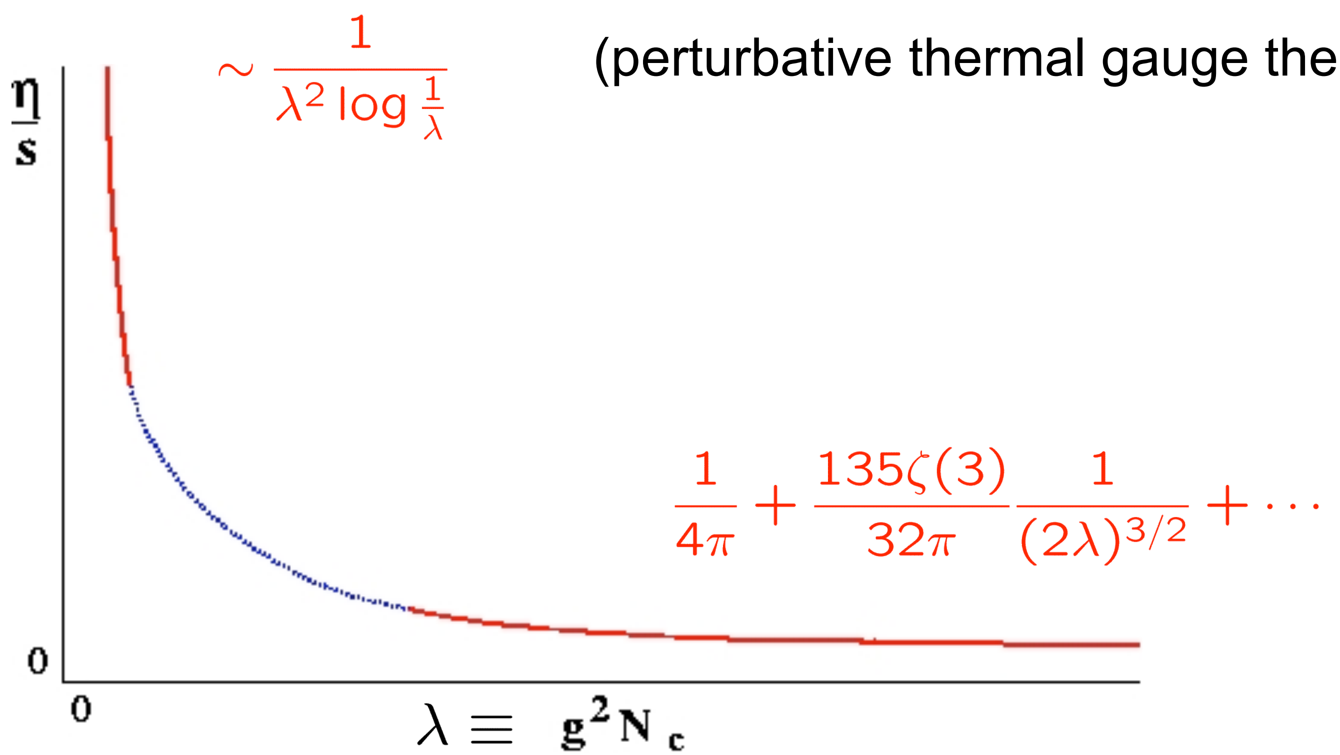
✓ **Bulk viscosity for non-conformal theories**

$$\frac{\zeta}{\eta} = -\kappa \left(v_s^2 - \frac{1}{3} \right)$$

- in the limit $g^2 N = \infty$ $N = \infty$
- model-dependent

✓ **R-charge diffusion constant for N=4 SYM:** $D_R = \frac{1}{2\pi T}$

Shear viscosity in $\mathcal{N} = 4$ SYM



Correction to $1/4\pi$: A.Buchel, J.Liu, A.S., hep-th/0406264

Shear viscosity at non-zero chemical potential

$\mathcal{N} = 4$ SYM $\Longleftrightarrow$ Reissner-Nordstrom-AdS black hole

$q_i \in U(1)^3 \subset SO(6)_R$ with three R charges

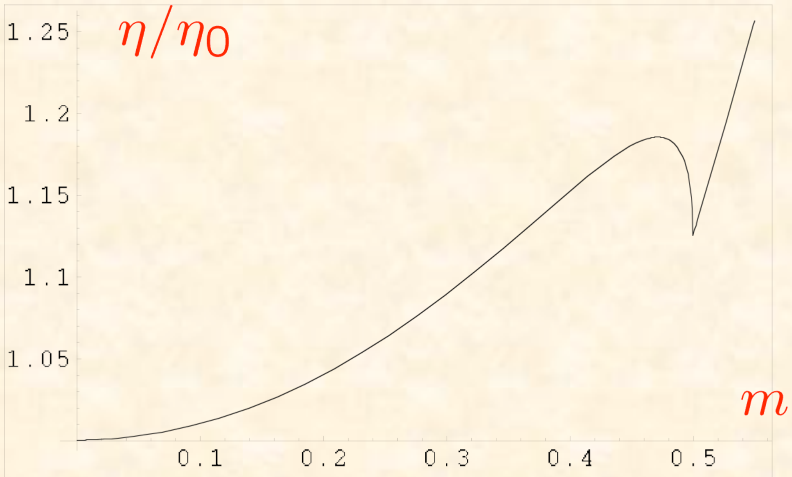
$Z = \mathrm{tr}\, e^{-\beta H + \mu_i q_i}$ (Behrnd, Cvetic, Sabra, 1998)

We still have $\frac{\eta}{s} = \frac{1}{4\pi}$

J.Mas
D.Son, A.S.
O.Saremi
K.Maeda, M.Natsuume, T.Okamura

$$\eta = \pi N^2 T^3 \frac{m^2(1 - \sqrt{1 - 4m^2} - m^2)^2}{(1 - \sqrt{1 - 4m^2})^3}$$

$m \equiv \mu / 2\pi T$



Thermal conductivity

Non-relativistic theory: $Q = -\kappa_T \nabla T$

Relativistic theory: $T^{0i} = -\kappa_T \left(\partial^i T - \frac{T}{\varepsilon + P} \partial^i P \right)$

Kubo formula: $\kappa_T = -\frac{(\varepsilon + P)^2}{\rho^2 T} \lim_{\omega \to 0} \frac{1}{\omega} \text{Im}\, G(\omega, 0)$

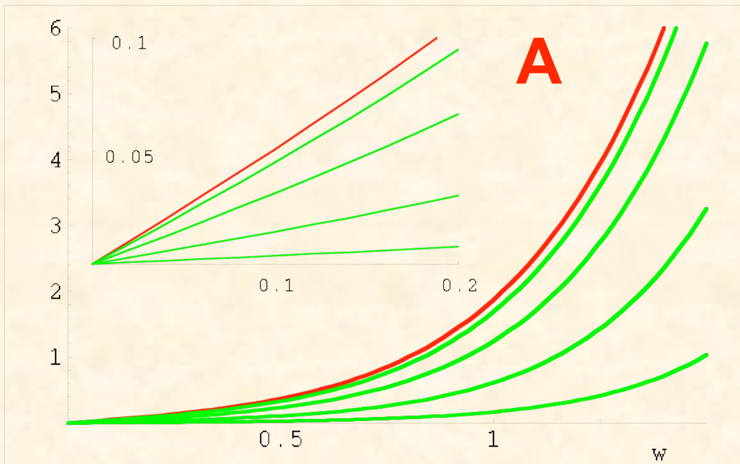
In $\mathcal{N} = 4$ SYM with non-zero chemical potential μ :

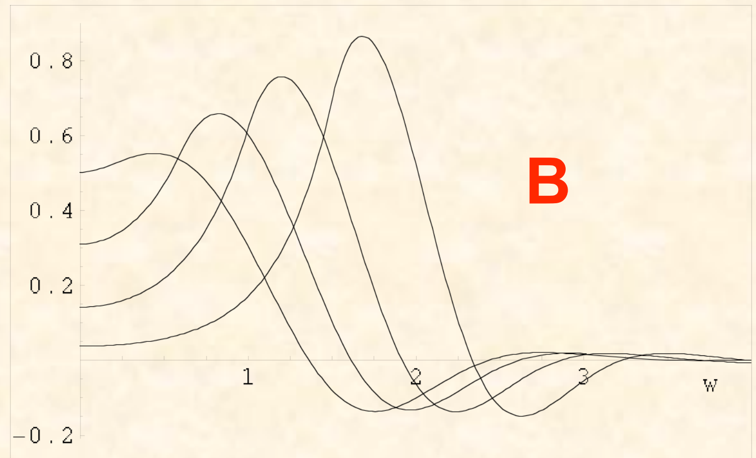
$$\frac{\kappa_T \ \mu^2}{\eta \ T} = 8\pi^2$$

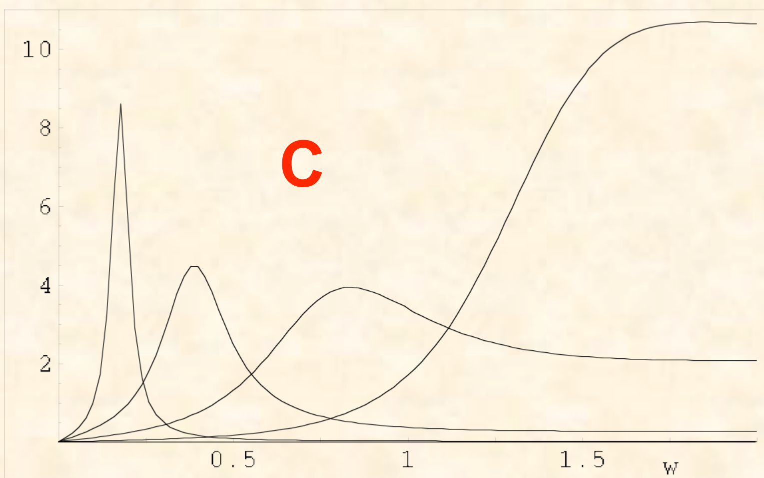
One can compare this with the Wiedemann-Franz law for the ratio of thermal to electric conductivity: $\frac{\kappa_T \ e^2}{\sigma \ T} = \pi^2/3$

Spectral function and quasiparticles

$$\chi_{\mu\nu,\alpha\beta}(k) = \int d^4x\, e^{-ikx} \langle \left[T_{\mu\nu}(x) T_{\alpha\beta}(0) \right] \rangle$$





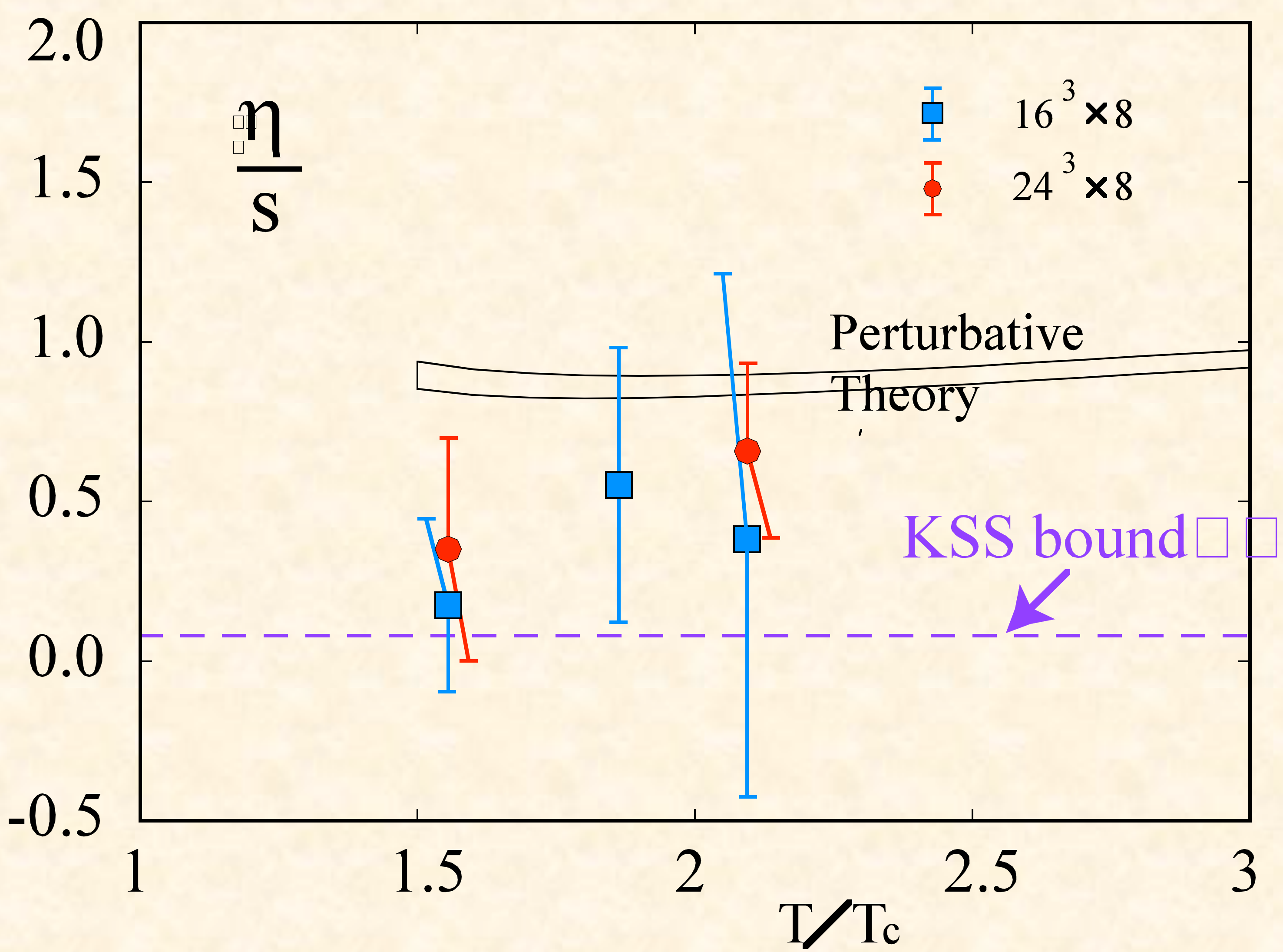


A: scalar channel

B: scalar channel - thermal part

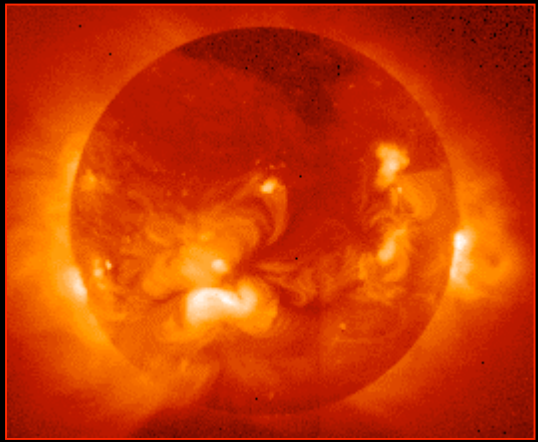
C: sound channel

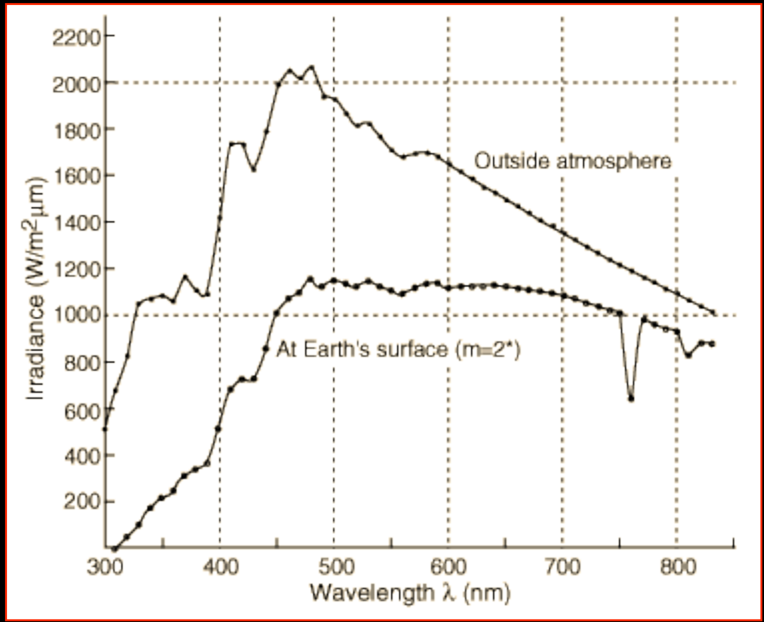
Lattice test of the viscosity/entropy bound $\eta/s \geq 1/4\pi$?

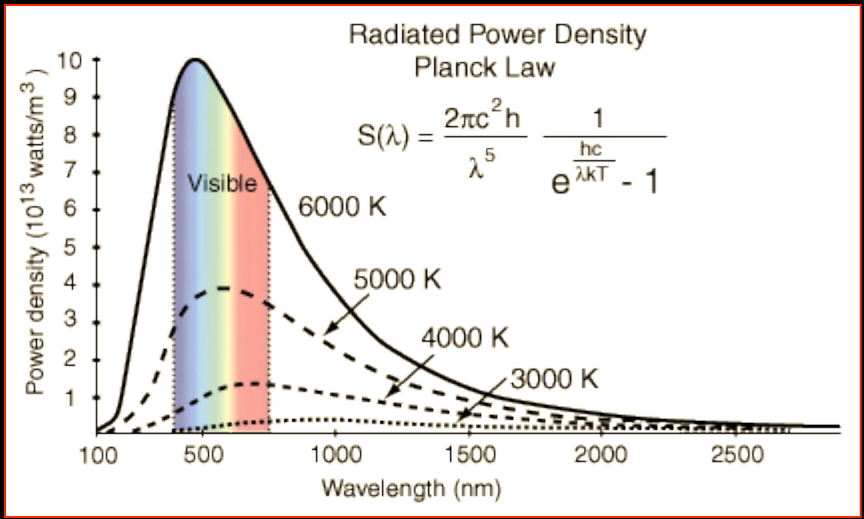


A.Nakamura, S.Sakai, hep-lat/0510039

Photon and dilepton emission from strongly coupled YM plasma







Computing the emission rate

1. In N=4 SYM, gauge $U(1)_R \subset SU(4)_R$ with $\alpha_{em} \ll 1$

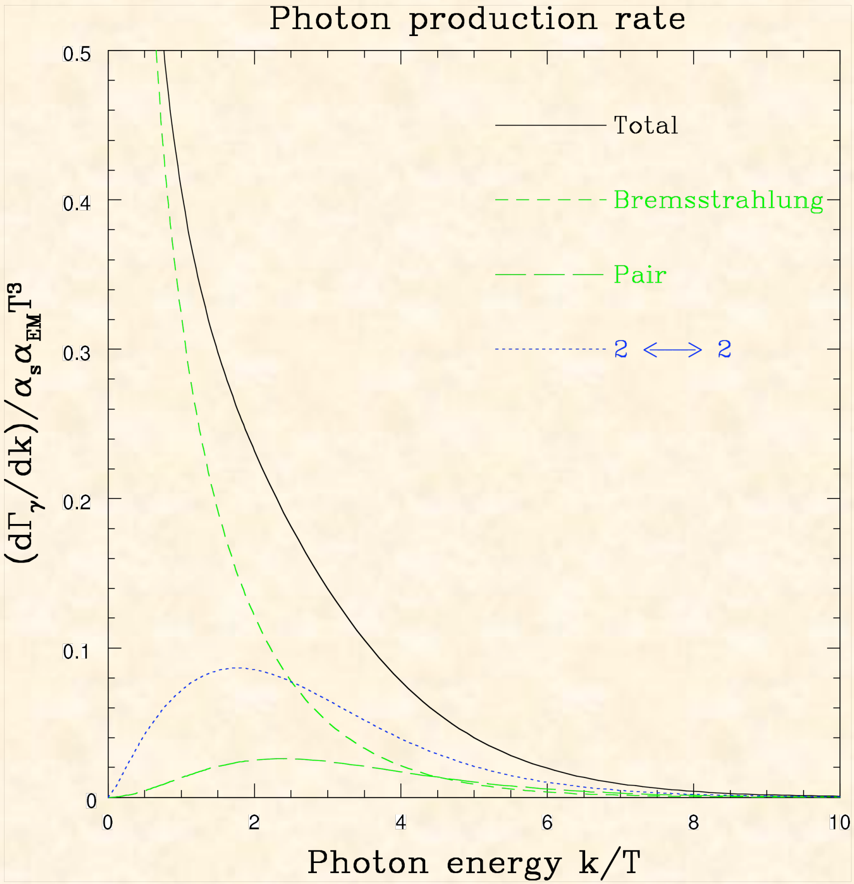
2. Cancel the anomaly by adding weakly interacting, non-thermal fermions

3. The emission rate is $\omega \frac{d\Gamma}{d^4x d^3q} = \frac{\alpha_{em}\, \eta^{\mu\nu}}{(2\pi)^2}\, \Pi_{\mu\nu}(\omega, q)$

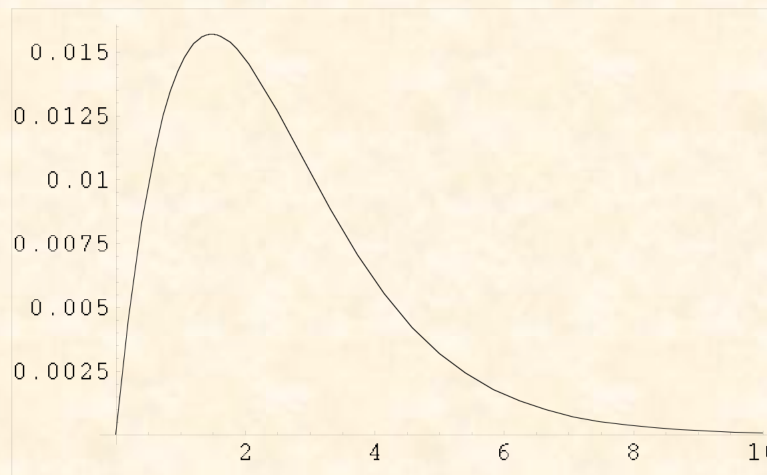
$$\Pi_{\mu\nu}(\omega, q) = \int d^4x e^{-i\omega t + iqx} \langle J_\mu(0)\, J_\nu(x) \rangle_T$$

4. The Wightman correlator is computed from gravity

Photoproduction rate



Perturbative QCD
(Arnold, Moore, Yaffe, 2001)



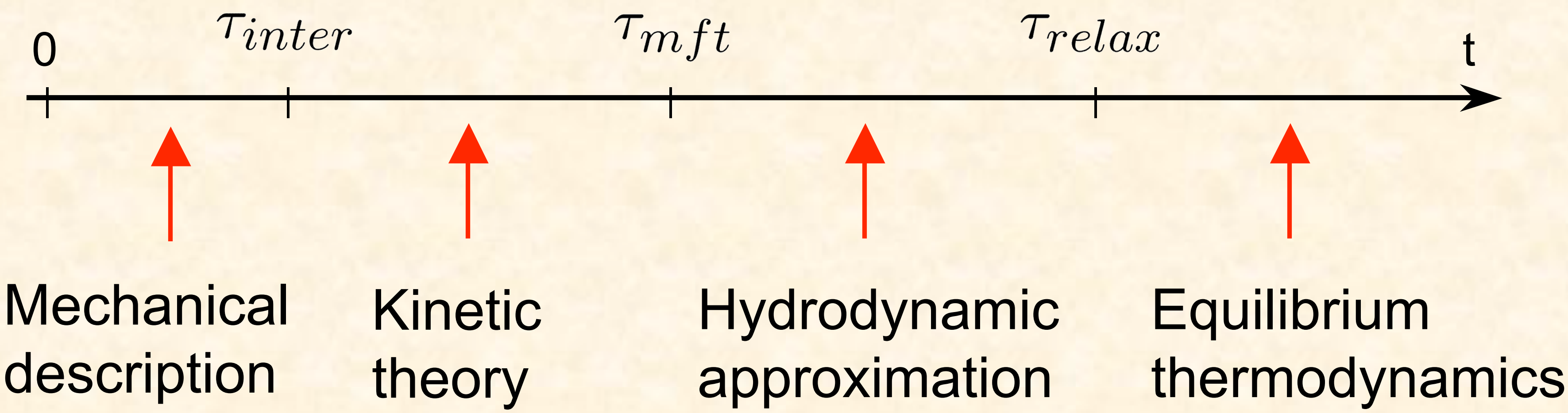
N=4 SYM ++

Outlook

- How universal is η/s ?
- How useful are the N=4 spectral functions for thermal QCD lattice simulations?
- Can we get a meaningful comparison of photon and lepton production rates obtained using pQCD, lattice, AdS/CFT, RHIC?

The hydrodynamic regime

Hierarchy of times (example)



Hierarchy of scales

$$l_{mfp} \ll l \ll L$$

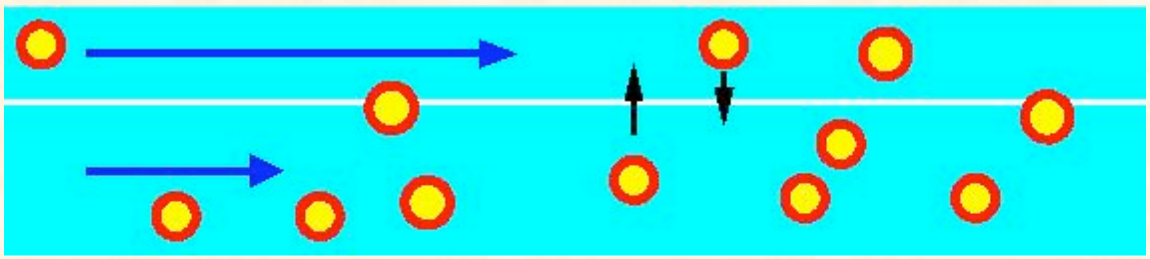
(L is a macroscopic size of a system)

What is viscosity?

Friction in Newton's equation: $$\frac{d(mv_i)}{dt} + \gamma v_i = F_i$$

Friction in Euler's equations

$$\frac{\partial(\rho v_i)}{\partial t} = -\frac{\partial}{\partial x^k}\left(P\delta_{ik} + \rho v_i v_k\right) + \frac{\partial}{\partial x^k}\sigma_{ik}^{fric}$$



$$\sigma_{ik}^{fric} \sim \partial v_i/\partial x^k$$

$$\sigma_{ik}^{fric} \sim \partial v_i/\partial x^k + \partial v_k/\partial x^i$$

$$\sigma_{ik}^{fric} = \eta\left(\frac{\partial v_i}{\partial x^k} + \frac{\partial v_k}{\partial x^i} - \frac{2}{d}\delta_{ik}\frac{\partial v_l}{\partial x^l}\right) + \zeta\delta_{ik}\frac{\partial v_l}{\partial x^l} + \cdots$$

Viscosity of gases and liquids

Gases (Maxwell, 1867): $\eta \sim \rho \, \bar{v} \, l_{mfp} \sim \frac{m_o \bar{v}}{\sigma} \sim \frac{m_o^{1/2}}{\sigma} \sqrt{T}$

Viscosity of a gas is

- independent of pressure
- scales as square of temperature
- inversely proportional to cross-section

Liquids (Frenkel, 1926): $\eta \sim A(P,T) \, \exp \frac{W}{T}$

- W is the "activation energy"
- In practice, A and W are chosen to fit data

Computing transport coefficients from "first principles"

Fluctuation-dissipation theory
(Callen, Welton, Green, Kubo)

Kubo formulae allows one to calculate transport coefficients from microscopic models

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d^3x e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle$$

In the regime described by a gravity dual the correlator can be computed using AdS/CFT

Universality of shear viscosity in the regime described by gravity duals

$$ds^2 = f(w)\left(dx^2 + dy^2\right) + g_{\mu\nu}(w)dw^\mu dw^\nu$$

$$\left.\begin{aligned} \eta &= \lim_{\omega\to 0}\frac{1}{2\omega}\int dt\,dx e^{i\omega t}\langle\left[T_{xy}(t,x),T_{xy}(0,0)\right]\rangle \\ \sigma_{abs} &= -\frac{16\pi G}{\omega}\,\mathrm{Im}\,G^R(\omega) \\ &= \frac{8\pi G}{\omega}\int dt\,dx e^{i\omega t}\langle\left[T_{xy}(t,x),T_{xy}(0,0)\right]\rangle \end{aligned}\right\} \quad \eta = \frac{\sigma_{abs}(0)}{16\pi G}$$

Graviton's component h_y^x obeys equation for a minimally coupled massless scalar. But then $\sigma_{abs}(0) = A_H$.

Since the entropy (density) is $s = A_H/4G$ we get

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Three roads to universality of η/s

- **The absorption argument**

  D. Son, P. Kovtun, A.S., hep-th/0405231

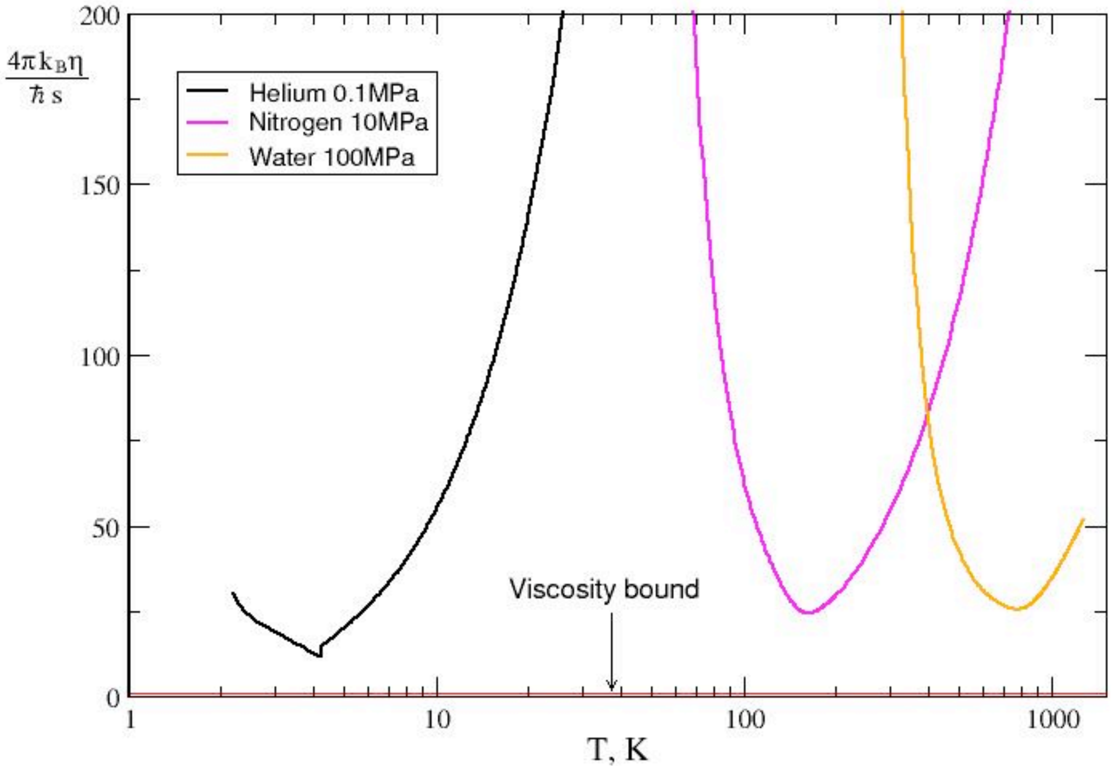
- **Direct computation of the correlator in Kubo formula from AdS/CFT** A.Buchel, hep-th/0408095

- **"Membrane paradigm" general formula for diffusion coefficient + interpretation as lowest quasinormal frequency = pole of the shear mode correlator + Buchel-Liu theorem**

  P. Kovtun, D.Son, A.S., hep-th/0309213, A.S., to appear,
  P.Kovtun, A.S., hep-th/0506184, A.Buchel, J.Liu, hep-th/0311175

A viscosity bound conjecture

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \approx 6.08 \cdot 10^{-13}\, K \cdot s$$



P.Kovtun, D.Son, A.S., hep-th/0309213, hep-th/0405231

Hydrodynamics as an effective theory

Thermodynamic equilibrium: $\langle T^{00}\rangle = \epsilon\,,\ \langle T^{0i}\rangle = 0$

$$T^{ij} = P(\epsilon)\delta^{ij}$$

Near-equilibrium: $T^{00} = \epsilon + \tilde{T}^{00}$

$$T^{ij} = P\delta^{ij} + \frac{\partial P}{\partial \epsilon}\tilde{T}^{00} + \tilde{T}^{ij}$$

$\frac{\partial P}{\partial \epsilon} = v_s^2$

$$\tilde{T}^{ij} = -\frac{1}{\epsilon + P}\left[\eta\left(\partial_i \tilde{T}^{0j} + \partial_j \tilde{T}^{0i} - \frac{2}{3}\delta^{ij}\partial_k \tilde{T}^{0k}\right) + \zeta\delta^{ij}\partial_k \tilde{T}^{0k}\right] + \cdots$$

Eigenmodes of the system of equations $\partial_\mu T^{\mu\nu} = 0$

Shear mode (transverse fluctuations of $\tilde{T}^{0i}$): $\omega = -\frac{i\eta}{\epsilon + P}q^2$

Sound mode: $\omega = v_s q - \frac{i}{2}\frac{1}{\epsilon + P}\left(\zeta + \frac{4}{3}\eta\right)q^2$

For CFT we have $\zeta = 0$ and $\epsilon = 3P$ ⇒ $v_s = 1/\sqrt{3}$

Two-point correlation function of stress-energy tensor

Field theory

Zero temperature: $\langle T_{\mu\nu} T_{\alpha\beta} \rangle_{T=0} = \Pi_{\mu\nu,\alpha\beta} F(k^2) + Q_{\mu\nu,\alpha\beta} G(k^2)$

Finite temperature: $\langle T_{\mu\nu} T_{\alpha\beta} \rangle_{T} = S^{(1)}_{\mu\nu,\alpha\beta} G_1(\omega, q) + S^{(2)}_{\mu\nu,\alpha\beta} G_2(\omega, q)$

$+ \; S^{(3)}_{\mu\nu,\alpha\beta} G_3(\omega, q) + S^{(4)}_{\mu\nu,\alpha\beta} G_4 + S^{(5)}_{\mu\nu,\alpha\beta} G_5$

Dual gravity

- Five gauge-invariant combinations Z_1, Z_2, Z_3, Z_4, Z_5 of $h_{\mu\nu}$ and other fields determine G_1, G_2, G_3, G_4, G_5
- Z_1, Z_2, Z_3, Z_4, Z_5 obey a system of coupled ODEs
- Their (quasinormal) spectrum determines singularities of the correlator

Classification of fluctuations and universality

$$ds^2 = \frac{r^2}{R^2}\left(-f(r)dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{R^2}{r^2 f} dR^2$$

$\delta g_{\mu\nu} \sim e^{-i\omega t + iqz} h_{\mu\nu}(r)$ O(2) symmetry in x-y plane

Shear channel: h_{tx} h_{zx} h_{ty} h_{zy} → Z_1

Sound channel: h_{tt} h_{tz} h_{zz} $h_{xx} + h_{yy}$ → Z_2

Scalar channel: h_{xy} $h_{xx} - h_{yy}$ → Z_3

Other fluctuations (e.g. $\delta\varphi_1, \ldots \delta\varphi_n$) may affect sound channel

But not the shear channel → universality of η/s

Gauge-invariant variables for a gravity dual to a conformal theory

$$ds^2 = a(r)\left(-f(r)dt^2 + dx^2 + dy^2 + dz^2\right) + b(r)dr^2$$

$$h_{\mu\nu} \to h_{\mu\nu} - \nabla_\mu \xi_\nu - \nabla_\nu \xi_\mu$$

Shear: $Z_1 = q\, H_{tx} + \omega\, H_{zx}$

Sound: $Z_2 = q^2 f H_{tt} + 2\omega q H_{tz} + \omega^2 H_{zz} + q^2 f \left(1 + \frac{a f'}{a' f} - \frac{\omega^2}{q^2 f}\right) H$

Scalar: $Z_3 = H_{xy}$

$H_{ij} = h_{ij}/a \qquad h_{\mu\nu} \sim e^{-i\omega t + iqz} \qquad H = \left(h_{xx} + h_{yy}\right)/2a$

Bulk viscosity and the speed of sound in $\mathcal{N}=2^*$ SYM

$\mathcal{N}=2^*$ is a "mass-deformed" $\mathcal{N}=4$ (Pilch-Warner flow)

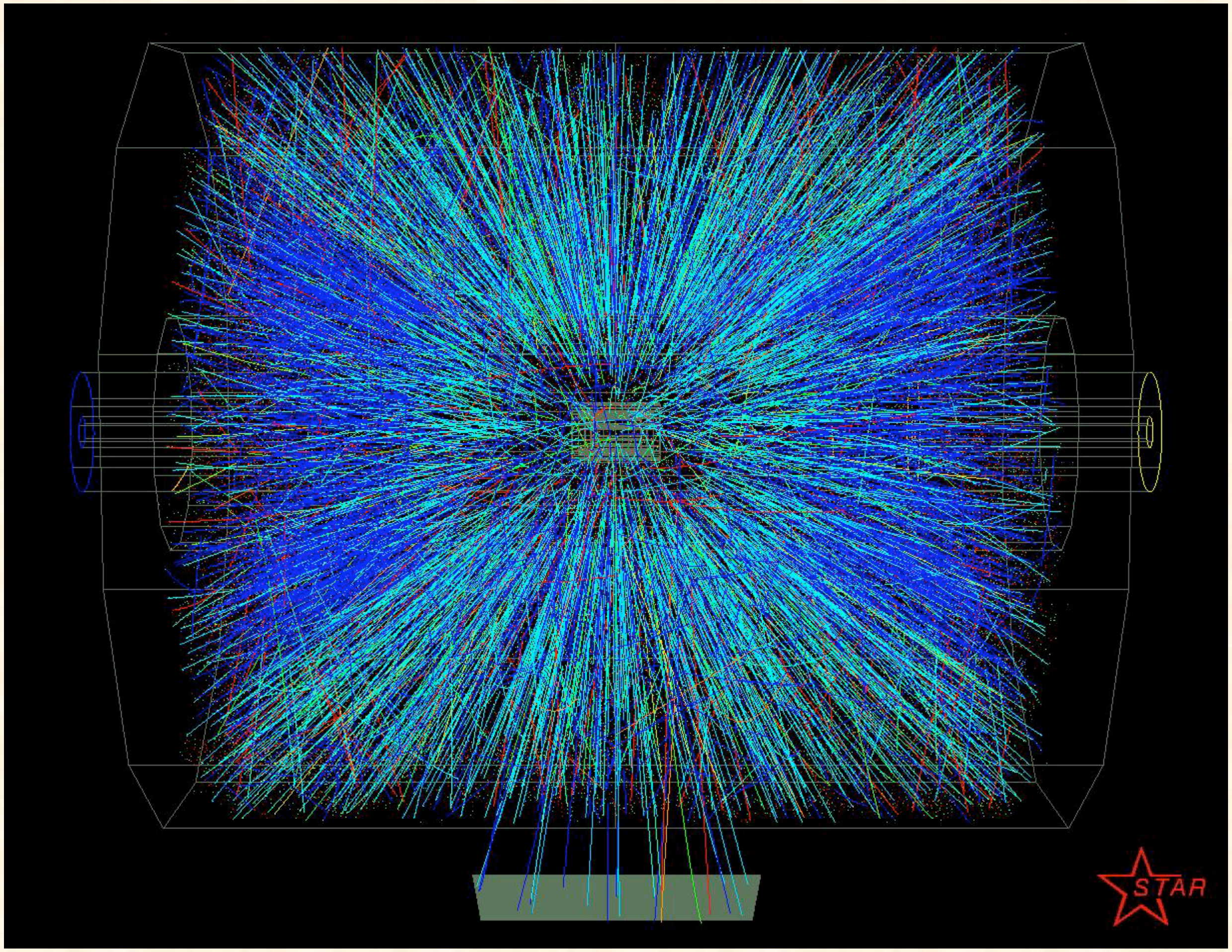
- Finite-temperature version: A.Buchel, J.Liu, hep-th/0305064
- The metric is known explicitly for $m/T \ll 1$
- Speed of sound and bulk viscosity:

$$v_s = \frac{1}{\sqrt{3}}\left(1 - \frac{\left[\Gamma\left(\frac{3}{4}\right)\right]^4}{3\pi^4}\left(\frac{m_f}{T}\right)^2 - \frac{1}{18\pi^4}\left(\frac{m_b}{T}\right)^4 + \cdots\right)$$

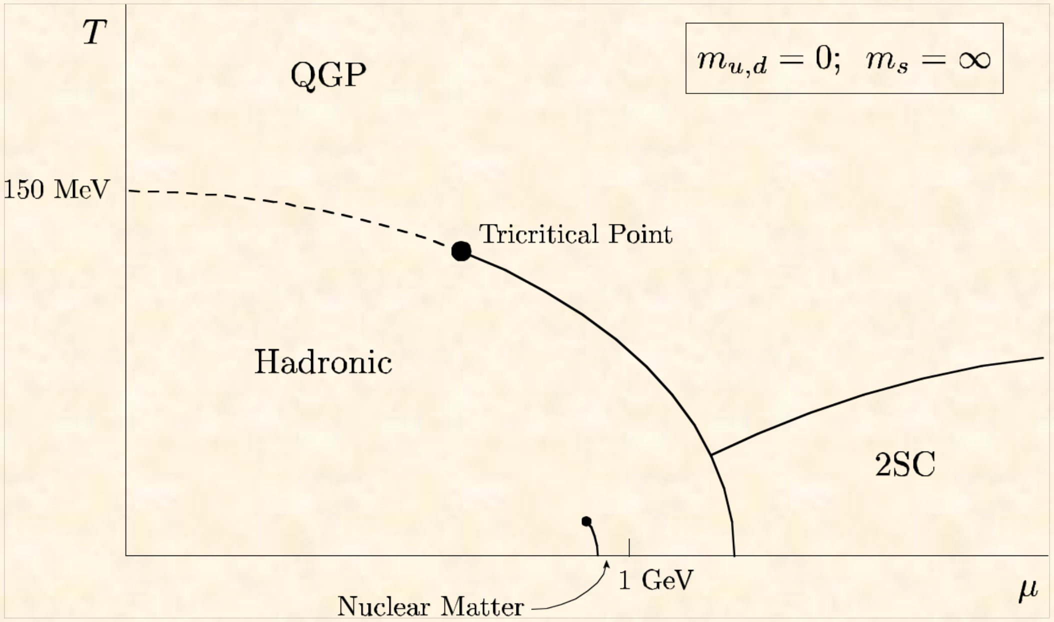
$$\frac{\zeta}{\eta} = \beta_f^\Gamma \frac{\left[\Gamma\left(\frac{3}{4}\right)\right]^4}{3\pi^3}\left(\frac{m_f}{T}\right)^2 + \frac{\beta_b^\Gamma}{432\pi^2}\left(\frac{m_b}{T}\right)^4 + \cdots$$

$$\frac{\zeta}{\eta} = -\kappa\left(v_s^2 - \frac{1}{3}\right)$$

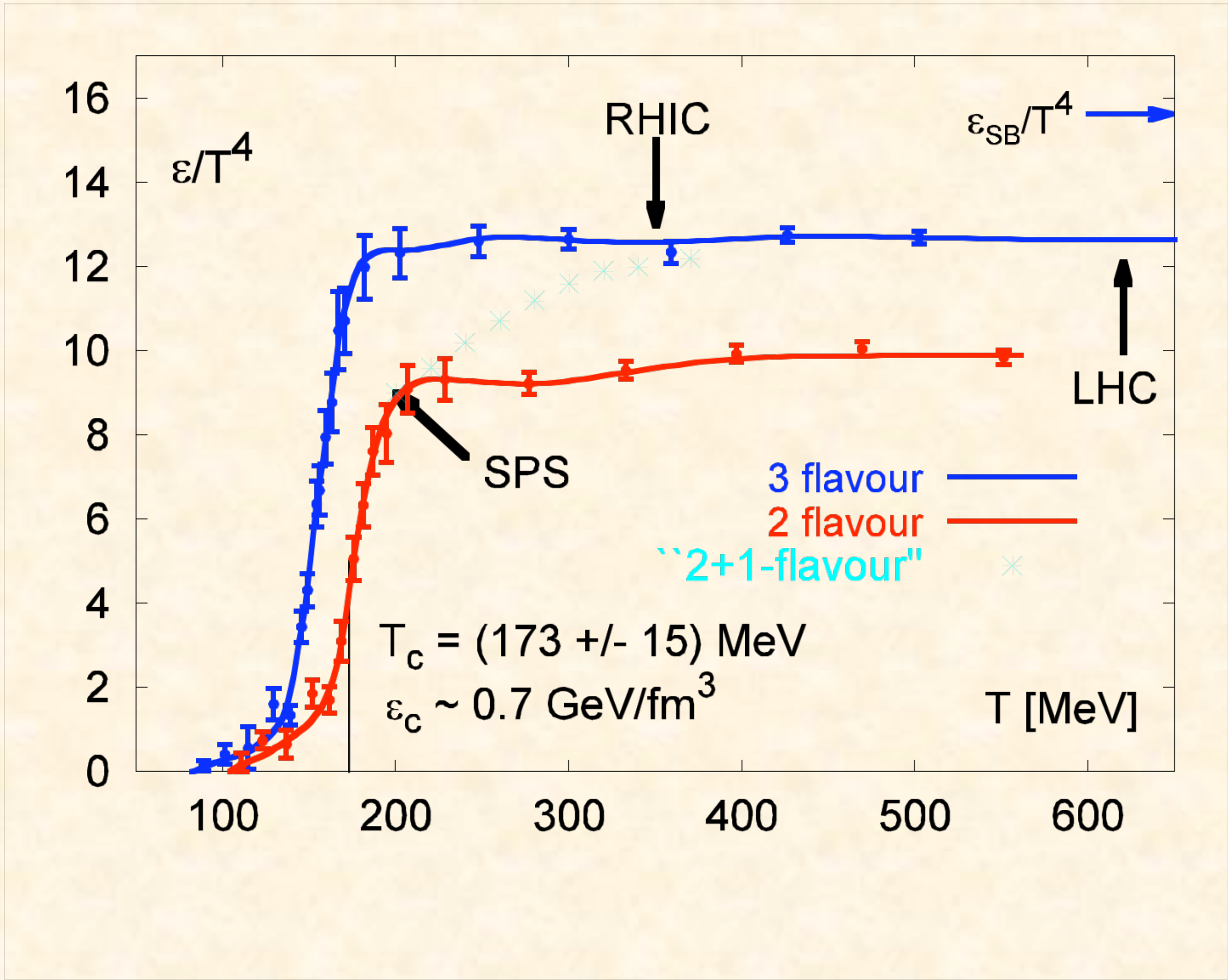
Heavy ion collisions: RHIC/LHC



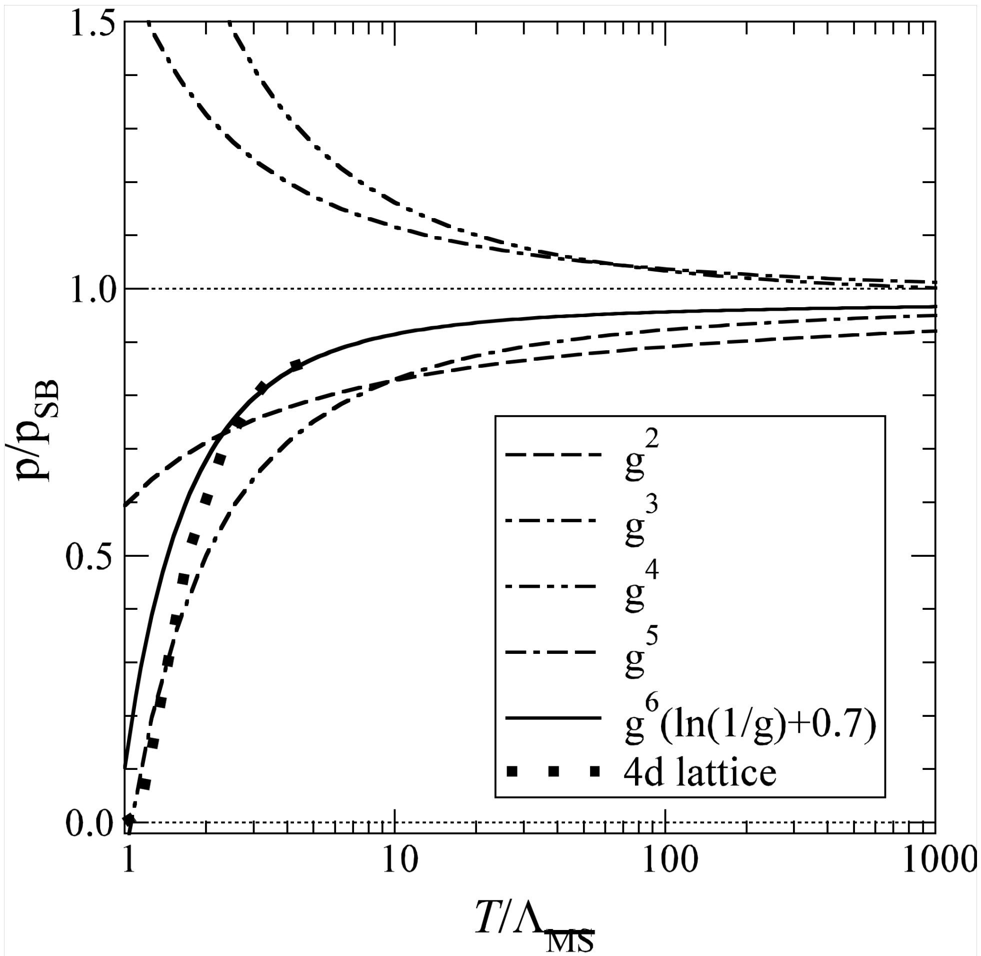
QCD phase diagram



QCD deconfinement transition



Pressure in perturbative QCD



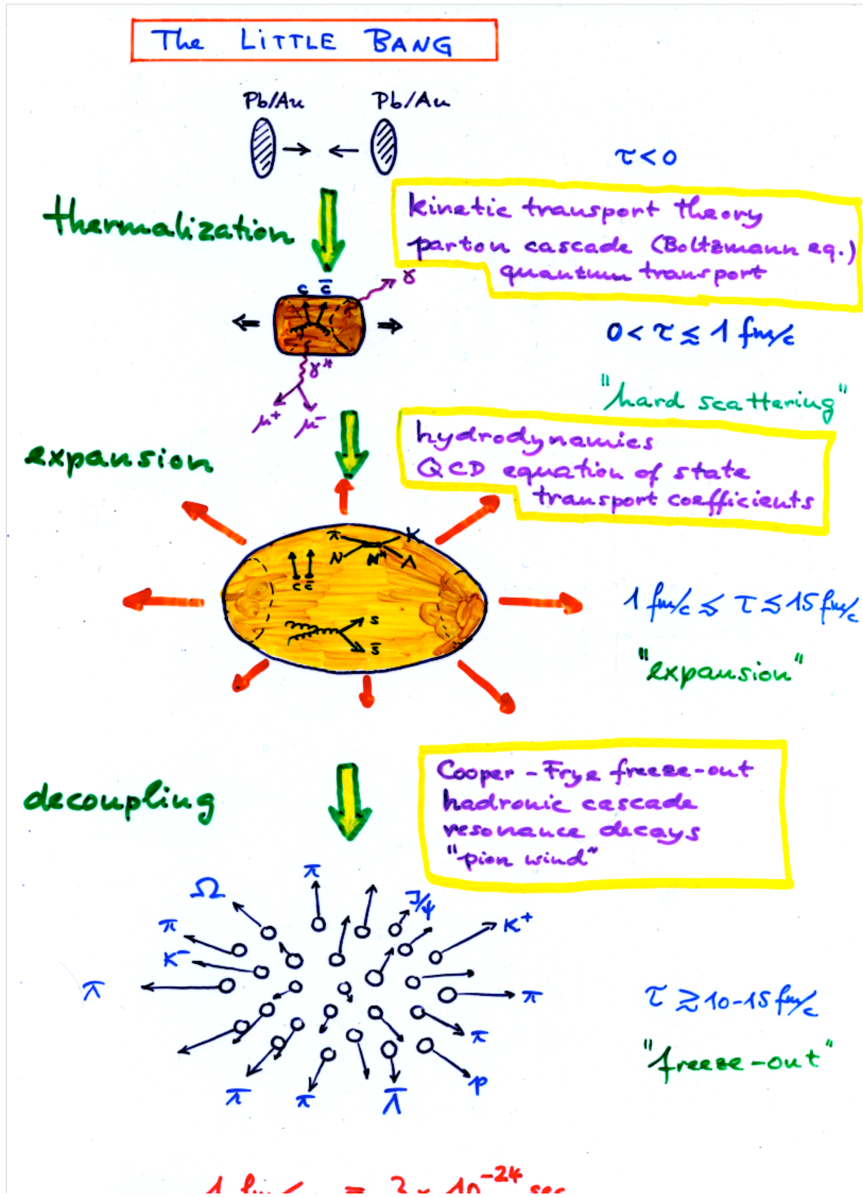
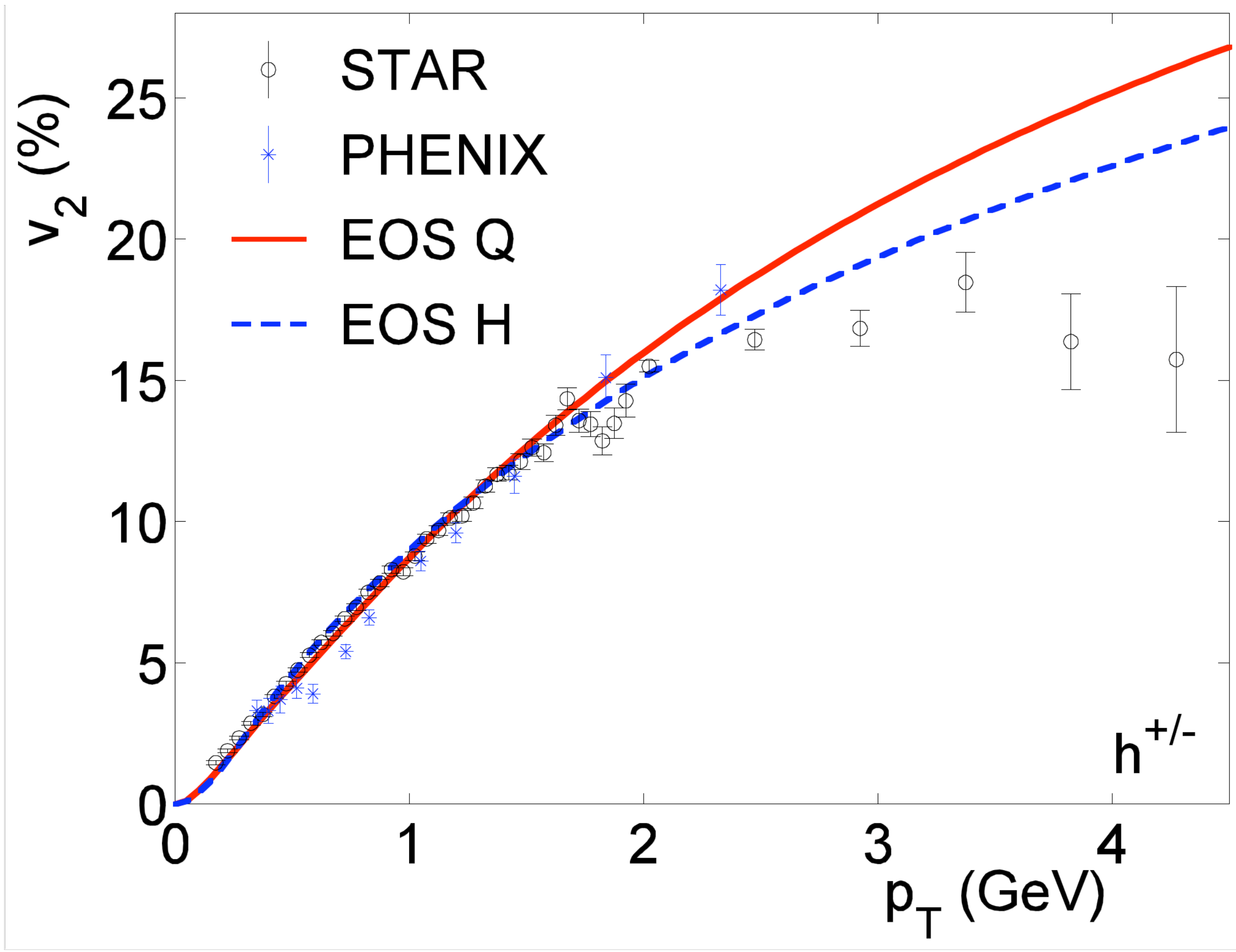
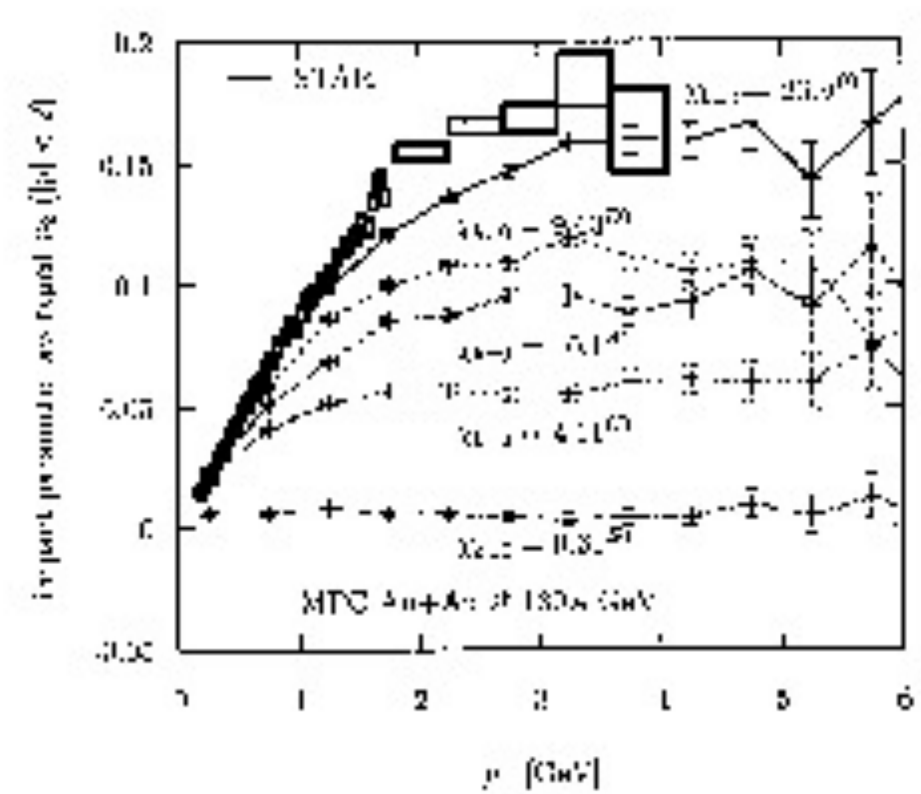


Figure from: U.Heinz, “Concepts of heavy-ion physics”, hep-ph/0407360

Elliptic flow at RHIC



Effect of viscosity on elliptic flow



Conclusions

- AdS/CFT gives insights into physics of thermal gauge theories in the nonperturbative regime
- Generic hydrodynamic predictions can be used to check validity of AdS/CFT
- General algorithm exists to compute transport coefficients and the speed of sound in any gravity dual
- Model-independent statements can presumably be checked experimentally