Engineering the Standard Model in M-theory: A Local Origin of Three Families from E_8

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The Physics and Mathematics of G_2 Compactifications
MCTP, Ann Arbor, Michigan

4^{th} May 2007

[arXiv:0704.0444] and [arXiv:0704.0445]
[arXiv:0705.0XXX]

Outline

Model Building in M-Theory (*really*)

Gauge theory with massless charged matter is known to arise in M-theory two ways, both geometric in origin:

- boundaries in the compactification manifold [e.g. Hořava & Witten]
- or singularities (known as geometrical engineering):
  - gauge theory: 3-d (co-dim 4) 'ADE' singularities [Atiyah & Witten]
  - charged matter: isolated (co-dim 7) enhanced singularities living the ADE singularities [Witten, Acharya & Witten]

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Naïvely Engineering the Standard Model in M-Theory

What would the Standard Model look like in M-theory?

Within the G_2 manifold there must be:

- 3-d SU_3 and SU_2 singularities
- which meet at exactly three points (one for each Q_L)
- and one additional conical singularity for every known (or expected) matter field

Seems to be wishful thinking at best, and terribly ad hoc at worst:

- Why those singularities?
- Why at those relative positions?

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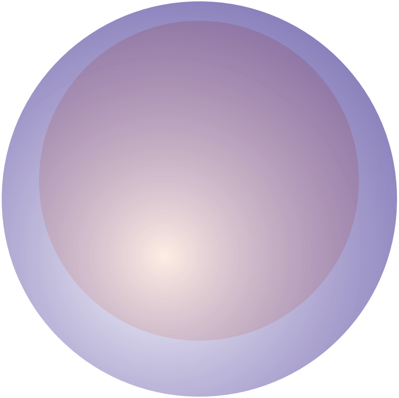

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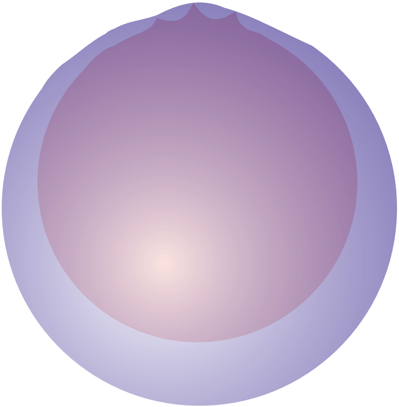

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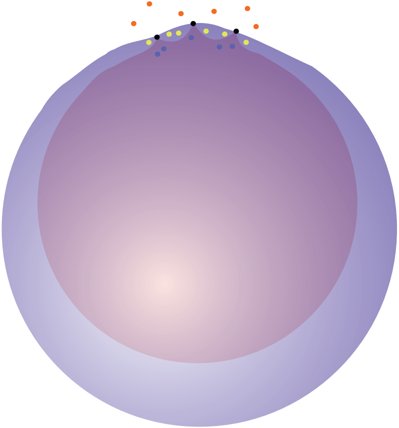

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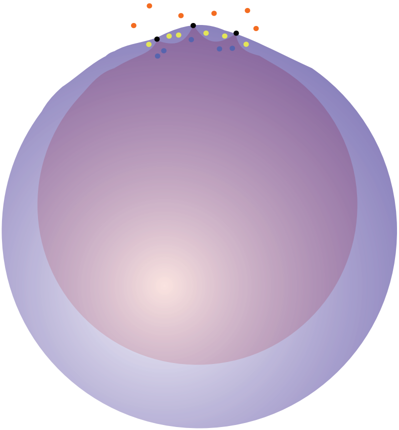

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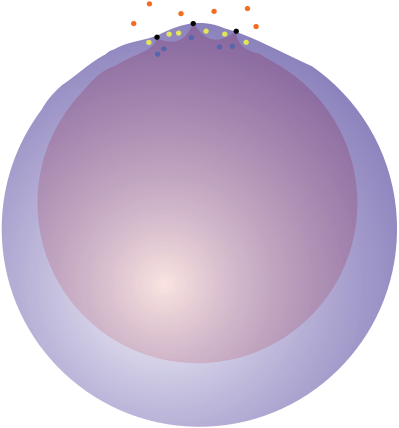

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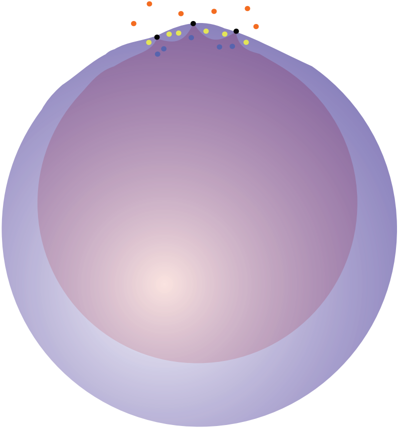

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A G_2 manifold engineered to give grand unification can be unfolded in a way analogous to symmetry breaking:

[JB,to appear, arXiv:0704.0444 arXiv:0704.0445]

Unfolding a $\mathbf{27}$ of E_6

Features of the unfolded geometry:

- local geometry (topology) containing all unfolded singularities is known
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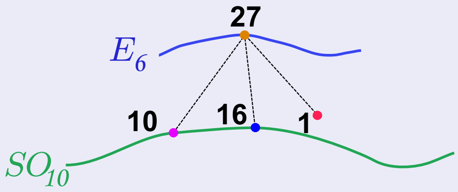

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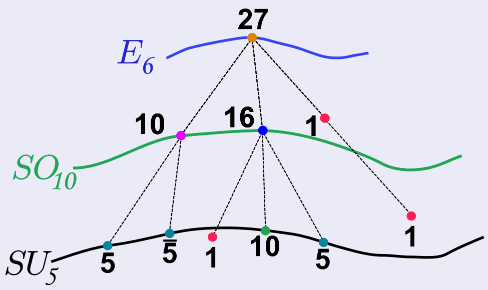

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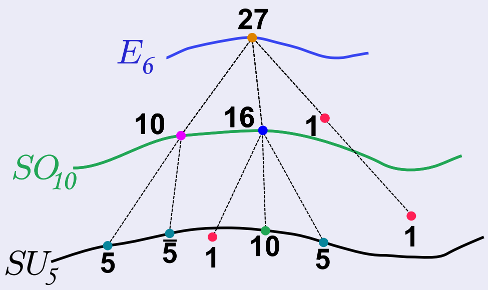

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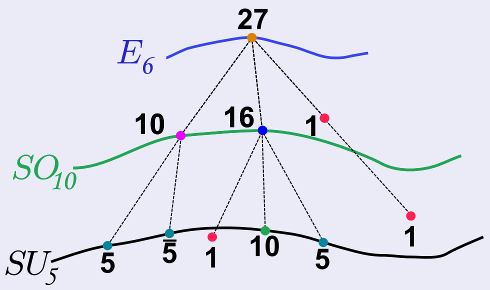

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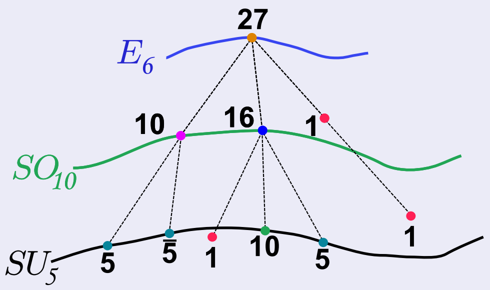

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Engineering Gauge Theory in M-theory

M-theory on:		Heterotic string on:
$K3$	$\Longleftrightarrow$	T^3
$\downarrow$	fibre-wise duality	$\downarrow$
Q		Q

$\therefore$ There are places in $K3$ moduli space with non-Abelian gauge theory (and possibly charged matter).

- Gauge theory arises when the (generic) $K3$ fibres develop isolated ADE-type singularities

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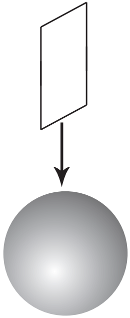

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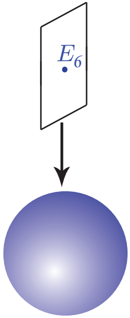

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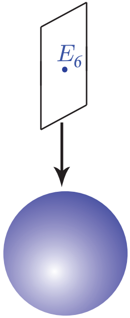

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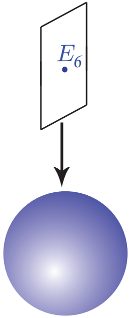

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[Katz & Vafa, Witten, Acharya & Witten]

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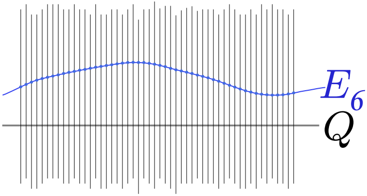

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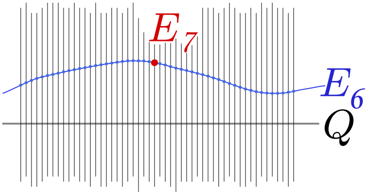

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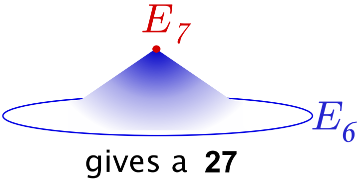
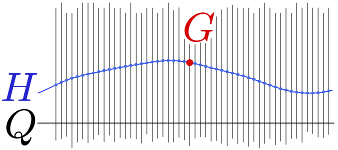


Representations Resulting from ADE Resolutions

The enhancement of an H-type singularity into one of type G at an isolated point is called the resolution $G \to H$.



Representation from $G \to H$

When $G \supset H \times U(1)$, $G \to H$ gives rise to matter charged under H; the representation follows from the $U(1)$-charged parts of the decomposition of the adjoint of G into $H \times U(1)$.

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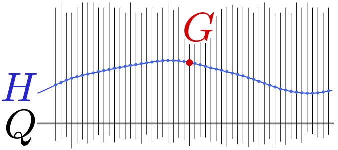

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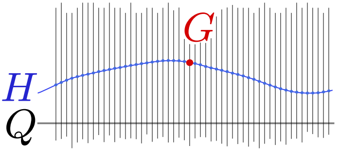

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- $SU_{n+1} \to SU_n \implies \mathbf{n}$ of SU_n
- $E_6 \to SO_{10} \implies \mathbf{16}$ of SO_{10}
- $SO_{2n} \to SU_n \implies [\mathbf{n} \times \mathbf{n}]_a$ of SU_n
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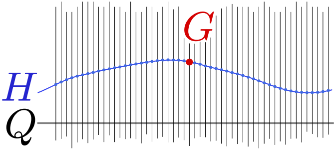

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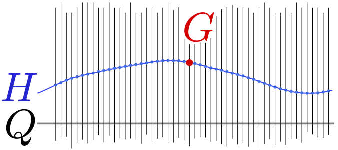

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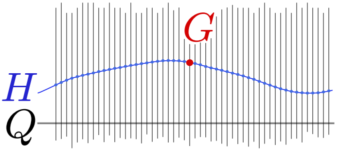

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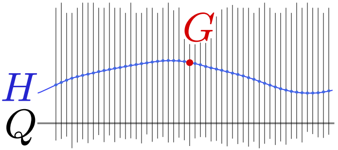

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- $SO_{2n} \to SU_n \implies [\mathbf{n} \times \mathbf{n}]_a$ of SU_n
- $E_6 \to SO_{10} \implies \mathbf{16}$ of SO_{10}
- $E_7 \to E_6 \implies \mathbf{27}$ of E_6

A Provocation:

- $E_8 \to E_6 \times SU_2 \implies (\mathbf{1}, \mathbf{2}) \oplus (\mathbf{27}, \mathbf{1}) \oplus (\mathbf{27}, \mathbf{2})$ of $E_6 \times SU_2$.

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E_6	$x^2 = y^3 + z^4$
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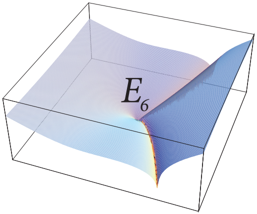

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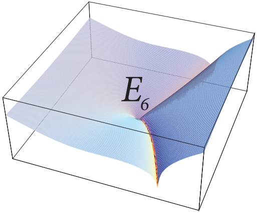

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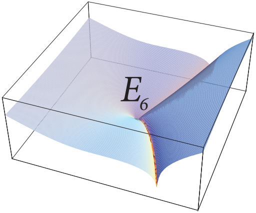

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But not easy to upgrade to 3-d deformations applicable to M-theory.

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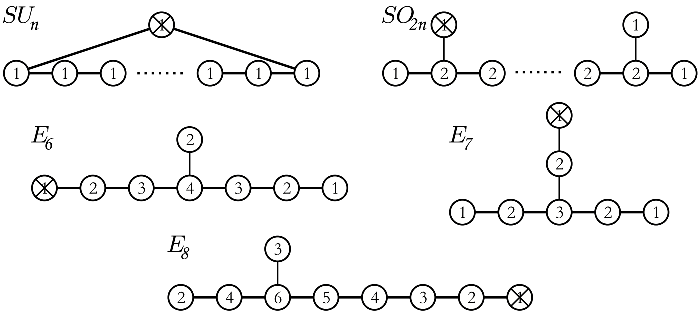

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- Consider n hypermultiplets $\{\Phi_i\}$ $i=0,\ldots,(n-1)$ charged under $K\equiv U(1)^{n-1}$ gauge theory as follows:

	Φ_0	Φ_1	Φ_2	$\cdots$	Φ_{n-1}
$U(1)_1$	-1	1	0	$\cdots$	0
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The vacuum manifold of this Σ-model is denoted $\mathbb{H}^n//K$.

- Specified by applying all D/F-term constraints (setting the FI-parameter for each $U(1)_i$ to zero)
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The FI-parameter for $U(1)_m$ can be written

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- By relaxing the D-term constraints on the Σ-model, we can unfold an SU_n singularity into its subgroups.
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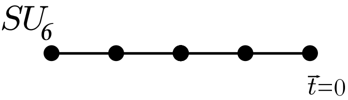

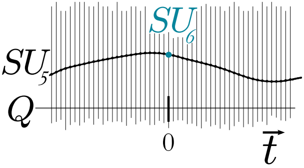

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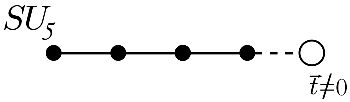

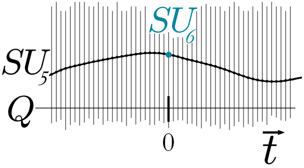

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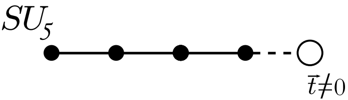

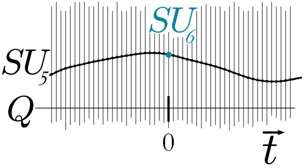

This is how a $\mathbf{5}$ of SU_5 is engineered in M-theory.

[Witten
Acahrya & Witten]

Unfolding Gauge Theory in M-Theory

Consider now varying the type of singularity of the fibres independent of the location on Q:

- e.g. consider relaxing the D-term constraint on $\vec{t}_2 \equiv \vec{s}$ of an SU_5 singularity:

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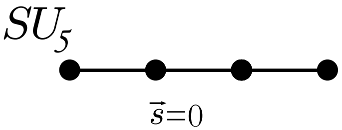

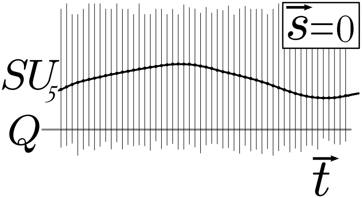

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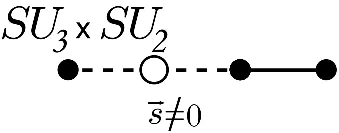

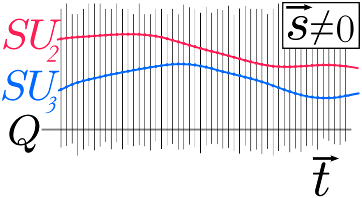

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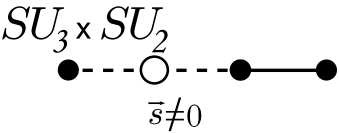

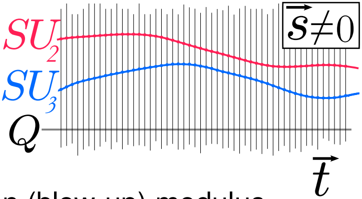

We can envision $\vec{s}$ as a deformation (blow-up) modulus changing the type of gauge theory.

Unfolding a $\mathbf{5}$ of SU_5 into $SU_3 \times SU_2$

A natural question to ask is: what happens when a fibration with an isolated enhanced singularity is 'globally' unfolded?

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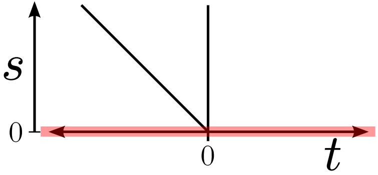

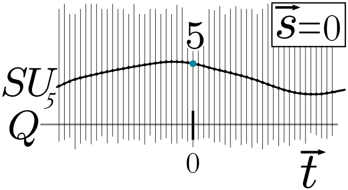

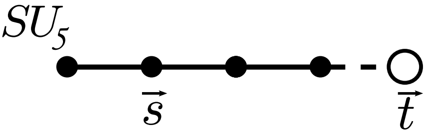

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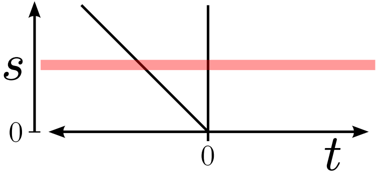

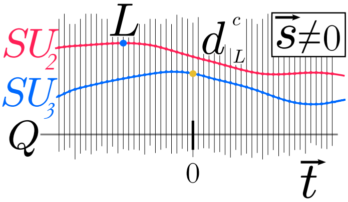

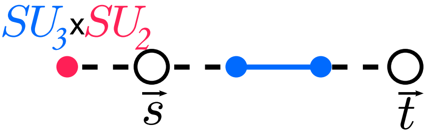

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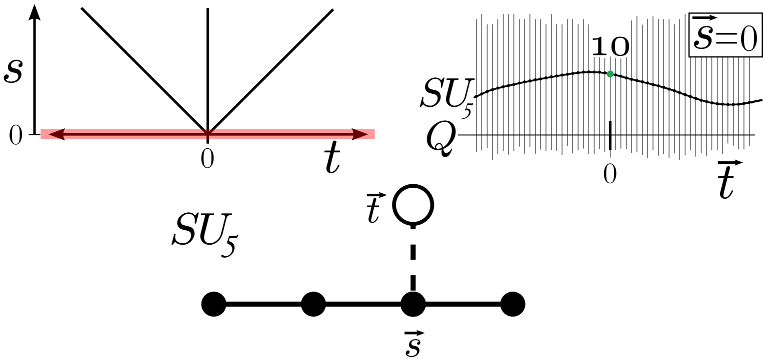

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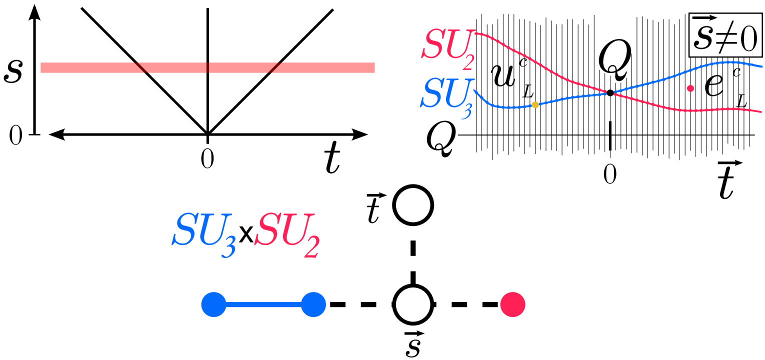

Unfolding Three Families out of E_8

- Recall: $E_8 \to E_6 \times SU_2 \implies (\mathbf{1},\mathbf{2}) \oplus (\mathbf{27},\mathbf{1}) \oplus (\mathbf{27},\mathbf{2})$ of $E_6 \times SU_2$.
- Therefore, unfolding this singularity manifestly produces three families.

Let's see how this looks in detail.

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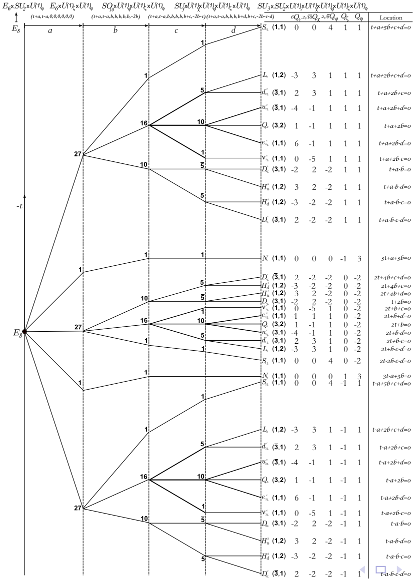
E6×SU2×U(1)φ E6×U(1)ζ×U(1)φ SO10×U(1)ψ×U(1)ζ×U(1)φ SU5×U(1)χ×U(1)ψ×U(1)ζ×U(1)φ SU3×SU2×U(1)Y×U(1)χ×U(1)ψ×U(1)ζ×U(1)φ
(t+a,t-a,0,0,0,0,0,0) (t+a,t-a,b,b,b,b,b,-2b) (t+a,t-a,b,b,b,b,b+c,-2b-c) (t+a,t-a,b,b,b,b+d,b+c,-2b-c-d)
E8 a b c d
6QY 2√10Qχ 2√6Qψ Qζ Qφ Location
S (1,1) 0 0 4 1 1 t+a+5b+c+d=0
L (1,2) -3 3 1 1 1 t+a+2b+c+d=0
d^c (3̄,1) 2 3 1 1 1 t+a+2b+c=0
u^c (3̄,1) -4 -1 1 1 1 t+a+2b+d=0
Q (3,2) 1 -1 1 1 1 t+a+2b=0
e^c (1,1) 6 -1 1 1 1 t+a+2b-d=0
ν^c (1,1) 0 -5 1 1 1 t+a+2b-c=0
D (3,1) -2 2 -2 1 1 t+a-b=0
Hu (1,2) 3 2 -2 1 1 t+a-b-d=0
Hd (1,2) -3 -2 -2 1 1 t+a-b-c=0
D^c (3̄,1) 2 -2 -2 1 1 t+a-b-c-d=0
N (1,1) 0 0 0 -1 3 3t+a+3b=0
D^c (3̄,1) 2 -2 -2 0 -2 2t+4b+c+d=0
Hd (1,2) -3 -2 -2 0 -2 2t+4b+c=0
Hu (1,2) 3 2 -2 0 -2 2t+4b+d=0
D (3,1) -2 2 -2 0 -2 t+2b=0
ν^c (1,1) 0 -5 1 0 -2 2t+b+c=0
e^c (1,1) -1 1 1 0 -2 2t+b+d=0
Q (3,2) 1 -1 1 0 -2 2t+b=0
u^c (3̄,1) -4 -1 1 0 -2 2t+b-d=0
d^c (3̄,1) 2 3 1 0 -2 2t+b-c=0
L (1,2) -3 3 1 0 -2 2t+b-c-d=0
S (1,1) 0 0 4 0 -2 2t-2b-c-d=0
N (1,1) 0 0 0 1 3 3t-a+3b=0
S (1,1) 0 0 4 -1 1 t-a+5b+c+d=0
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-t

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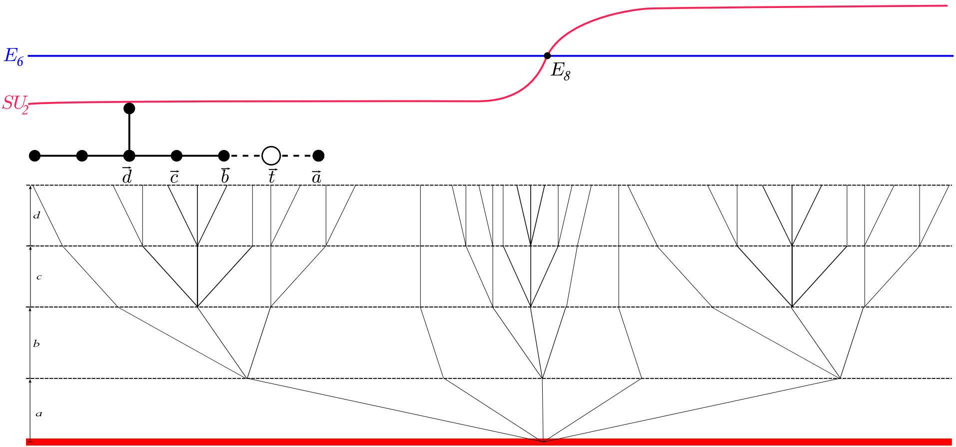

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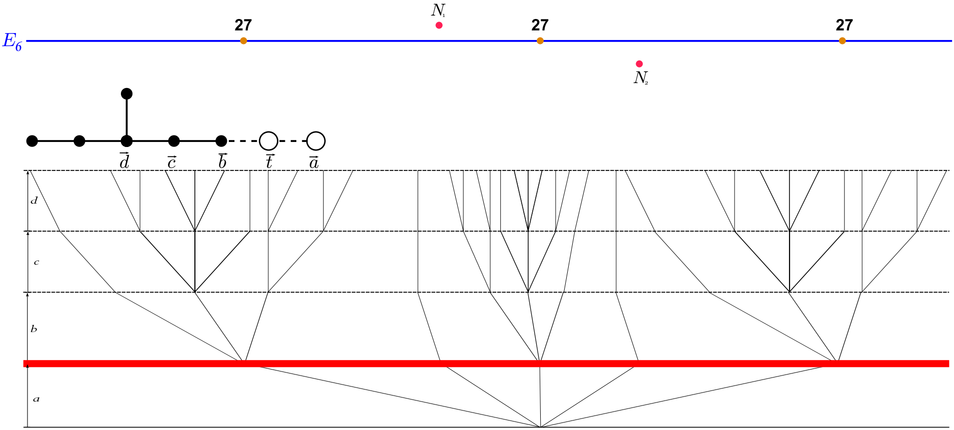

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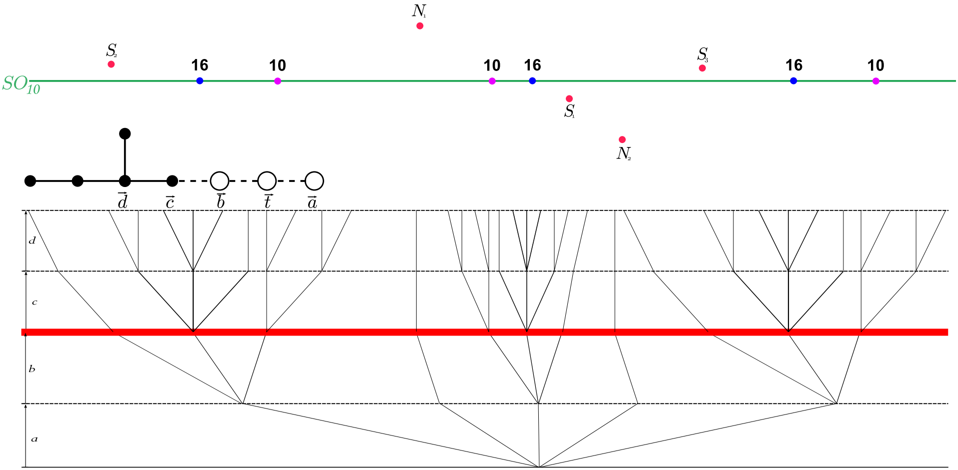

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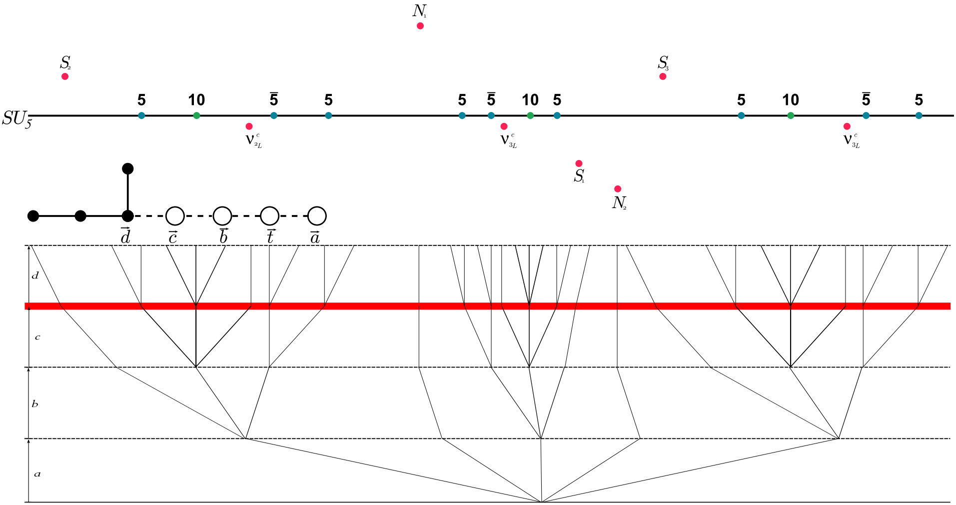

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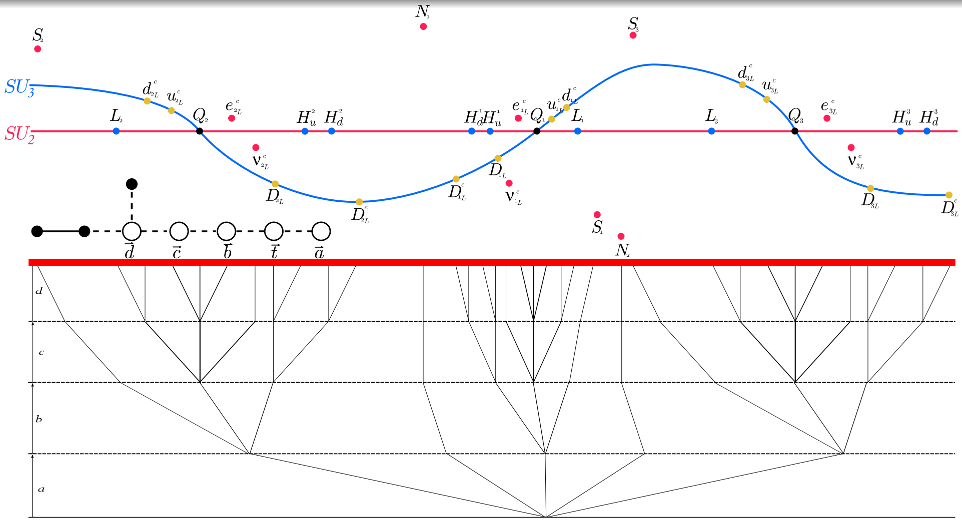

Unfolding Three Families out of E_8

Features of and questions about the resulting Standard-Model like model:

- $U(1)$-charges allow only terms of the form $\mathbf{27}_1\mathbf{27}_2\mathbf{27}_3$ can appear in the superpotential
  - 'families' in the colloquial sense must be linear combinations of the $\mathbf{27}$'s
- fate of extra $U(1)$ symmetries not yet clear
- E_6-like exotics: six Higgs coloured triplets
  - no obvious solution to the 3-2 splitting problem
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Further Questions and Future Directions

For M-theory experts:

- How are the parameters deforming the geometry controlled—fixed—physically?
- What stabilizes $SU_3 \times SU_2$ from unfolding further?
- How can geometric unification be connected with gauge coupling unification?

For mathematicians:

- Can these local geometries be shown (explicitly) to be G_2?
- How can these patches be glued into a compact G_2 manifold?

For E_6 Grand Unification experts

- Is the complexity of the spectrum rich enough to avoid the usual problems?

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