Role of Global Warming on Frequency of Record Temperature Events

Large Deviations Conference, MCTP, June 2007

collaborator: Mark Petersen (LANL)

thanks to CHMI & Phil Jones (EAU) for data

Facts about urban temperatures

- annual pattern & long-term trends
- daily temperature distribution

Tutorial: evolution of record temperature events

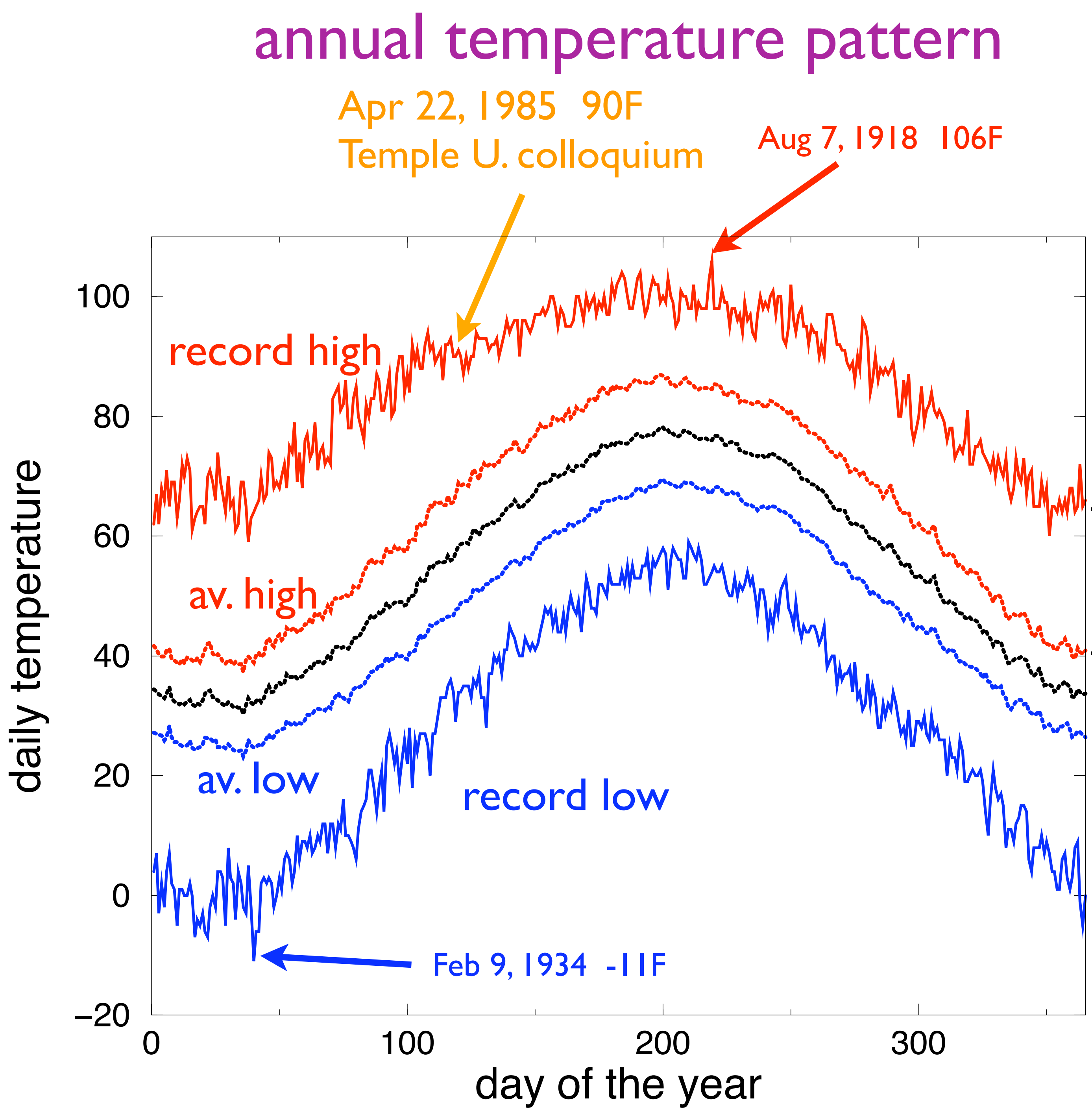
- magnitude of successive records
- time between successive records

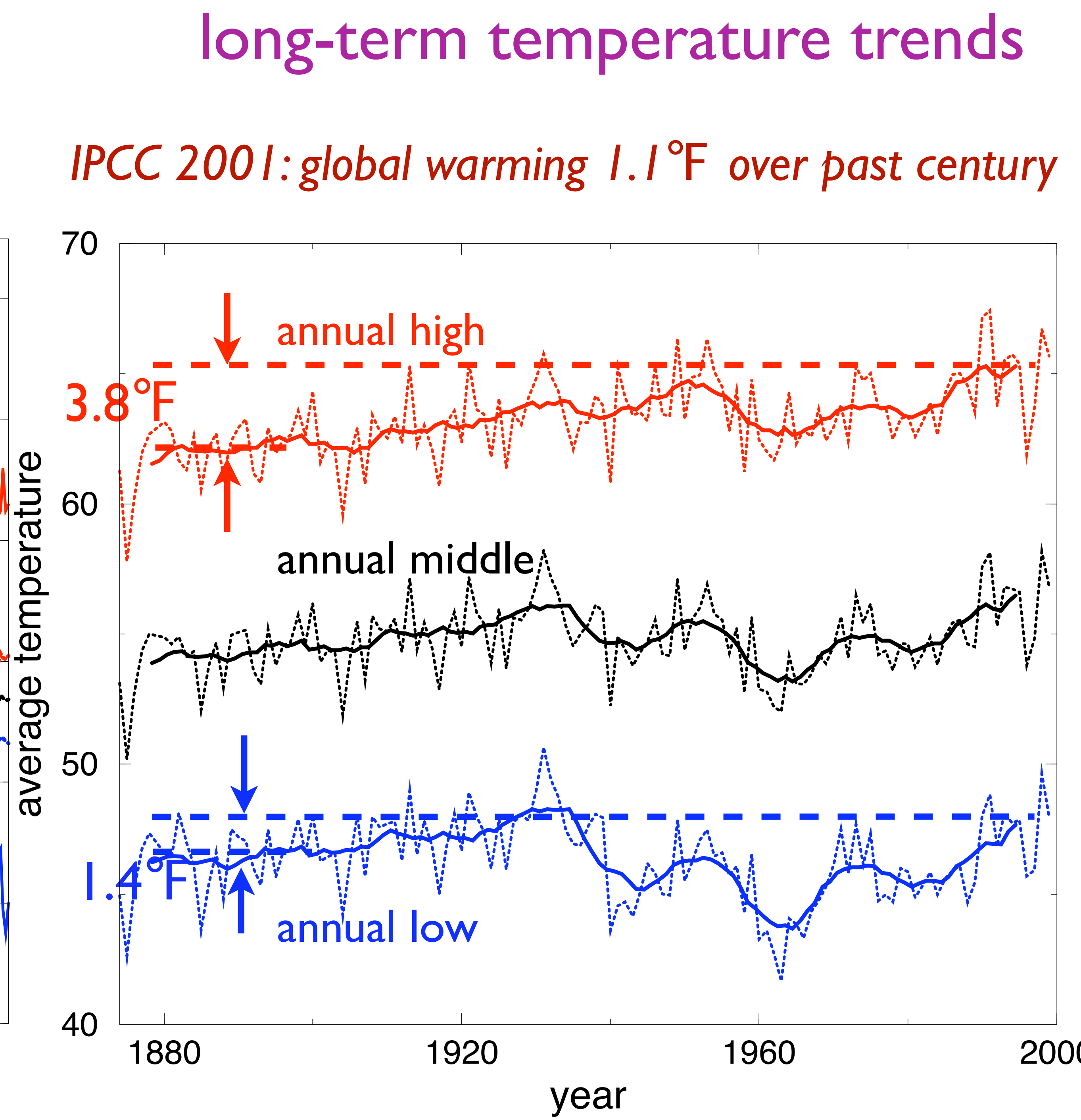
Role of global warming on records

Summary & Outlook

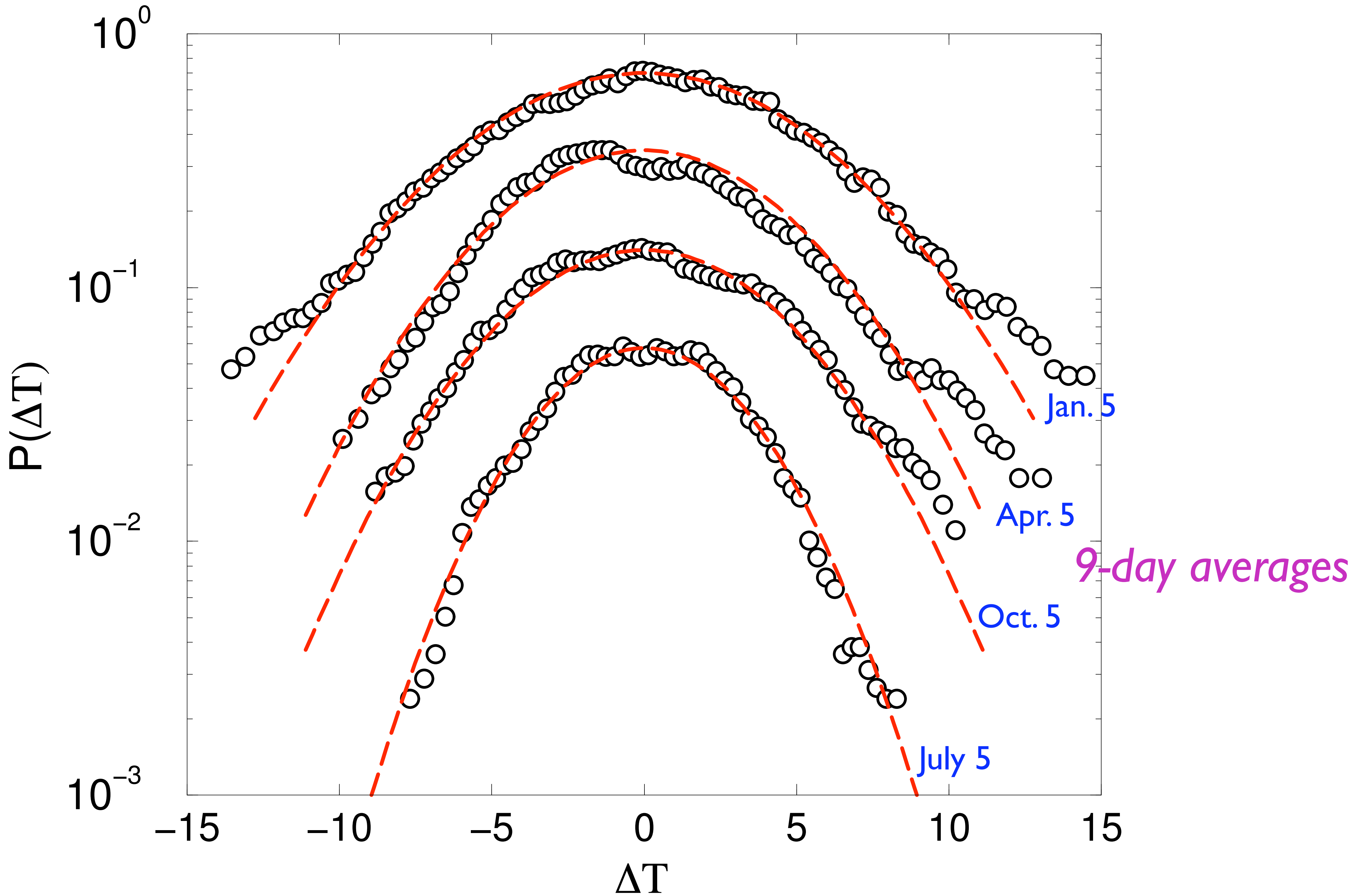
- role of inter-day temperature correlations
- seasonal effects
- asymmetry in high & low temperatures
- changing variability

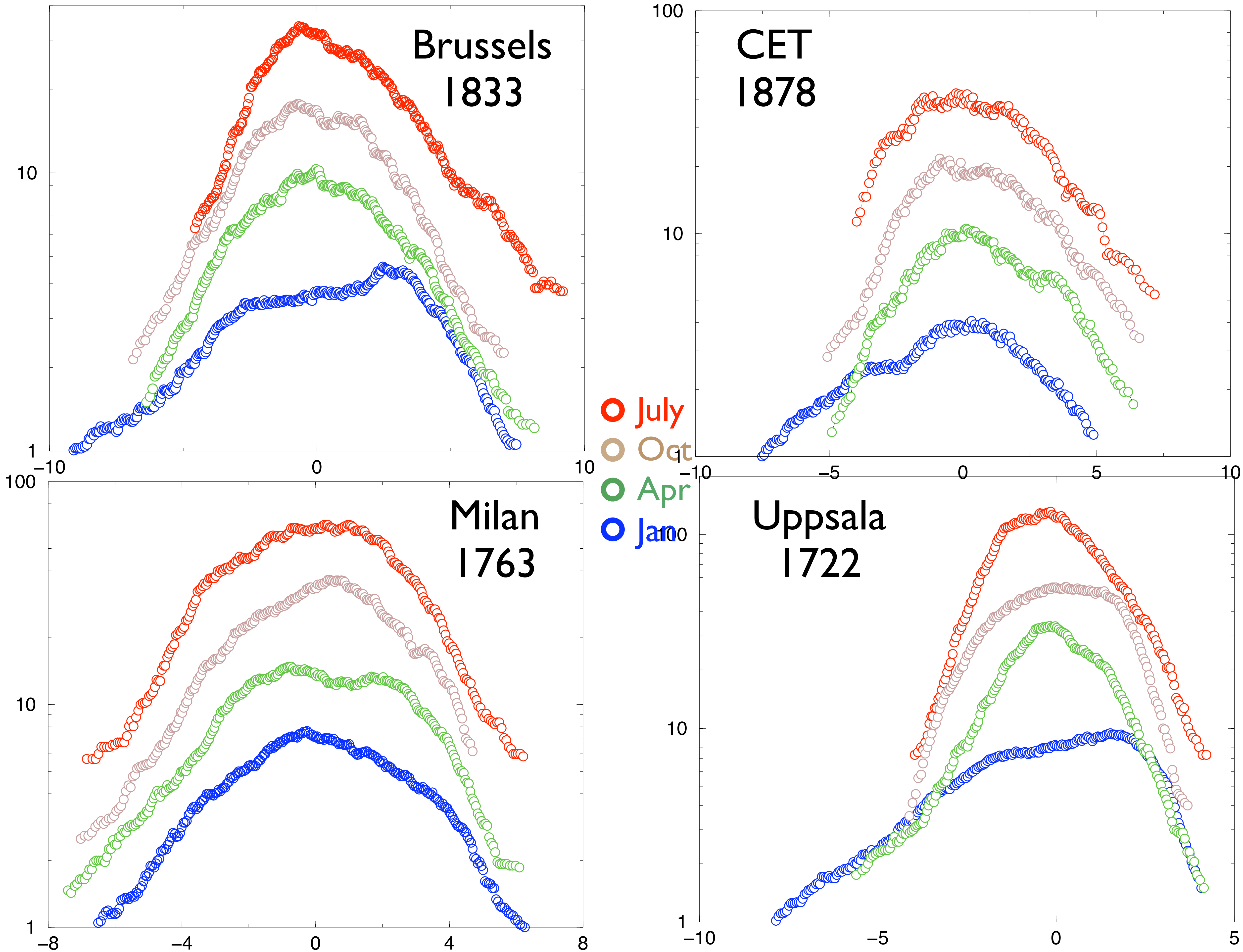
Philadelphia Temperatures





Daily Temperature Distribution



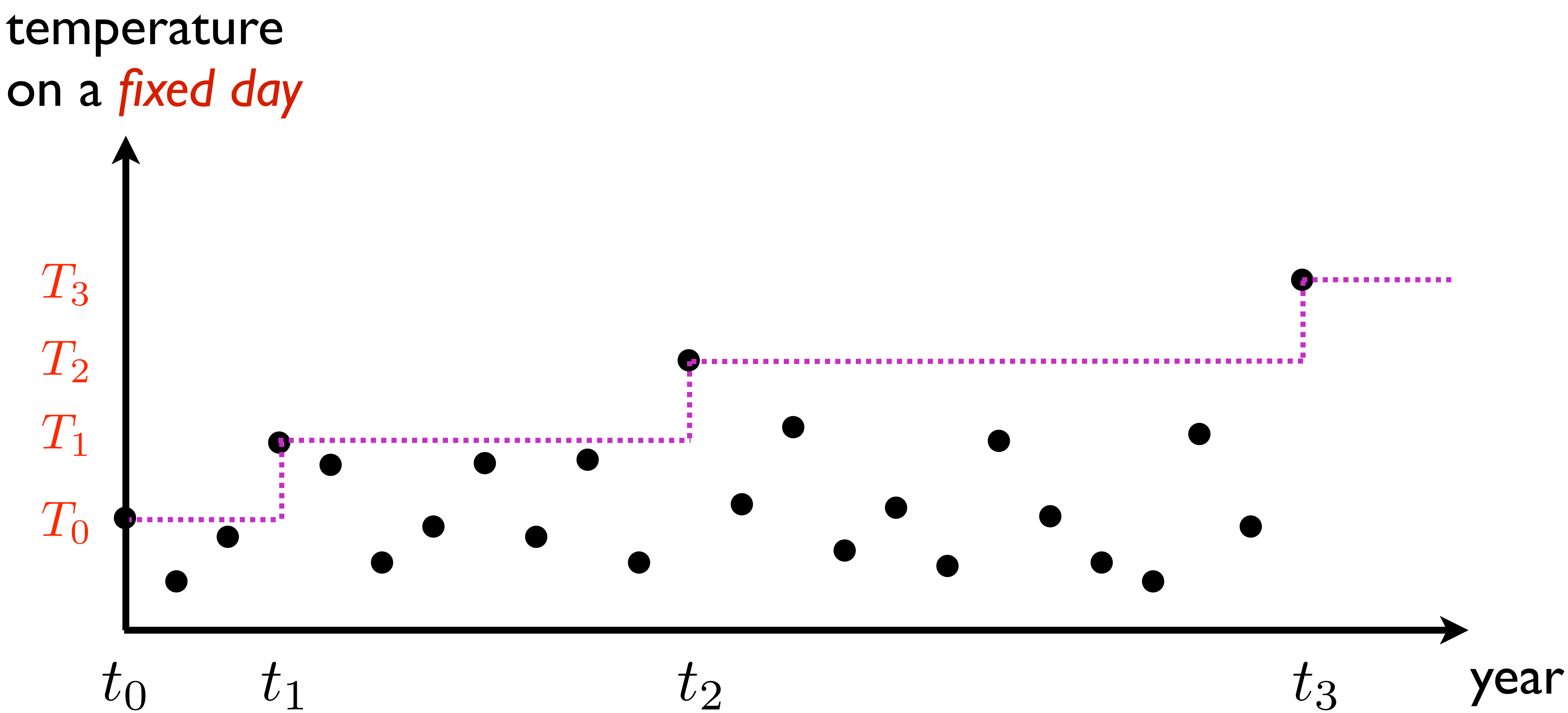


Brussels
1833
CET
1878
Milan
1763
Uppsala
1722
July
Oct
Apr
Jan

Evolution of temperature records

(Arnold et al., *Records*, 1998)

basic assumption: *daily temperatures are iid continuous variables*



Goal: compute $T_k, \mathcal{P}_k(T)$ $t_k, p_k(t)$

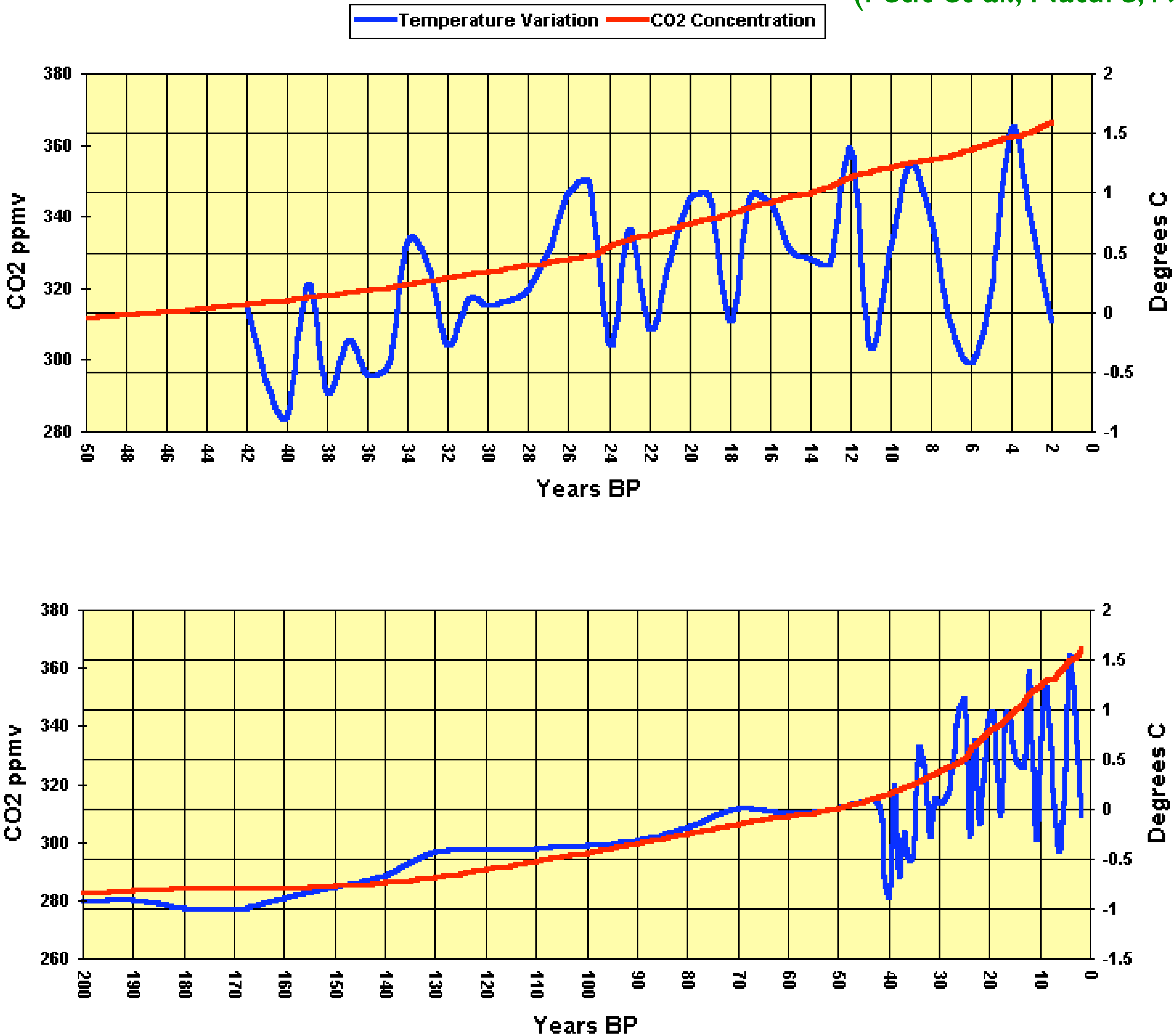
distribution independent!

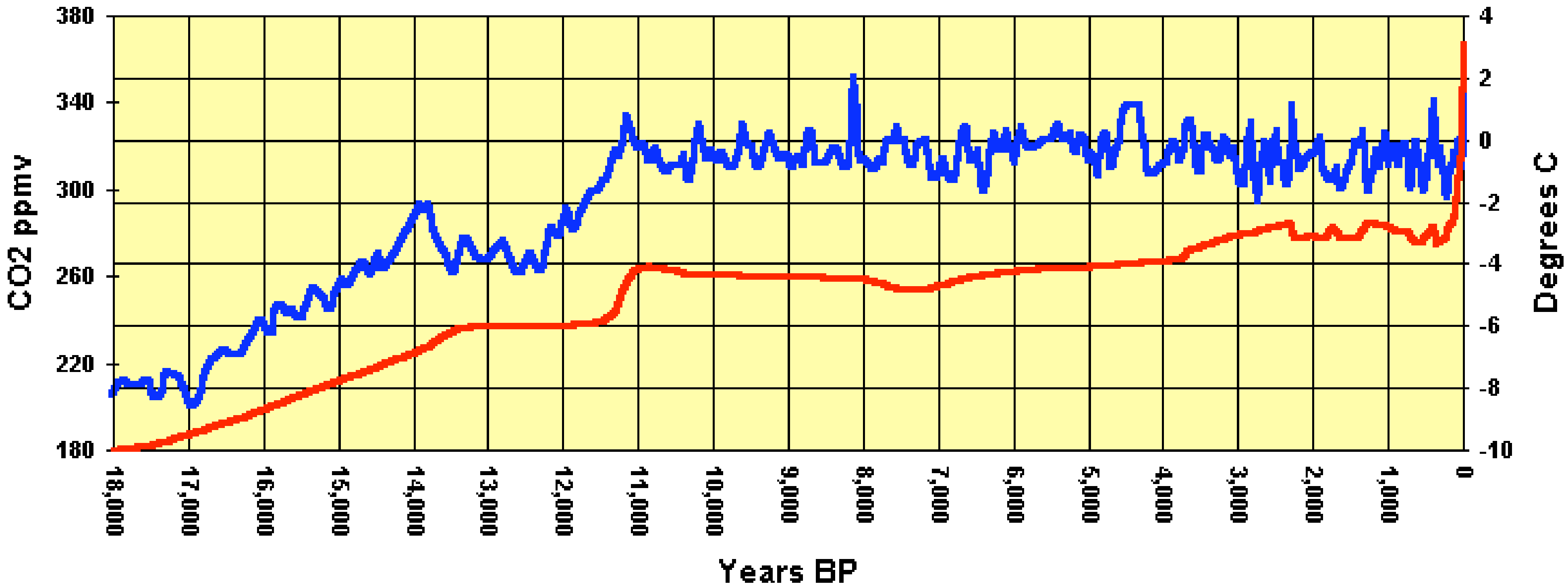
Disclaimers

126-285 years of data---climatologically puny

Antarctic Ice Core Data

(Petit et al., Nature, 1999)





380
340
300
260
220
180
CO2 ppmv
4
2
0
-2
-4
-6
-8
-10
Degrees C
18,000
17,000
16,000
15,000
14,000
13,000
12,000
11,000
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9,000
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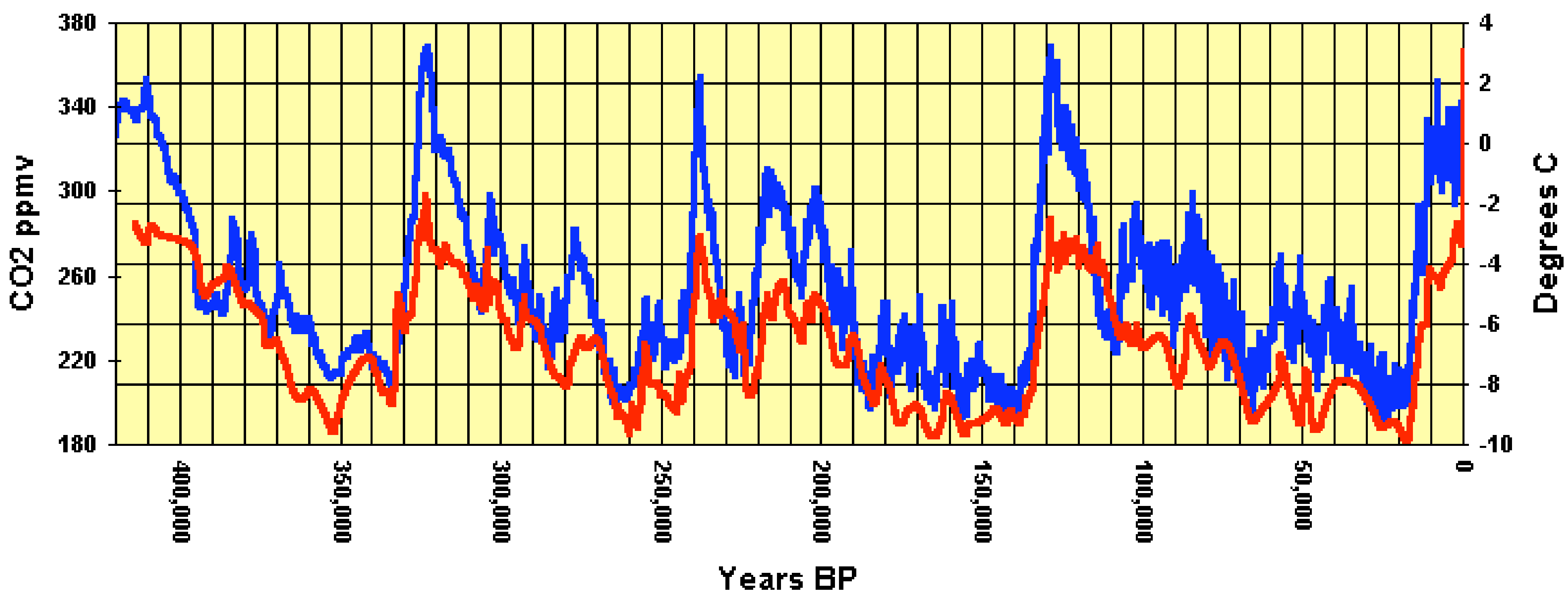
7,000
6,000
5,000
4,000
3,000
2,000
1,000
0
Years BP



380
340
300
260
220
180
CO2 ppmv
4
2
0
-2
-4
-6
-8
-10
Degrees C
400,000
350,000
300,000
250,000
200,000
150,000
100,000
50,000
0
Years BP

Disclaimers

126-285 years of data---climatologically puny

data from a limited number of stations (12)

no control for possible urban heat island effect

record temperatures → most data discarded

unknown data quality and accuracy:

- only daily high & low are reported
- reported accuracy of 1°F (or 1°C or even 0.1°C)

few records → asymptotic analysis questionable

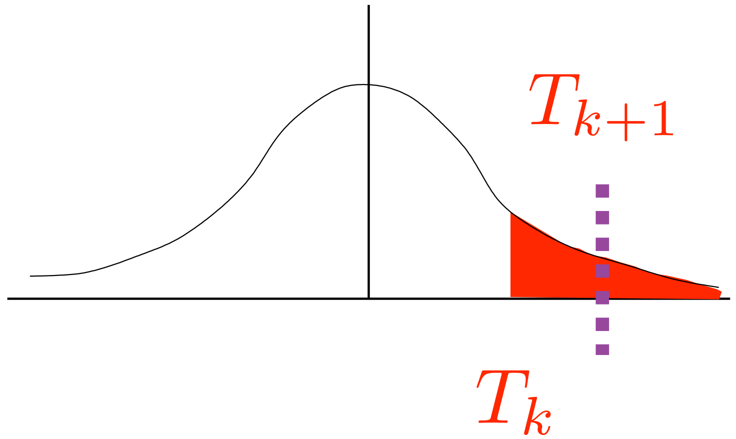
.....

Evolution of *Typical* Record Temperature

$$p(T) \equiv \text{daily temperature distribution}$$

$$T_0 = \int_0^\infty T\, p(T)\, dT$$

$$T_{k+1} = \frac{\displaystyle\int_{T_k}^\infty T\, p(T)\, dT}{\displaystyle\int_{T_k}^\infty p(T)\, dT}$$



Record Temperature Distributions

prob. temperature > T

$$p_>(T) \equiv \int_T^\infty p(T')\,dT'$$

prob. temperature < T

$$p_<(T) = 1 - p_>(T)$$

probability distribution of k^{th} record

previous record T' < T

next n temperatures < T'

last temperature = T

$$\begin{aligned}
\mathcal{P}_k(T) \quad &= \quad \left(\int_0^T \mathcal{P}_{k-1}(T') \sum_{n=0}^{\infty} [\, p_<(T')\,]^n \, dT' \right) p(T)\,, \\
&= \quad \left(\int_0^T \frac{\mathcal{P}_{k-1}(T')}{p_>(T')} \, dT' \right) p(T)\,.
\end{aligned}$$

Record Time Evolution

$q_n(T_k) \equiv$ prob. $(k+1)^{\mathrm{st}}$ record n years after k^{th} at T_k

$$= p_<(T_k)^{n-1}\, p_>(T_k)$$

n-1 non-records   record

expected time between k^{th} and $(k+1)^{\mathrm{st}}$ records at T_k:

$$t_{k+1} - t_k = \sum_{n=1}^{\infty} n\, p_<^{n-1}\, p_> = \frac{1}{p_>(T_k)}$$

finite for given T_k

waiting time distribution for k^{th} record: averaged over T_k

$$Q_n(k) \equiv \int_0^\infty \mathcal{P}_k(T)\, q_n(T)\, dT$$

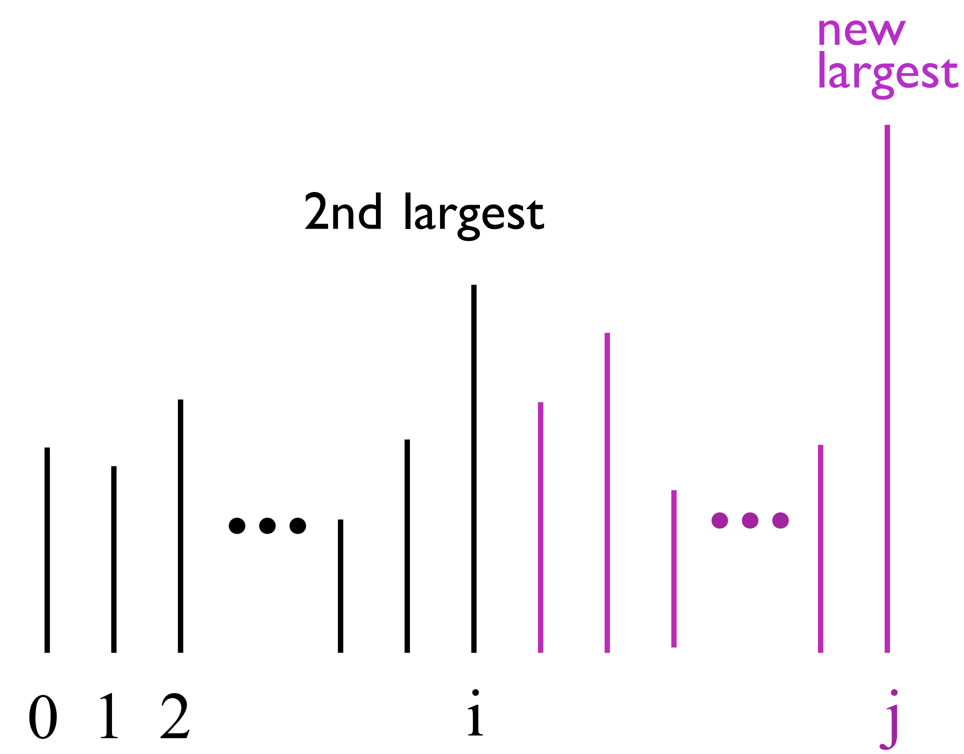
$$= \int_0^\infty \mathcal{P}_k(T)\, p_<(T)^{n-1}\, p_>(T)\, dT$$

infinite waiting time in general

Record Time Statistics

(Glick 1978; Sibani et al 1997, Krug & Jain 05, Majumdar)

define $\sigma_i = \begin{cases} 1 & \text{if record in } i^{\text{th}} \text{ year} \\ 0 & \text{otherwise} \end{cases}$



then $\langle \sigma_i \rangle = \dfrac{1}{i+1}$ $\quad \langle \sigma_i \sigma_j \rangle = \langle \sigma_i \rangle \langle \sigma_j \rangle$

therefore $\langle n(t) \rangle = \sum_{i=1}^{t} \langle \sigma_i \rangle \sim \ln t$ $\quad P(n) = \dfrac{(\ln t)^n}{n!} e^{-\ln t}$

probability that current record is broken in ith year:

$$= \frac{1}{i(i+1)} \rightarrow \text{time until next record} = \infty$$

Records with Gaussian Temperature Distribution

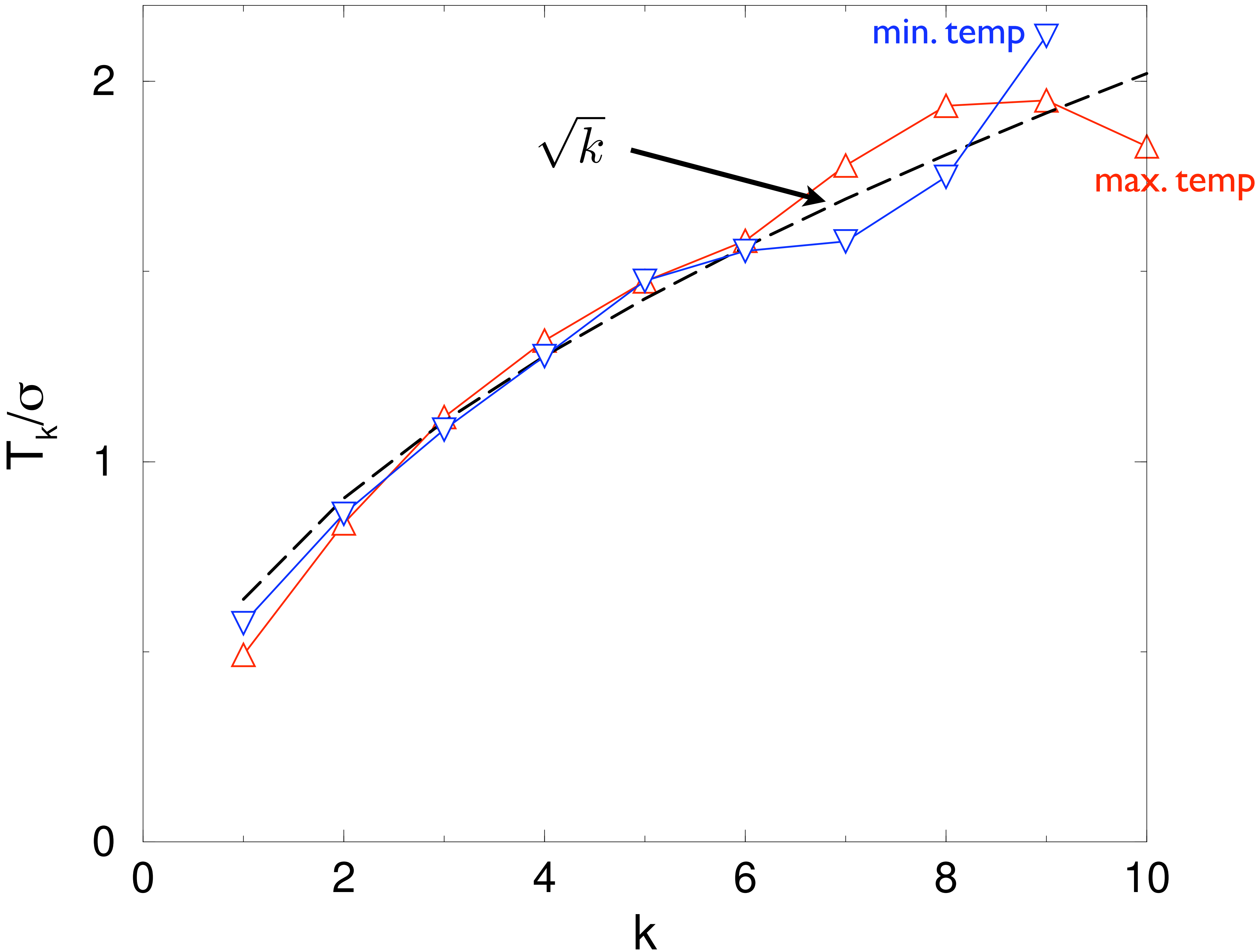
$$p(T) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-T^2/2\sigma^2} \qquad T_{k+1} = \frac{\int_{T_k}^{\infty} T\, p(T)\, dT}{\int_{T_k}^{\infty} p(T)\, dT}$$

$$T_0 = 0 \qquad T_1 = \sqrt{\frac{2}{\pi}}\,\sigma$$

$$\begin{aligned} T_{k+1} &= \frac{\int_{T_k}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}}\, T\, e^{-T^2/2\sigma^2}\, dT}{\int_{T_k}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-T^2/2\sigma^2}\, dT} \\ &= \frac{T_k\, e^{-T_k^2/2\sigma^2}}{\mathrm{erfc}\left(T_k/\sqrt{2\sigma^2}\right)} \\ &\sim T_k \left(1 + \frac{\sigma^2}{T_k^2}\right) \qquad k \to \infty \end{aligned}$$

$$\frac{\partial T}{\partial k} \sim \frac{\sigma^2}{T} \quad \to \quad T_k \sim \sqrt{2k\sigma^2}$$

Philadelphia Record Temperature Values



Records If Global Warming Is Occurring

assume temperature distribution *as a soluble example only*

$$p(T;t) = \begin{cases} e^{-(T-vt)} & T > vt \\ 0 & T < vt \end{cases}$$

exceedance probability

$$\begin{aligned} p_>(T_k; t_k + j) &= \int_{T_k}^{\infty} e^{-[T-v(t_k+j)]}\, dT \\ &= e^{-(T_k - vt_k)}\, e^{jv} \equiv X\, e^{jv} \end{aligned}$$

prob of record at year n

$$\begin{aligned} q_n(T_k) &\equiv p_>(T_k)\, p_<(T_k)^{n-1} \\ &\to e^{nv}\, X \prod_{j=1}^{n-1} (1 - e^{jv}\, X) \end{aligned}$$

(over)estimate of record time

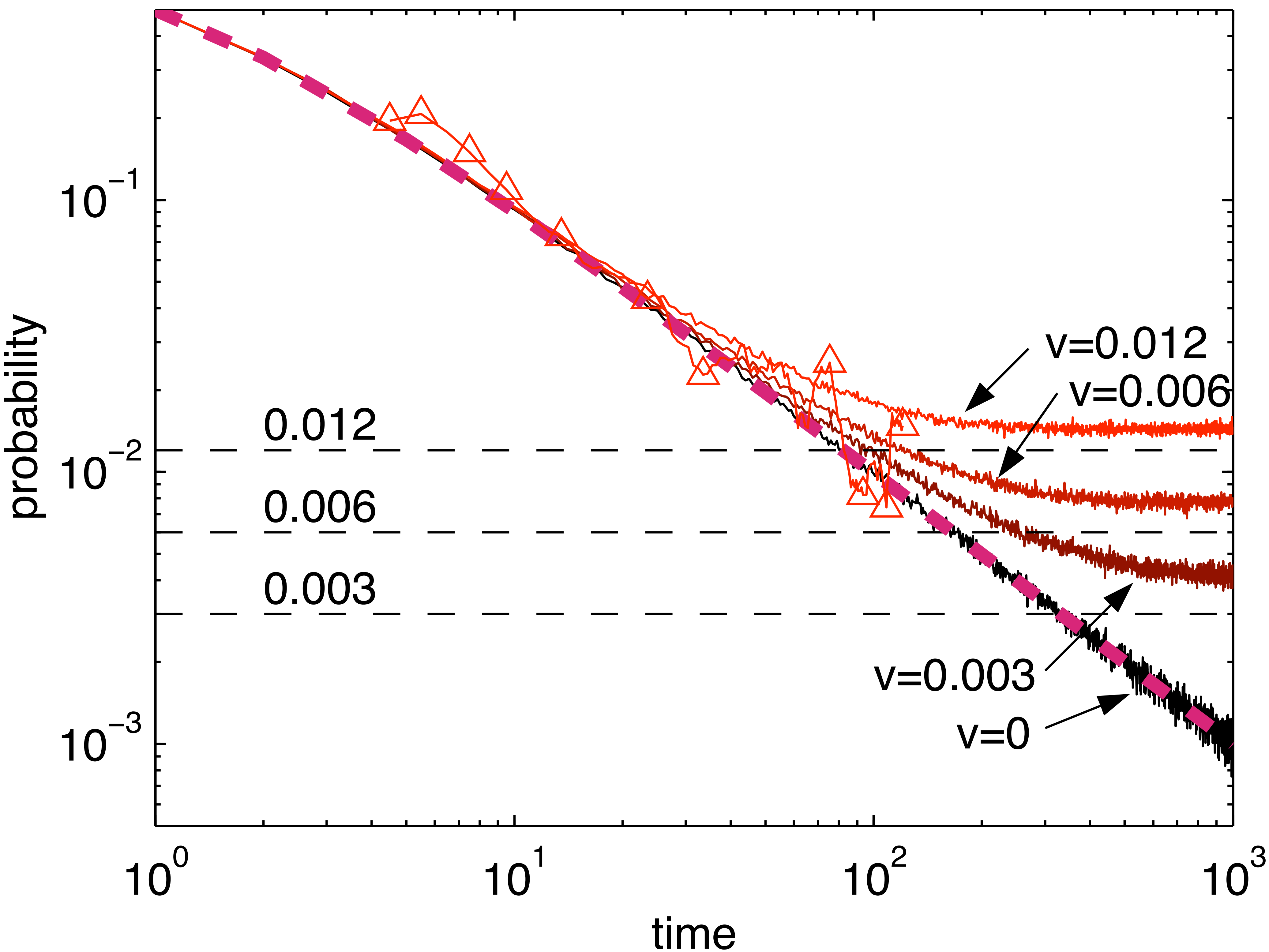
$$e^{jv} X = 1 \;\to\; (t_{k+1} - t_k)v = T_k - vt_k$$

$$\to t_k \sim \frac{k}{v}$$

records ultimately occur at constant rate

(Ballerini & Resnick, 85; Borokov, 99)

Frequency of Record High Temperatures

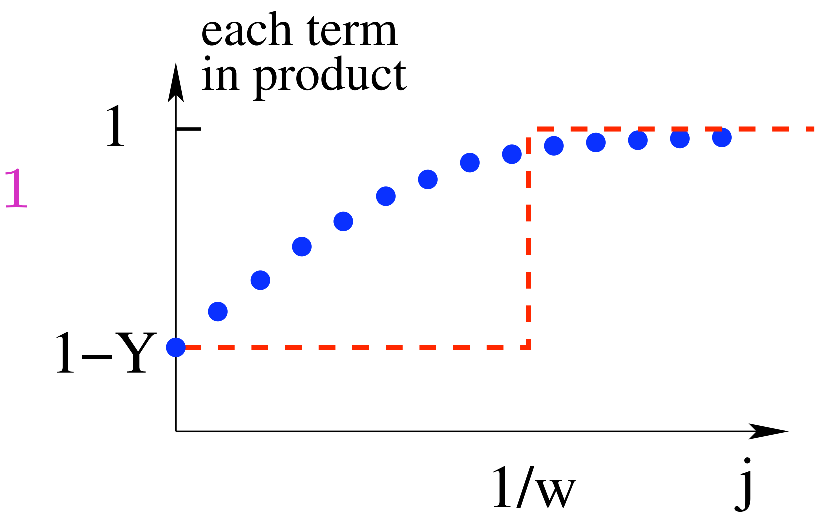


Global Cooling *(low temperature records in warming)*

$$q_n(T_k) \equiv p_>(T_k)\, p_<(T_k)^{n-1}$$
$$\rightarrow e^{-nw}\, Y \prod_{j=1}^{n-1} (1 - e^{-jw}\, Y)$$

with $w = |v| > 0$

$Y = e^{-(T_k + w t_k)}$



$$q_n(T_k) \sim \begin{cases} (1-Y)^n\, e^{-nw}\, Y & n < n* \sim w^{-1} \\ (1-Y)^{1/w}\, e^{-nw}\, Y & n > n* \end{cases}$$

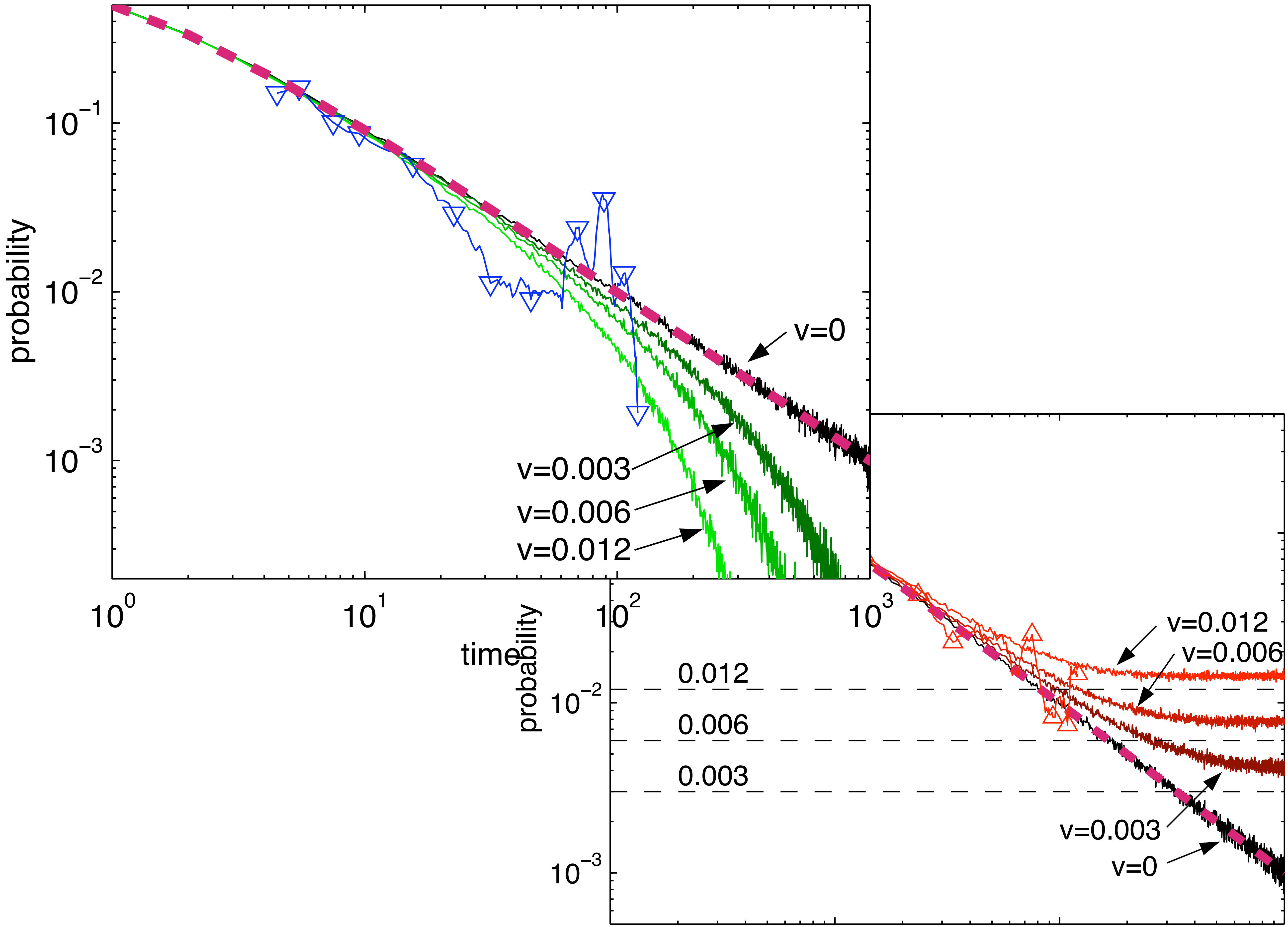
$$\rightarrow t_{k+1} - t_k \sim e^{[T_k + w t_k]} \qquad (e^{T_k} \text{ for } w = 0)$$

continuum limit: $\dfrac{\partial t}{\partial k} \sim e^{k + wt}$

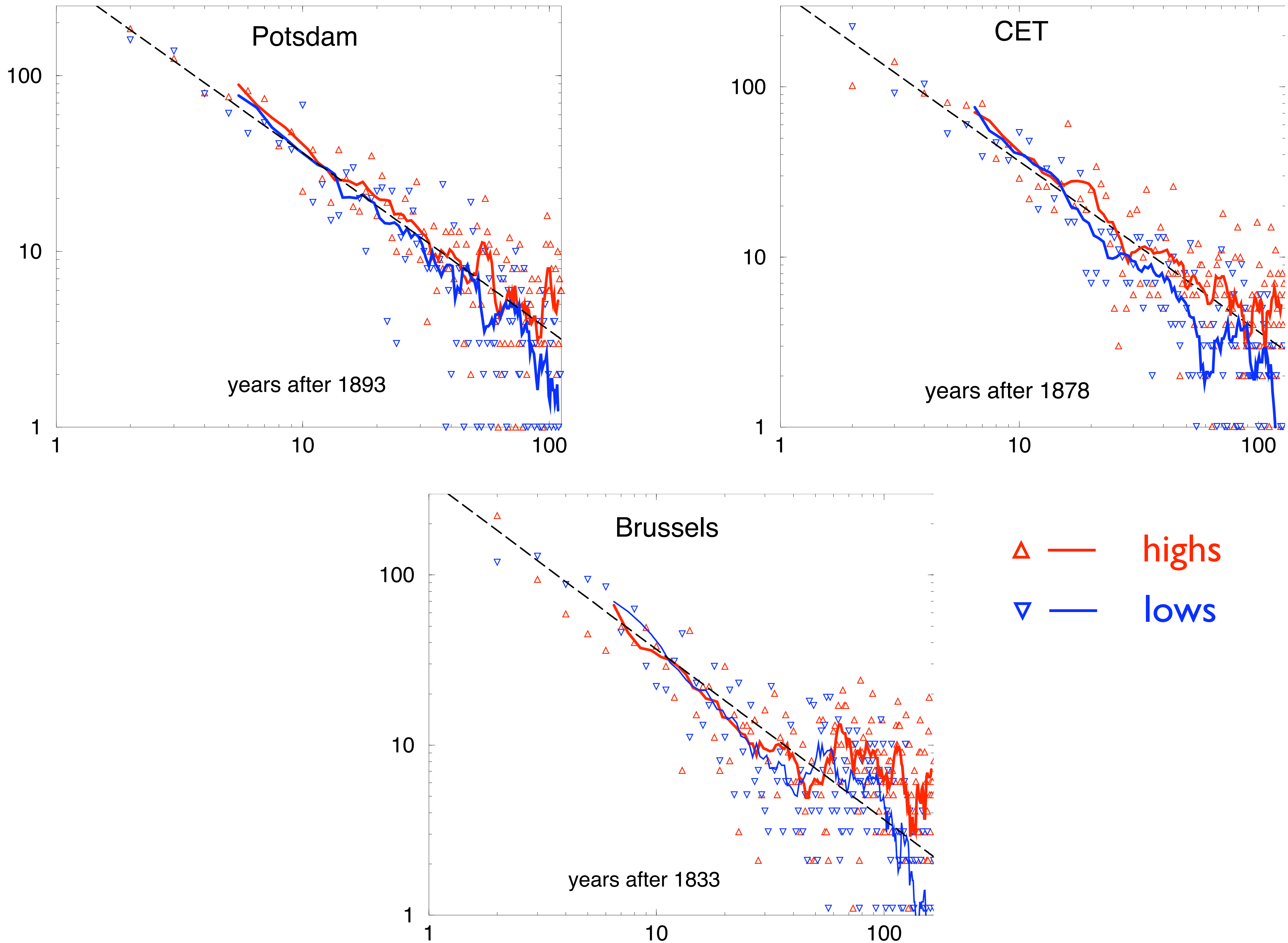
$$\underbrace{(1 - e^{-wt})}_{<0} = w(e^k - 1) \approx wt$$

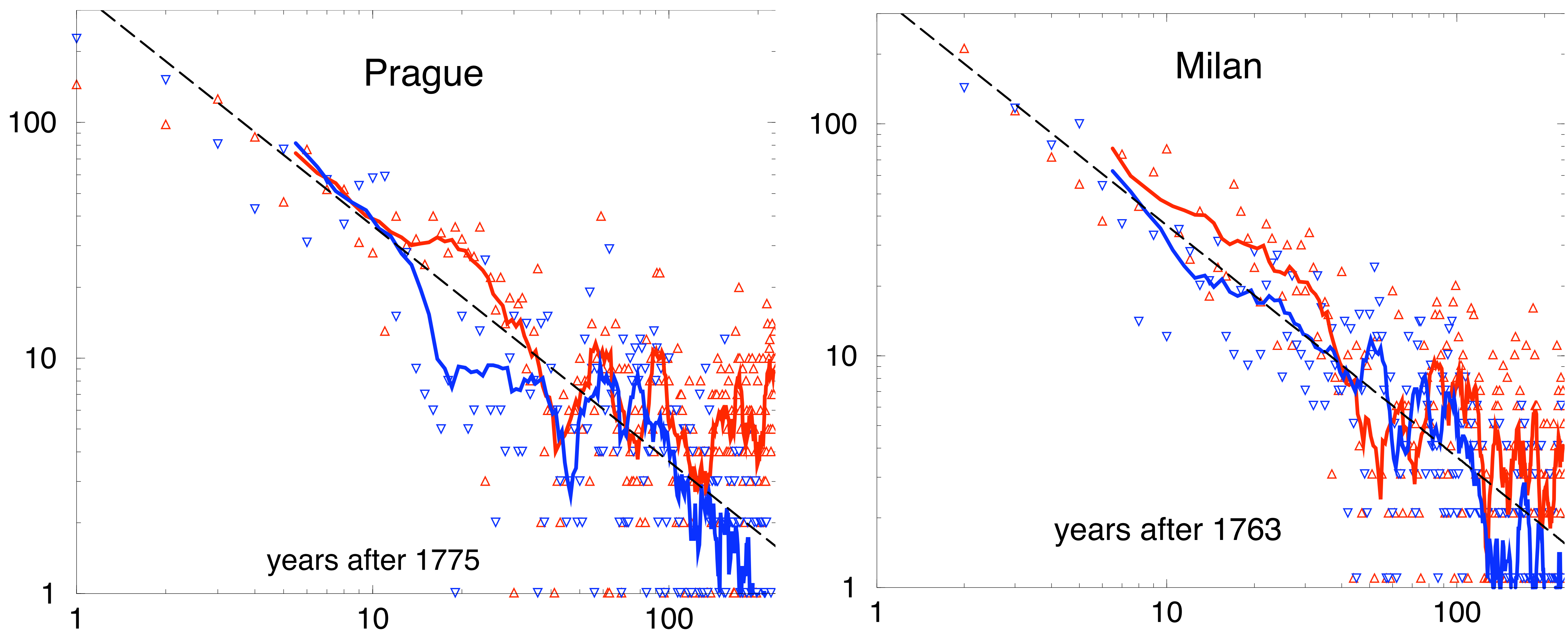
no solution for $t_k > \frac{1}{w}$

Frequency of Record Low Temperatures



More Record Frequencies




Prague
100
10
1
1
10
100
years after 1775
Milan
100
10
1
1
10
100
years after 1763

Summary & Outlook

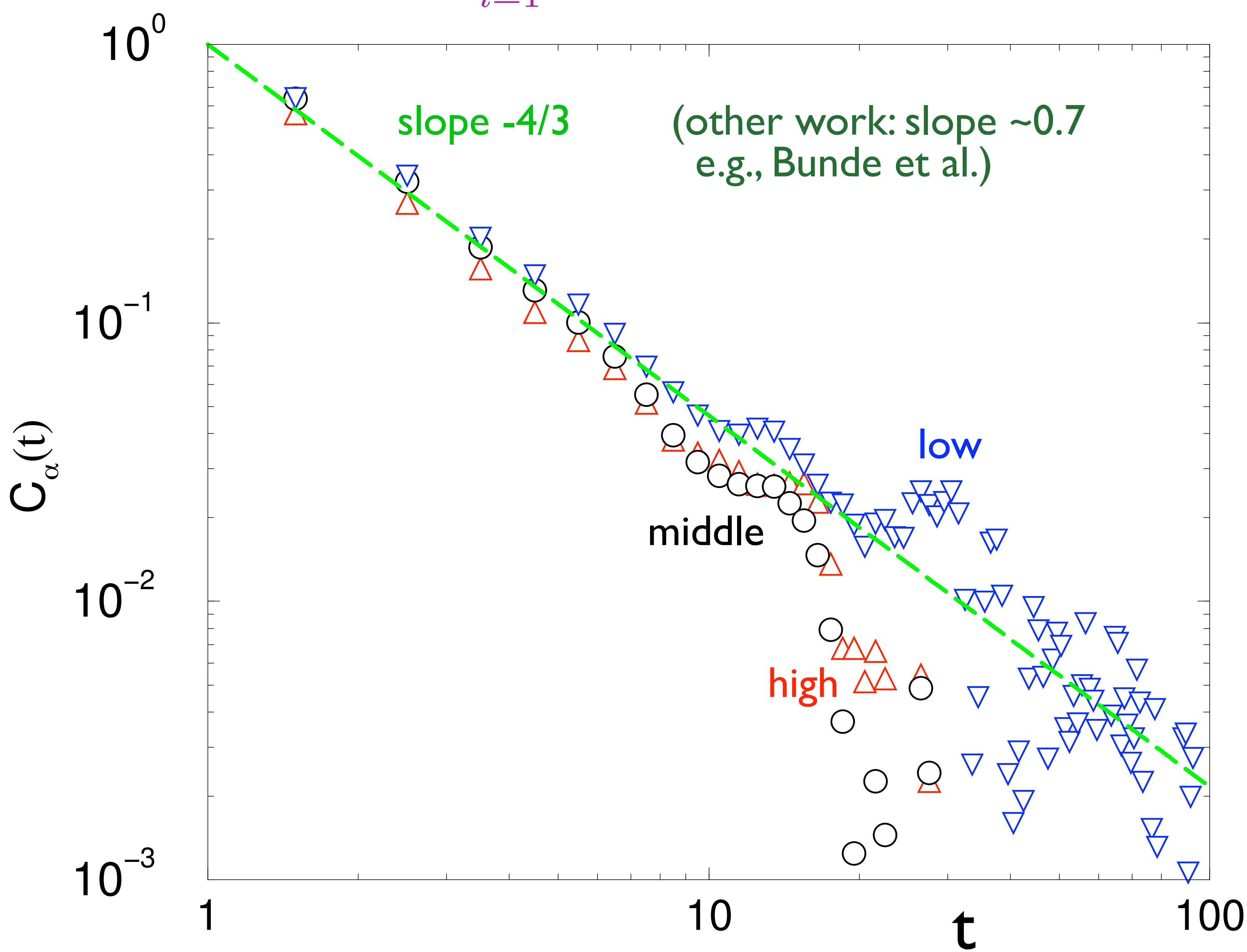
Extreme value considerations give record temperature and time statistics

Current global warming rate seems to significantly affect record temperature statistics

Inter-day temperature correlations do not appear to affect record temperature statistics

Inter-day Temperature Correlations

$$C_\alpha(t) = \frac{1}{365}\sum_{i=1}^{365} \frac{\langle T_i T_{i+t}\rangle - \langle T_i\rangle\langle T_{i+t}\rangle}{\langle T_i^2\rangle - \langle T_i\rangle^2} \quad \alpha = \text{low, middle, high}$$



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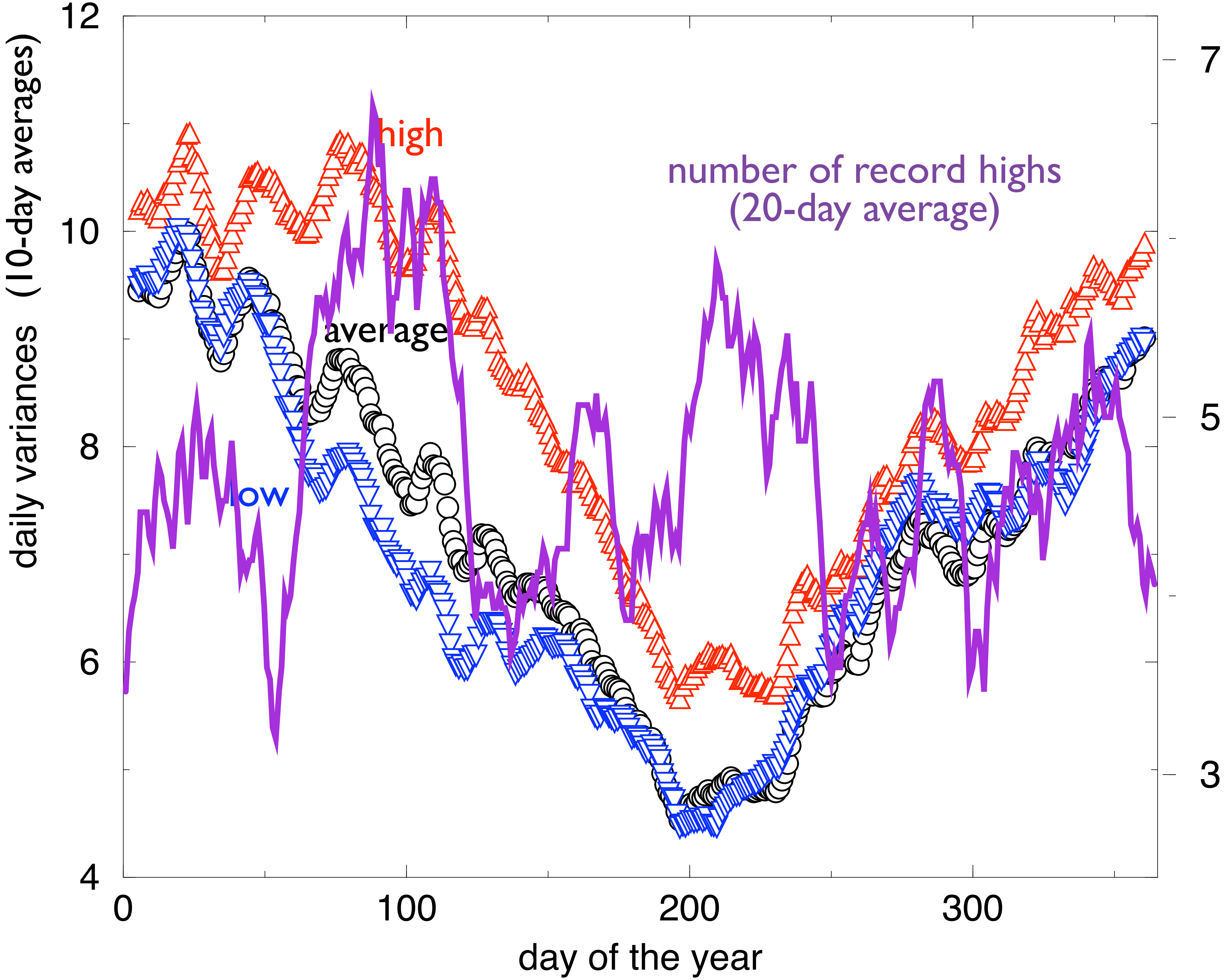
Open issues: Seasonal effects

more record highs in spring

Seasonal Variance & Record Numbers



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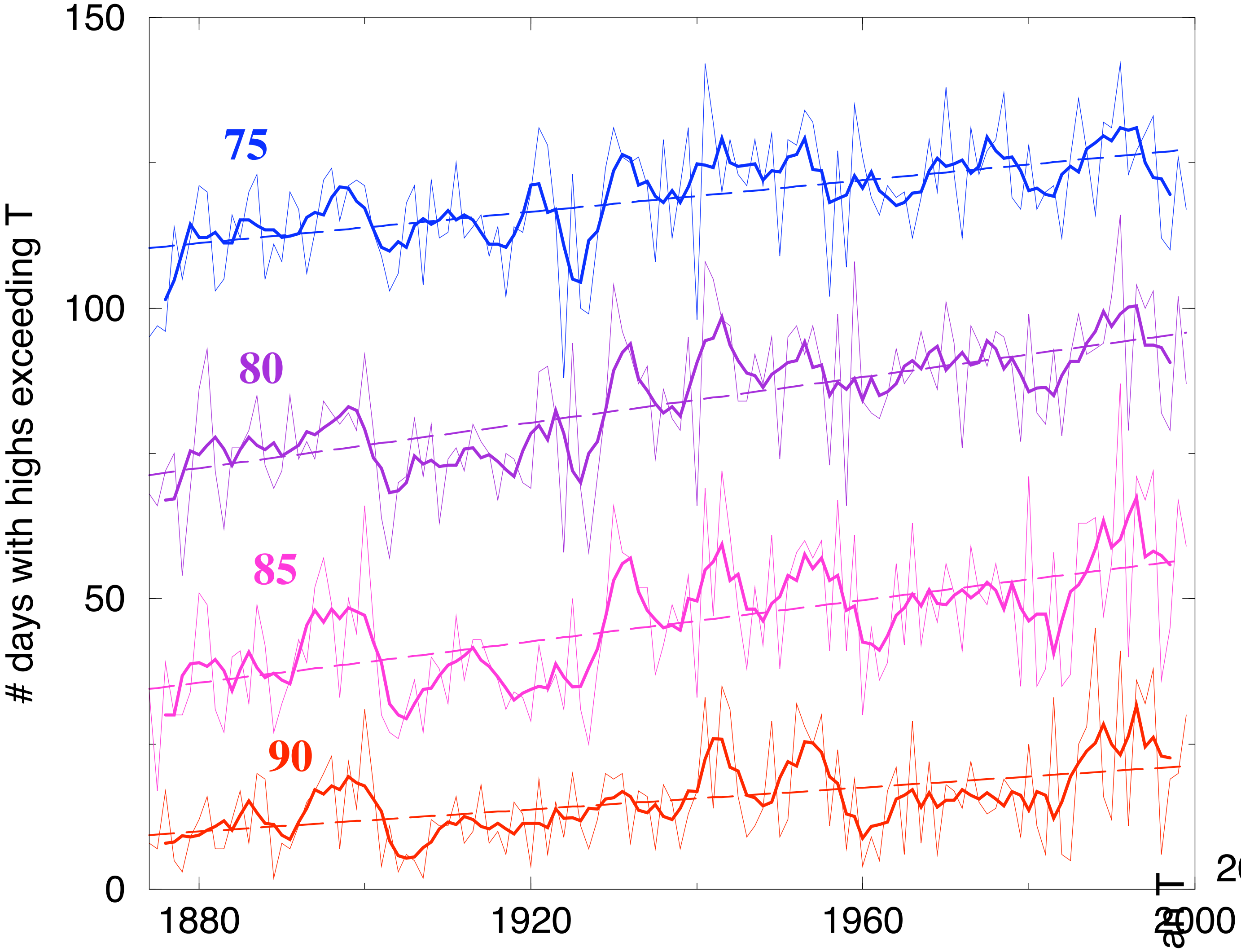
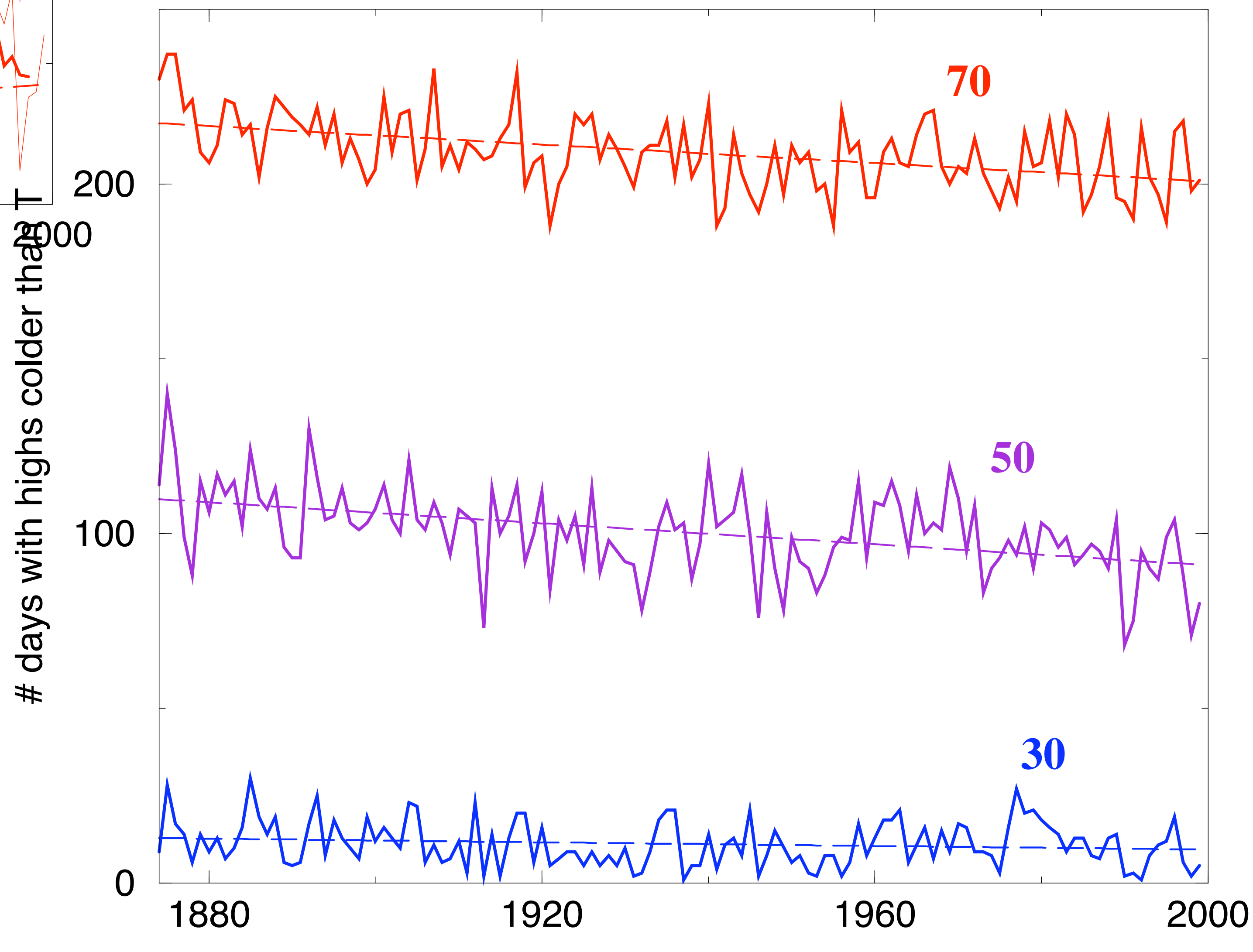
Day/night asymmetry

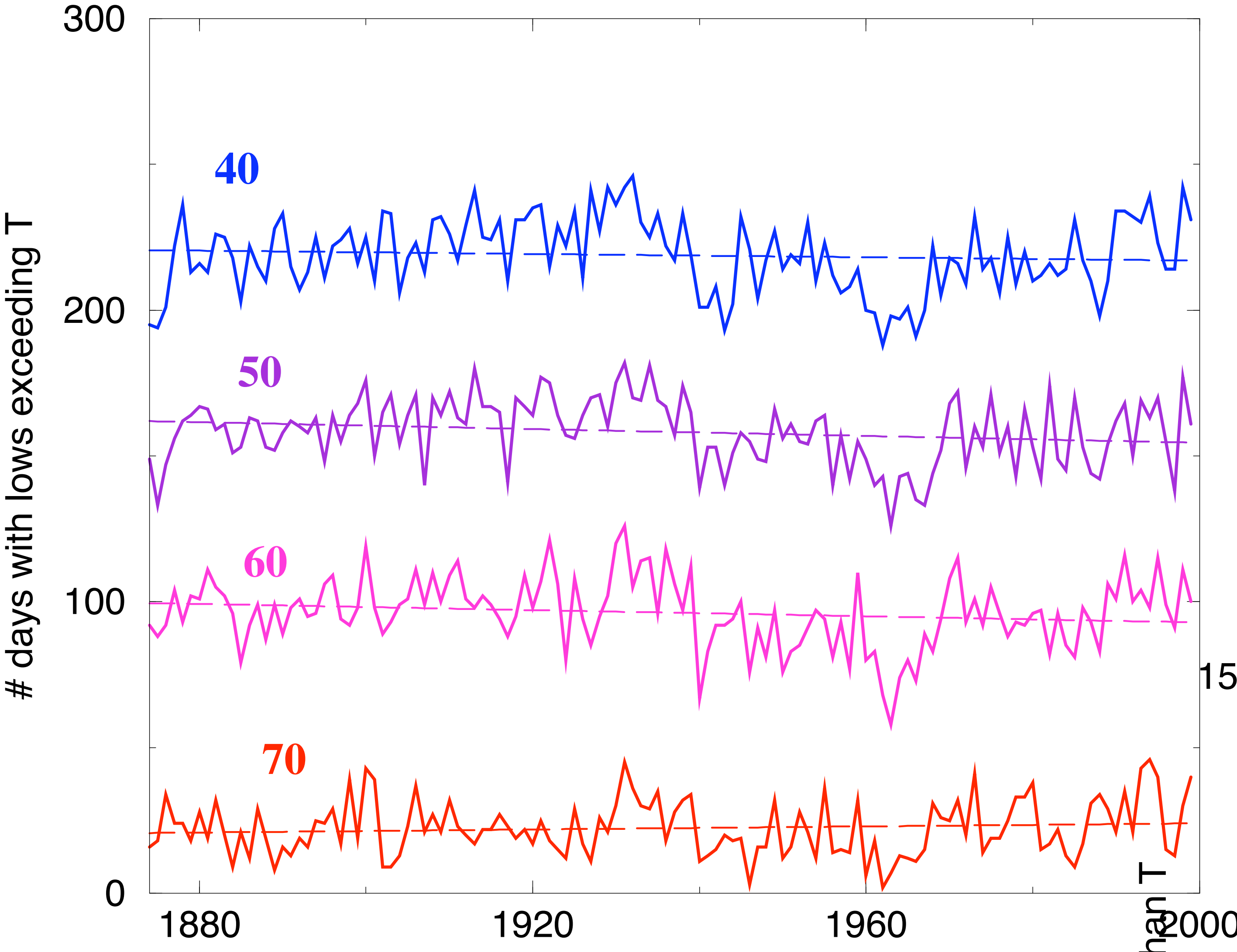
more hot days....



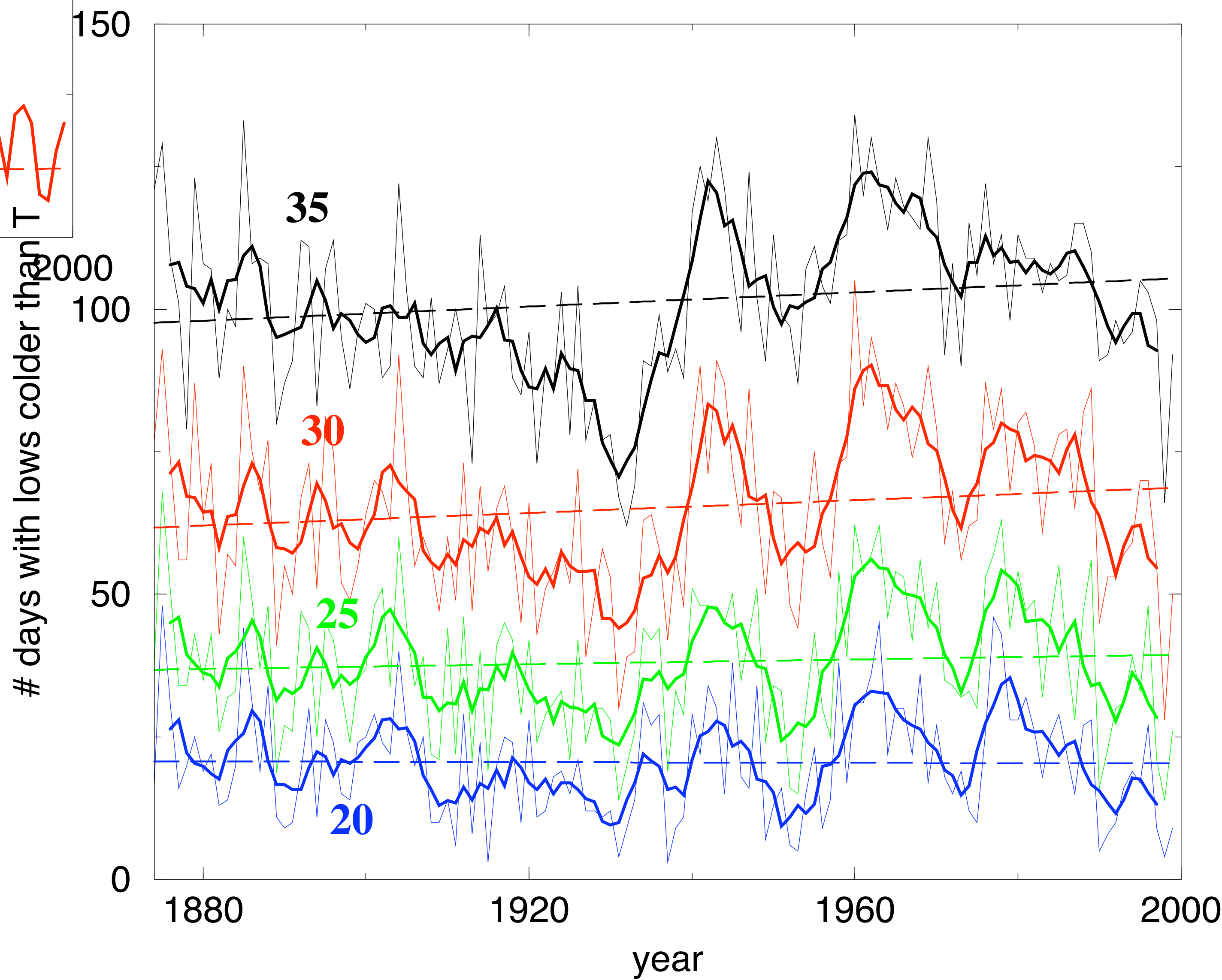
& fewer cold days....



....but fewer hot nights



....and more cold nights



Summary & Outlook

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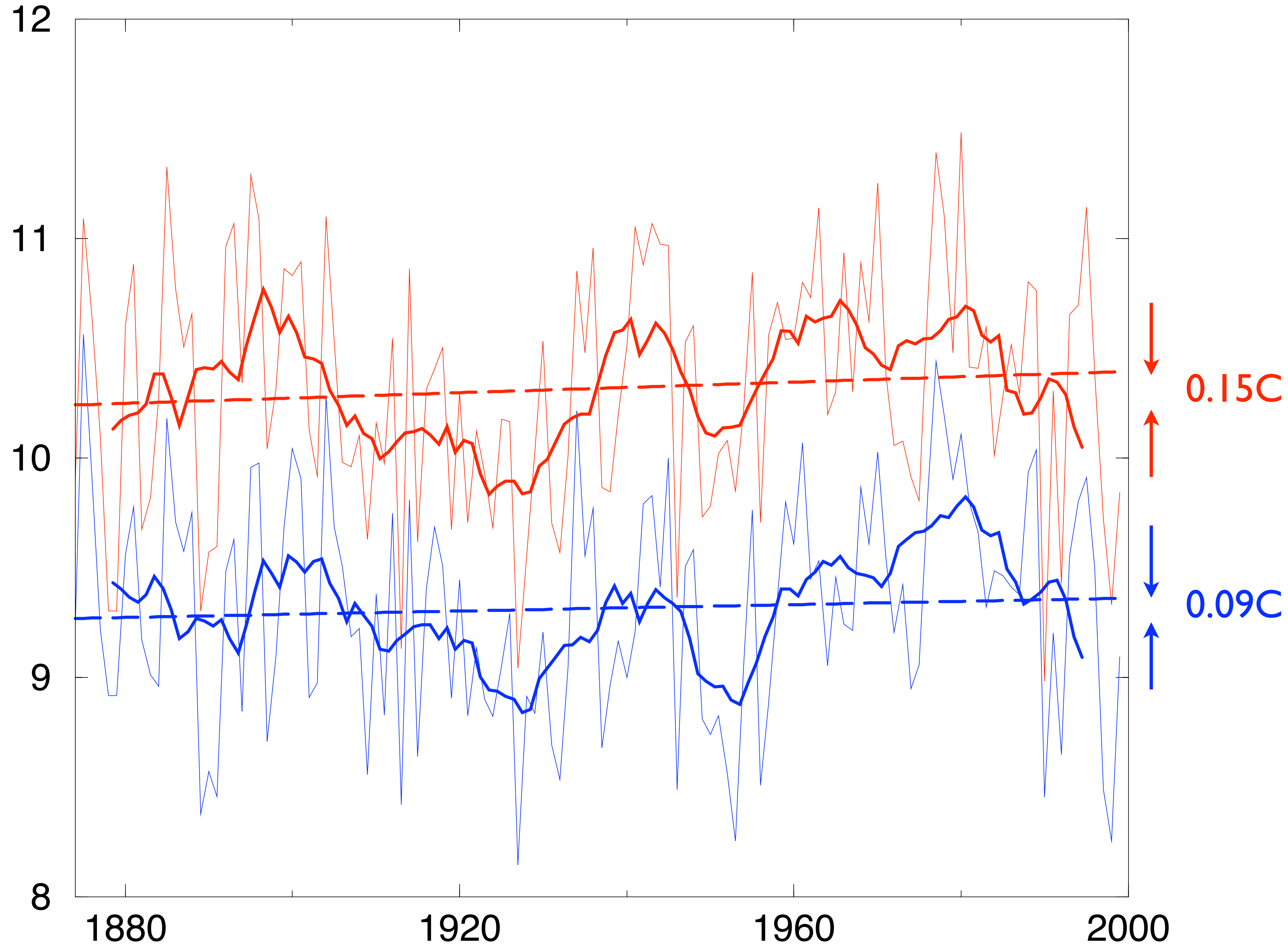
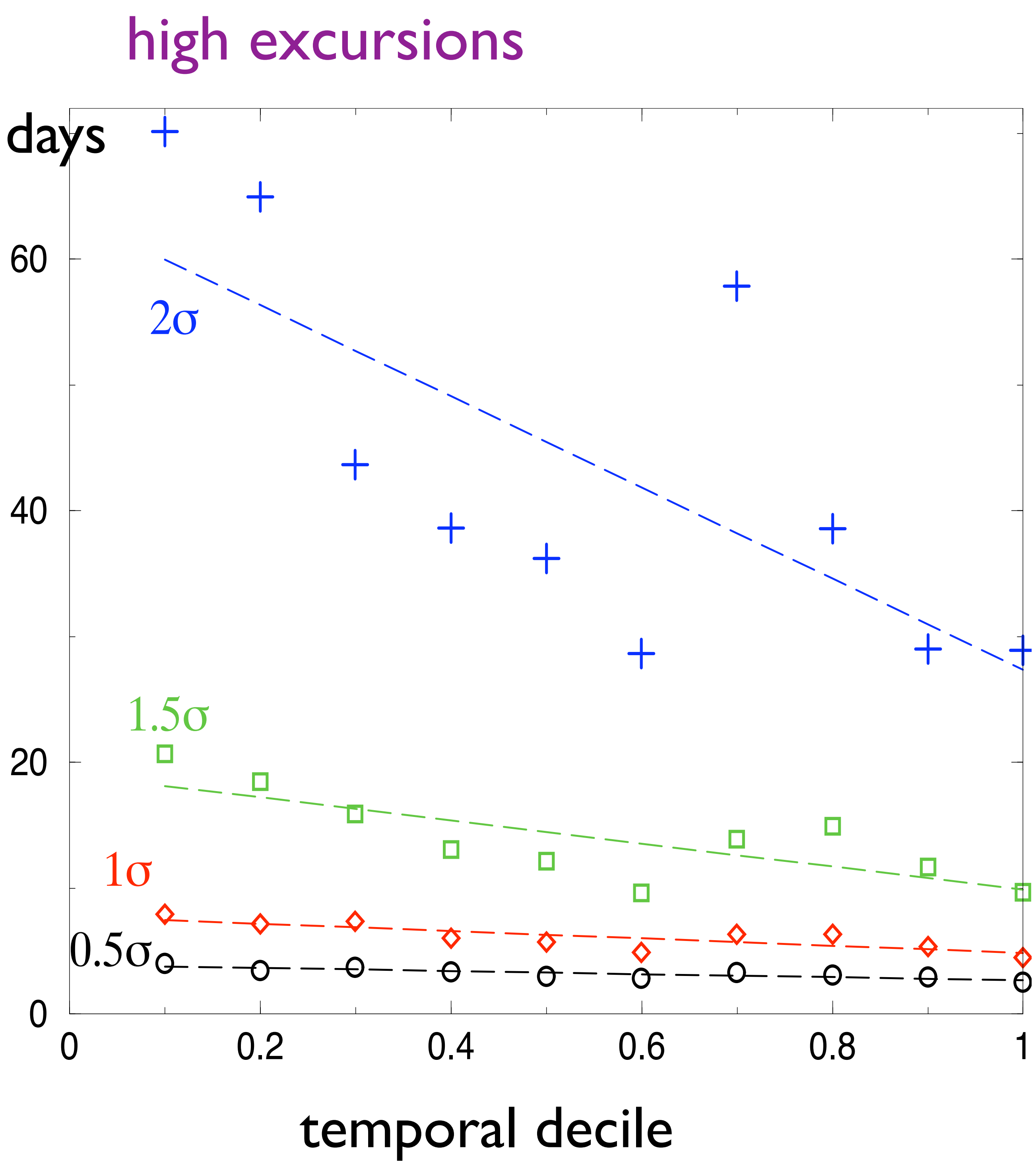
Open issues:

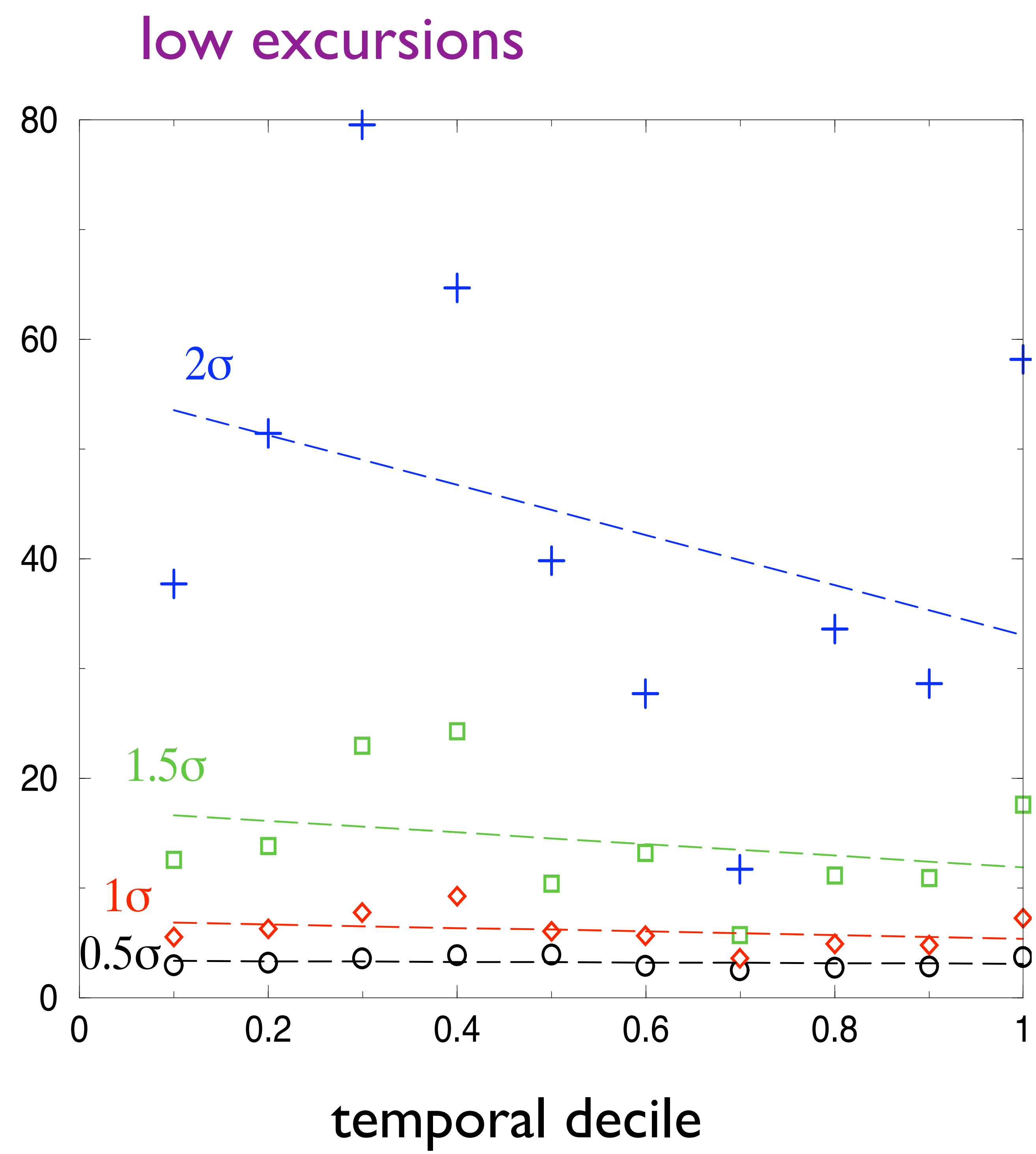
- Seasonal effects
  - *more record highs in spring*
- Day/night asymmetry
- Changing variability

Annual Variability *Slowly* Increasing



Decreasing Time Between Threshold Events





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- Day/night asymmetry
- Changing variability
- Comprehensive data & theory validation