The LHC Inverse Problem

Kane Symposium
MCTP. 2007

Lian-Tao Wang
Princeton University

Work in collaboration with:
1. N. Arkani-Hamed, G. Kane, J. Thaler: hep-ph/0512190
2. N. Arkani-Hamed, B. Knuteson, S. Mrenna,
   P. Schuster, J. Thaler, N. Toro,... (to appear)

- Outline

- Motivation
- Supersymmetry and the Inverse Problem

  degeneracies.

- OSET

  brief, *Nima's Talk*

- My beginning with Gordy:

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⟶ Great tradition.

⟶ Thesis subject.

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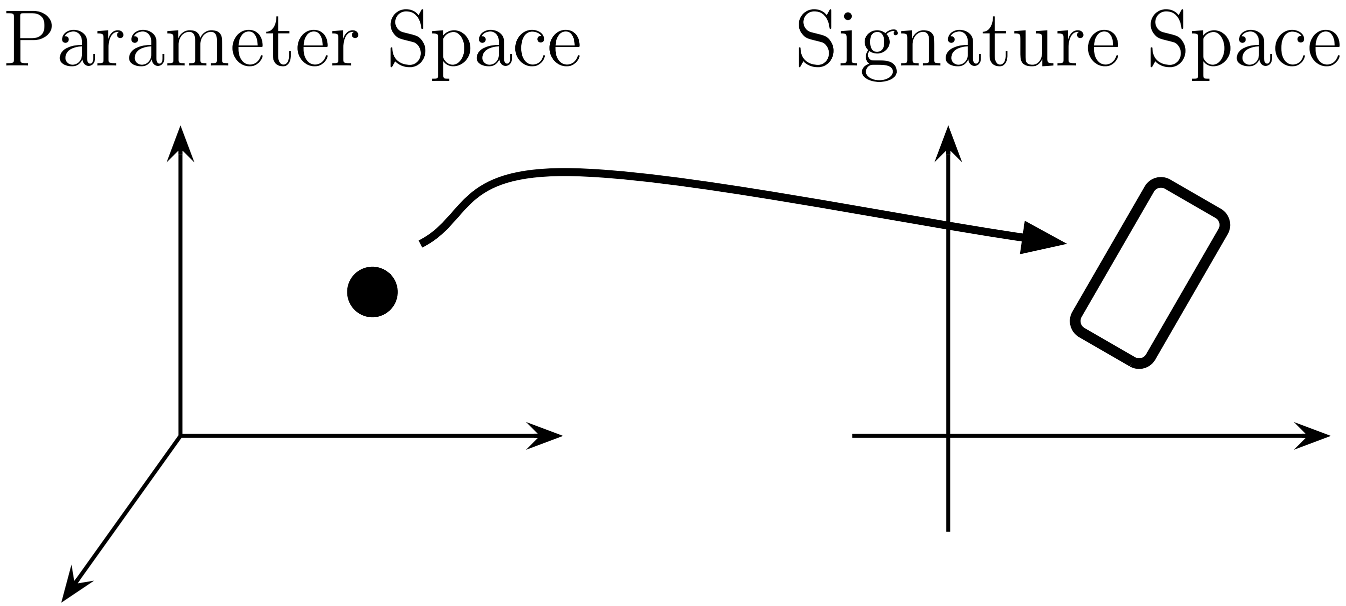
(more iterations)

- Much work have been focused on:
  Forward Direction



$\mathcal{L}_{\text{New Physics}} \rightarrow$ LHC signal

Many benchmark studies have been carried out. *

Standard tools developed
Valuable experience has been accumulated

*For example, detailed study SPS points in supersymmetry

The *inverse* problem: LHC data $\rightarrow \mathcal{L}_{\text{New Physics}}$

General Character of the inverse map (One-to-One?)

How can we extract as much information as possible about the underlying theory from data?

- What to expect

In order to study the inverse problem, it is important to understand what to expect first.

Focus of decades model building and anticipation

new physics ↔ the mystery of the weak scale

$\longrightarrow$ new (BSM) information about the fundamental theory

- Possible outcomes at LHC:

1. New physics is responsible for the weak scale:

   Big discovery, complicated physics

   rest of the talk: challenges, crucial steps in understanding new physics

2. Very special new physics (hard to miss): $Z' \to \ell^+\ell^-\dots$

3. Very hard: $\delta\lambda_h$, $\mathcal{O}^{(n>4)}$ (Flavor, EDM, $g_\mu - 2$) ...

- Mystery of the weak scale

1. (big) Hierarchy problem: $\Lambda_{\text{EW}} << M_{\text{Pl}}$

a. How to generate a scale very different from the Planck scale

QCD: Logarithmic running → exponential separation of scales.

b. How to stabilize such a scale

Two classes of ideas: supersymmetry, compositeness

Here, I will focus on supersymmetry.

Lessons generic.

- Many SUSY breaking scenarios/models:

GMSB, AMSB, gaugino-MSB, ...

Gravity mediation with flavor symmetries.

special relations between soft parameters

e.g., focus point, AMSB+moduli...

Various string theory constructions...

Fine arts of supersymmetry breaking have been developed. Many of them very sophisticated.

No clear winner...

Lesson from two decades of model building and experimental search:

If supersymmetry is responsible for the weak scale, its implementation is probably subtle, and/or complicated, and/or strange.

It is interesting to explore radically new ideas in model building.

On the other hand, it is hard to be sure that we are getting closer to the answer by refining our current setups.

Is little hierarchy (flavor, EWPT...) solved by mechanism or an 1% accident?

Good news: LHC will teach us a lot about it!

• Learning supersymmetry breaking from the LHC

Starting Point:

Low energy effective Lagrangian with the possibility of incorporating all possible supersymmetry breaking models.

$$\mathrm{MSSM} + \mathrm{V(h)}$$

Measurement of parameters in it will lead us toward specific supersymmetry breaking scenarios.

The supersymmetry LHC inverse problem: how well can we extract information about MSSM Lagrangian from the LHC data

- MSSM parameter space at the LHC: $D_{\text{para}} \sim 20$

Largest cross section, important links in decay chains...

$$M_{\tilde{g}} \quad M_2 \quad M_1 \quad \mu$$

$$m_{\tilde{q}_L}^{1,2} \quad m_{\tilde{q}_R}^{1,2} \quad m_{\tilde{t}_{1,2}} \quad m_{\tilde{b}_{1,2}} \quad A_t \quad A_b$$

$$m_{\tilde{\ell}_L}^{1,2} \quad m_{\tilde{\ell}_R}^{1,2} \quad m_{\tilde{\tau}_{1,2}}$$

$$\tan\beta$$

$$m_h \quad m_A \quad m_H$$

• LHC^{-1} MSSM: What do we want to know?

1. Gauginos: one of the most direct links to fundamental theory

$$M_1,\ M_2,\ M_3 \leftarrow \int d\theta^2 f_a(\{S, T_i ...\}) W^\alpha W_\alpha$$

dilaton domination

$f_a = S,\ F_S$ dominate $\qquad M_1 : M_2 : M_3 \sim 1 : 2 : 6$

anomaly mediation

$f_a \propto \beta_a \log(\sqrt{\Phi\Phi^\dagger})\ F_\Phi \neq 0,\ F_S ... = 0 \qquad M_1 : M_2 : M_3 \sim 3.3 : 1 : 8.8$

many others...

Gauge coupling unification ↔ gaugino mass unification?

Generic gaugino mass patterns ↔ subtle supersymmetry breaking

2. 3rd generation:

How special are they? Unified (b vs τ...)?

3. Is there an approximate GUT? $SU(2)_R$?

$m_{\tilde{\ell}}$ vs $m_{\tilde{q}}$, m_{left} vs m_{right}

4. μ, mysterious vector mass for $H_u H_d$

related to supersymmetry breaking?

We will ask these questions when studying the inverse problem.

- Signature space: what do we observe at the LHC?

jet, b-flavored jet, $e^{\pm}$, $\mu^{\pm}$, $\tau^{\pm}$, γ, $\not{E}_T$,

$\rightarrow$ $\mathcal{O}(10)$objects per event

P^i_μ,

flavor, charge

Typical SUSY signal:

$$\begin{aligned} pp &\rightarrow \text{[colored superpartners]} + \text{jets} \\ &\rightarrow \text{more jets} + \text{(leptons/photon)} + \not{E}_T \end{aligned}$$

• MSSM signature space at the LHC

1. Number counting: (145 used in our study)

$n_j \times$ jet	b-jet non-b-jet
$+$	
$n_\ell \times$ lepton	ℓ all flavor and charge combo: e.g. $2\ell \rightarrow$21 comb.
$+$	
$n_\gamma \times \gamma$	

More inclusive: Nev, $N_{1\ell}$, ...

For example: lepton countings contain information such as

color/electroweak, additional source of lepton besides W/Z

2. Kinematical distributions

$\sum P_T$: sum over any combination of interests

$\not{E}_T$, P_T leading lepton, jet, b-jet...

M_{eff} many combinations:
3 hardest jets, 2 hardest b-jets....

m_{inv}: many combinations of objects

$$m_{\text{inv}}(i, j, ...),\ i,\ j = \ell,\ b,\ \text{jet}$$

e.g. m_{ℓ^+,ℓ^-}, m_{bb}, $m_{q\ell\ell}$, $m_{q\ell}$, $m_{qq\ell\ell}$...

Most of Distributions charge/flavor specific, $\mathcal{O}(10)$ bins each

Total: $\mathcal{O}(10^3)$ observables, 1808 used in our study

- Statistical Method

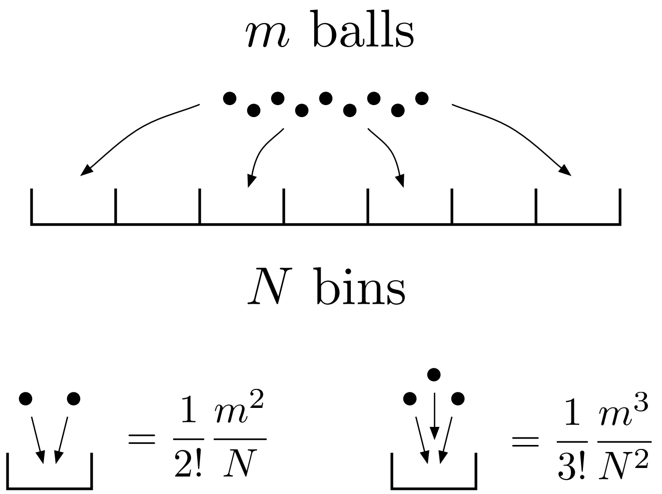
1. Simulate a relatively small number, m, of models.

2. Pair wise comparison of the resulting signatures, $m(m-1)/2$ pairs.

• Power of the Statistical method

Signature bin: a volume in the signature space with finite size determined by the experimental error bar.

The signature space spanned by MSSM are divided into $N_{\text{sig.}}$ bins.



$= \frac{1}{2!}\frac{m^2}{N}$

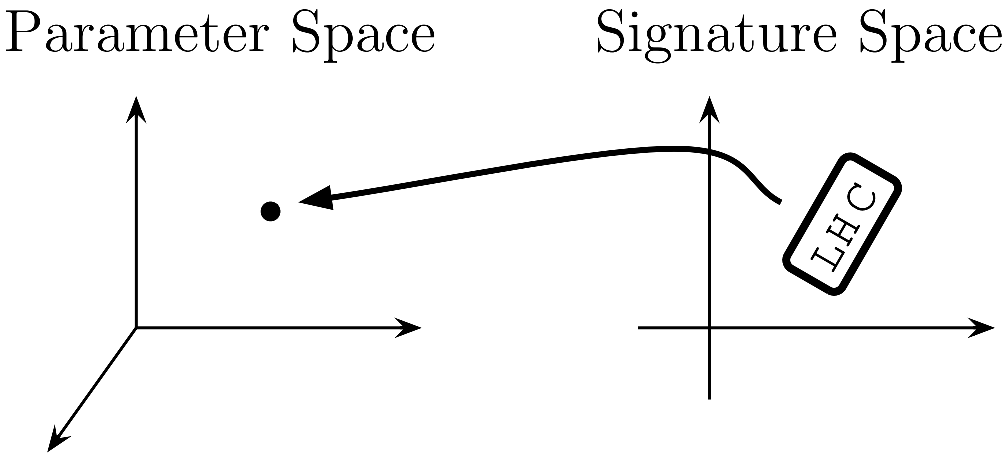
$= \frac{1}{3!}\frac{m^3}{N^2}$

Simulating $m(<< N_{\text{sig.}})$, the chance of getting a degenerate pair is $< \#\text{doubles} > \sim \frac{m^2}{N_{\text{sig}}}$

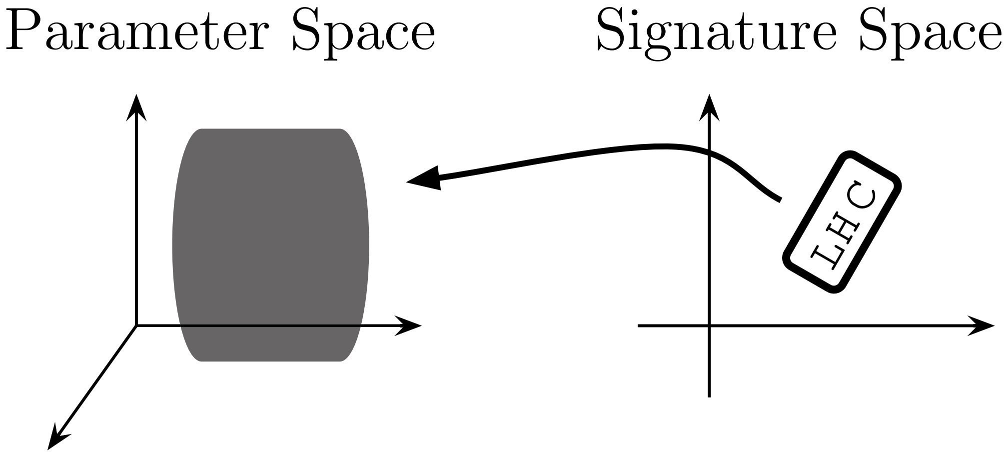
We only need to simulate $m \sim \sqrt{N_{\text{sig}}}$ in order to probe degeneracies!

We also obtain an estimate of N_{sig} by counting doubles as a function of m.

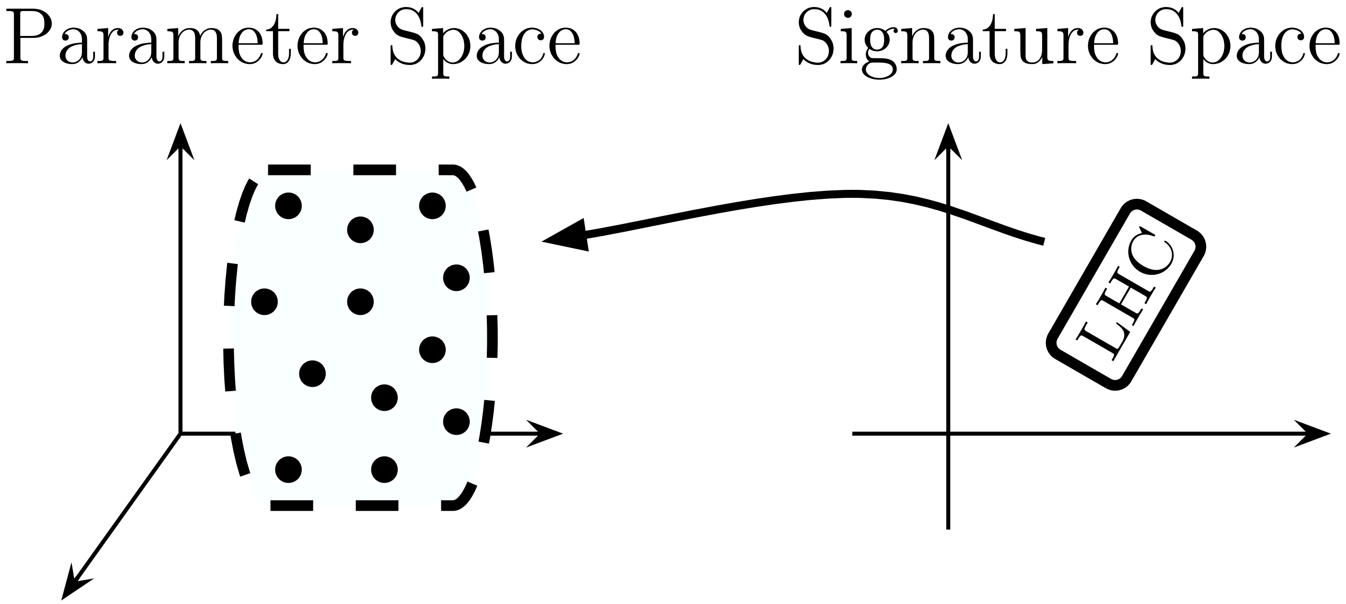
Best of all Possible Worlds



Worst of all Possible Worlds



- The Character of the Inverse Map*



Degeneracies!

Many small footprints in a large overall region.
Interesting structures of degeneracy.

*N. Arkani-Hamed, G. L. Kane, J. Thaler, LW: hep-ph/0512190

• Our result

10 fb^{-1} pure signal, strong cuts

15% error on total rate, $\sqrt{n}$ error for others

Require 10% accuracy on electroweak-ino, squarks, gluino, etc.

1. <number of degeneracies> $\sim \mathcal{O}(10-100)$

   Not unique! Not impossibly large either.

2. A lot of cliffs $\mathcal{O}(10^3) \longrightarrow$ Islands are small

   Good sensitivity but qualitative different degeneracies.

- Dimension of the Signature space

How does number of signature bins scale with size of the signature bin $|\delta s|$?

$$N_{\mathsf{sig}} \sim |\delta s|^{-D}$$

$$D_{\mathsf{sig}} \sim \text{Dim. of Sig. space}$$

A fit to our data set show that

$$D_{\mathsf{sig}} \sim (7) < \# \text{ of parameters}$$

$\longrightarrow$ degeneracies!

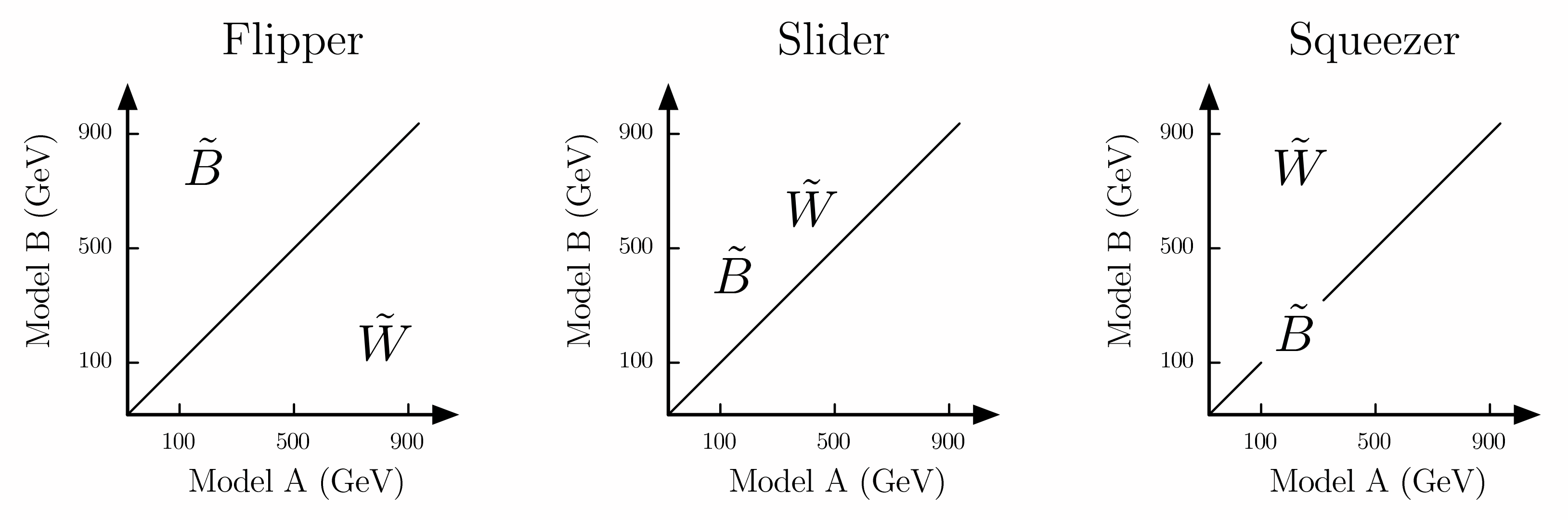
• Classification of degeneracies

Electroweak-ino

flipper: Mass, but not mass eigenstate

slider: only measure mass differences

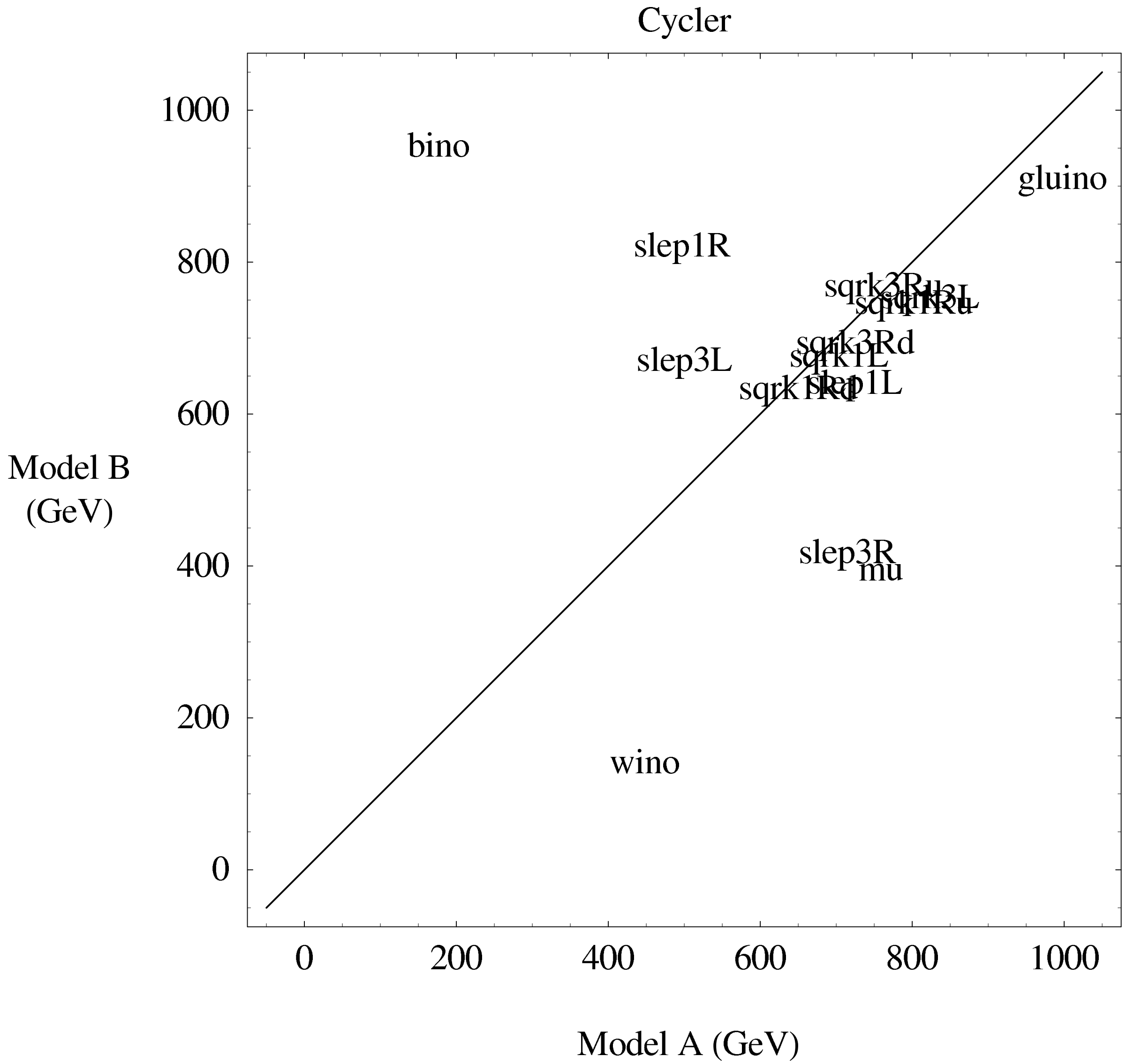
squeezer: can not measure soft stuff



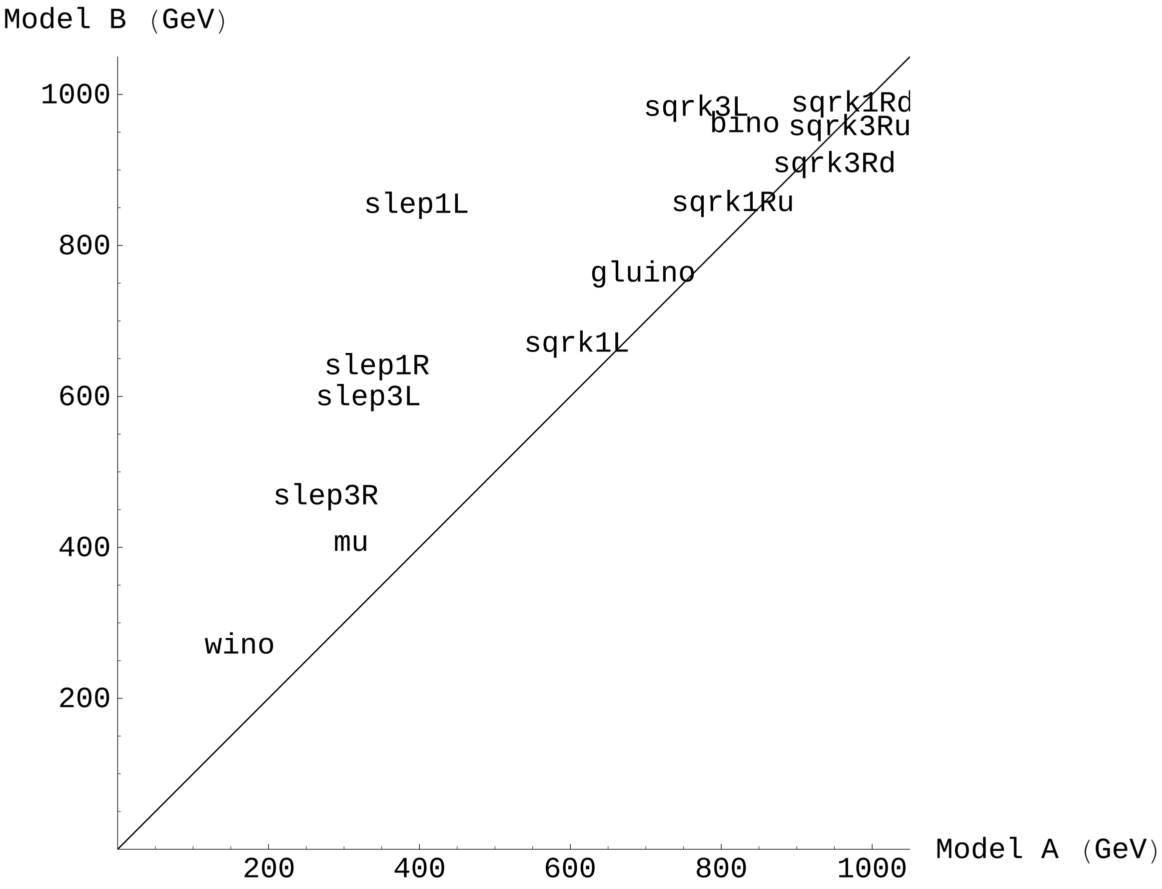
sfermion: LR split...

With light slepton: LR+ino combination flippers

1. flipper:

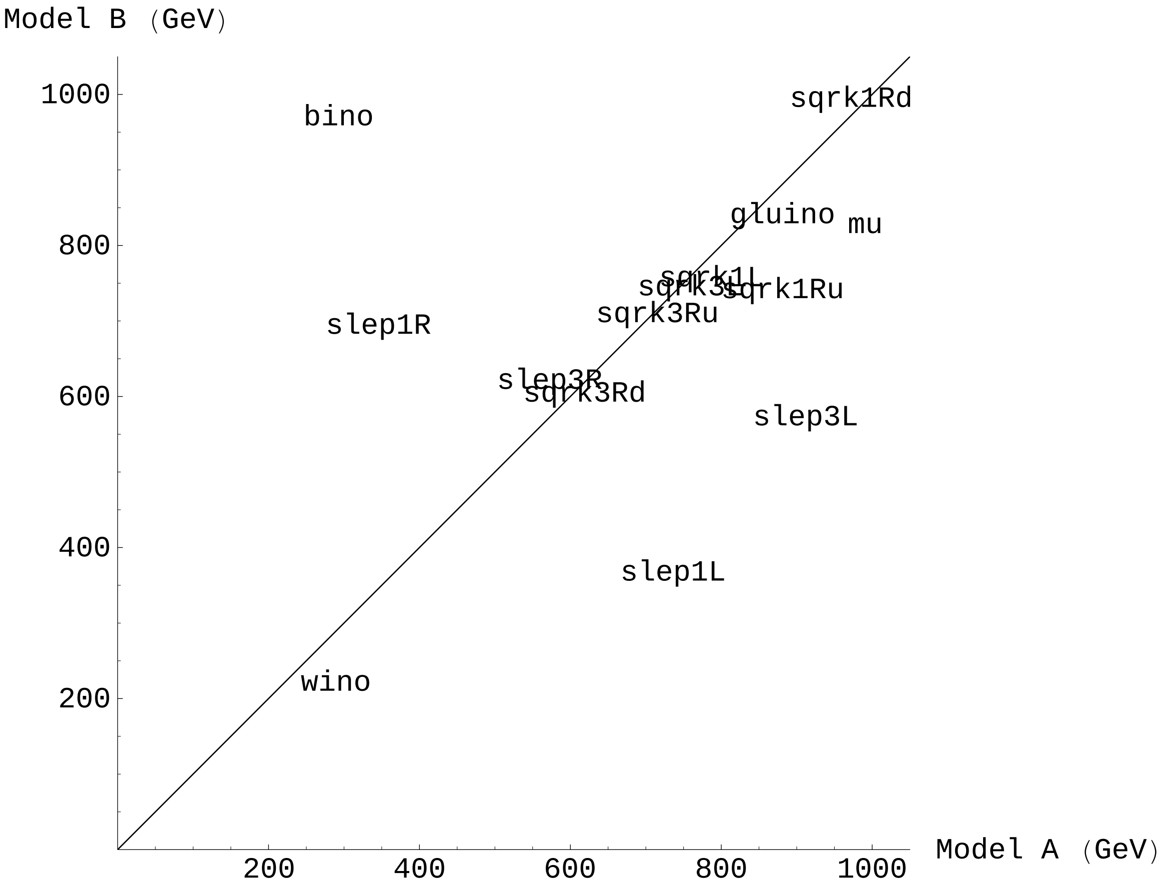

Cycler
1000
800
600
400
200
0
Model B (GeV)
bino
gluino
slep1R
sqrk3Ru
sqrk3L
sqrk1Ru
sqrk3Rd
slep3L
sqrk1L
sqrk1Rd
slep1L
slep3R
mu
wino
0
200
400
600
800
1000
Model A (GeV)

Slider:



Model B (GeV)
1000
800
600
400
200
Model A (GeV)
200
400
600
800
1000
sqrk3L
bino
sqrk1Rd
sqrk3Ru
sqrk3Rd
slep1L
sqrk1Ru
gluino
sqrk1L
slep1R
slep3L
slep3R
mu
wino

Squeezers: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1000
800
600
400
200
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600
800
1000
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bino
sqrk1Rd
gluino
mu
sqrk1L
sqrk3L
sqrk1Ru
sqrk3Ru
slep1R
slep3R
sqrk3Rd
slep3L
slep1L
wino

- Why are there degeneracies?

Colored particle → jets + ... :

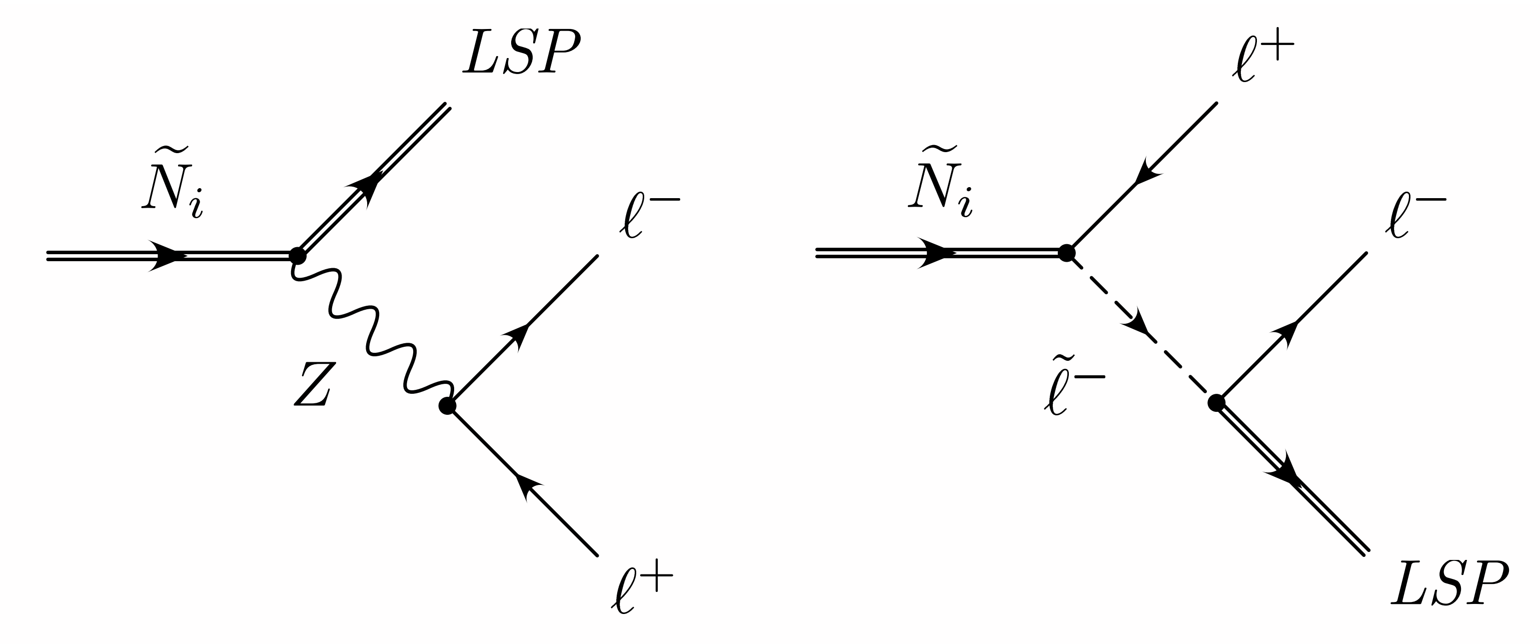
No flavor/charge information from the jets, except b-tag.

Electroweak-ino decay:

most of information in $W/Z \rightarrow$ leptons

h difficult: b-tagging, SUSY background

On-shell slepton in the decay chain will help!



on-shell slepton enhances significantly lepton signatures

- More charge and flavor information
- More handles on ino identities

However, no really good reason to expect this is generically true

• Limitation of our Study

SM background, dependence on luminosity...

Model dependence.

Repeat study for more models?

Model dependent piece and model independent pieces.

OSET: *Nima's Talk.*

Model independent approach

Provides a natural separation for degeneracies of different types.

- Multiple channels, different channels
- Multiple solutions for the same set of channels

OSET and Degeneracies: examples

Continuous deformations

Fix mass differences, slide the mass scale.

Easy to identify.

Only map to discrete solutions in model context.

e.g.: fermionic vs bosonic mother particles.

detailed kinematical distribution may tell them apart.

Two channels (trees) compensate each other

```
GN     : charge=0 color=8 mass=820
CP     : charge=3 color=0 mass=502
N2     : charge=0 color=0 mass=500
MPT    : pdg=1000022 charge=0 color=0 mass=200
....
GN > N2 u ubar
GN > CP ubar d
N2 > MPT Z0
CP > MPT W+
...
=================================================

GN     : charge=0 color=8 mass=820
CP     : charge=-3 color=0 mass=202
N2     : charge=0 color=0 mass=500
MPT    : pdg=1000022 charge=0 color=0 mass=200
....
GN > N2 u ubar
N2 > MPT Z0
N2 > CP W+
CP > MPT mu+ nu
...
```

Each has a SUSY solution. A bino and wino flipper. (soft muons...)

Seems to be one channel

```
GN      : charge=0 color=8 mass=820
QP      : charge=2 color=3 mass=780
CP      : charge=3 color=0 mass=202
MPT     : pdg=1000022 charge=0 color=0 mass=200
....
GN > QP ubar
QP > MPT u
QP > CP dbar
CP > MPT u dbar
...
```

1. $M_2 = 200 \ldots.$
2. We could trivially split the neutral channel.

   $\rightarrow M_1 \sim M_2$, a bino-wino squeezer.

One channel, more solutions

```
GN      : charge=0 color=8 mass=820
N2      : charge=0 color=0 mass=500
CP      : charge=-3 color=0 mass=202
MPT     : pdg=1000022 charge=0 color=0 mass=200
....
GN > N2 u ubar
N2 > MPT Z0
N2 > CP W+
CP > MPT u dbar
...
=================================================
```

Two potential solutions

1. $M_1 = 500$, $\mu = 200\ldots$
2. $M_2 = 500$, $\mu = 200$, … $\tilde{C}_2$ suppressed.

• Conclusions

If new physics is complicated, we expect degeneracies similar to the MSSM example.

Disentangle degeneracies requires hard work

New observables....

Model independent approach, such as OSET, will be very useful in understanding new physics.

Quick understanding of qualitative pictures.

More effective way to characterize data.

Natural separation of different types of degeneracies.

→ Different strategies.

• OSET view of degeneracy

Many more “trivial” degeneracies

Sliding scales, adding many more similar channels.

Reduced by Model context, simplicity...

Distcrete degeneracies

Different collection of trees → same signature

More signatures, kinematical details...

Different model points → same trees

More signatures, information about more trees.

Understanding is preliminary. More studies necessary.

LHC is coming.

Exciting discoveries.

A lot of work to do!

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Happy birthday, Gordy.

- backup