ACIP Summer Lectures on Extra Dimensions  

Reference: General overview: PDG (Giudice and Wells) 

Today:
- Why extra dimensions
- The (semi) simplest ED toy model: 1 scalar bulk field
- Extended toy model: 1 bulk scalar, 1 brane scalar
- Overview on ED models
  - only gravity in the bulk (ADD models - Lect 2 and 3)
  - SM fields in the bulk (UED models - Lect 4 and 5)
  - warped compactification (RS models - Lect 6 and 7)

Introduction: Why extra dimensions?

The top-down argument:
- ultimate goal: a theory describing the SM (GFT) and gravity.
- Best candidate: Super String Theory
  ⇒ D = 10, but we only see 3+1 → have to "hide" 6 dims.

Possibility I: Make the EDs compact.
Then, at large distances = low energies, only the non-compact dimensions are apparent.
II) Suppose we (the SM) live on a 4D subspace (a "brane"). Then, all fields apart from gravity are 4D.
In flat ED, this means that the ED must be compact, but for general metrics there are examples where gravity is "pulled" towards the brane and the ED can be non-compact (see eg. RS II).

The catch: the most natural size for compact ED's is $M_{Pl}^{-1}$
  $\rightarrow$ they only become apparent at $M_{Pl}$
  $\rightarrow$ no experimental signatures

- analogously in a brane world scenario, mass hierarchies are typically dictated by $10^{-6} M_{Pl} \rightarrow k$ (curvature scale of the EDs)

In an ultimate theory, the stability of the geometry ($\rightarrow$ size of extra dimensions, curvatures, shape) is derived from the theory, and the low energy theory is the SM + GR (and SM + GR only).
$\rightarrow$ ambitious.

For approaches see eg:
  - KKLT 0301240
  - Large volume flux compact (Aquino et al) 0505076
A bottom-up approach:

Let's assume there are ED at "low" energies and approach physics from an effective field theory (EFT) point of view.

- use the ingredients (motivated from string theory)
  - compact extra dimensions
  - branes
  - fields living on either of them
to see whether
  - they can solve theoretical problems of the SM
  - they can reformulate " " " " geometrically
  - they make novel predictions for experiments
  - and to study the actual experimental bounds on them.

[Comparing SUSY from Strings to ED from Strings]

<table>
<thead>
<tr>
<th>SUSY</th>
<th>ED</th>
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<tbody>
<tr>
<td>$\mathcal{g}_{Pl} = 10^{-18}$</td>
<td>String Theory ($D=10, N=4$ SUSY)</td>
</tr>
<tr>
<td>$\mathcal{g}_{GUT} = 10^{16}$</td>
<td>String Theory (Some ED is large; 0-dim Field theory with/without SUSY)</td>
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<td>SUSY $\sim 1 TeV$</td>
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<td>$SM, 4D, N=0$ SUSY</td>
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SM: 4D
The (semi) simplest ED toy model

Coordinates: \( X^M = (x^\mu, y) \)

Conventions:
\[
\mathcal{G}_{MN} = \text{diag} (-1, -1, -1, -1, -1, -1, -1, -1)
\]
\( M = 0, 1, 2, 3, 5 \)
\( \mu = 0, 1, 2, 3 \)

Massive interacting scalar field in 5D: \( M_5 \times S^1 \)

\[
S = \int \frac{1}{2} g_{\mu\nu} \partial \phi \partial \phi - m^2 \phi^2 - \frac{1}{4} \phi^4
\]

Mass dimensions:
\[
\begin{align*}
\left[ S \right] &= 0 \\
\left[ S \partial_5 \phi \right] &= -5 \\
\left[ \partial_5 \phi \phi \right] &= +1 \\
\Rightarrow \int \partial_5 \phi = \frac{2}{3} \\
\Rightarrow \left[ m^2 \right] &= -1
\end{align*}
\]

\[
S = \int ds \times \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} \int 5 \phi \dot{\phi} \partial_5 \phi - m^2 \phi^2 - \frac{1}{4} \phi^4
\]

Vary the action:
\[
\Rightarrow \text{EOM} \quad \left[ \Box - \frac{1}{2} \partial_5 \phi + m^2 \right] \phi = 0
\]

Separate:
\[
\Phi(x, y) = \sum \phi_\mu(x) f_\mu
\]

In EOM:
\[
\Rightarrow \left( \Box + m^2 \right) \frac{\phi_\mu}{f_\mu} = -M_\mu = \frac{2}{\partial_5} \frac{f_\mu}{f_\mu}
\]

Solutions:
\[
f_\mu = N_\mu \left( \sin (M_5 y) + b \cos (M_5 y) \right)
\]

Boundary conditions:
\[
f_\mu(y) = f_\mu(y + 2\pi R) \Rightarrow M_\mu = \frac{\pi}{R}
\]
A few remarks:

From a SD field, we get a whole set of fields $\phi_{1\ldots K}$.

\[ A_{\mu} = \sum_{n+m-p=0} A_{\mu}^{(n,m,p)} \]

if $n+m-p=0$ for some $n, m, p$.

\[ \text{in total, find} \]

\[ \sum_{n+m-p=0} A_{\mu}^{(n,m,p)} = \sum_{m,n,p} A_{\mu}^{(n,m,p)} \]

\[ \Rightarrow \]

\[ N_0 = \frac{1}{2eR}, \quad N_1 = \frac{1}{4eR} \]

So, solve $g \Phi = \frac{\mathcal{N}_0}{\mathcal{N}_1}$, $\mathcal{N}_0$, $\mathcal{N}_1$, $\mathcal{N}_2$}

\[ f(x) = \frac{\mathcal{N}_0}{\mathcal{N}_1} \tilde{f}(x), \quad \mathcal{N}_0, \mathcal{N}_1, \mathcal{N}_2 \]

so solve for $g$.

To find the 4D effective theory, plug solutions into the

\[ \text{Lindic terms} \]
The "smoking gun" signal for such a model would be
- detecting effects of the higher KK modes coupling to the SM
or, if the field is a 5D extension of an SM field:
  - detecting "heavy siblings" of SM fields (same charge, same spin, higher masses)
or, if the field is the graviton: modifications of GR at small scales.

Aside on the 4D-5D coupling relation:

We saw that $\hat{A} = -1$ for the 5D coupling and $\hat{A}^{\mu\nu\rho\sigma}_{\text{4D}} = 0$ for the 4D coupling and

$$\hat{A}^{\mu\nu\rho\sigma}_{\text{4D}} = \frac{A}{2\pi R} \quad \text{for} \quad m+m-p-q = 0 \quad \text{or permutations}$$

In an EFT, all parameters with mass dimension "naturally" take a value of the order of the cutoff scale $M^*$ of the EFT (unless suppressed by symmetries).

Here $\hat{A} \sim (M^*)^{-1} \Rightarrow \hat{A}^{\mu\nu\rho\sigma}_{\text{4D}} \sim \frac{1}{2\pi R M^*}$

We need $\frac{1}{2\pi R} \ll M^*$ for our 5D description to be valid.

Two ways of interpreting (↑)

If we assume $\hat{A}$ to be of natural size, $\hat{A}^{\mu\nu\rho\sigma}_{\text{4D}} \ll 1$. i.e., we get a suppression from geometry.

If we know $\hat{A}^{\mu\nu\rho\sigma}_{\text{4D}}$ from measurement not to be tiny → the cut-off $M^*$ is low.
Extension of the toy model

This was a single 4D field. Now see what happens if we couple a 5D to a 4D field.

\[ y = 0 \]

\[ \phi \quad (5D) \quad \phi \quad (4D) \quad \phi \quad (4D \ at \ y = \pi R) \]

\[ \lambda \quad (4D) \quad \lambda \quad (5D) \quad \lambda \quad (4D) \quad \lambda \quad (5D) \]

\[ S = \int d^4x \ \frac{1}{\pi R} \ \phi^* \ n^\phi \ + \ \delta (y) \left[ \bar{\psi}_i \ 4_i \psi_i - \lambda \ 4_i \bar{\psi}_i \right] \]

\[ + \ \delta (y - R) \left[ \bar{\psi}_i \ 4_i \bar{\psi}_i - \bar{\lambda} \ \phi \bar{\psi}_i \bar{\psi}_i \right] \]

again \[ \phi \ (x, y) = \sum_n \ \phi_n (x) \ \bar{\phi}_n (y) \]

but this time we have to specify boundary conditions at \[ y = 0 \] and \[ y = \pi R \].
If we choose Neumann b.c.'s \[ \left. \frac{\partial \phi}{\partial y} \right|_{y=0,\pi R} = 0 \], then the zero mode survives.

\[ \{f_n\} = \frac{1}{\pi R} \ \cos (M_n y) \]

\[ M_n = \frac{n}{R} \]

\[ n = 0, 1, 2, \ldots \]

\[ S = \int d^4x \ \frac{1}{\pi R} \ \sum_n \left( \frac{n}{R} \ \phi_n (x) \ \bar{\phi}_n (x) + \left( \frac{n}{R} \right)^2 \ \phi_n (x) \ \bar{\phi}_n (x) \right) \]

\[ + \ i \ \bar{\psi}_i \ 4_i \psi_i + i \ \bar{\psi}_i \ 4_i \bar{\psi}_i \]

\[ + \ \sum_n \ \lambda_n \ \phi_n (x) \bar{\psi}_i \bar{\psi}_i + \ \sum_n \ \bar{\lambda}_n \ \phi_n (x) \bar{\psi}_i \bar{\psi}_i \]
where \( \lambda_i^{(n)} = \frac{\lambda_i}{F_{n}} \)

- again we have the volume "dilation" (\( \lambda_i^{(n)} \) is naturally small)
- Note that the brane field couples to the whole tower of KK-modes
- Note that (at tree level) there are naturally no couplings between \( y_i \) and \( y_n \) due to localization.
  Even when loops are taken into account, the couplings will be volume suppressed

(as the loop has to "span" through the bulk)

Classification of ED Theories

**I only gravity in the bulk** ADO models

How large can the extra-dimension(s) be?  

\[
S_D = \int d^{4-D} x \, d^{D-4} y \, \sqrt{-g} \, \frac{1}{2} M_D^{0-2} R_D \\
\Rightarrow S_4 = \int d^4 x \, \sqrt{-g} \, \frac{1}{2} M_{pl}^2 R_{4D} \\
\Rightarrow M_{pl}^2 = M_D^{0-2} (\pi R)^{D-4}  
\]

=> In the presence of EDs, the true Planck scale is smaller than \( M_{pl} \)

(alternatively)

1) modification of Newton's law in 4D: \( V(r) = \frac{m_1 m_2}{M_{pl}^2} \frac{1}{r} \)

in D-dim: \( V(r) = \frac{m_1 m_2}{m_{min}} \frac{1}{r^{D-3}} \)
at large distances: \[ V(r) = \frac{\hbar m}{M_D^{D-1} R^{D-4}} \frac{1}{r} \]

\[ M_{PL}^2 = M_D^{D-1} [IR^{D-4}] \]

How are KK gravitons seen?

i) Modifications of Newton's Law

Gravity is only tested down to length scales of \( \mathcal{O}(1 \text{ mm}) \) 

\( \Rightarrow \) the extra dimension must be smaller than \( R \sim 1 \text{ mm} \).

Using \( R \sim 1 \text{ mm} \) c-uppper bound on \( R \rightarrow \) lower bound on \( M_D \)

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
D & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline
\hline
5 & 10^{-5} & 1 & 10^{-3} & \cdots & \cdots & \cdots \\
\hline
\end{array}
\]

\( \Rightarrow M_D = \begin{array}{|c|c|c|c|c|c|}
\hline
5 & 6 & 7 & 8 & 5 & 10 \\
\hline
5 & 6 & 7 & 8 & 5 & 10 \\
\hline
\end{array} \)

ii) Astrophysical bounds:

The KK gravitons couple to the SM particles, e.g. \( \nu, e^\pm \)

The O(\text{TeV}) couples with \( G_N \) determine the coupling of the KK modes (of the same order).

Apart from gravity mediation, the additional particles contribute to cooling of stars/supernovae.

Weaker coupling \( \rightarrow \) longer free path, freely escape

Lots of KK modes \( \rightarrow \) energy loss enhanced with loss by graviton.

Below temperature of star/SN.

iii) Phenomenology physics at colliders?

Lectures II and III
II. Standard model fields in the bulk

Now, the picture is

\[ \text{SM fields} \]

"We are not localized in the ED, we are trappable of the lowest excitations of extra dimensional fields."

For every SM field in the bulk, \( \ell \ell \)-modes appear at \( M \sim \frac{1}{R} \)

The most obvious bound: We did not produce them at LEP / Fermilab

\[ \Rightarrow \quad R \gtrsim (1 \text{ TeV})^{-1} \]

Further bounds arise from electroweak precision tests and FCNC constraints.

The rough picture: The additional \( \ell \ell \)-modes have SM charges \( \Rightarrow \) they contribute to loop corrections

[Box diagrams: \( b \rightarrow q \gamma \rightarrow FCNC \) corrections]

If all SM fields are in the extra-dimension, the \( \ell \ell \)-modes can be produced only pairwise [a remnant of the SO momentum conservation] \( \rightarrow \) the lightest \( \ell \ell \)-mode particle is stable \( \rightarrow \) Dark Matter
Randall-Sundrum Models

so far, we assumed flat extra dimensions but really, we only need our 4D world flat.
A metric \( ds^2 = f(r_m) \eta_{\mu \nu} dx^\mu dx^\nu + dy_m dy_m \)
contains flat 4D "slices" we could live on.

RS:

consider a slice of AdS$_5$

\[
ds^2 = e^{-2kr} \eta_{\mu \nu} dx^\mu dx^\nu + dy^2
\]

in flat extra dimensions, we found KK modes \( \sin(kh), \cos(kh) \)
— no distinction between the "left" and the "right" brane.

With the \( y \)-dependence in the metric, the wavefunctions change.
Performing the KK-decomposition for the gravity, one finds that the massless mode is exponentially localized towards the "Planck brane" while KK modes are localized towards the SM brane.
Thus we can "see" why gravity is so weak:

The overlap of the massless graviton with the SM is exponentially small. [Pictorial why of describing the set to the hierarchy problem]

(bounds on the model again arise from modification of gravity).

But the only dimension full operator in the SM is the Higgs mass term.

In order to geometrically reinterpret the hierarchy problem, we only need the Higgs to be localized on the brane.

As in flat ED, can study SM fields in the bulk. The main new feature: localization of zero modes does matter.

Example: hierarchy for Yukawa couplings can be achieved by localizing fermions differently:

\[ \begin{align*}
 & H \\
 & u \\
 & t
\end{align*} \]

The top has a large mass

\[ \rightarrow \text{IR localized} \]

\[ u \rightarrow \text{small} \]

\[ u \text{ IR localized} \]

\[ y_t \text{ is small because the } t-u \text{ overlap at the IR is small} \]