MCTP Summer Lectures on Extra Dimensions

Lecture 1

Reference: General overview: PDG (Giudice and Wells)
W.M. Yao et al. J. Phys G33: 1-1232, 2006

Today:

- Why extra dimensions
- The (semi) simplest ED toymodel: 1 scalar bulk field
- extended toy model: 1 bulk scalar, 1 brane scalar
- Overview on ED models
  - only gravity in the bulk (ADD models - Lect 2 and 3)
  - SM fields in the bulk (UED models - Lect 4 and 5)
  - warped compactification (RS models - Lect 6 and 7)

Introduction: Why extra dimensions?

The top-down argument:

- ultimate goal: a theory describing the SM (QFT) and gravity.
- Best candidate: Super String Theory

$\Rightarrow D = 10$, but we only see $3+1$ $\rightarrow$ have to "hide" 6 dims.

Possibility I: Make the EDs compact.
Then, at large distances $\hat{=}$ low energies,
only the non-compact dimensions are apparent.

II) Suppose we (the SM) live on a 4D subspace (a "brane").
Then, all fields apart from gravity are 4D.
In <u>flat</u> ED, this means that the ED must be compact, but for general metrics, there are examples where gravity is "pulled" towards the brane and the ED can be non-compact (see eg. RS II).

The catch: · the most natural size for compact ED's is M_{PL}^{-1}

→ they only become apparent at M_{PL}

→ no experimental signatures

· analogously in a brane world scenario, mass hierarchies are typically dictated by $e^{\pm \ldots}$ $M_{PL} \leftrightarrow k$ (curvature scale of the EDs)

In an ultimate theory, the stability of the geometry (→ size of extra dimensions, curvatures, shape) is <u>derived</u> from the theory, and the low energy theory is the SM + GR (and SM + GR, only).
→ ambitions.

For approaches see eg:

· KKLT th/0301240

· Large-volume flux compact. (Quevedo et al) th/0505076

A bottom-up approach:

Let's assume there are ED at "low" energies and approach physics from an effective field theory (EFT) point of view.

→ use the ingredients (motivated from string theory)

- compact extra dimensions
- branes
- fields living on either of them

to see whether

- they can solve theoretical problems of the SM
- they can reformulate " " " " " geometrically
- they make novel predictions for experiments.

and to study the actual experimental bounds on them.

[Comparing SUSY from Strings to ED from Strings]

SUSY		ED
$M_{Pl} \sim 10^{18}$ GeV	String Theory ($D=10$, $N=4$ SUSY)	String Theory
	ED are integrated out; can break some of the SUSY	(Some) ED is large; D-dim Field theory with/without SUSY
$M_{GUT} \sim 10^{16}$ GeV	4D, $N=1$ SUSY	
SUSY ~ 1 TeV	SM, 4D, "$N=0$ SUSY"	SM: 4D

The (semi) simplest ED toy model

conventions:

coords: $X^M = (x^\mu, y)$

$g_{MN} = \mathrm{diag}\,(+1,-1,-1,-1,-1)$

$M = 0,1,2,3,5$

$\mu = 0,1,2,3$

massive interacting scalar field in 5D: $M_4 \times S^1$, $y \in [0, 2\pi R)$

$$S = \int d^5x \, \tfrac{1}{2}\partial_M \phi \partial^M \phi - \hat{m}^2 \phi^2 - \hat{\lambda}\phi^4$$

mass dimensions: $[S] = 0$

$[\int d^5x] = -5$

$[\partial_M] = +1 \Rightarrow [\phi] = 3/2 \Rightarrow [\hat{\lambda}] = -1$

$[\hat{m}^2 = 2]$

$$S = \int d^5x \, \tfrac{1}{2}\partial_\mu\phi \partial^\mu \phi - \tfrac{1}{2}\partial_5\phi\partial_5\phi - \hat{m}^2\phi^2 - \hat{\lambda}\phi^4$$

vary the action

$$\Rightarrow \text{EOM} \quad (\Box - \partial_5^2 + \hat{m}^2)\,\phi = 0$$

separate: $\Phi(x,y) = \sum_n \phi^{(n)}(x) f_n$

in EOM $\Rightarrow \dfrac{(\Box + \hat{m}^2)\,\phi^{(n)}(x)}{\phi^{(n)}(x)} = -M_n^2 = \dfrac{\partial_5^2 f_n}{f_n}$

solutions: $f_n = N_n \left(\sin(M_n y) + b_n \cos(M_n y)\right)$

boundary conditions: $f_n(y) = f_n(y + 2\pi R) \Rightarrow \boxed{M_n = \dfrac{n}{R}}$

so soln's are

$$\{f_n\} = \left\{ N_n \sin\left(\frac{ny}{2\pi R}\right),\ N_n \cos\left(\frac{ny}{2\pi R}\right) \right\}$$

to find the 4D effective theory, plug solutions into the action:

from the kinetic terms

$$\int d^4x \left(\int dy \sum_{n,m} f_n f_m \right) \phi^n(x)\, \phi^m(x)$$

$$\Rightarrow \quad N_n = \frac{1}{\sqrt{\pi R}}, \quad N_0 = \frac{1}{\sqrt{2\pi R}}$$

in total, find

$$S_{4D} = \int d^4x \sum_n \frac{1}{2} \partial_\mu \phi^{(n)} \partial^\mu \phi^{(n)} - \left(\left(\frac{n}{R}\right)^2 + m^2 \right) \phi^{(n)2}$$
$$- \sum_{m,n,p,q} \lambda_{mnpq}\, \phi^{(n)} \phi^{(m)} \phi^{(p)} \phi^{(q)}$$

where $\boxed{\lambda_{mnpq} = \frac{\hat{\lambda}}{2\pi R}}$ if $n+m-p-q=0$ [or some permutation]

<u>A few remarks:</u>

masses $\mathcal{O}(1/R)$ • From a 5D field we get a whole set of fields $\phi^{(n)}(x)$ (the Kaluza-Klein tower) with masses $m_n^2 = \left(\frac{n}{R}\right)^2 + m^2$

(*) • the 4D couplings are related to the 5D coupling by the appropriate power of (πR)

• We obtain "selection rules" for couplings λ_{mnpq} [discrete version of momentum conservation]

The "smoking gun" signal for such a model would be

- detecting (effects of) the higher KK modes coupling to the SM
- or, if the field is a 5D extension of an SM field: detecting "heavy siblings" of SM fields [same charge, same spin, higher masses]
- or, if the field is the graviton: modifications of GR at small scales.

(*) Aside on the 4D - 5D coupling relation

We saw that $[\hat{\lambda}] = -1$ for the 5D coupling and $[\lambda_{mnpq}] = 0$ for the 4D coupling and

$$\lambda_{mnpq} = \begin{cases} \frac{\hat{\lambda}}{2\pi R} & \text{for } m+n-p-q = 0 \text{ or permutations} \\ 0 & \text{otherwise.} \end{cases}$$

In an EFT, all parameters with mass dimension "naturally" take a value of the order of the cutoff scale M^* of the EFT (unless suppressed by symmetries).

(†) Here $\hat{\lambda} \sim (M^*)^{-1} \Rightarrow \lambda_{mnpq} \sim \begin{cases} \frac{1}{2\pi R M^*} \\ 0 \end{cases}$

We need $\frac{1}{2\pi R} < M^*$ for our 5D description to be valid.

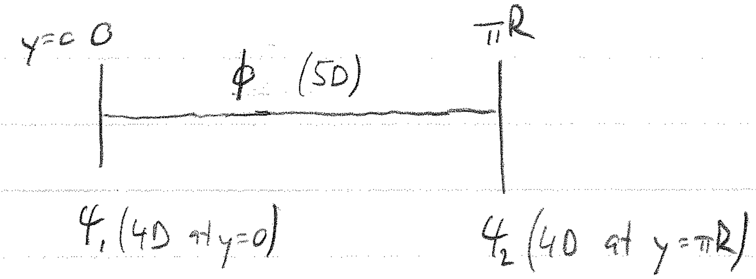
Two ways of interpreting (†)

I If we assume $\hat{\lambda}$ to be of natural size, $\lambda_{mnpq} \ll 1$. I.e, we get a suppression from geometry

II If we know λ_{mnpq} from measurement not to be tiny $\rightarrow$ the cutoff M^* is low

Extension of the toymodel

This was a single ED field. Now see what happens if we couple a 5D to a 4D field.



$$[-5] \qquad [1][3/2][1][3/2] \qquad [1][3/2][1][3/2][-\tfrac{1}{2}][3/2][3]$$

$$S = \int d^5x \; \frac{1}{2}\partial_M \phi \partial^M \phi + \delta(y)\left[\bar\psi_1 i\gamma\psi_1 - \lambda_1 \phi \bar\psi_1 \psi_1\right] + \delta(y-\pi R)\left[\bar\psi_2 i\gamma\psi_2 - \lambda_2 \phi \bar\psi_2 \psi_2\right]$$

again $\phi(x,y) = \sum_n \phi^n_{(x)} f_n(y)$

but this time we have to specify boundary conditions at $y=0$ and πR. If we choose Neumann b.c.'s $\partial_y f_n\big|_{0,\pi R} = 0$, then the zero mode survives

$$\rightarrow \{f_n\} = \frac{1}{\sqrt{\pi R}} \cos(M_n y), \quad M_n = \frac{n}{R} \qquad n = 0,1,2,\ldots$$

$$\Rightarrow S = \int d^4x \; \frac{1}{2}\sum_n \left(\partial_\mu \phi^{(n)}_{(x)} \partial^\mu \phi^{(n)}_{(x)} + \left(\frac{n}{R}\right)^2 \phi^{(n)2}\right)$$

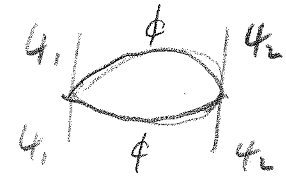
$$+ i\bar\psi_1\gamma\psi_1 + i\bar\psi_2\gamma\psi_2$$

$$+ \sum_n \lambda_1^{(n)} \phi^{(n)}_{(x)} \bar\psi_1\psi_1 + \sum_n \lambda_1^{(n)} \phi^{(n)}_{(x)} \bar\psi_2\psi_2$$

where $\lambda_1^{(n)} = \dfrac{\hat{\lambda}_1}{\sqrt{\pi R}}$

- again we have the volume "dilution" ($\lambda^{(n)}$ is naturally small)
- Note that the brane field couples to the whole tower of KK-modes
- Note that (at tree level) there are naturally no couplings between ψ_1 and ψ_2 <u>due to localization</u>.
  Even when loops are taken into account, the couplings will be volume suppressed

(as the loop has to "span" through the bulk)



Classification of ED Theories

<u>I only gravity in the bulk</u> ADD models

How large can the extra-dimension(s) be ?

$$S_D = \int d^4x\, d_y^{D-4} \sqrt{-g}\, \tfrac{1}{2} M_D^{D-2} R_D$$

$$\hookrightarrow S_4 = \int d^4x \sqrt{-g}\, \tfrac{1}{2} M_{pl}^2 R_4$$

SM | gravity, D-dim, 4D

$$\Rightarrow \boxed{M_{pl}^2 = M_D^{D-2} (\pi R)^{D-4}}$$

$\Rightarrow$ In the presence of EDs, the <u>true</u> Planck scale is <u>smaller</u> than M_{pl}

(alternatively) i) modification of Newton's law : in 4D : $V(r) = \dfrac{m_1 m_2}{M_{pl}^2} \dfrac{1}{r}$

in D-dim : $V(r) = \dfrac{m_1 m_2}{M_D^{D-2}} \dfrac{1}{r^{D-3}}$

at large distances: $V(r) = \frac{m_1 m_2}{M_D^{D-2} (2\pi R)^{D-4}} \frac{1}{r}$

$$\Rightarrow \boxed{M_{Pl}^2 = M_D^{D-2} (2\pi R)^{D-4}}$$

How are KK gravitons seen?

i) Modifications of Newton's Law

Gravity is only tested down to length scales of $\mathcal{O}(.1\,\mathrm{mm})$

$\Rightarrow$ the extra dimension must be smaller than $R \sim .1\,\mathrm{mm}$

using $2\pi R = .1\,\mathrm{mm}$ $\leftarrow$ upper bound on R $\rightarrow$ lower bound on M_D

$D =$	5	6	7	8	9	10
$\Rightarrow M_D$ (TeV) $=$	$1 \cdot 10^5$	1	10^{-3}	...	...	...

using $M_D = 1$ TeV	5	6	7	8	9	10
$2\pi R$ in mm	$3 \cdot 10^{14}$	.1	$9 \cdot 10^{-7}$	$2 \cdot 10^{-9}$	$7 \cdot 10^{-11}$	$7 \cdot 10^{-12}$

ii) Astrophysical bounds:

The KK gravitons couple to the SM particles, e.g. $\gamma, e^{\pm}$

The 0-mode couples with G_N $\Rightarrow$ determines the coupling of the KK modes (of the same order).

Apart from gravity mediation, the additional particles contribute to cooling of stars / supernovae.

weaker coupling $\rightarrow$ longer free path, freely escape

lots of KK-modes $\rightarrow$ energy loss enhanced w.r.t. loss by graviton.

below temperature of star/SN

iii) Planck scale physics at colliders?

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$\rightarrow$ Lectures II and III

II. Standard model fields in the bulk

Now, the picture is

"We are not localized in the ED, We are made of the lowest excitations of extra dimensional fields."

For every SM field in the bulk, kk-modes appear at $M \sim \frac{1}{R}$

The most obvious bound: We did not produce them at LEP / Fermilab

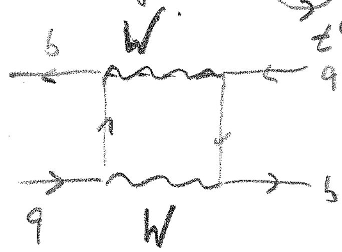
(naively) $\Rightarrow \quad R \gtrsim (1\ \mathrm{TeV})^{-1}$

Further bounds arise from electroweak precision tests and FCNC constraints.

The rough picture: The additional kk modes have SM charges $\Rightarrow$ they contribute to loop corrections

[gauge boson loops eg 

$\Rightarrow$ EWPT corrections]

[box diagrams 

$\rightarrow$ FCNC corrections]

ED
ed IV + V

If all SM fields are in the extra-dimension, the kk-modes can be produced only pairwise [a remnant of the 5D momentum conservation] $\rightarrow$ the lightest kk particle is stable $\rightarrow$ Dark Matter (and bounds from its relic density)

III

Randall-Sundrum Models

so far, we assumed flat extra dimensions

but really, we only need our 4D world flat.

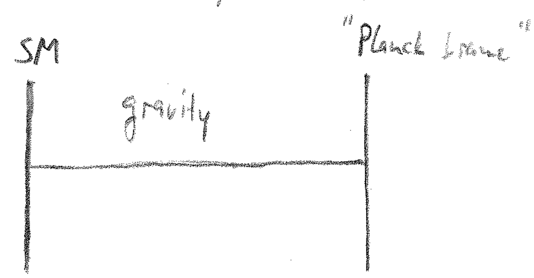
A metric $ds^2 = f(y_m)\, \eta_{\mu\nu}\, dx^\mu dx^\nu + dy_m dy^m$

contains flat 4D "slices" we could live on.

RS:

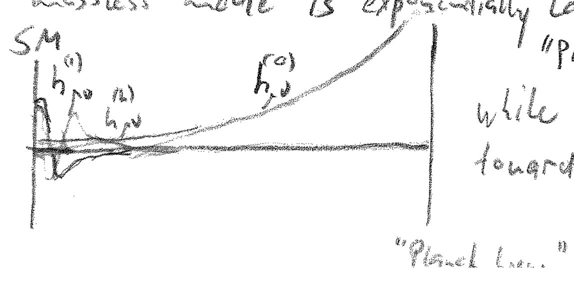
consider a slice of AdS_5

$$ds^2 = e^{-2ky}\, \eta_{\mu\nu}\, dx^\mu dx^\nu + dy^2$$



in flat extra dimensions, we found KK modes ~ $\sin(M_n y)$, $\cos(M_n y)$
— no distinction between the "left" and the "right" brane.

With the y-dependence in the metric, the wavefunctions change.
Performing the KK-decomposition for the graviton, one finds that the massless mode is exponentially localized towards the "Planck brane"
while KK modes are localized towards the SM brane.



Thus we can "see" why gravity is so weak:

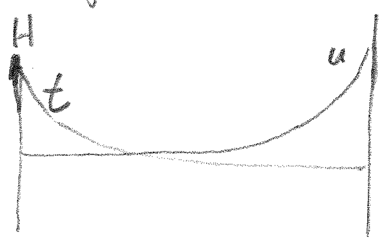
The overlap of the massless graviton with the SM is exponentially small. [Pictorial way of describing the sol^n to the hierarchy problem]

(bounds on the model again arise from modification of gravity).

But the only dimension full operator in the SM is the Higgs mass term.
In order to geometrically reinterpret the Hierarchy problem, we only need the Higgs to be localized on the brane.

As in flat ED, can study SM fields in the bulk. The main new feature: localization of zero modes does matter.

Example: Hierarchy for Yukawa couplings can be achieved by localizing fermions differently:



the top has a large mass
→ IR localized
u " small "
UV localized
→ Y_{tu} is small because the t-u overlap at the IR is small

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