MCTP Summer Lectures on Extra Dimensions — Lecture 2

"Largest extra dimensions" (ADD Models ; only gravity in the bulk)

some References for Lectures 2 and 3

- Reviews and Lecture notes:

  Csaki hep-ph/0404096 [TASI 04 Lectures , pp 2-16]

  Kribs hep-ph/0605325 [TASI 04 Lectures, ED Phenomenology]

  Hewett SLAC-PUB-12188 [Les Houches 05, ED Phenomenology]

- original papers

  Arkani-Hamed, Dimopoulos, Dvali hep-ph/9803315 ["The ADD paper"]

  (hep-ph/9804398 ["Ideas on a String embedding"]

  hep-ph/9807344 [ADD Pheno]

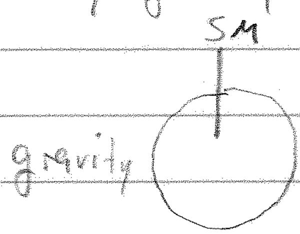
  Giudice, Rattazzi, Wells hep-ph/9811291 ["turning ~ into ="]

  hep-ph/0112161 ["Transplankian Physics"]

ADD models: The basic setup

SM localized on a 4D subspace ("brane")

only gravity in D dimensions ("bulk")



$d = D-4$ dim. compact space

(1) $$S = -\int d^4x\, d^dy \; -M_D^{D-2}\sqrt{-g}\,\mathcal{R}_D + g_{MN}T^{MN})$$

where $T_{MN} = \eta^\mu_M \eta^\nu_N T_{\mu\nu}(x)\,\delta(y)$ is the SM energy momentum tensor

Two main steps:

1| find the kaluza-klein spectrum of the bulk graviton

2| find the couplings to the SM fields

- Reminder from last Lecture: relations of D dim to 4D couplings [see e.g. Csaki ph/0404096]

(2) $S_D = \int d^4x \, d^d y \sqrt{-g} \, \frac{M_D^{D-2}}{2} R_D$, where M_D is the D dim reduced Planck mass

(3) $S_4 = \int d^4x \sqrt{-g_4} \, \frac{M_{pl}^2}{2} R_4$

(4) $\Rightarrow M_{pl}^2 \sim M_D^{D-2} \cdot \mathrm{Vol}\,(\text{compact space})$

$= M_D^{D-2} \, (2\pi R)^{D-4}$ for a D dim torus

- Another route to the same result: [ADD ph/9803315]

The gravitational law is nothing but Gauss Law.

$\int_V dV \, \rho = \oint_{\partial V} d\vec{S} \cdot \vec{F}$, where ρ is the mass distrib.

Consider a sphere around a point like mass

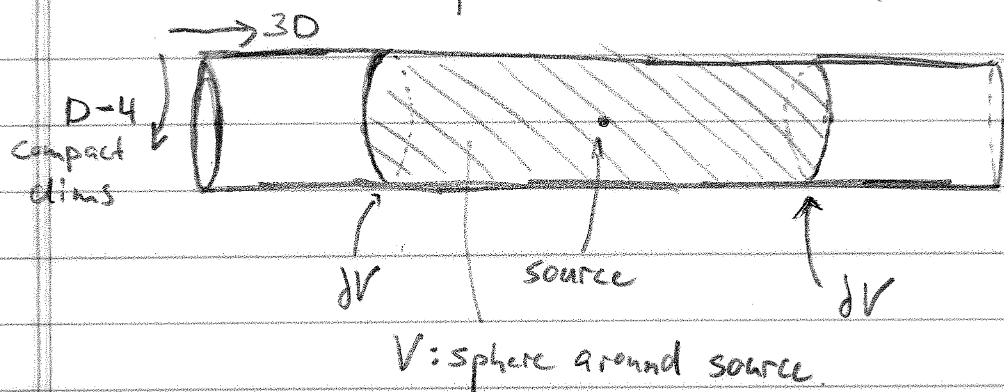
$\Rightarrow \text{const} = \oint_{\partial V} d\vec{S} \cdot \vec{F}$

in a flat, non-compact space with D-1 space dims

$\partial V \sim r^{D-2} \Rightarrow \vec{F} \sim r^{2-D} \Rightarrow V \sim r^{3-D}$

(2) $V(r) \sim \frac{m_1 m_2}{M_D^{D-2}} \frac{1}{r^{D-3}}$

now suppose we take a space with d compact dims.
and consider a sphere with $r \gg 2\pi R$, the size of the compact dim



Now, ∂V only scales like $\frac{1}{r^2}$ as the flux lines "saturate" the extra dimension

$$\Rightarrow \quad \vec{F} \sim \frac{1}{(2\pi R)^d} \frac{1}{r^2}$$

$$\Rightarrow \quad V \sim \frac{1}{(2\pi R)^d} \frac{1}{r}$$

$$V(r) \sim \frac{m_1 m_2}{M_D^{D-2}} \frac{1}{(2\pi R)^{D-4}} \frac{1}{r}$$

match to 4D potential $V(r) = \frac{m_1 m_2}{M_{PL}^2} \frac{1}{r}$

$$\Rightarrow \quad M_D^{D-2} (2\pi R)^{D-4} \sim M_{PL}^2$$ as before in (4)

In toy example 2 in Lecture 1 we saw, that the 4 dim - D dim scaling relations also hold for couplings of brane-to-bulk fields.

by analogy: If the <u>fundamental</u> massscale of the D dim theory is M_D, the Kaluza-Klein graviton modes couple to the SM fields on the brane with $M_4^2 = M_D^{D-2}\, Vol_d$

... but we only saw this in an example with scalars. For the graviton, this has been worked out in detail in

Giudice, Rattazzi, Wells; hep-ph/9811291

[see also Csaki; hep-ph/0404096 for a pedagogical review]

<u>Step I: graviton kk decomposition</u>

(5) action: $S = - \int d^4x\, d^dy \sqrt{-g}\, M_D^{D-2} R_D + \underbrace{g_{MN} T^{MN}}_{\text{boundary interactions}}$

(6) $\rightarrow$ Einstein eqns: $\mathcal{G}_{MN} \equiv R_{MN} - \frac{1}{2} g_{MN} R = - \frac{T_{MN}}{M_D^{D-2}}$

expand the metric around the flat metric

(7) $$g_{MN} = \eta_{MN} + 2\, M_D^{-D/2-1} \underbrace{h_{MN}}_{\text{graviton in D dim}}$$

back into Einstein eqns and only keep terms linear in h

$$\Box h_{MN} - \partial_M \partial^P h_{PN} - \partial_N \partial^P h_{PM} + \partial_M \partial_N h^P{}_P - \eta_{MN} \Box h^P{}_P + \eta_{MN} \partial^P \partial^Q h_{PQ} = - \frac{T_{MN}}{M_D^{D/2+1}}$$

now we have system of coupled 2^{nd} order linear DE for h.

$\rightarrow$ separable in (x_μ, y_i).

Boundary conditions for compactification on a torus T^{D-4}:

$h_{AB}(x, y_i)$ is periodic under $y_i \rightarrow y_i + 2\pi R$

(8) $$\Rightarrow h_{AB} = \sum_{n_1, \ldots n_d = -\infty}^{\infty} \frac{h^{(n)}_{AB}(x)}{\sqrt{V_d}} e^{i \frac{n_j y_j}{R}}$$

The further programme:

- plug h_{AB} into the action (5) and integrate over the compact EDs

$\rightarrow$ a set of Einstein eqns at each KK level

- find the right linear combinations of fields to decouple the EOM for the graviton (spin 2), vectors (spin 1) and scalars (spin 0) where all spins are wrt. the <u>4D</u> Lorentz group

<u>Aside on the 4D gravitons, vectors and scalars in h_{MN}</u>

from the 4D perspective, can split $h_{MN}(x)$ into 3 classes:

$h^{(\vec{n})}_{\mu\nu} \rightarrow 1$ 4D 2-tensor per KK level $\vec{n}$

$h^{(\vec{n})}_{\mu i} \rightarrow d$ 4D vectors

$h^{(\vec{n})}_{ij} \rightarrow \frac{d(d+1)}{2}$ scalars

but these are not the physical degrees of freedom, yet.

Lets count degrees of freedom:

	massless h_{MN} in D dim	"massless" DOF in 4D	massive DOF 4D
	$h_{MN} \rightarrow \frac{D(D+1)}{2}$ DOF	$h_{\mu\nu}^{(\vec{n})} \longrightarrow 2$ DOF	$h_{\mu\nu}^{(\vec{n})} \longrightarrow 5$ DOF
	4D coord inv.: $-D$ DOF	$(D-4)\, h_{\mu i} \longrightarrow 2(D-4)$ DOF	$(D-3)\, h_{\mu i} \rightarrow (D-3)\cdot 3$ DO
	freedom $\square\, \varepsilon_M = 0$ $\quad -D$ DOF	$\frac{(D-4)(D-3)}{2} h_{ij} \rightarrow \frac{(D-4)(D-3)}{2}$ DOF	$\frac{(D-3)(D-5)}{2} h_{ij} \rightarrow \frac{(D-3)(D-5)}{2}$
total	$\frac{D(D-3)}{2}$ DOF	$\frac{D(D-3)}{2}$ DOF	$\frac{D(D-3)}{2}$

rewriting $\qquad\qquad$ "eating"

$$h_{MN} \rightarrow \left(\begin{array}{c|c} h_{\mu\nu} & h_{i\nu} \\ \hline h_{\mu j} & h_{ij} \end{array}\right)$$

to go from "massless 4D" to massive via eating the Goldstones from the gauge invariants:

- the $(D-4)$ massless vectors eat $(D-4)$ scalars to get their 3rd DOF
- the graviton eats one massive vector to get its additional 3 DOF

in practice, find EOMs

identifies $\frac{1}{M_{Pl}}$

graviton $\quad \left(\Box + \frac{n^i n_i}{R^2}\right) G^{(n)}_{\mu\nu} = \frac{1}{\sqrt{V_d M_D^{D-2}}}\left[-T_{\mu\nu} + \left(\frac{\partial_\mu \partial_\nu}{n^i n_i/R^2} + \eta_{\mu\nu}\right)\frac{T^{\lambda}{}_{\lambda}}{3}\right]$

"radion" $\quad \left(\quad " \quad\right) H^{(n)} = \frac{\kappa}{3\sqrt{V_d} M_D^{D-3}} T^{\mu}{}_{\mu}$

vectors $\quad \left(\quad " \quad\right) V^{(n)}_{\mu j} = 0$

scalars $\quad \left(\quad " \quad\right) S^{(n)}_{jk} = 0$

where $\quad G^{(n)} = h^{(n)}_{\mu\nu} + \frac{\kappa}{3}\left(\eta_{\mu\nu} + \frac{\partial_\mu \partial_\nu}{n_i n^i/R^2}\right) H$

$$H^{(n)} = \frac{1}{\kappa} h^{(n)j}{}_{j} \qquad \left[\kappa = \sqrt{\frac{3(D-4)}{D-2}}\right]$$

The upshot

- the KK gravitons indeed couple with $\frac{1}{M_{Pl}} \equiv \frac{1}{\sqrt{V_d M_D^{D-2}}}$
- there is an additional scalar degree of freedom (radion) which couples to the brane fields [SM], too!
- there are additional vectors and scalars which are decoupled from the brane.
- with the explicit KK decomposition, and the action, all Feynman rules are determined $\rightarrow$ can calculate processes like $e^+e^- \rightarrow G^{(n)}_{\mu\nu}$

What can and what cannot be (easily) calculated in ADD models?

We saw that we are equipped with the full 4D effective action at tree level now

→ all Feynman rules are determined

→ can calculate $SM \to h^{(n)}_{\mu\nu}$ processes

- kk graviton production in colliders
- " in stars / supernovae

but there is a strong limitation. The fundamental scale of the theory now is $M_D \ll M_{Pl}$.

An intuitive way to see, that gravity becomes important at these scales [i.e. the EFT description breaks down]: [ADD ph/9807344]

We saw that the graviton kk modes couple with $\frac{1}{\sqrt{V_d M_D^{D-2}}}$,

but there are many of them! the mass spectrum is

$m^2_{h^{\vec{n}}} = \frac{n_i n^i}{R^2}$ ⟹ the "graviton state density"

scales like $\rho(E) \sim (ER)^{D-4}$,

consider the rate for emitting any kk graviton

$$\Gamma \sim \frac{1}{M_{Pl}^2} (ER)^{D-4} \sim \frac{1}{V_d M_D^{D-2}} (ER)^{D-4} \sim \frac{1}{M_D^2} \left(\frac{E}{M_D}\right)^{D-4}$$

$\Rightarrow$ at $E \approx M_D$, the <u>total</u> effect of D-dim gravity is unsuppressed.

So at $E \ll M_D$, the effective coupling is small ("IR softness") but at $E \approx M_D$, gravity becomes strong $\rightarrow$ need the <u>full</u> UV complete description.

"Problem": There are hints in the SM beyond the TeV scale and bounds on operators which are stronger.

The SM is renormalizable $\rightarrow$ no need for self consistency to include higher dimensional operators
But for any non-renormalizable Theory (like our D dim model) the generation of higher dimensional operators are expected if they are allowed by the (local) symmetries.

dangerous operators: <u>dim 5</u> $\quad \frac{1}{\Lambda} \bar{L}_R \phi^2 L_L \qquad \Lambda \gtrsim 10^{14}$ GeV
[Buchmüller, Wyler Nucl Phys. B268:621] [violates L, induces Majorana mass]

<u>dim 6</u> $\quad \frac{1}{\Lambda^2}\, qqql$ [violates B $\rightarrow$ proton decay $\Lambda \gtrsim 10^{16}$ GeV]

$\frac{1}{\Lambda^2}\, qqqq$ [FCNC's $\Lambda \gtrsim$ TeV]

$\frac{1}{\Lambda^2}$ [Electro Weak Precision $\Lambda \gtrsim$ TeV

When taking $M_D \sim 10\,\text{TeV}$, one has to assume that
- the suppression of B and L violating operators comes from the embedding theory (not approachable from EFT)
- FCNC's, EWTP start becoming dangerous

...and some of the virtues of the (MS)SM are lost
- Gauge coupling unification is not described
- Baryogenesis / Leptogenesis
- the reheating temperature must be very low in order to not upset BBN (more (hopefully) next Lecture)

Allowing for larger $M_D \leftrightarrow$ smaller extra dimensions of course mildens the problems but also reduces the virtues

- Higgs fine-tuning is not solved/reformulated geometrically anymore
- experimental signatures, i.e. testable differences from the SM reduce.

Next time: experimental signatures of and constraints on ADD models