Phenomenology of ADD models

SomeRefs

General Reviews
- PDG (Giudice and Wells) W.M.Yao et al. JPhys G33:1-1232, 2006
- Kribs, TASI 2004 lectures, hep-ph/0605325
- Hewett, Les Houches 2005 lectures SLAC-pub-12188
- ADD hep-ph/9807344

Papers
- Adelberger et al. hep-ph/0307284
- Callen, Perelstein ph/9903422
- Hanhart, et al. ph/0102063
- Hall, Smith ph/0904267
- Homma et al., Raffelt ph/0103201
- ph/0110067
- Arkani-Hamed et al. ph/9807344
- Giudice, Kaloper, Wells ph/9811291

Today: Experimental constraints on ADD models from:

- Modification of the Gravitational force law
- Astrophysics
- Cosmology
- Particle Colliders
Modifications of the gravitational force law

E. Akolkar et al. hep-ph/0307284

Last time we saw for an ADD model with \( D = d + 1 \) dimensions (or compact dimensions with radius \( R \))

for distances \( r \ll R \) :

\[
V(r) \sim \frac{m_1 m_2}{M_{D}^{D-2}} \frac{1}{r^{D-3}}
\]

for distances \( r \gg R \) :

\[
V(r) \sim -\frac{m_1 m_2}{M_{D}^{D-2}} \frac{1}{(2\pi)^{D/2} \Gamma} = \frac{m_1 m_2}{M_{pl}} \frac{1}{r} = -G \frac{m_1 m_2}{r}
\]

and hence \( M_{pl} = M_{D}^{D-2} (2\pi)^{D/2} \)

Conventionally, modifications of the force law are parameterized as

\[
V(r) = -G \frac{m_1 m_2}{r} \left[ 1 + \alpha e^{-r/\lambda} \right]
\]

(1)

[To get a feeling for \( \alpha, \lambda \): The exchange of a scalar particle with mass \( \frac{\lambda}{R} \) and interaction strength \( G \) induces the above correction to the gravitational potential.]

From (1), modifications due to the heavier KK modes are exponentially suppressed \( \rightarrow \) consider only the lightest KK excitations.

They have mass \( \frac{\lambda}{R} \rightarrow \lambda = R \)

\( \alpha \) is determined by the coupling strength of the lightest KK mode, which is \( G \), then coupling of a massive spin 2 field contributes a factor of \( \frac{4}{3} \), and an additional factor arises from the degeneracy of the first KK mode.

\( \rightarrow \alpha = \frac{4}{3} \) \( \cdot \) \( \delta \) for \( d \)-dim torus, \( \delta = 2d \) \( \cdot \) \( (d-2) \)-dim sphere, \( \delta = d+1 \)

Note: This is not including the \( \delta \)-term (would contribute another \( \frac{4}{3} \)) which needs to be known for stability of the ED.
The currently best bounds:

**Eot-Wash experiment (low frequency torsion oscillator)**

see: Kapon et al. hep-ph/0611233

- URL: [www.npl.washington.edu/eotwash/experiments/shortrange/sr.html](http://www.npl.washington.edu/eotwash/experiments/shortrange/sr.html)

for $\alpha = 2$ (tors): $R < 37 \mu m \, @ \, 95\% \, c.l.$

$\to M_D > 3.6 \, TeV$

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11. **Astrophysics bounds on ADD models**

The coupling of individual graviton $kk$ modes is small, but for large $R$, the $kk$ spectrum is very dense.

**Bounds from production of $kk$-modes**

Reminder from last lecture:

For a production process at energy $E$, the couplings of the individual $kk$ modes are $\sim \frac{1}{M_{kk}}$.

the number of states with mass $\leq E$ (which can be produced on shell) scales like $N(E) \sim (ER)^{D-4}$

$\Rightarrow$ the production amplitude scales like $A \sim \frac{1}{M_{kk}(ER)}^{D-4}$

the production rate scales like $\Gamma \sim \frac{1}{M_{kk}^2(ER)}^{D-4} \sim \frac{1}{M_D^2} \left( \frac{E}{M_D} \right)^{D-4}$

so the production rate is enhanced by the multiplicity of $kk$ modes.
Once the kk modes are produced, their individual coupling is \( \frac{1}{M_D} \) very weak → long free path in a plasma.

Consider energy loss processes in stars or supernovae.

Graviton \( \text{kk} \) modes, when produced, can carry away energy freely and contribute to cooling of stars or energy deposition of supernovae.

But stars still burn, and for SN1987, the neutrino flux has been measured and it matches the energy deposition.

→ Bound on the energy carried away by graviton \( \text{kk} \) modes.

From the rate estimate we see that the rate grows with \( E \)^{-0.4} → expect the strongest bounds from the most energetic objects if measured with comparable precision.

→ Bounds from SN1987

[Cullen-Perczelstein, hep-ph/9903422]

process: \( N+N \rightarrow N+N+G \) "Gravit's-trolling"

\[ \text{kk graviton} \]

- calculate the matrix element \( \frac{\langle N \mid M^{(j)} \mid N+G \rangle}{M_N-M_G} \) for production of the \( j \)-th node

- energy loss is given by the phase space integral

\[ E \sim \frac{1}{\mathcal{S}} \int \mathcal{S}(p_1+p_2-p_3-p_4-p_5) E_g f_1 f_2 (1-f_3)(1-f_4) \]

where \( \mathcal{S} \) symmetry factor

\[ E_g \] energy of the produced graviton kk mode

\( f_i(T) \) are the nucleon distribution functions.

Result: for 2ED \( M_D \sim 50 \text{ TeV} \)

<table>
<thead>
<tr>
<th>Cullen-Perczelstein</th>
<th>Heubert et al. ph/0102063</th>
</tr>
</thead>
<tbody>
<tr>
<td>Result: for 2ED</td>
<td>( M_D \geq 50 \text{ TeV} )</td>
</tr>
<tr>
<td>3ED</td>
<td>( M_D \geq 4 \text{ TeV} )</td>
</tr>
<tr>
<td>4ED</td>
<td>( M_D \geq 1 \text{ TeV} )</td>
</tr>
</tbody>
</table>
Bounds from kk-graviton decay

The kk gravitons are produced in stellar objects and they are very long lived, so they stay around and eventually decay.

* For decays into photons, find $\tau_{\gamma\gamma} \approx 3 \cdot 10^8 \text{ yr} \left(\frac{100 \text{ MeV}}{m_{\text{KK}}}\right)^3$

the photon background spectrum for $E < 100 \text{ MeV}$ is very well matched by EGRET.

Hannestad, Raffelt [ph/0103201]

consider kk-gravitons produced by all supernovae and their decay into photons and compare this spectrum to the measured photon background spectrum.

find $2\text{ED}: m_D > 84 \text{ TeV}$
$3\text{ED}: m_D > 7 \text{ TeV}$

* Another bound [Hannestad, Raffelt ph/0110067]

For the production of the heaviest kinematically accessible kk modes, no energy is left to go into kinetic energy

$\Rightarrow$ the heavy kk modes are trapped by the gravitational field of SN remnants

$\Rightarrow$ expect a photon excess [different spectrum as above because of the "kinetic selection"] from the direction of a neutron star.

Result $2\text{ED}: m_D > 200 \text{ TeV}$
$3\text{ED}: m_D > 16 \text{ TeV}$
III Bounds on ADD models from Cosmology

[Hall, Smith ph/9904267]

The fundamental cut-off of ADD models is $M_D \leq M_{pl}$

$\Rightarrow$ The early universe cosmology at scales $E > M_D$ drastically changes compared to standard 4D physics.

$\Rightarrow$ Challenges for inflation, baryogenesis, dark matter...

[see ADD ph/9807344
[Arkani-Hamed, Dimopoulos, Randall, Rissel ph/9809124, ph/9903224]

even below $M_D$, cosmology is affected by the presence of the graviton KK modes.

But we have a very good understanding of Big Bang Nucleosynthesis

$\Rightarrow$ don’t want modifications below $T \sim 1$ MeV

In Standard cosmology the cooling of the universe is dominated by Hubble expansion

$$\frac{d \rho}{dt} = -3 H \rho - \frac{3}{M_{pl}^2} \frac{T^4}{3}$$

In ADD, cooling by KK gravitons emission competes with a rate

$$\frac{d \rho}{dt} = -\frac{T^{D-4}}{M_{pl}^{D-2}}$$

these rates are equal for

$$T_X = 10^{6(D-4)^{-}}$$

$\Rightarrow \begin{cases} 10 \text{ MeV} & 2 \text{ED} \\ 10 \text{ GeV} & 6 \text{ED} \end{cases}$
Overclosure from KK gravitons

Due to the high multiplicity of KK gravitons, decay into them is quite probable, but they all couple very weakly → long lifetime

KK gravitons freeze out early and store energy density

\[ \text{estimate } S_{\text{grav}} \sim T_{*} n_{\text{grav}} \sim \frac{T_{*} Q_{D+1}}{M_{\text{Pl}}} \]

if \[ \frac{S_{\text{grav}}}{T_{*}^{4}} > 3 \times 10^{-9} \text{GeV} \]
gravitons overclose the universe

plugging in the \[ M_{D} \approx M_{\text{Pl}} \text{ TeV} \]

\[ \Rightarrow \text{need } T_{*} \leq 10^{10} \frac{(10^{-15})^{-1}}{D-2} \text{ MeV} \frac{M_{D}}{\text{TeV}} \text{ to prevent overclosure.} \]

We know from BBN constraints that we need \[ T_{*} > 1 \text{ MeV} \]

\[ \Rightarrow M_{D} > 10 \text{ TeV} \text{ for 2ED} \]

[Calculation \rightarrow \text{Hall, Smith} : M_{D} > 6.5 \text{ TeV} \text{ for 2ED}]

Diffuse \( \mu \)-ray background from KK gravitons \[ \text{[Hall, Smith]} \]

Assume that at \( T_{*} \), no gravitons KK modes are excited

[Conservation assumption]

Then gravitons KK modes are produced from processes \( SM SM \rightarrow G \)

(e.g. \( \nu \bar{\nu} \rightarrow G \))

and the gravitons KK modes become populated.

Gravitons decay into photons with \[ \Gamma(G \rightarrow \gamma \gamma) = \frac{m_{G}^{3}}{8 \pi^{2} M_{\text{Pl}}^{4}} \]

\[ = \text{can calculate the } \mu \text{-ray spectrum from such decays} \]

to get a bound \[ M_{D} > 110 \text{ TeV for 2ED} \]
IV Particle Colliders

missing energy signals

In particle colliders, KK gravitons show up in processes like

e^+e^- \rightarrow \mu^+\mu^-.

Again, for the individual KK gravitons, the cross section is tiny, but for large \( R \rightarrow \) high multiplicity, the combined signal becomes relevant.

For an explicit and detailed calculation see Giudice, Rattazzi, Wells

hep-ph/9811291

currently best bounds [LEP Exotica Working Group, LEP Exotica WG 2004-03]

\begin{tabular}{c|c|c|c|c|c}
D-4 & 2 & 3 & 4 & 5 & 6 \\
\hline
M_D \text{ in TeV} & 1.6 & 1.10 & 0.94 & 0.77 & 0.66 \\
\end{tabular}

virtual exchange of KK gravitons

see Giudice, Rattazzi, Wells:

the coupling of the gravitons is dictated by \( \mathcal{L} \supset \frac{1}{M_D^{D-1}} F_{AB} T^{AB} \),

virtual graviton exchange at tree level induces operators

\[ \mathcal{L} = \frac{4\pi}{\Delta_{\Lambda}} \mathcal{F}^{\mu \nu \rho} T_{\mu \nu} F_{\rho} \]

where \( \Lambda_{\mu} \) is a cutoff parameter

\[ \frac{4\pi}{\Lambda_{\mu}} = \sum_{D=2}^{D-2} \right\} \mathcal{F}^{\mu \nu \rho} T_{\mu \nu} F_{\rho} \]

GRW: \( \Lambda_{\mu} = \frac{\sum \Delta_{D=2} \left\{ \mathcal{F}^{\mu \nu \rho} T_{\mu \nu} F_{\rho} \right\}}{2} \quad \text{cutoff} \quad \text{Planck mass} \)

\rightarrow new contributions to \( e^+e^- \rightarrow \mu^+\mu^- , WW, ZZ, \ell\ell \)
Current bounds on $\Lambda_4$

$\Lambda_4 > 1.29 \ (1.12) \ TeV$ from LEP for constructive (destructive) interference

$\Lambda_4 > 1.43 \ (1.27) \ TeV$ from Tevatron

For a comprehensive review of ADD phenomenology and further Refs see Hewett, SLAC-Pub-12188

For an introduction to kmibs Lep-ph/0605325
For a summary of the latest most relevant bounds: PDG (Gunion and Wells)

Some general remarks

From the experimental bounds it seems that the strongest bound arises from the photon excess from KK gravitons captured by neutron stars, so why worry about the other bounds?

All these bounds have been calculated under very specific assumptions about a) the model
b) "initial conditions" at high energies

Varying them can vary the bounds relative to each other.

Examples: i) For tests of Gravity we saw that the result depends mainly on the lightest KK mode and its degeneracy.
Results from Astrophysics and Particle Physics depend on all KK modes accessible

Suppose we compactify on $S^n$ instead of $T^n$
This changes the 1st $k k$ mode degeneracy by a factor of $O(10)$, but leaves the $k k$ mode density merely unchanged.

b) Suppose we take a 5D dim model with one 4D SM brane and one 4D "hidden" brane.

```
  SM
  \_
  hidden
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New $k k$ graviton modes can also decay into "hidden" fields and the photon production from $k k$ modes is reduced:
- weakening the bounds from photons from supernovae and neutron stars
- also weakening the bounds from relic photons
- but the particle bounds are unchanged as SM-hidden interactions are suppressed by the special separation of the SM and the hidden brane in the extra dimension.

- bounds from overclosure should only be marginally be affected.
  $k k$ gravitons contribute energy density, which now might get transferred to hidden particles, but it is still present in non-SM particles.

c) dropping the assumption that at some temperature $T^*$, no $k k$-gravitons are excited [by building an early cosmology model from which the $k k$-gravity density $g(t)$ is calculable], strengthens bounds from overclosure and from relic photons but does not modify astro and particle constraints.
d) In the particle constraints, we assumed that the operators included are of the form

$$\mathcal{L} = \pm \frac{4\pi}{\Lambda_H} TrI\nu T^\nu{\bar{\nu}}$$, i.e. dimension 8,

and that they respect all symmetries of the SM.

But the cut off of the theory is low, and lepton - or baryon number violating operators could be induced at loop level, which gives additional (very strong) constraints from particle physics.