MCTP summer Lectures on Extra Dimensions

Lecture 4

Today: Standard model fields in extra dimensions [and UED models]

References:

Review article: Hooper, Profumo hep-ph/0701157

papers:

Appelquist, Cheng, Dobrescu hep-ph/0012100
Buras, Spranger, Weiler hep-ph/0212143
Mück, Pilaftsis, Rückl hep-ph/0110391
Cheng, Matchev, Schmaltz hep-ph/0204342

[for NDA
Manohar, Georgi Nucl Phys B234, 189
Papucci hep-ph/0408058]

Plan for today:

- Overview and outlook
- scalars in ED (reminder from 1^{st} Lecture)
- fermions " "
- gauge fields
- Putting things together: The UED model [at tree level]
- UED as an effective field theory
- UED at loop level: The minimal UED model
- Dark Matter from MUED

Overview:

So far we assumed the Standard Model to be confined to a 4D subspace. What if Standard Model fields propagate in the (or at least some) extra dimensions as well?

We saw in the toy example in Lecture 1, that for an extra dimensional field, we get a whole Kaluza Klein tower of fields in the 4D effective theory with masses

$$m_n^2 = \left(\frac{n}{R}\right)^2 + m_0^2 \quad [n \text{ is the KK level}, \; m_0 \text{ the 5D mass}]$$

$\Rightarrow$ every SM field (identified with the lightest effective field) gets a tower of partner particles with the same charges (and the same spin) as the SM field.

BUT: No "heavy electron partners, W partners, ..." have been observed at LEP, Fermilab, ...

$\Rightarrow$ (naively) $\frac{1}{R} \gtrsim 1$ TeV

Gravity is still in the bulk $\Rightarrow$ $M_{Pl}^2 \sim M_D^{D-2} R^{D-4}$ still holds

$\Rightarrow$ $D=5 \longleftrightarrow M_D \lesssim 2 \cdot 10^{11}$ GeV

so effects of gravity becoming strong are phenomenologically irrelevant. (and the hierarchy problem is <u>not</u> addressed in this model).

[We will see however that there is a cutoff at a much lower scale which is of a different nature: The higher dimensional field theory becomes non-perturbative.]

Outlook

- We will do the KK decomposition of scalars, fermions and gauge fields in 5 dimensions.
- We will find, that compactification on S^1 does not provide chiral fermions and leads to an unwanted scalar field A_5 (from $A_M = (A_\mu, \underline{A_5})$)

- This can be resolved by compactifying on an "orbifold" $S^1/\mathbb{Z}_2$ (an interval with symmetric end points)

- We will find that the 4D effective couplings obey selection rules (inherited from 5D momentum conservation, now broken by the orbifold fixed points) which forbid interactions of the 1st KK modes with only 0-modes.
  - → The lightest particle at the first KK level is stable (Dark matter candidate)
  - → KK particles can only be produced pairwise (experimental constraints are weaker than expected)

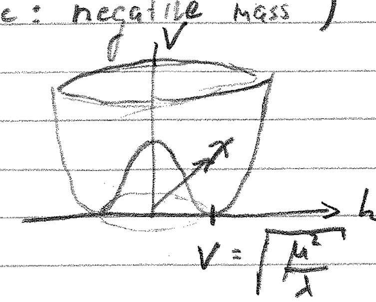
- For the final part we will discuss some issues and results from loop corrections of the model and see, which is the dark matter candidate.

Scalars in the bulk [reminder from Lecture 1]

Lets look at the Higgs

$$S = \int d^5x \, (\partial_M H)^\dagger (\partial^M H) + \mu^2 |H|^2 - \lambda |H|^2$$ (note: negative mass^2)

expand around the minimum



$$H = \sqrt{\tfrac{1}{2}} \left[v + h(x,y) + i\chi(x,y) \right]$$

$$S = \int d^5x \, \tfrac{1}{2} \partial_M h \partial^M h - \frac{m_H^2}{2} h^2 + \tfrac{1}{2} \partial_M \chi \partial^M \chi \qquad [m_h^2 = 2\mu^2]$$
+ interactions

EOM: $\left(\Box - \partial_5^2 + m_H^2\right) h = 0$

Separate: $\dfrac{(\Box - m_H^2)\, h^{(n)}(x)}{h^{(n)}(x)} = -M_n^2 = \dfrac{\partial_5^2 f_n(y)}{f_n(y)}$

$$\Rightarrow f_n = N_n \left(\sin(M_n y) + b_n \cos(M_n y) \right)$$

Impose boundary conditions: __on S^1__: $f_n(y) = f_n(y + 2\pi R)$

$\Rightarrow$ · both, sin and cos solutions are allowed

· note, that $f_0 = \text{const.}$ satisfies EOM __and__ boundary condition

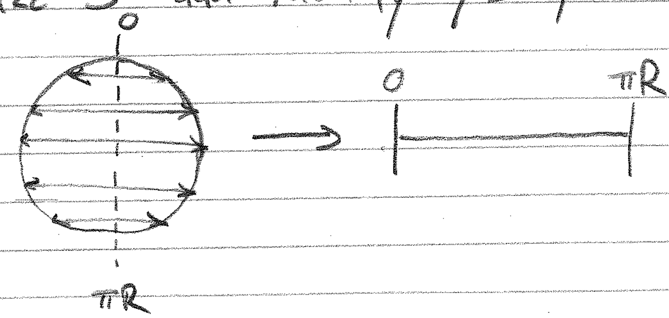
$$\Rightarrow m_0^2 = M_0^2 + m_H^2 = m_H^2$$

$$m_n^2 = M_n^2 + m_H^2 = \left(\frac{n}{R}\right)^2 + m_H^2$$

[Normalization and couplings follow from plugging sol^{ns} into 5D action and integrating over y]

On $S^1/\mathbb{Z}_2$:

what is $S^1/\mathbb{Z}_2$? Take S^1 and identify $y \equiv -y$



we demand, that S is invariant under $y \to -y$

this does not mean, that $H(y) = H(-y)$ though

action only contains $|H|^2 \Rightarrow H(y) = -H(-y)$ is also possible

for H and χ, we have the general sol^n as before, but

for the b.c.'s, we have $H(y) = H(y + 2\pi R)$

and one choice of $H(y) = \pm H(-y)$

we want the zero mode solution to survive $\Rightarrow$ choose $H(y) = + H(-y)$

then $\begin{cases} f_0 = \text{const} \\ f_n = \cos(M_n y) \end{cases}$

for the scalars, we were not forced to compactify on $S^1/\mathbb{Z}_2$,

but lets look at fermions.

Fermions in the bulk [Appelquist et al hep-ph/0012100
Buras et al hep-ph/0212143]

$S = \int d^5x \, i \, \bar{\psi} \gamma^M \partial_M \psi$ where ψ is a 5D spinor.

In 4D, the Lorentz group has chiral representations, satisfying

$$\left. \begin{array}{l} P_R \psi_R = \psi_R \\ P_L \psi_L = \psi_L \end{array} \right\} \text{ where } P_{R,L} = (1 \pm \gamma_5)/2$$

and no 4D Lorentz transformation changes/mixes ψ_R, ψ_L

In 5D, γ_5 is part of the Clifford algebra, and Lorentz trafos involving the y direction do mix ψ_R, ψ_L.

We can nevertheless perform the KK-decomposition (same procedure as before)

find $$\psi(x,y) = \frac{1}{\sqrt{2\pi R}} \left(P_L \psi(x) + P_R \psi(x) \right) + \frac{1}{\sqrt{\pi R}} \sum_n \Big(\cos(\tfrac{n}{R} y) (P_L + P_R) \psi^{(n)}(x)$$
$$+ \sin(\tfrac{n}{R}) (P_L + P_R) \psi^{(n)}(x) \Big)$$

Problem: the zero mode contains both chiralities.

But in the SM, fermions are left or right handed (and not Dirac)

on $S^1/\mathbb{Z}_2$, fermions have to satisfy $\psi(y) = \pm \gamma_5 \psi(-y)$ as additional (boundary) condition.
This projects out half of the modes and gives

$$\psi^+ = \frac{1}{\sqrt{2\pi R}} P_R \psi^+(x) + \frac{1}{\sqrt{\pi R}} \sum_n \left(\cos(\tfrac{ny}{R}) P_R \psi^+ + \sin(\tfrac{ny}{R}) P_L \psi^+ \right)$$
$$\psi^- = \frac{1}{\sqrt{2\pi R}} P_L \psi^-(x) + \frac{1}{\sqrt{\pi R}} \sum_n \left(\cos(\tfrac{ny}{R}) P_L \psi^- + \sin(\tfrac{ny}{R}) P_R \psi^+ \right)$$

- Now, the 0-mode is chiral
- The KK modes are Dirac (and they must be, because they have a mass term: $\int d^5x\, \bar{\psi}\gamma^5 \partial_5 \psi \Rightarrow \int d^4x \left(\frac{n}{R}\right)\left(\bar{\psi}_L^{(n)} \psi_R^{(n)} + h.c.\right)$)

Gauge fields in the bulk

$$S = \int d^5x \, -\frac{1}{4g_5^2} F_{MN}F^{MN}$$

$$= \int d^5x \, \frac{1}{4g_5^2}\Big(\underbrace{-F_{\mu\nu}F^{\mu\nu}}_{\text{kin. term } A_\mu} + \underbrace{2\,\partial_5 A_\mu \partial_5 A_\mu}_{\text{"mass term } A_\mu\text{"}} + \underbrace{2\,\partial_\mu A_5 \partial^\mu A_5}_{\text{kin term } A_5} - \underbrace{4\,\partial_\mu A_5 \partial_5 A^\mu}_{\text{mixing term } A_\mu \leftrightarrow \partial_\mu A_5}\Big)$$

the mixing can be canceled by choosing an R_ξ type gauge
[compare to R_ξ gauges in spont. broken gauge theories
cf eg Cheng and Li, Gauge Theory of elementary particle physics, chpt 9.2]

$$S_{GF} = -\frac{1}{2g_5^2\xi}\Big[\partial^\mu A_\mu - \xi \underbrace{\partial_5 A_5}_{\uparrow\ \text{"Goldstone boson"}}\Big]^2$$

$$S + S_{GF} = \int d^5x \underbrace{\frac{1}{4g_5^2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2g_5^2\xi}\left(\partial^\mu A_\mu\right)^2}_{\text{gauge field } A_\mu} \underbrace{- \frac{\xi}{2g_5^2}\left(\partial_5 A_5\right)^2 + \frac{1}{2g_5^2}\partial_\mu A_5 \partial^\mu A_5}_{\text{scalar field } A_5}$$

Now, can again derive the EOM and separate to find

on S^1: 0-modes and sin and cos solutions for both, A_μ and A_5

on $S^1/\mathbb{Z}^2$: 0-mode and cos sol^{ns} for one field
sin " for the other one.

Aside

Why are the $\mathbb{Z}_2$ parities of A_μ and A_5 opposite?

Look at gauge transformations $A_\mu \to A_\mu + \partial_\mu \theta$

as the gaugefields, the gauge transformation $\theta(x,y)$ must have a definite $\mathbb{Z}_2$ parity

$$\theta(x,y) = \pm\, \theta(x,-y)$$

suppose A_μ is even $\quad A_\mu \to A_\mu + \partial_\mu\theta \Rightarrow \theta$ is even

but

$$\begin{array}{ccc} A_5 & \longrightarrow & A_5 + \partial_5\theta \\ {\scriptstyle y\to -y}\downarrow & & \downarrow{\scriptstyle y\to -y} \\ \pm A_5 & \longrightarrow & \pm A_5 - \partial_5\theta \end{array}$$

$\Rightarrow A_5$ must be odd

same goes through if A_μ is odd $\Rightarrow \theta$ odd $\Rightarrow A_5$ even

We want a gaugefield 0-mode $\Rightarrow$ choose A_μ even
A_5 odd

$\Rightarrow$ 0-mode $A_\mu^{(0)}$ $\quad m_0^2 = 0$

KK-modes $A_\mu^{(n)}, A_5^{(n)}$ $\quad m_n^2 = \left(\frac{n}{R}\right)^2$

at the KK level, the A_5 provides the goldstone-mode which is eaten in order to make the KK gauge field massive.

Final ingredient: spontaneous symmetry breaking in 5D

[qualitatively; for details, see Muck, Pilaftsis, Ruckl hep-ph/0110391]

$$S = \int d^5x \, -\frac{1}{4g_2^2} W^a_{MN} W^{a\,MN} - \frac{1}{4g_Y^2} B_{MN} B^{MN}$$

$$+ (D_M H)^\dagger (D^M H) + \mu^2 H^\dagger H - \lambda |H|^4$$

H, W^a_μ, B_μ even, W^a_5, B_5 odd

0-mode: $h \longrightarrow$ higgs field $m_H = \sqrt{2}\,\mu$

$\left.\begin{array}{l} W^\pm_\mu \\ \chi^\pm \end{array}\right\} \longrightarrow W^\pm$ eat $\chi^\pm$ $m_{W^\pm} = \frac{g_2 v}{2}$

$\left.\begin{array}{l} W^3_\mu \\ B_\mu \end{array}\right\} \longleftrightarrow \left\{\begin{array}{l} A_\mu \text{ photon massless} \\ Z_\mu \end{array}\right.$

$\left.\begin{array}{l} Z_\mu \\ \chi^3 \end{array}\right\} \longrightarrow Z$ eats χ^3 $m_Z = \frac{\sqrt{g_2^2 + g_Y^2}\, v}{2}$

kk-modes:

$h \longrightarrow$ Higgs

$\left.\begin{array}{l} W^\pm_\mu \\ W^\pm_5 \\ \chi^\pm \end{array}\right\} \longrightarrow$ massive $W^\pm_\mu$, two linear combinations of $\chi^\pm$, $W^\pm_5$ eaten, two physical scalars $a^\pm$ $(m^{(n)}_{a^\pm} = m^{(n)}_W)$

$\left.\begin{array}{l} W^3_\mu \\ B_\mu \\ W^3_5 \\ B_5 \\ \chi^3 \end{array}\right\} \longrightarrow$ massive Z_μ, A_μ, to linear combinations eaten, one physical scalar a^0 $(m^{(n)}_{a^0} = m^{(n)}_Z)$

Taking everything together : 5D Universal Extra Dimension Models

- Take all SM fields in the bulk and compactify on $S^1/\mathbb{Z}_2$

$$S = \int d^5x \; -\frac{1}{4g_3^2} G^A_{MN} G^{AMN} - \frac{1}{4g_2^2} W^a_{MN} W^{aMN} - \frac{1}{4g_Y^2} B_{MN} B^{MN}$$

$$+ (D_M H)^\dagger (D^M H) + \mu^2 |H|^2 - \lambda |H|^4$$

$$+ i \bar{\psi} \gamma^M D_M \psi + (\lambda_E \bar{L} E H + \lambda_u \bar{Q} u \tilde{H} + \lambda_D \bar{Q} D H + h.c.)$$

$$\psi : \text{fermions} \begin{cases} SU(2) \text{ doublets} : Q, L \\ \quad " \quad \text{singlets} : u, D, E \end{cases}$$

$$\tilde{H} = i\sigma^2 H$$

- kk masses of all states are $m_\phi^{(n)^2} = \left(\frac{n}{R}\right)^2 + m_0^2$

- 4D couplings (at tree level) are related to 5D couplings via rescaling by the appropriate powers of R

- couplings (at tree level) respect kk-number conservation [i.e. they are 0 unless there exists a combination for which

$$\pm n_{1,in} \pm \ldots \pm n_{k,in} = \pm m_{1,out} \pm \ldots m_{k,out}$$

- At each kk-level, every SM field has a kk excitation with the same SM charges and the same spin
<u>plus</u> 3 additional scalar modes $a^\pm, a^0$

What is the lightest kk particle (LKP)?

At tree level : $\gamma^{(1)}$ and $\nu^{(1)}$ both have mass $\frac{1}{R}$

(and so does the kk graviton $h^{(1)}_{\mu\nu}$)

<u>but this is subject to loop corrections</u>

UED as an effective field theory

UED is a higher dimensional model ⇒ even if we ignore gravity, the theory is non-renormalizable and only makes sense as an effective field theory.

So what is the cutoff ?

This can be estimated by finding the energy scale at which all loop orders contribute corrections of the same order of magnitude, also known as Naive Dimensional Analysis (NDA)

The principle is well known in 4D theories [see Manohar, Georgi, Nucl. Phys. B234, 189]. For higher dimensional theories, there are a few subtleties which are resolved [see Papucci hep-ph/0408058]

From NDA on UED, one finds $\Lambda \sim 50 \frac{1}{R}$

A physical way to "see" that the cutoff is low: Think in terms of the 5D theory: the gauge couplings have mass dimension → they have powerlike RG running (Linear for 5D theories), so they become strong quickly

From the 4D effective perspective: the 4D effective couplings run logarithmically, <u>BUT</u> at each KK-level the RG flow goes through a mass threshold above which new particles contribute to the running. Summing all contributions then yields the expected running from 5D.

As the cutoff is so low ($\sim$50 TeV for $R = (1\,\text{TeV})^{-1}$) we should expect threshold corrections and allow for <u>all</u> operators allowed by all symmetries [with unknown parameters as they follow from the UV completion which is not specified].

We should also expect these operators to be induced by loop-corrections.

UED at Loop Level [Cheng, Matchev, Schmaltz hep-ph/0204342]

Cheng, Matchev and Schmaltz performed the calculation of the one-loop corrections to UED.

The crucial point (technically):

We considered the 5D action, but we also have boundaries at 0 and πR.
An operator like
$$S_0 = \int d^5x \, \frac{r_B}{4} B_{\mu\nu} B^{\mu\nu} \left(\delta(y) + \delta(y - \pi R) \right) \quad \text{with coeff } r_B$$

violates 5D Lorentz invariance, (but the $S^1/\mathbb{Z}_2$ compactification does anyway!)
but respects 4D Lorentz invariance, gauge symmetry and the $\mathbb{Z}_2$ parity

$\rightarrow$ should be induced, and it is

Example from (CMS):
$$\delta \mathcal{L} \supset \left(\frac{\delta(y) + \delta(y - \pi R)}{2} \right) \frac{\pi R g^2}{64 \pi^2} \ln\left(\frac{\Lambda^2}{\mu^2} \right) \left[\bar{\psi}_+ i \not{\partial} \psi_+ + 5 (\partial_5 \bar{\psi}_-) \psi_+ + h.c. \right]$$

from the calculation of the fermion self energy.

The divergence can be dealt with by a boundary localized counterterm, but the boundary contribution induces corrections to the masses.

For full results, see Cheng, Matchev, Schmaltz. Here, let me just pick out some results for the electro-weak gauge bosons, as the KK photon was one of the LKP candidates.

Radiative corrections to B and W can be split as

$$\hat{\delta}_{B,W} = \delta_{B,W} + \bar{\delta}_{B,W}$$

: $\delta_{B,W}$ are corrections from the bulk counter term (same as for S^1) and well determined

$\bar{\delta}_{B,W}$ are corrections from the boundary counter term. Only determined up to threshold corrections.

$$\mathcal{M}_{B^{(1)}W^{3(1)}} = \begin{pmatrix} \left(\frac{1}{R}\right)^2 + \frac{g_Y^2 v^2}{4} + \hat{\delta} m_{B^{(1)}} & \frac{1}{4} g_Y g_L v^2 \\ \frac{1}{4} g_Y g_L v^2 & \frac{1}{R^2} + \frac{g_L^2 v^2}{4} + \hat{\delta} m_{W^{(1)}} \end{pmatrix}$$

the rotation angle needed is the Weinberg angle for the 1st KK mode

at tree level,

$$\mathcal{M}_{B^{(1)}W^{3(1)}} = \frac{1}{R}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \mathcal{M}_{BW,SM}$$

$\Rightarrow$ the Weinberg angle for all KK modes is the same

At loop level $\hat{\delta} m_{B^{(1)}} \neq \hat{\delta} m_{W^{(1)}} \neq 0$, so the Weinberg angle changes from KK level to KK level.

To determine the Weinberg angles, one has to make an assumption about the threshold corrections in order to fix $\bar{\delta}$ in $\hat{\delta}$

Cheng, Matchev, Schmaltz assume that all threshold corrections vanish at the cut off scale Λ

With this assumption, one finds that $\hat{\delta} m_B < \hat{\delta} m_W$, the Weinberg angle becomes small ($\sin^2(\theta^{(1)}) \sim 10^{-2} - 10^{-3}$ depending on R)

and the KK "photon" is almost identical to the KK-B_μ.

All other KK modes are heavier $\Rightarrow$

$\Downarrow$

$$\boxed{\text{Assuming all threshold corrections to vanish at the cutoff } \Lambda \text{ the LKP is almost purely the } B_\mu^{(1)}}$$