Today: Standard model fields in extra dimensions (and UED models)

References:
Review article: Hooper, Procura hep-ph/0701157

Papers:
Apelein, Chang, Dobrescu hep-ph/0012100
March, Pilaftsis, Ruckl hep-ph/0110381
Cheng, Matchev, Schmaltz hep-ph/0204341

[for NDA]
Manohar, Georgi Nucl. Phys. B234, 189
Papucci hep-ph/0408058

Plan for today:

- Overview and outlook
- Scalars in ED (reminder from 1st lecture)
- Fermions
- Gauge fields
- Putting things together: The UED model [at tree level]
- UED as an effective field theory
- UED at loop level: The minimal UED model
- Dark Matter from MUED
Overview:

So far we assumed the Standard Model to be confined to a 4D subspace. What if Standard Model fields propagate in 5D (or at least some) extra dimensions as well?

We saw in the toy example in lecture 1, that for an extra-dimensional field, we get a whole Kaluza-Klein tower of fields in the 4D effective theory with masses

\[ m_n = \left( \frac{n}{R} \right)^2 m_0 \]  

\[ m_0 \text{ the 4D mass} \]

\[ m_n \text{ is the KK level} \]

⇒ every SM field (identified with the lightest effective field) gets a tower of partner particles with the same charges (and the same spin) as the SM field.

BUT: No "heavy electron partners, W partners, ..." have been observed at LEP, Fermilab, ...

⇒ (naively) \[ \frac{1}{R} \gg 1 \text{ TeV} \]

Gravity is still in the bulk ⇒ \[ M_{Pl} \sim M_0 R^{D-2} \] still holds

⇒ \( D = 5 \) ⇔ \[ M_0 \lesssim 2 \cdot 10^{11} \text{ GeV} \]

So effects of gravity becoming strong are phenomenologically irrelevant. (and the hierarchy problem is not addressed in this model).

[We will see however that there is a cutoff at a much lower scale which is of a different nature: The higher dimensional field theory becomes non-perturbative.]
Outlook

- We will do the KK decomposition of scalars, fermions and gauge fields in 5 dimensions.
- We will find, that compactification on $S'$ does not provide chiral fermions and leads to an unwanted scalar field $A_5$ (from $A_5 = (A_\mu, A_5)$)
- This can be resolved by compactifying on an "orbifold" $S'/\mathbb{Z}_2$ (an interval with symmetric end points)
- We will find that the 4D effective couplings obey selection rules (inherited from 5D momentum conservation, now broken by the orbifold fixed points) which forbid interactions of the 1st KK modes with only 0-modes.

→ The lightest particle at the first KK level is stable
  (Dark matter candidate)

→ KK particles can only be produced pairwise
  (Experimental constraints are weaker than expected)

- For the final part, we will discuss some issues and results from loop corrections of the model and see, which is the dark matter candidate.
Scalars in the bulk [Reminder from Lecture 1]

Let's look at the Higgs

\[ S = \int d^5x \left( \frac{1}{2} (\partial^a H)(\partial^a H) + \mu^2 |H|^2 - \lambda |H|^4 \right) \]  
\[ \text{(note: negative mass)} \]

expanded around the minimum

\[ H = \frac{1}{\sqrt{2}} \left( V + h(x) + i \times (x \times) \right) \]

\[ V = \sqrt{\frac{m^2_H}{\lambda}} \]

\[ S = \int d^5x \left( \frac{1}{2} \frac{\mu^2}{2} \frac{m^2_H}{2} \frac{h^2}{2} + \frac{1}{2} \frac{\mu^2}{2} \frac{m^2_H}{2} \frac{\partial^a h(x)^2}{\partial^a h(x)} \right) \]

+ interactions

EOM: \( (\square - \frac{\partial^2}{\partial s^2} + m^2_H) h = 0 \)

\[ \text{Separate:} \quad \left( \frac{\Box}{\partial^2} - m_H^2 \right) h (x) = m_H^2 \]  
\[ h \bigg|_{x} = \frac{d^L f(y)}{f(y)} \]

\[ \Rightarrow f_n = n \left( \sin (M_n y) + b_n \cos (M_n y) \right) \]

Impose boundary conditions: on \( S' \): \( f_n (y) = f_n (y + 2\pi R) \)

\[ \Rightarrow \text{both, \( \sin \) and \( \cos \) solutions are allowed} \]

\[ \text{note, that} \quad f_0 = \text{const.} \quad \text{satisfies EOM and boundary condition} \]

\[ \Rightarrow m_0 = M_0 + m_H^2 = m_n^2 \]

\[ m_n = M_n + m_H^2 = (\frac{m}{R})^2 + m_H^2 \]

[Normalization and couplings follow from plugging solutions into 5D action and integrating over \( y \)]
On $S^1/\mathbb{Z}_2$:

What is $S^1/\mathbb{Z}_2$? Take $S^1$ and identify $y \rightarrow -y$.

We demand that $S$ is invariant under $y \rightarrow -y$.

This does not mean that $H(y) = H(-y)$, though.

The action only contains $[H^2 = \pi]$, $H(-y) = -H(y)$ is also possible.

For $H$ and $\pi$, we have the general solution as before, but for the b.c.s. we have $H(y) = H(y + 2\pi)$ and one choice of $H(y) = \pm H(-y)$.

We want the zero mode solution to survive $\Rightarrow$ choose $H(y) = +H(-y)$.

Then $\int f_n = \text{const}$

$\int f_n = \cos (M_0 y)$

For the scalars, we were not forced to compactify on $S^1/\mathbb{Z}_2$.

But let's look at fermions.
Fermions in the bulk [Appelquist et al. hep-ph/0012100
Barroso et al. hep-ph/0212143]

\[ S = \sum_{\delta} \mu^\delta \, \delta_{\mu^4} \quad \text{where} \ 4 \ \text{is a 5D spinor.} \]

In 4D, the Lorentz group has chiral representations satisfying
\[ P_R \, 4_R = 4_R \]
\[ P_L \, 4_L = 4_L \]
where \( P_{RL} = (1 + \gamma_5)/2 \)

and no 4D Lorentz transformation changes/mixes \( 4_R, 4_L \)

In 5D, \( \gamma_5 \) is part of the Clifford algebra, and Lorentz trasnsformations involving the \( y \) direction do mix \( 4_R, 4_L \).

We can nevertheless perform the \( kk \)-decomposition (same procedure as before)

\[ 4(x,y) = \frac{1}{2\pi R} \left[ P_L \, 4_{(n)} + P_R \, 4_{(n)} \right] + \frac{1}{2\pi R} \, \sum_n \left[ \cos \left( \frac{n}{R} y \right) (P_L + P_R) \, 4_{(n)}^{(n)} + \sin \left( \frac{n}{R} y \right) (P_L + P_R) \, 4_{(n)}^{(n)} \right] \]

Problem: the zero mode contains both chiralities.

But in the SM, fermions are left or right handed
(and not Dirac)

on \( S/\mathbb{Z}_2 \), fermions have to satisfy \( 4^+(y) = \pm \gamma_5 \, 4^-(\pm y) \)
as additional (boundary) condition.

This projects out half of the modes and gives

\[ 4^+ = \frac{1}{2\pi R} \, P_R \, 4^+(x) + \frac{1}{2\pi R} \, \sum_n \left[ \cos \left( \frac{n}{R} y \right) P_R \, 4^+ + \sin \left( \frac{n}{R} y \right) P_L \, 4^+ \right] \]
\[ 4^- = \frac{1}{2\pi R} \, P_L \, 4^-(x) + \frac{1}{2\pi R} \, \sum_n \left[ \cos \left( \frac{n}{R} y \right) P_L \, 4^- + \sin \left( \frac{n}{R} y \right) P_R \, 4^- \right] \]
Now, the O-mode is chiral.

The kl modes are Dirac (and they must be, because they have a mass term: $S d^4 x p^5 d s^4 \Rightarrow S d^4 x \left( \frac{1}{R} \left( F_+^{(n)} F_+^{(n)} + b.c. \right) \right)$)

Gauge fields in the bulk

$$S = \int d^5 x \left( -\frac{1}{4 g_5^2} F_{M N} F^{M N} \right)$$

$$= \int d^5 x \left( \frac{1}{4 g_5^2} \left( \frac{F_{M N} F^{M N}}{n} + 2 d s A_M \cdot d s A^M - 4 d s A_s \cdot d s A^M \right) \right)$$

Kin-term $A_M$, "mass term $A_s$", Kin-term $A_s$, mixing term $A_s = \frac{1}{2} d^4 A_s$.

The mixing can be canceled by choosing an R3-type gauge.

[Compare to R3 gauges in spontaneous broken gauge theories, eg. Cheng and Li, Gauge Theory of Elementary Particle Physics, chpt 9.2]

$$S_{GF} = -\frac{1}{2 g_5^2} \left( d^4 A_M - 3 d s A_s \right)^2$$

"Goldstone boson"

$$S + S_{GF} = \int d^5 x \left( \frac{1}{4 g_5^2} F_{M N} F^{M N} + \frac{1}{2 g_5^2} \left( d^4 A_M \right)^2 - \frac{3}{2 g_5^2} \left( d s A_s \right)^2 + \frac{1}{2 g_5^2} d^4 A_s d^4 A_s \right)$$

gauge field $A_M$, scalar field $A_s$.

Now, can again derive the EOM and separate to find on $S': O$-modes and sin and cos solutions for both, $A_M$ and $A_s$.

on $S' / \mathbb{Z}^2$: $O$-mode and cos sols for one field

sin $\pi$ for the other one.
Why are the $\mathbb{Z}_2$ parities of $A_\mu$ and $A_5$ opposite?

Look at gauge transformations: $A_\mu \rightarrow A_\mu + \mu \Theta$

as the gauge fields, the gauge transformation $\Theta(x,y)$ must have a definite $\mathbb{Z}_2$ parity

$$\Theta(x,y) = \pm \Theta(x,-y)$$

Aside

Suppose $A_\mu$ is even $A_\mu \rightarrow A_\mu + \mu \Theta \Rightarrow \Theta$ is even

but

$A_5 \rightarrow A_5 + \delta_5 \Theta$

$$\begin{align*}
   y &\rightarrow -y \\
\delta_5 &\rightarrow -\delta_5
\end{align*}$$

$$\pm A_5 \rightarrow \mp A_5 - \delta_5 \Theta$$

$\Rightarrow A_5$ must be odd

Same goes through if $A_\mu$ is odd $\Rightarrow \Theta$ odd $\Rightarrow A_5$ even

We want a gauge field 0-mode $\Rightarrow$ choose $A_\mu$ even

$A_5$ odd

$\Rightarrow$ 0-mode $A_\mu^{(0)} \rightarrow m_\epsilon = 0$

KK-modes $A_\mu^{(n)}$, $A_5^{(n)}$

$$m_\epsilon^n = \left(\frac{n}{R}\right)^2$$

at the KK level, the $A_5$ provides the Goldstone mode

which is eaten in order to make the $KK^0$ gauge field massive.
Final ingredient: spontaneous symmetry breaking in SD

[qualitatively; for details, see Mack, Pilaftsis, Ruch, hep-ph/0110351]

\[ S = \frac{1}{4g_2^2} W_{MN}^a W^{a MN} - \frac{1}{4g_4^2} B_{MN}^a B^{MN} + (D^a_M H)(D_M^a H) + \mu^2 H^+ H = \lambda |H|^4 \]

H, \ W^3, B_5 even, \ W_5, B_5 odd

0-mode: \( h \rightarrow \) higgs field \( m_h = \sqrt{2} \mu \)

\( W_3^\pm \rightarrow W^\pm \) e \( m_{W^\pm} = \frac{g_1 V}{2} \)

\( \chi_3 \rightarrow 2 \mu \) photon massless

\( B_5 \rightarrow Z_\mu \) e \( m_\chi = \frac{g_5^2 - g_\mu^2}{2} V \)

KK-modes:

\( h \rightarrow \) higgs

\( W_3^\pm \rightarrow \) massive \( W_3^\pm \) two linear combinations of \( \chi_3^\pm \) \( W_5^\pm \) e \( \chi_3^\pm \) two physical scalars \( a^\pm \) \( (m_{a^\pm} = m_{W_3}^2) \)

\( W_5^\pm \rightarrow \) massive \( Z_\mu, A_\mu \) to linear combinations eaten

\( B_5 \rightarrow \) one physical scalar \( a^0 \) \( (m_{a^0} = m_\chi^2) \)
Taking everything together: 5D Universal Extra Dimension Models

- Take all SM fields in the bulk and compactify on $S^1/\mathbb{Z}_2$.

$$\mathcal{S} = \frac{1}{4g_5^2} G_{MN}^A G^{A}_{MN} + \frac{1}{4g_5^2} W^a_{MN} W^{a MN} - \frac{1}{4g_5^4} R_{MN}^B B^{MN}$$

$$+ (D^a H)^T (D^a H) + \mu |H|^2 - \lambda |H|^4$$

$$+ i \frac{4}{3} \mu^a D_m H + (\lambda_1 \bar{E} L H + \lambda_2 \bar{Q} u H + \lambda_3 \bar{D} \bar{D} H + h.c.)$$

- Fermions $\{ SU(2) \text{ doublets: } Q, L \}$

- Singlets: $U, D, E$

- $\tilde{H} = i \sigma^2 \tilde{H}$

- KK masses of all states are $m_{\phi}^2 = \frac{m^2}{R^2} + m_o^2$

- 4D couplings (at tree level) are related to 5D couplings via rescaling by the appropriate powers of $R$.

- Couplings (at tree level) respect KK-number conservation, i.e., they are 0 unless there exists a combination for which

$$\pm m_{1, \text{in}} \pm m_{2, \text{in}} \pm \cdots m_{k, \text{in}} = \pm m_{1, \text{out}} \pm \cdots m_{l, \text{out}}$$

- At each KK-level, every SM field has a KK excitation with the same SM charges and the same spin plus 3 additional scalar modes $\phi^0, \phi^+,

What is the lightest KK particle (LKP)?

At tree level: $\nu^{(1)}$ and $\nu^{(0)}$ both have mass $m_{\nu} = \frac{1}{R}$

(and so does the KK graviton $h^{(0)}_{\mu\nu}$)

but this is subject to loop corrections.
UED as an effective field theory

UED is a higher dimensional model – even if we ignore gravity, the theory is non-renormalizable and only makes sense as an effective field theory.

So what is the cutoff $\Lambda$?

This can be estimated by finding the energy scale at which all loop orders contribute corrections of the same order of magnitude, also known as Naive Dimensional Analysis (NDA).

The principle is well-known in 4D theories [see Manohar, Georgi, Nucl. Phys. B234, 189]. For higher dimensional theories, there are a few subtleties which are resolved [see Popescu, hep-ph/0408058].

From NDA on UED, one finds $\Lambda \sim 50/R$

A physical way to "see" that the cutoff is low: Think in terms of the 5D theory: the gauge couplings have mass dimension $\rightarrow$ they have power-like RG running (linear for 5D theories), so they become strong quickly.

From the 4D effective perspective: the 4D effective couplings run logarithmically. But at each KE level the RG flow goes through a mass threshold $\lesssim 246$ which new particles contribute to the running. Summing all contributions then yields the expected running from 5D.

As the cutoff is so low ($50$ TeV for $R = (1$ T.eV$)^5$) we should expect threshold corrections and allow for all operators allowed by all symmetries [with unknown parameters as they follow from the UV completion which is not specified].

We should also expect these operators to be induced by loop corrections.
UED at loop level \ ([Cheng, Matchev, Schwall, hep-ph/0204342]\)

Cheng, Matchev and Schwall performed the calculation of the one-loop corrections to UED.

The crucial point (technically):

We considered the 5D action, but we also have boundaries at 0 and πR.
An operator like
\[ S_0 = \int d^5x \frac{1}{4} B_{\mu\nu} B^{\mu\nu}(F(x) + F(\pi R)) \]
with coeff T_0

violates 5D Lorentz invariance (but the 5/2 compaction does anyway!)
but respects 4D Lorentz invariance, gauge symmetry and the \( Z_2 \) parity

→ Should be induced, and it is

Example From CMS:
\[
\delta \mathcal{L} = \frac{\bar{f}(x) \cdot \bar{f}(y-R)}{2} \tau R^4 \frac{\pi R^4}{64 \pi^2 \ln(\frac{A^2}{R^4})^2} \left[ -\frac{1}{4} \frac{1}{5} \pi^4 + \frac{1}{4} \delta(x-y) \delta(x-y) \right]
\]
from the calculation of the fermion self energy.

The divergence can be dealt with by a boundary localized counter-term,
but the boundary contribution induces corrections to the masses.

For full results, see Cheng, Matchev, Schwall. Here, let me just pick out some results for the electroweak gauge bosons, as the KK photon was one of the LKP candidates.

Radiative corrections to \( B \) and \( W \) can be split as
\[
\delta_{\text{ew}} = \delta_{\text{ew}} + \delta_{\text{ew}}
\]
\( \delta_{\text{ew}} \) are corrections from the bulk counter term (same as for \( S_0 \)) and well determined
\( \delta_{\text{ew}} \) are corrections from the boundary counter term. Only determined up to threshold corrections.
\begin{align*}
M_{B^0W^{30}} &= \left( \frac{1}{4R^4} + \frac{1}{R^2} + \frac{1}{R^4} \right) \left( \frac{1}{4} \hat{S} \gamma^5 \nabla^2 \right) \\
&= \left( \frac{1}{4R^4} + \frac{1}{R^2} + \frac{1}{R^4} \right) \left( \frac{1}{4} \hat{S} \gamma^5 \nabla^2 \right)
\end{align*}

The rotation angle needed is the Weinberg angle for the 1\textsuperscript{st} KK mode at tree level,

\[ M_{B^0W^{30}} = \frac{1}{R} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + M_{B^0W^{30}, SM} \]

\[ \Rightarrow \text{the Weinberg angle for all KK modes is the same} \]

At loop level, \( \hat{S}_{m_0^3} \neq \hat{S}_{m_0^3} \neq 0 \), so the Weinberg angle changes from KK level to KK level.

To determine the Weinberg angles, one has to make an assumption about the threshold corrections \( \delta \) in order to fix \( \hat{S} \) in \( \hat{S} \).

Cheng, Nastarova, Schwetz assume that all threshold corrections vanish at the cut off scale \( \Lambda \).

With this assumption, one finds that \( \hat{S}_{m_0^3} < \hat{S}_{m_0^3} \), the Weinberg angle becomes small

\[ (\sin^2(\Theta_W) \sim \mathcal{O}(10^{-2} \ldots 10^{-3}) \text{ depending on } R) \]

and the KK "photon" is almost identical to the KK-B\( \gamma \).

All other KK modes are heavier \( \Rightarrow \)

Assuming all threshold corrections to vanish at the cutoff \( \Lambda \),

the LKP is almost purely the B\( \gamma \).