Optical Semiconductor Dots for Quantum Information I

- Theory here. Phys fr experimental view - Prof Steel
- Qubit: a spin in a semiconductor quantum dot
- Preparation of a quantum state: optical initialization
- Quantum operations by optical control
  - universal if arbitrary one qubit rotations plus an entangling two-qubit operation
- Scaling up to a useful system, eg quantum computer
- Dissipative effects (tomorrow afternoon session)
  - Spin relaxation and decoherence

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Three Kinds of Semiconductor Quantum Dots

Self-assembled quantum dots (3-10 nm) \( \Delta E \sim 100 \text{ meV} \)

Interface fluctuation quantum dots (30x40x3 nm\(^3\)) \( \Delta E \sim 10 \text{ meV} \)

Gated quantum dots (100 nm) \( \Delta E \sim 1 \text{ meV} \)

InAs lattice mismatch

GaAs

Electrodes

Gated quantum dots

GaAs
**Experimentalists’ view of Quantum Dots**

**Self-assembled quantum dot**


**Interface fluctuation quantum dot**

**Gated quantum dot**

A real system: electron spin in a quantum dot

MBE-grown InAs/GaAs Dots

Cross-sectional STM image

Quantum Dot: height ~ 2-4 nm
width ~ 10-30 nm

Optical spectrum provides excellent measure of charge state

NRL group: D. Gammon, A. S. Bracker, M. F. Doty, M. Scheibner, E. A. Stinaff, J. G. Tischler, M. E. Ware

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Theorists’ view of quantum dots

Square-well quantum dots

Lateral harmonic well quantum dots

Electron

Conduction band

Empty

spin qubit

Occupied

Valence band

\[ E \]

\[ > 100 \text{meV} \]

\[ < 0.1 \text{meV} \]

\[ 1.25 \text{eV} \]

\[ > 100 \text{meV} \]

\[ < 0.1 \text{meV} \]

\[ 1.25 \text{eV} \]

\[ E \]

\[ p \]

\[ s \]

\[ 50 \text{meV} \]

\[ x, y \]

Spin qubit is robust

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p

s

x, y
Is an electron in a dot isolated?

N+1 electron problem

• lattice symmetry ==> band gap
• e-e interaction ==> 
  – renormalized mass
  – dielectric screening of interaction
  – e in filled valence band inert
  – life time infinite w/o excitations across the gap
• e-phonon interaction ==> finite life time, prolonged at low T
• Exciton (e-h) is an exact excited state w/o radiative interaction.
• Confinement of an electron in a quantum dot
• Residual decoh - op gen phonons & local electron polarizations

Kohn PR (1960), Sham, PR (1966), Sham and Rice, PR (1966).
Multi-particle states

allowed $\Gamma > \gamma$ forbidden

A. Kastler (1952)

- Expt of SAQD InAs in GaAs
- Resonant laser excitation for a time ($\sim 300$ ms) $>> 1/\gamma$ (1 $\mu$s) but less than $T_1$ due to tunneling
- Fidelity 0.998 at 0.3T (or spin $T \sim 20$ mK for Zeeman $\sim 4$K) - at op temp of 4K, B$\sim 62T$, it would takes forever at spin flip rate $1/T_1$ to equilibrate

Atatüre, Dreiser, Badolato, Högele, Karrai, Imamoğlu, Science 2006
Fast spin initialization in a singly charged quantum dot

- Population of $|x->$ states as a function of time.
- The blue dash curves and the solid lines are the analytical and numerical results, respectively.
- Near-unity fidelity is approached around 10 ns.
- The Rabi frequency is taken to be equal to the trion decay rate.

Theory: Emary, Xudong Xu, Steel, Saikin, Sham, PRL 2007

Experiment (in Steel's lectures): Xu, Wu, Sun, Huang, Cheng, Steel, Bracker, Gammon, and Emary, Sham, PRL 2007
Quantum Operation on a Single Qubit

Not q-op

Optical Excitation by Fermi golden rule

Rabi rotation

Precession not Rabi

A coherent b pulse of the duration may rotation the spin in an angle $\alpha$ about a fixed axis in the rotating frame

$$\hat{H} = B\sigma_z + b[\cos(\omega t)\sigma_x + \cos(\omega t)\sigma_y]$$

$$R(\alpha, n) = \begin{bmatrix} \cos(\frac{\alpha}{2}) - in_z \sin(\frac{\alpha}{2}) & -i(n_x - in_y) \sin(\frac{\alpha}{2}) \\ -i(n_x + in_y) \sin(\frac{\alpha}{2}) & \cos(\frac{\alpha}{2}) + in_z \sin(\frac{\alpha}{2}) \end{bmatrix}$$
Theory

One electron spin state in a SAQD

Single particle levels
Spin and trion states

Spin State to Trion

(1) Adiabatic NR Raman Spin-flip Process

One electron spin state in a SAQD

Or (2) direct microwave

Optical

Trion

Magnetic field in x

Arbitrary rotation of the spin state -- single qubit gate

Single Electron Spin Coherence: Raman Quantum Beats

Charged Exciton System

Neutral Exciton System

$T_2^* > 10 \text{ nsec at } B=0$

Petta et al. Science 2005
Bracker et al. 2005

Field (T)

$\hbar \gamma_s (\mu eV)$
Control of single spin with single optical pulse

S. Economou, L. J. Sham, Yanwen Wu, and D. G. Steel, PRB 2006

Rabi rotation
Spin Coherence Modulated Trion Excitation (taking background into account)

Calculated

Contol at the \(-Z\)

Control at the \(-Y\)

Control at the \(+Z\)

Measured

\(\theta_c \sim \pi\)

\(\tau_c\)

Expt

Yanwen Wu, Erik Kim, Xiaodong Xu, Jun Cheng, Steel, Bracker, Gammon, Economou & Sham, PRL 2007
An entangling gate with two interacting qubits

In the computational basis: $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$

General SWAP

$$S(\alpha, \gamma) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(\alpha/2) & -\sin(\alpha/2) & 0 \\
0 & \sin(\alpha/2) & \cos(\alpha/2) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$$

SWAP ($\alpha = \pi$)

$\sqrt{\text{SWAP}}$ ($\alpha = \pi/2$) entangling

Controlled-NOT Gate

Rot q2 conditional on q1

$$\text{CROT}_{12} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix}$$
Controlling spin interaction between two electrons in two dots

Continuum or tunnel exciton

Single particle levels

ORKKY or Bloombergen-Roland

Excited e wf covers both dots

Theory

J\textsubscript{1}s\textsubscript{1}.s\textsubscript{c}

Pochung Chen, Piermarocchi, Sham, & Steel, PRL 02

Emary & Sham, PRB 07

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Optical control of two dot-spins via 2 trions

Spin and trion states

Two trions have Coulomb interaction

Cf. ORKKY: less demand on dot fabrication, more on optics
**TABLE I: Gates, pulses, and time-consumption required for factoring 15 with Shor’s quantum algorithm**

<table>
<thead>
<tr>
<th>Scenario</th>
<th># of one-bit gates</th>
<th># of swap gates</th>
<th># of phase gates</th>
<th># of pulses</th>
<th>time-consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>a=4</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>48</td>
<td>0.8 ns</td>
</tr>
<tr>
<td>a=13 (Toffoli gate)</td>
<td>19</td>
<td>8</td>
<td>15</td>
<td>159</td>
<td>1.2 ns</td>
</tr>
<tr>
<td>a=13 (S- Toffoli gate)</td>
<td>12</td>
<td>6</td>
<td>7</td>
<td>102</td>
<td>1.0 ns</td>
</tr>
</tbody>
</table>

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*All one-bit gates between two controlled gates are counted as one gate requiring 4 pulses which can be done within 10 ps*

*b* including 21 pulses for initialization

*c* including the time for initialization, estimated as 100 ps per bit

To built a scalable system: Qubit conversion

• CQED and Q-Net pioneered by
• Control - deterministic in CQED
  – Adiabatic control: Duan, Kuzmich & Kimble, PRA 67, 032305 (2003)
  – Non-adiabatic: Yao, Liu & Sham PRL 95, 030504 (2005)
Deterministic process driven by \textbf{Y pulse (2)}

1. Pulse moves $|T+\rangle$ to resonance with $|-\rangle |C\rangle$
2. \textbf{Y pulse (macro) transforms} $|+\rangle$ to $|T+\rangle$
3. $|T+\rangle$ evolves to $|-\rangle |C\rangle$ generating a photon
4. \textbf{Photon (micro) moves along wave guide}

Spin-photon state swap $|+\rangle |\text{vac}\rangle \rightarrow |-\rangle |\alpha\rangle$

Cavity containing a cluster of dots

Selection rules

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Consequences of spin-photon swap

\[ |+\rangle |\text{vac}\rangle \rightarrow |-\rangle |\alpha\rangle \]

\[ [\beta_+ |+\rangle + \beta_- |\text{vac}\rangle] |\text{vac}\rangle \rightarrow |-\rangle [\beta_+ |\alpha\rangle + \beta_- |\text{vac}\rangle] \]

- A stationary qubit & a flying qubit exchanging info.
- Initialization
  - Reduce an unpolarized state to a spin state, \( |-\rangle \), say.
  - Basic process: Wave guide serves as entropy dump
- Entanglement of a spin and a photon

\[ |+\rangle |\text{vac}\rangle \rightarrow [|+\rangle |\text{vac}\rangle + |-\rangle |\alpha\rangle]/\sqrt{2} \]

Liu, Yao, Sham, PRB 72, 081306 (R) (2005)
Quantum Non-Demolition (QND) Measurement of n Spins

- Projective measurement
  - If there is no photon output, the spin state is $|\rightarrow\rangle$.
  - If there is a photon, the spin state is $|\leftarrow\rangle$.
- QND - The spin state unchanged between measurements.
  - Hence, can be cycled many times to collect photons.
- Nonideal measurements can be analyzed by POVM.
Send: Optical control of the spin qubit in the dot via trion & cavity mode generates a photon wave packet
  – entangling the spin qubit with the (0,1) photon states
Receive: reverse optical pulse to absorb photon completely
  – net: entangling sender spin qubit with receiver spin qubit
Basis for distributed computation to scale up a Q computer

Cavity and wave guide in photonic lattice

Photonic double-heterostructure

Q = 6 x 10^5

c = 250 nm

Bong-Shik Song, Susumu Noda, Takashi Asano, Yoshihiro Akahane, Nature Materials 05
Evidence for Strong Coupling CQED

Yoshie, Schere, Hendrickson, Khitrova, Gibbs, Ruppe, Ell, Shchekin, Deppe, Nature 04