Solving the LHC Inverse Problem with Dark Matter Observations

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The “LHC Inverse Problem” – in One Slide

⇒ Basic premise: multiple SUSY parameter sets likely to fit the LHC data

• After many years of data-taking at LHC we have a well-defined “signature space” bounded only by experimental errors

• A global fit to a “minimal” multi-parameter MSSM model is performed

\[
\begin{align*}
&\{ \tan \beta, \mu, M_1, M_2, M_3 \\
&m_{Q_{1,2}}, m_{U_{1,2}}, m_{D_{1,2}}, m_{L_{1,2}}, m_{E_{1,2}} \\
&m_{Q_3}, m_{U_3}, m_{D_3}, m_{L_3}, m_{E_3} \\
&m_A, A_t, A_b, A_\tau \}
\end{align*}
\]

• Several (isolated) points in “parameter space” will be good fits to the observed data
This conjecture was explicitly verified by brute force

- 45,000 SUSY models sampled and 10 fb\(^{-1}\) of simulated data generated

\[
\left\{ \begin{array}{c}
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\end{array} \right\} = \text{“model”}_i
\]

- Over 1800 observables \( s_j \) constructed – some examples:
  - Number of events with 2 jets and 2 leptons
  - \( M_{\text{eff}} \) of everything in the event for events with 5\( ^+ \) jets and no leptons
  - Invariant mass of the two hardest non-b-tagged jets

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- The set \{s$_j$\} for each model defines a point in “signature space”

- Distance between model$_A$ & model$_B$ in signature space can be defined by

\[
(\Delta S_{AB})^2 = \frac{1}{N_{\text{sig}}} \sum_j \left( \frac{s_j^A - s_j^B}{\sigma_{AB}^j} \right)^2
\]

- If $(\Delta S_{AB})^2 \leq (\Delta S_{AB})_{\text{min}}^2$, the pair $(A,B)$ was defined to be degenerate
The “LHC Inverse Problem” – Is it a Problem?

- Out of 45,000 test models they found 276 degenerate pairs
- Even found triplets, quartets, etc. [therefore only 378 unique models]
- Degeneracy inevitable!
- Estimate $\mathcal{O}(10 – 100)$ twins for any given parameter set

⇒ How should we attack this important problem?
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(1) Make better use of LHC data itself
   - Better choice of observables
   - Use exclusive measurements/reconstruct decay chains
   - Simply wait for more integrated luminosity
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⇒ But let’s go ahead and take the LHC Inverse Problem at face value
(2) Wait for the ILC to rescue us?


- Good News: When charged superpartners accessible, pairs generally separable

- Bad News: Only 57 pairs distinguishable at $5\sigma$ at $\sqrt{s} = 500$ GeV ILC (63 at $3\sigma$ level)

- Worse News: The earliest we can expect the ILC is 2019...
The Search for Solutions

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(3) Use dark matter observables as a discriminant

- Many experiments taking data now or in near future
- WIMP signal rates strongly sensitive to things LHC has difficulty seeing (like LSP wavefunction)
Given a WIMP signal can we say definitively that it is consistent with only one of our post-LHC models?

⇒ Dark matter arena very different from collider studies!

- Variety of experiments and detection methodologies
- Backgrounds to WIMP signals less well modeled and understood
- Theoretical assumptions often the biggest source of uncertainty
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  • Backgrounds to WIMP signals less well modeled and understood
  • Theoretical assumptions often the biggest source of uncertainty

⇒ One major uncertainty → number density of WIMPs $n_\chi = \rho_\chi/m_\chi$
  • We have little idea what this should be...
    but rotation curves give some indication
  • Assume a local density normalized by $(\rho_\chi)_0 = 0.3 \text{ GeV/cm}^3$
  • Indirectly related to thermal relic density $\Omega_\chi h^2$
  • Impacts direct detection nuclear recoil rates
  • Annihilation rates and branching fraction into interesting final states

⇒ We performed all calculations using DarkSUSY
Thermal Relic Density

\[ 0.086 < \Omega \chi h^2 < 0.119 \]

- **WMAP 2\(\sigma\) region**
  \[ \Omega \chi h^2 \leq 0.025 \]
  225 models

- **WMAP 3yr 2\(\sigma\)**
  \[ 0.025 \leq \Omega \chi h^2 \leq 0.119 \]
  8 models

- **\(\Omega h^2\)_{min}**
  \[ \Omega \chi h^2 > 0.119 \]
  145 models
Thermal relic density sensitive to small variations in SUSY parameters

Lots of ways to alter standard predictions for $\Omega_\chi h^2$

⇒ Our approach is to consider two possibilities

1. Assume $(\rho_\chi)_0 = 0.3$ GeV/cm$^3$ regardless of $\Omega_\chi h^2$ prediction
2. Rescale $(\rho_\chi)_0$ by a factor $r_\chi = \text{Min}(1, \Omega_\chi h^2/0.025)$
Defining “Physical” Models

⇒ Thermal relic density not the only problem these models have...

<table>
<thead>
<tr>
<th></th>
<th>Models</th>
<th>Pairs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial Set</strong></td>
<td></td>
<td></td>
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<tr>
<td>PYTHIA chargino warnings</td>
<td>378</td>
<td>276</td>
</tr>
<tr>
<td><strong>Relic Density</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Omega\chi h^2 &gt; 0.1189$</td>
<td>145</td>
<td>116</td>
</tr>
<tr>
<td>$\Omega\chi h^2 &lt; 0.0250$</td>
<td>224</td>
<td>164</td>
</tr>
<tr>
<td><strong>Additional Constraints</strong></td>
<td></td>
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<tr>
<td>$m_h &lt; 114.4$</td>
<td>43</td>
<td>52</td>
</tr>
<tr>
<td>$\text{Br}(B \to X_s \gamma) &gt; 4.45 \times 10^{-4}$</td>
<td>101</td>
<td>98</td>
</tr>
<tr>
<td>$(a_\mu)_{\text{SUSY}} &gt; 4.7 \times 10^{-9}$</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td><strong>Visible at 500 GeV ILC</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Remove PYTHIA chargino warnings</td>
<td>190</td>
<td>173</td>
</tr>
<tr>
<td><strong>All Physical Conditions Satisfied</strong></td>
<td>127</td>
<td>77</td>
</tr>
</tbody>
</table>
Direct Detection
What does it mean to distinguish between degenerate models?

- Values of $s_i^A$ and $s_i^B$ need to be large enough that both are detectable above the relevant background for the experiment in question.
- Values of $s_i^A$ and $s_i^B$ need to be sufficiently separated to give a statistically significant difference when measured with respect to the appropriate mutual error $\sigma_i^{AB}$. 
Direct Detection Experiments

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⇒ Direct detection experiments look for scattering of WIMPS from heavy nuclei

- The actual observable ($s_i$) is the number of recoil events observed over some time interval
- The rate for such events is approximated by

$$\frac{dR}{dE} \sim \sum_i \Phi \sigma^\text{SI}_\chi \frac{\sigma^\text{SI}}{M_i} = \sum_i \frac{\langle v_\chi \rangle \rho_\chi \sigma^\text{SI}_\chi}{m_\chi M_i},$$

with $M_i$ being the mass of $i$-th nucleus and $\langle v_\chi \rangle \simeq 270\text{km/s}$
Red circles
\(\Omega \chi h^2 < 0.025\)

Green triangles
\(0.025 \leq \Omega \chi h^2 \leq 0.1189\)

Blue triangles
\(\Omega \chi h^2 > 0.1189\)

<table>
<thead>
<tr>
<th>Name</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDMS II</td>
<td>3.75 kg Ge</td>
</tr>
<tr>
<td>XENON10</td>
<td>5.4 kg Xe</td>
</tr>
<tr>
<td>ZEPLIN II</td>
<td>7.2 kg Xe</td>
</tr>
<tr>
<td>SuCDMS (Soudan)</td>
<td>7.5 kg Ge</td>
</tr>
<tr>
<td>SuCDMS (SNOlab)</td>
<td>27 kg Ge</td>
</tr>
<tr>
<td>SuCDMS (DUSEL)</td>
<td>1140 kg Ge</td>
</tr>
<tr>
<td>EDELWEISS-2</td>
<td>9 kg Ge</td>
</tr>
<tr>
<td>XENON100</td>
<td>170 kg Xe</td>
</tr>
<tr>
<td>XENON1T</td>
<td>1000 kg Xe</td>
</tr>
<tr>
<td>LUX</td>
<td>350 kg Xe</td>
</tr>
<tr>
<td>ZEPLIN III</td>
<td>8 kg Xe</td>
</tr>
<tr>
<td>ZEPLIN IV</td>
<td>1000 kg Xe</td>
</tr>
</tbody>
</table>

Sensitivity curves taken from: http://dmtools.berkeley.edu/limitplots/
Direct Detection: HEP Theory Style

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Sensitivity curves taken from: [http://dmtools.berkeley.edu/limitplots/](http://dmtools.berkeley.edu/limitplots/)
Three Problems with Previous Figure

(1) Direct detection experiments measure recoil rates, not cross-sections

- Calculation of integrated event rate depends on experimental configuration

\[ R = \int_{E_{\text{min}}}^{E_{\text{max}}} \frac{dR}{dE} dE \]

\[ R_1 \text{ (Xenon)} : 5 \text{ keV} \leq E_{\text{recoil}} \leq 25 \text{ keV} \]

\[ R_2 \text{ (Germanium)} : 10 \text{ keV} \leq E_{\text{recoil}} \leq 100 \text{ keV} \]

- The quantity \( \sigma^{\text{SI}}_{\chi i} \) is inferred from an assumed \( \rho_\chi \)

- After rescaling by \( r_\chi = \text{Min}(1, \Omega_\chi h^2/0.025) \) none of these models would have produced more than one or two events in current experiments
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(2) Where do sensitivity curves come from?

- Are \( N \) events enough for a discovery? Depends on backgrounds!

- Primary backgrounds: nuclear recoils/ionization charge induced by radioactive decays/cosmic rays

- Expectation:

  \[ \sim \mathcal{O}(1) \text{ events/experiment/year in germanium;} \]
  \[ \sim \mathcal{O}(10) \text{ events/experiment/year in liquid xenon} \]
Three Problems with Previous Figure

(3) Where are the error bars?

- We have already touched on two sources of error:
  - Experimental error/background – assumed small or even negligible
  - Model-dependence (halo dependence) – potentially large
- But also theoretical errors associated with calculation of cross-section

\[
\sigma_{\chi^i}^{\text{SI}} = \frac{\mu_{i\chi}^2}{\pi} \left| Z G_s^p + (A - Z) G_s^n \right|^2
\]

\[
G_s^N = \sum_{q=u,d,s,c,b,t} \langle N|\bar{q}q|N \rangle \times (\text{SUSY parameters})
\]

- Large uncertainties in these matrix elements

\[
\langle N|\bar{q}q|N \rangle = f_q^N \frac{m_N}{m_q}; \quad f_q^N = f_q^N (\Sigma_{\pi N}, \sigma_0, \frac{m_u}{m_d}, \frac{m_s}{m_d})
\]

\[
\Sigma_{\pi N} = 64 \pm 8 \text{ MeV} \quad \Rightarrow \quad \delta f_q^N \approx 20\% - 30\%
\]

\[
\sigma_0 = 36 \pm 7 \text{ MeV} \quad \Rightarrow \quad \delta \sigma_{\chi^i}^{\text{SI}} \text{ theor} \lesssim 50\%
\]

- This error source dwarfs all others!

Ellis, Olive & Savage, PRD 77 (2008) 065026
1. Counts $N_A$ and $N_B$ ($N_i = \text{rate}_i \times \text{exposure}$) must both exceed $N_{\text{min}}$ events.

2. The two quantities $N_A$ and $N_B$ must differ by at least $5\sigma^{AB}$.

3. For the moment assume statistical errors only: $\sigma^{AB} = \sqrt{N_A + N_B}$.
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**Graph Description**

The graph shows the percentage of events distinguished with respect to exposure in kg-years for different isotopes (Xe and Ge). The x-axis represents exposure in kg-years, and the y-axis shows the percentage of distinguished events. Two thresholds are highlighted: $N > 10$ and $N > 100$. The curves indicate the progression of events distinguished over exposure time for each isotope.
Direct Detection Experiments: Near Future

⇒ NOTE: We assume 200 days of data-taking per calendar year using 80% of nominal target mass
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Effect of inflating the experimental error

- Introduce a scaling parameter $f$: $\sigma^{AB} = \sqrt{(1 + f)(N_A + N_B)}$

<table>
<thead>
<tr>
<th></th>
<th>Require 100 Events</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Xenon</td>
<td>Germanium</td>
</tr>
<tr>
<td></td>
<td>$f = 0$</td>
<td>$f = 0.5$</td>
</tr>
<tr>
<td>1 ton-yr</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3\sigma$</td>
<td>79</td>
<td>71</td>
</tr>
<tr>
<td>$5\sigma$</td>
<td>52</td>
<td>43</td>
</tr>
<tr>
<td>5 ton-yr</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3\sigma$</td>
<td>199</td>
<td>182</td>
</tr>
<tr>
<td>$5\sigma$</td>
<td>170</td>
<td>159</td>
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Direct Detection: Less Naive

⇒ Effect of inflating the experimental error

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⇒ Effect of the theoretical error on nuclear matrix elements

• Allow for a theoretical error on the count rate of $X\%$ and $f = 0$

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<tr>
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## Summary of Direct Detection Capability

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<tr>
<td>Direct detection, xenon</td>
<td>48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct detection, germanium</td>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
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- **Conservative**
  - Density rescaled
  - $N \geq 100$ recoil events
  - Error $f = 0.2$
  - 100 kg-years Ge, 1 ton-year Xe

- All cases assume perfect knowledge of nuclear matrix elements!
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- **Moderate**
  - Density rescaled
  - \( N \geq 10 \) recoil events
  - Error \( f = 0 \)
  - 100 kg-years Ge, 1 ton-year Xe

- **Optimistic**
  - Density \textbf{not} rescaled
  - \( N \geq 10 \) recoil events
  - Error \( f = 0 \)
  - 1 ton-year Ge, 5 ton-years Xe

\textbf{All cases assume perfect knowledge of nuclear matrix elements!}
Indirect Detection

![Graph showing photon flux versus photon energy with curves for different neutralino contributions.](image)

![Diagram of a satellite or observatory setup.](image)

![Image of a ground-based observatory.](image)
Indirect Detection

⇒ Indirect detection experiments look for products of LSP annihilation

- Many possible signals: neutrinos, gamma rays, anti-matter
- Gamma rays special: travel directly from source, relatively easy to detect
- Can therefore focus search in direction of expected high density areas – like galactic center

⇒ Halo profiles especially important in this situation

- Annihilation rates scale like the square of the density
- We observe the entire line-of-sight to the galactic center – therefore need to know the halo profile $\rho_{\chi}(r)$
- Many possible profiles suggested in literature; each can be summarized by one parameter $\overline{J}(\Delta \Omega)$

$$\overline{J}(\Delta \Omega) \equiv \frac{1}{\Delta \Omega} \int_{\Delta \Omega} d\Omega' \, J(\psi') ; \quad J(\psi) = \frac{1}{8.5 \text{ kpc}} \int_{\text{l.o.s.}} ds(\psi) \left( \frac{\rho_{\chi}(r)}{0.3 \text{ GeV/cm}^3} \right)^2$$
Indirect Detection Experiments: Experiments

⇒ Two types of signal: continuous spectrum and mono-energetic lines

\[
d\Phi_\gamma / dE_\gamma = 0.94 \times 10^{-13} \sum_i dN_i^{\gamma} / dE_\gamma \left( \frac{\langle \sigma_i v \rangle}{10^{-29} \text{ cm}^3 \text{ s}^{-1}} \right) \left( \frac{100 \text{ GeV}}{m_\chi} \right)^2 J(\Delta \Omega) \Delta \Omega
\]

• Typical sensitivities require \( \Phi_{\text{min}} \sim 10^{-10} \) photons/cm\(^2\)/sec

⇒ We therefore consider the NFW profile model


• Plain vanilla version gives \( J(10^{-5} \text{ sr}) = 1.3 \times 10^4 \)

• With effects of adiabatic compression \( J(10^{-5} \text{ sr}) = 1.0 \times 10^6 \)

⇒ Two classes of experiments: satellite telescopes and earth-based atmospheric Cherenkov Telescopes (ACTs)

<table>
<thead>
<tr>
<th></th>
<th>( E_{\text{min}} )</th>
<th>( E_{\text{max}} )</th>
<th>( \sigma_E / E )</th>
<th>( A_{\text{eff}} )</th>
<th>( \Delta \Omega )</th>
</tr>
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<tbody>
<tr>
<td>GLAST</td>
<td>50 MeV</td>
<td>300 GeV</td>
<td>10%</td>
<td>( 1 \times 10^4 ) cm(^2)</td>
<td>( 1 \times 10^{-5} ) sr</td>
</tr>
<tr>
<td>ACT</td>
<td>100 GeV</td>
<td>10 TeV</td>
<td>15%</td>
<td>( 3 \times 10^8 ) cm(^2)</td>
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</tbody>
</table>

• Continuous spectrum cuts off quickly at \( E_\gamma \approx m_\chi \)

• For our models \( 98 \text{ GeV} \leq m_\chi \leq 557 \text{ GeV} \) with 85% having \( m_\chi \leq 300 \text{ GeV} \)

• Thus we choose to only consider GLAST for the continuous spectrum
Indirect Detection Experiments: Backgrounds

⇒ No way to distinguish photons from WIMP annihilation and those from generic astrophysical sources

\[
\frac{d\Phi^{bkg}_{\gamma}}{dE_{\gamma}} = 9 \times 10^{-11} \times \left( \frac{E_{\gamma}}{1 \text{ GeV}} \right)^{-2.7} \text{ photons/cm}^2/\text{s}/\text{GeV}
\]

⇒ New wrinkle: both ACTs and satellites (EGRET) observe an excess of gamma ray photons from the galactic center!

- EGRET data only covers low energy range...

\[
\frac{d\Phi^{EG}_{\gamma}}{dE_{\gamma}} = 2.2 \times 10^{-7} \times \exp \left( -\frac{E_{\gamma}}{30 \text{ GeV}} \right) \times \left( \frac{E_{\gamma}}{1 \text{ GeV}} \right)^{-2.2} \text{ photons/cm}^2/\text{s}/\text{GeV}
\]

- ACT data covers much higher energy range....

⇒ Our treatment: consider both the “low” background and the EGRET normalized “high” background

- Conservative: EGRET data likely consistent with point sources
- Might be possible to remove with angular information

_Dodelson, Hooper & Serpico_, PRD 77 (2008) 063512_
Nominal Reach – NFW w/o Rescaling

\[ \Phi_\gamma \text{ [photons/cm}^2\text{/sec]} \]

\[ \text{NFW Without Rescaling} \]

\[ m_\chi \text{ [GeV]} \]

High Background
Low Background
Nominal GLAST Reach
Our Requirements

1. Use DarkSUSY to compute \( d\Phi / dE_\gamma \) over range \( 1 \text{ GeV} \leq E_\gamma \leq 200 \text{ GeV} \)

2. Differential rate integrated over six energy bins

\[
\begin{align*}
1 - 10 \text{ GeV} & \quad 60 - 100 \text{ GeV} \\
10 - 30 \text{ GeV} & \quad 100 - 150 \text{ GeV} \\
30 - 60 \text{ GeV} & \quad 150 - 200 \text{ GeV}
\end{align*}
\]

3. Require \( N_\gamma > 100 \) for both models in the model pair, where \( N_\gamma \) is photon count over the full energy range \( 1 \text{ GeV} \leq E_\gamma \leq 200 \text{ GeV} \)

4. Require excess over background in multiple, adjacent energy bins:
\[
N_i > 2 \sqrt{N_i^{\text{bkg}}} \quad \text{for three of the six energy bins}
\]

5. To be distinguishable, we also require
\[
|N_i^A - N_i^B| > 5 \sqrt{N_i^A + N_i^B + 2N_i^{\text{bkg}}} \]
holds for at least three adjacent bins, simultaneously.
Separating Models at GLAST

NFW + Adiabatic Compression

No rescaling

With rescaling

Solid = low background
Dashed = high background

% Distinguished

Exposure [m²-years]

Number Distinguished

0 1 2 3 4 5
Halo Profile Uncertainties

⇒ “Benchmark” halo profiles cover a wide range in $\bar{J}(10^{-5} \text{ sr})$

<table>
<thead>
<tr>
<th>Model</th>
<th>$r_0$ (kpc)</th>
<th>$a$ (kpc)</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\bar{J}(10^{-5} \text{ sr})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFW</td>
<td>8.0</td>
<td>20.0</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>$1.2644 \times 10^4$</td>
</tr>
<tr>
<td>NFW + AC</td>
<td>8.0</td>
<td>20.0</td>
<td>0.8</td>
<td>2.7</td>
<td>1.45</td>
<td>$1.0237 \times 10^6$</td>
</tr>
<tr>
<td>Moore + AC</td>
<td>8.0</td>
<td>28.0</td>
<td>0.8</td>
<td>2.7</td>
<td>1.65</td>
<td>$3.0896 \times 10^8$</td>
</tr>
</tbody>
</table>

• If truly discrete choices, LHC + GLAST/ACT will clearly choose one
• 252 of 276 pairs gave predictions for $\Phi^\text{int}_\gamma$ within factor of 10 of each other
⇒ But presumably also small perturbations near these benchmarks
• Let $(\delta \bar{J})_\text{theor} = \epsilon \times \bar{J}(\Delta \Omega)$ and compute $\delta \Phi^\text{int}_\gamma$ as a function of $\epsilon$
• Assume NFW + adiabatic, no halo rescaling, 3 m$^2$-years of exposure
• How many can be separated at the 3$\sigma$ level now?

<table>
<thead>
<tr>
<th>Background:</th>
<th>low</th>
<th>high</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon = 0$</td>
<td>160</td>
<td>145</td>
</tr>
<tr>
<td>3 m$^2$ yr</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\epsilon = 0.05$</td>
<td>102</td>
<td>22</td>
</tr>
</tbody>
</table>

• Zero pairs separated at $\epsilon = 0.26$ (high bkgrnd) or $\epsilon = 0.32$ (low bkgrnd)
Monochromatic Signals

- Monochromatic gamma ray signals a “smoking gun” for dark matter
  - Loop-induced diagrams provide annihilation into $\gamma\gamma$ and $\gamma Z$ final states
  - Monoenergetic signals with $E_{\gamma\gamma} = m_\chi$ and $E_{\gamma Z} = m_\chi - M_Z^2/4m_\chi$
  - Energy range makes this signal relevant for ACTs, not GLAST
  - Easy to pick out over background, but branching fractions reduce rate by factors of $10^3 - 10^4$
  - This signal only visible for NFW + AC case or NFW without rescaling

- Backgrounds
  - To be conservative we rescale the background rate to the higher value as seem by HESS and other experiments
    \[
    \frac{d\Phi^{ACT}}{dE_\gamma} = 1.0 \times 10^{-8} \times \left( \frac{E_\gamma}{1 \text{ GeV}} \right)^{-2.25} \text{ photons/cm}^2/\text{s}/\text{GeV}
    \]
    - Integrating background flux over signal width of $\pm \sigma_E^{ACT} = 15\%$
      gives $\mathcal{O}(1)$ event/m$^2$-year at $E_{\gamma\gamma} = 200$ GeV
Monochromatic Signals: Our Analysis

- Requiring $N_\gamma > 5 \sqrt{N_{\text{bkgrnd}}}$ not restrictive, but energy resolution the major limiting factor
- For the vast majority of the models cannot resolve $\gamma\gamma$ and $\gamma Z$ lines with energy resolution of 15%
- Consider $\overline{E} = (E_{\gamma\gamma} + E_{\gamma Z})/2$ and demand at least 10 photons within $\pm \sigma_{\overline{E}}$ for discovery
- Require the two values $\overline{E}^A$ and $\overline{E}^B$ be separated by $n \sigma_{\overline{E}}$, where $\sigma_{\overline{E}} = 0.15 \overline{E}$
• Requiring $N_\gamma > 5\sqrt{N_{\text{bkgrnd}}}$ not restrictive, but energy resolution the major limiting factor

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• Require the two values $\bar{E}^A$ and $\bar{E}^B$ be separated by $n \sigma_{\bar{E}}$, where $\sigma_{\bar{E}} = 0.15 \bar{E}$

⇒ Because often $m^A_\chi \simeq m^B_\chi$ difficult to produce a separation

<table>
<thead>
<tr>
<th>ACT Exposure</th>
<th>Rescaled 1σ</th>
<th>Rescaled 2σ</th>
<th>Rescaled 3σ</th>
<th>Not Rescaled 1σ</th>
<th>Not Rescaled 2σ</th>
<th>Not Rescaled 3σ</th>
</tr>
</thead>
</table>
Summary of Indirect Detection Capability

<table>
<thead>
<tr>
<th></th>
<th>Conservative</th>
<th>Moderate</th>
<th>Optimistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gamma rays, continuum</td>
<td>56</td>
<td>115</td>
<td>158</td>
</tr>
<tr>
<td>Gamma rays, monochromatic</td>
<td>23</td>
<td>34</td>
<td>36</td>
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⇒ All estimates assume NFW profile model with adiabatic compression and imagine 5 m²-years of exposure for GLAST towards the galactic center

**Conservative** Density is rescaled and “high” background rate used; 1000 m²-years for ACT experiment

**Moderate** Density is rescaled and “low” background rate used; 2500 m²-years for ACT experiment

**Optimistic** Density is *not* rescaled and “low” background rate used; 10000 m²-years for ACT experiment
# Summary of Distinguishability

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<td>Direct detection, xenon</td>
<td>48</td>
<td>112</td>
<td>224</td>
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<td>Direct detection, germanium</td>
<td>4</td>
<td>14</td>
<td>147</td>
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<tr>
<td><strong>All Pairs, All Signals</strong></td>
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<td>186</td>
<td>245</td>
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- Total number of degenerate pairs = 276
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- Total number of degenerate pairs = 276
- Total number of “physical” pairs = 77
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<td>62</td>
<td>81</td>
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</tbody>
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- Total number of degenerate pairs = 276
- Total number of “physical” pairs = 77
- Pairs inaccessible at the ILC = 103

⇒ And all before the ILC is even a year old!
Conclusions

• Degenerate points in SUSY parameter space are a real possibility in a post-LHC world

• Dark matter observations truly are “orthogonal” to collider observations

• Current and near-future experiments will be remarkably sensitive to dark matter signals

• If theoretical errors were negligible a large fraction of degenerate models could be resolved well before the ILC ever turns on

• To realize this potential theoretical inputs will need to be known to at least the 5-10% level