ResBos
(Resummation for Bosons)
for Higgs Physics

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• What’s it for?
• Where is it?
• For Higgs physics
• Limitations
ResBos

Initial state QCD soft gluon resummation and
Final state QED corrections

In collaboration with

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What’s it for? An Example

- Transverse momentum of Drell-Yan $V H$ including QCD Resummations.

- Kinematics of Leptons from the decays ($W^\pm$ included)

- $V H$ including QCD Resummations.
$P \bar{P} \rightarrow W^+ X$

$\sqrt{S} = 1.8$ TeV

Cross Section

Matching

Perturbative

non-perturbative

$\frac{d\sigma}{dq_T}$ [pb/GeV]

$q_T$ [GeV]

$\sim 90\%$
Fixed order pQCD prediction

\[ \sigma = \frac{1}{2S} \int \frac{d\xi_A}{\xi_A} \frac{d\xi_B}{\xi_B} f_{i/A}(\xi_A, \mu) f_{i/B}(\xi_B, \mu) \cdot d\hat{\sigma} \]

\[ d\hat{\sigma} = \left[ M \right]^2 (2\pi)^4 \delta^{(4)}(q - k - l) \frac{d^3q}{(2\pi)^3 2q_0} \]

\[ \frac{d\sigma}{dq_T^2 dy dQ^2} = \frac{1}{S} \int \frac{d\xi_A}{\xi_A} \frac{d\xi_B}{\xi_B} f_{i/A}(\xi_A, \mu) f_{i/B}(\xi_B, \mu) \]

\[ \cdot \left( \frac{\pi^2}{Q^2} \right) \left[ M \right]^2 \cdot \delta \left( 1 - \frac{x_A}{\xi_A} \right) \cdot \delta \left( 1 - \frac{x_B}{\xi_B} \right) \]

\[ \cdot \delta \left( q_T^2 \right) \cdot \delta \left( Q^2 - M_W^2 \right) \]

\[ Q \equiv \sqrt{Q^2} = \sqrt{q^2}, \mu = Q = M_W, x_A = \frac{Q}{\sqrt{S}} e^y, x_B = \frac{Q}{\sqrt{S}} e^{-y} \]
\[ \frac{d\sigma}{dq_T^2dydQ^2} = \int \frac{d\xi_A}{(\xi_A S + U - Q^2)} \left( \frac{dsd\hat{\sigma}}{d\hat{t}} \right) \cdot f_{j/A}(\xi_A, \mu) \cdot \delta(Q^2 - M_W^2) \]

\[ f_{j/B}(\xi_B, \mu) = \left( \frac{-Q^2 - \xi_A(T - Q^2)}{\xi_A S + U - Q^2} \right) \cdot \delta(Q^2 - M_W^2) \]

\[ f_{j/A}(\xi_A, \mu) = \left( \frac{-Q^2 - \xi_B(U - Q^2)}{\xi_B S + T - Q^2} \right) \cdot \delta(Q^2 - M_W^2) \]

\[ T = Q^2 - \sqrt{q_T^2 + Q^2} \sqrt{S} \ e^{-y} \]

\[ U = Q^2 - \sqrt{q_T^2 + Q^2} \sqrt{S} \ e^{y} \]

\[ \hat{s} = \xi_A \xi_B S \]

\[ \hat{t} = \xi_A (T - Q^2) + Q^2 \]

\[ M = \frac{1}{16\pi^2} |M|^2 \]

(For simplicity, only consider qq→Wg)
• Virtual Corrections

• Real emission contributions
Perturbative Part:

- Higher order in $\alpha_s^{(n)}$
  - Less sensitive to Factorization Scale $\mu$

- High $q_T$ and smaller $y$ (i.e. more central)
  - PDF (parton distribution function) better known

- With larger Luminosity
  - Test QCD in one large scale problem (i.e. $q_T \sim Q$)

- Up to now, most of the Data used in Testing QCD were
  One large scale observables, e.g., Jet-$P_T$.

- Observables involving Multiple Scales, e.g., $q_T$ of W-Boson with mass $M_W$, can only be accurately described in QCD after including effects of Resummation.
Shortcoming of fixed order calculation

- Cannot describe data with small $q_T$ of W-boson.
- Cannot precisely determine $m_W$ at hadron colliders without knowing the transverse momentum of W-boson. Most events fall in the small $q_T$ region.

(at NLO)

Transverse momentum

$\delta \left( q_T^2 \right)$

$\frac{1}{q_T^2} \ln \left( \frac{Q^2}{q_T^2} \right), \frac{1}{q_T^2}$
QCD Resummation is needed

\[ \frac{d\sigma}{dQ_T} \quad (pb/GeV) \]

- Dashed: CSS (1,1,1)
- Solid: CSS (2,2,1)
- Perturbative
- Dotted: Pert (\(\alpha_s\))
- Dot-dashed: Pert (\(\alpha_s^2\))

Resummation
**ResBos** is also needed for Rapidity distributions

[Graphs showing corrected asymmetry and charge asymmetry with different curves labeled NNLL/NLO, NLO, LO. The black curve is from the ResBos calculation.]
What’s QCD Resummation?

• Perturbative expansion

\[
\frac{d\hat{\sigma}}{dq_T^2} \sim \alpha_s \{ 1 + \alpha_s + \alpha_s^2 + \cdots \}
\]

• The singular pieces, as \(\frac{1}{q_T^2}\) (1 or log’s)

\[
\frac{d\hat{\sigma}}{dq_T^2} \sim \frac{1}{q_T^2} \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} \alpha_s^{(n)} \ln^{(m)} \left( \frac{Q^2}{q_T^2} \right) \\
\sim \frac{1}{q_T^2} \left\{ \alpha_s \left( L + 1 \right) \\
+ \alpha_s^2 \left( L^3 + L^2 + L + 1 \right) \\
+ \alpha_s^3 \left( L^5 + L^4 + L^3 + L^2 + L + 1 \right) \\
+ \cdots \right\}
\]

Resummation is to reorganize the results in terms of the large Log’s.
Resummed results:

\[ \frac{d\sigma}{dq_T^2} \sim \frac{1}{q_T^2} \{ \left[\alpha_s(L+1) + \alpha_s^2(L^3 + L^3) + \alpha_s^3(L^5 + L^4) \right] + \cdots \} \]

\[ + \left[ + \alpha_s^2(L+1) + \alpha_s^3(L^3 + L^2) \right] + \cdots \]

\[ + \left[ + \alpha_s^3(L+1) \right] + \cdots \]

Determined by \( A^{(1)} \) and \( B^{(1)} \)

Determined by \( A^{(2)} \) and \( B^{(2)} \)

Determined by \( A^{(3)} \) and \( B^{(3)} \)

QCD Resummation

In the formalism by Collins-Soper-Sterman, in addition to these perturbative results, the effects from physics beyond the leading twist is also implemented as [non-perturbative functions].
CSS Resummation Formalism

\[ \frac{d\sigma}{dq^2_T dy dQ^2} = \frac{\pi}{S} \sigma_0 \delta (Q^2 - M_W^2). \]

\[ \left\{ \frac{1}{(2\pi)^2} \int d^2 b \ e^{i q_T \cdot b} \tilde{W} (b, Q, x_A, x_B) \right\} \text{[Non-perturbative functions]} \]

\[ + Y(q_T, y, Q) \text{[Non-perturbative functions]} \]

\[ \tilde{W} = e^{-S(b)} \cdot C \otimes f(x_A) \cdot C \otimes f(x_B) \]

\[ \sum_{j} \int_{x_A}^{1} \frac{d\xi_A}{\xi_A} \frac{C_q\left(\frac{x_A}{\xi_A}, b, \mu\right)}{\xi_A} f_{j_A}(\xi_A, \mu) \]

\[ \sum_{k} \int_{x_B}^{1} \frac{d\xi_B}{\xi_B} \frac{C_qk\left(\frac{x_B}{\xi_B}, b, \mu\right)}{\xi_B} f_{k_B}(\xi_B, \mu) \]

Sudakov form factor \[ S(b) = \int_{\xi_A}^{1} \frac{d\xi_A}{\xi_A} \ln\left(\frac{Q^2}{\mu^2}\right) A(\mu) + B(\mu) \]

[Non-perturbative functions] are functions of \((b, Q, x_A, x_B)\) which include QCD effects beyond Leading Twist.
• Example: for $W^\pm$

$$\sigma_0 = \left( \frac{4\pi^2 \alpha}{3} \sum_{jj'} Q^{(w)}_{jj'} \right), \quad Q^{(w)}_{jj'} = \frac{1}{4\sin^2 \theta_w} (kM)^2_{jj'}$$

The couplings of gauge bosons to fermions are expressed in the way to include the dominant electroweak radiative corrections. The propagators of gauge bosons also contain energy-dependent width, as done in LEP precision data analysis.

$$A \equiv \sum_{n=1}^{\infty} \left( \frac{\alpha_S}{\pi} \right)^n \cdot A^{(n)}, \quad B \equiv \sum_{n=1}^{\infty} \left( \frac{\alpha_S}{\pi} \right)^n \cdot B^{(n)}$$

Note:

$$C \equiv \sum_{n=0}^{\infty} \left( \frac{\alpha_S}{\pi} \right)^n \cdot C^{(n)}$$
As $q_T \to 0$

\[
\frac{d\sigma}{dq_T^2 dy dQ^2} \bigg|_{q_T \to 0} = \left(\frac{\pi}{s} \sigma_0\right) \cdot \delta(Q^2 - M_W^2) \cdot \left(\frac{1}{2\pi q_T^2}\right) \left(\frac{\alpha_s(Q)}{\pi}\right)
\]

\[
\cdot \left\{ f_{q/A}(x_A, Q) \left[ P_{\bar{q} \to q} \otimes f_{\bar{q}} \right]_{x_B, Q} \right. \\
+ \left[ P_{q \to q} \otimes f_q \right]_{x_A, Q} \cdot f_{\bar{q}/B}(x_B, Q) \\
+ f_{q/A}(x_A, Q) f_{\bar{q}/B}(x_B, Q) \cdot \left[ 2 \left(\frac{4}{3}\right) \ln \left(\frac{Q^2}{q_T^2}\right) + 2(-2) \right] \left\} \right.
\]

Exponentiate

To preserve transverse momentum conservation, we have to go to the impact parameter space (b-space) to perform resummation.
Diagrammatically, Resummation is doing

\[ \alpha_s^n \ln^m \left( \frac{Q^2}{q_T^2} \right) \]

Resum large terms

\[ \frac{d\sigma}{dq_T^2 dy} \bigg|_{q_T \to 0} \sim \frac{1}{q_T^2} \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} \alpha_s^n \ln^m \left( \frac{Q^2}{q_T^2} \right) \cdot C_m^n \]

Monte-Carlo programs ISAJET, PYTHIA, HERWIG contain these physics.

(Note: Arbitrary cut-off scale in these programs to affect the amount of Backward radiation, i.e. Initial state radiation.)
Kinematics of the radiated gluon, controlled by Sudakov form factor with some arbitrary cut-off. (In contrast to perform integration in impact parameter space, i.e., $b$ space.)

The shape of $q_T(w)$ is generated. But, the integrated rate remains the same as at Born level (finite virtual correction is not included).

* Recently, there are efforts to include part of higher order effect in the event generator.
Note that the integrated rate is the same as the Born level rate \( \alpha_s^{(0)} \) even though the \( q_T \) – distribution is different (i.e., not \( \delta(q_T^2) \) any more).

\[
\sigma = \int dq_T^2 \frac{d\sigma}{dq_T^2} \sim \int d^2 b \int e^{i\vec{q}_T \cdot \vec{b}} d^2 q_T \sigma_0 e^{-S(b)}
\]

\[
= \int d^2 b \delta^2(b) \sigma_0 \cdot e^{-S(b)}
\]

\[
= \sigma_0
\]

For C-Function = \( \delta \left( 1 - \frac{x_A}{\xi_B} \right) \)
To recover the “K-factor” in the NLO total rate

To include the C-Functions

\[
\frac{d\sigma}{dQ^2 dy} \sim \left( \frac{d\sigma}{dq_T dy dQ^2} \right)^2 + \text{Finite} + \text{Singular}
\]

The area under the \( q_T \) – curve will reproduce the total rate at the order \( \alpha_s^{(1)} \) if \( Y \) term is calculated to \( \alpha_s^{(1)} \) as well.
Include NNLO in high $q_T$ region

- To improve prediction in high $q_T$ region
- To speed up the calculation, it is implemented through K-factor table which is a function of $(Q, q_T, y)$ of the boson, not just a constant value.

ResBos predicts both rate and shape of distributions.
[non-perturbative function] is a function of \((b,Q,x_A,x_B)\), implemented to include effects beyond Leading Twist.

Until we know how to calculate QCD non-perturbatively, (Lattice Gauge Theory?), these functions can only be parameterized. However, the same functions should describe Drell-Yan, \(W^\pm, Z^0\) data.

- Test QCD in problems involving multiple scales.
- Measuring these non-perturbative functions may help in understanding the non-perturbative part of QCD.

[non-perturbative functions], dependent of \(Q, b, x_A, x_B\), is necessary to describe \(q_T\) – distribution of Drell-Yan, \(W^\pm, Z^0\) events.

\[
\exp \left[ -g_1 b^2 - g_2 b^2 \ln \left( \frac{Q}{2Q_0} \right) - g_3 b^2 \ln \left( 100x_A x_B \right) \right]
\]

New term with \(x\)-dependence

The coefficients \(g_1, g_2, g_3\) need to be determined by existing data.
Effects of Resummation on $W$ and $Z$ Boson physics

Mass information comes primarily from lepton $p_T$.
- Run 2 goal: calibrate $p_T$ to $\sim 0.01\%$

Additional information from $\nu p_T$ (inferred through measurement of hadronic recoil energy)

Use $Z$ decays to model boson $p_T$ distribution, detector response to hadronic recoil energy.

Combine lepton and neutrino $p_T$ to form transverse mass ($m_T$) for best statistical power.
Where is it?

- **ResBos**: http://hep.pa.msu.edu/resum/
- **Plotter**: http://hep.pa.msu.edu/wwwlegacy

ResBos-A (including final state NLO QED corrections)
http://hep.pa.msu.edu/resum/code/resbosa/
has not been updated.
Why? Because it was not used for Tevatron experiments.

The plan is to include final state QED resummation inside ResBos.
Physical processes included in ResBos

\[ W^\pm, \gamma, Z, H, \gamma\gamma, ZZ, WW \]

including gauge invariant set amplitude for Drell-Yan pairs

New physics: \( W', Z', H^+, A^0, H^0 \) …
### Physics processes inside ResBos

<table>
<thead>
<tr>
<th>Process</th>
<th>$A^{(i)}$</th>
<th>$B^{(i)}$</th>
<th>$C^{(i)}$</th>
<th>order of Pert. part</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A + B \to W^+ \to l^+ + \nu + X$</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>NNLO</td>
</tr>
<tr>
<td>$A + B \to W^- \to l^- + \bar{\nu} + X$</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>NNLO</td>
</tr>
<tr>
<td>$A + B \to Z^0 \to l^- + l^- + X$</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>NNLO</td>
</tr>
<tr>
<td>$A + B \to Z^0/\gamma^* \to l^+ + l^- + X$</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>NNLO</td>
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<td>$A + B \to \gamma^* \to l^+ + l^- + X$</td>
<td>3</td>
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</tr>
<tr>
<td>$A + B \to gg \to H^0 \to \gamma \gamma + X$</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>NNLO</td>
</tr>
<tr>
<td>$A + B \to gg \to H^0 \to Z^0 Z^0/W^+W^- \to 4l + X$</td>
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<td>1</td>
<td>NNLO</td>
</tr>
<tr>
<td>$A + B \to W^{++} \to W^+ + H^0 + X$</td>
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<td>1</td>
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<td>$A + B \to W^{--} \to W^- + H^0 + X$</td>
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<td>1</td>
<td>NLO</td>
</tr>
<tr>
<td>$A + B \to W^+W^- + X$ (upcoming)</td>
<td>3</td>
<td>2</td>
<td>1</td>
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#### New Physics (upcoming)

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<tr>
<td>$A + B \to Z' \to l^- + l^+ + X$</td>
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<td>1</td>
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<tr>
<td>$A + B \to bb \to A^0/H^0 + X$ (THDM)</td>
<td>3</td>
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ResBos for Higgs Physics

Quark initiated processes:

- Rate and shape:
  - at the same order of accuracy as Drell-Yan processes

Gluon initiated processes:

- Rate and shape:
  - at the same order of accuracy as Drell-Yan processes
  - consistent with NNLO QCD rate
  - include exact $\alpha_s^{(2)}$ contribution in high $p_T$
Gluon initiated processes for Higgs production in ResBos

New Physics (MSSM, THDM)
Shape changes from $\alpha_s^{(2)}$ and variation of scales

arXiv:0909.2305
Predict different shape
ResBos vs PYTHIA vs NLO

Consistent treatment of initial state parton mass with CTEQ6.6 PDFs, in GM scheme.

(see Sally Dawson’s talk)
Di-Photon Productions

### Theoretical predictions

**PYTHIA**
- \( qq \rightarrow \gamma \gamma \) and \( gg \rightarrow \gamma \gamma \) matrix elements.
- All-orders resummation to LL accuracy via parton shower.
- No fragmentation contributions included.

- Fixed-order NLO calculation (except for \( gg \rightarrow \gamma \gamma \), which is at LO)
- No resummation:
  - usually avoid divergence by requiring asymmetric \( p_{T\gamma 1} - p_{T\gamma 2} > 0 \).
- Single-photon fragmentation (to NLO) included.

**RESBOS** *PRD 76, 013009 (2007)*
- All-orders resummation (to NNLL accuracy) matched to NLO.
- Single-photon fragmentation included via parameterization that approximates rate predicted by NLO fragmentation functions.
Compare to CDF Run-2 di-Photon data

The cut $P_T < M$ is to suppress fragmentation contribution

(Data – theory)/theory vs. the diphoton transverse momentum for Higgs – like kinematics
Compare to CDF Run-2 di-Photon data

The cut $P_T < M$ is to suppress fragmentation contribution

(Data – theory)/theory vs. the diphoton azimuthal distance for Higgs – like kinematics
Large theoretical uncertainty in fragmentation contribution

\[ p\bar{p} \to \gamma X, \text{CDF Run-2, } 207 \text{ pb}^{-1} \]

- \( \Delta R_{\text{cone}} = 0.4, \Delta R_{\gamma} > 0.3 \)
- Resummed
- DIPHOX, \( E_T^{\gamma} = 1 \text{ GeV} \)
- DIPHOX, \( E_T^{\gamma} = 4 \text{ GeV}, \mu_F = Q/2 \)

\[ p\bar{p} \to \gamma X, \text{CDF Run-2, } 207 \text{ pb}^{-1} \]

- \( \Delta R_{\text{cone}} = 0.4, \Delta R_{\gamma} > 0.3 \)
- RESBOS, \( E_T^{\gamma} = 1 \text{ GeV} \)
- \( \mu_F = Q \) (lower), \( Q/2 \) (upper)
- DIPHOX, \( E_T^{\gamma} = 1 \text{ GeV} \)
- DIPHOX, \( E_T^{\gamma} = 4 \text{ GeV}, \mu_F = Q/2 \)

- Theoretical uncertainties are large in the region \( Q \lesssim 25 \text{ GeV}, Q_T \gtrsim 27 \text{ GeV}, \Delta \varphi < \pi/2 \), not relevant for the LHC Higgs searches; uncertainties are suppressed by a \( Q_T \leq Q \) cut
Limitations of ResBos

- Any perturbative calculation is performed with some approximation, hence, with limitation.
- To make the best use of a theory calculation, we need to know what it is good for and what the limitations are.

It does not give any information about the hadronic activities of the event.

It could be used to reweight the distributions generated by (PYTHIA) event generator, by comparing the boson (and its decay products) distributions to ResBos predictions.

This has been done for W-mass analysis by CDF and D0.)
Potential of ResBos yet to be explored

- E.g., in the measurement of forward-backward asymmetry in Drell-Yan pairs.

ResBos can be used for Matrix Element Method by including resummed $k_T$-dependent parton distribution functions together with higher order matrix element contributions.

For example: The coefficients in front of the complete set of angular functions are given by ResBos

$$\mathcal{L}_0 = 1 + \cos^2 \theta, \quad A_0 = \frac{1}{2} (1 - 3 \cos^2 \theta), \quad A_1 = \sin 2\theta \cos \phi, \quad A_2 = \frac{1}{2} \sin^2 \theta \cos 2\phi,$$

$$A_3 = 2 \cos \theta, \quad A_4 = \sin \theta \cos \phi.$$
Conclusion

• ResBos is a useful tool for studying electroweak gauge bosons and Higgs bosons at the Tevatron and the LHC.

• It includes not only QCD resummation for low $q_T$ region but also higher order effect in high $q_T$ region, with spin correlations included via gauge invariant set of matrix elements.

If you use it, I will keep providing the service to our community. Please send the request to me.
Backup Slides
ResBos vs D0 Run-2 $A_{FB}$ data

![Graph showing $A_{FB}$ vs $M_{Z/\gamma^*}$ (GeV)]