Nonlinear Saturation of the MTI and HBI in Dilute Magnetized Plasmas (i.e., clusters!)

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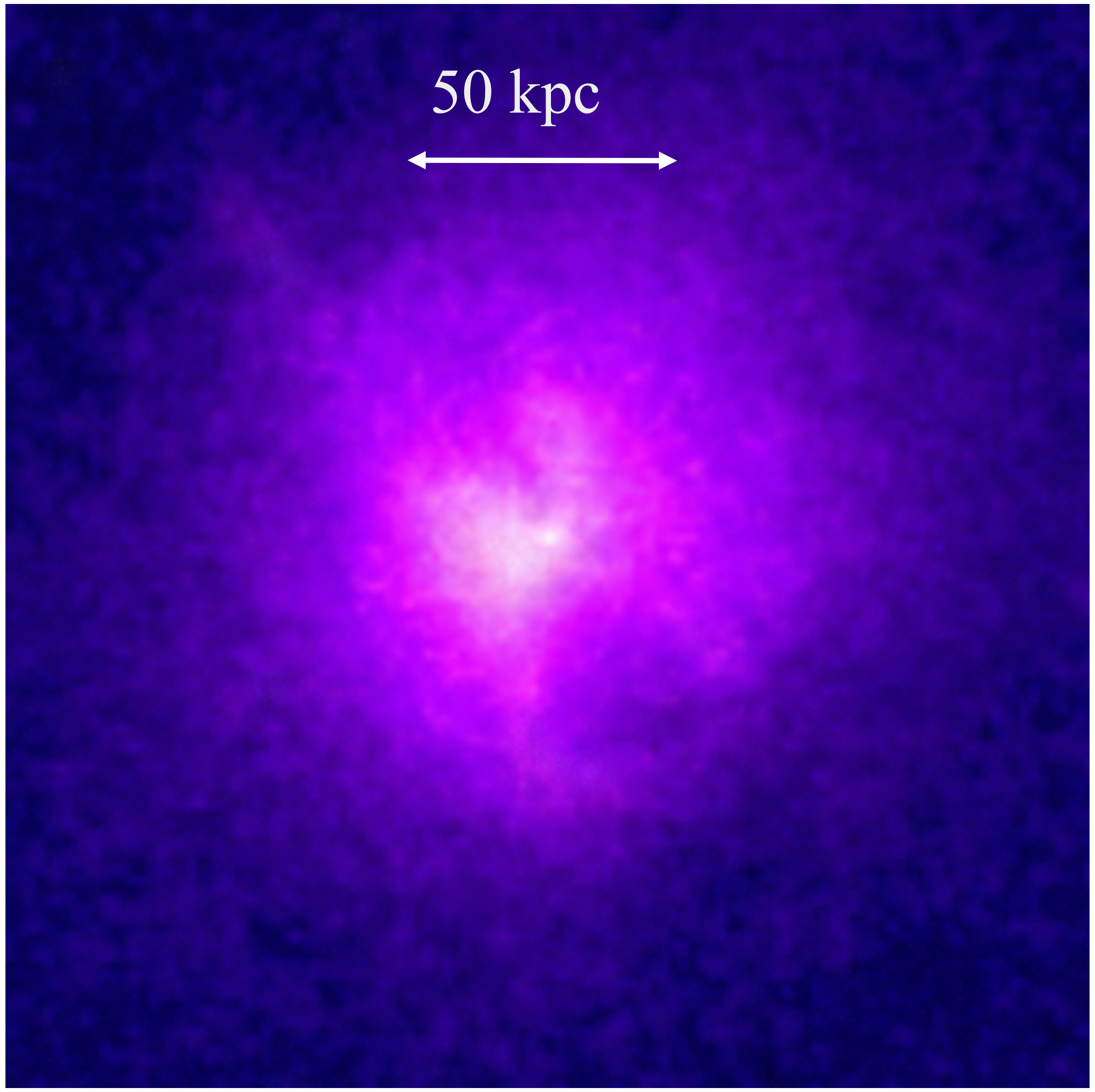
Motivation



Hydra A Cluster (Chandra)

T ~ 3 keV,   n ~ 10^{-2} cm^{-3}

Large electron mean free path:

$$\lambda_e \sim (0.1\,\mathrm{kpc}) \left(\frac{T}{3\,\mathrm{keV}}\right)^2 \left(\frac{n_e}{0.03\,\mathrm{cm}^{-3}}\right)^{-1}$$

$$r_g \sim (10^3\,\mathrm{km}) \left(\frac{T}{3\,\mathrm{keV}}\right)^{1/2} \left(\frac{B}{1\,\mu\mathrm{G}}\right)^{-1}$$

Conduction important on scales:

$$\boldsymbol{L \lesssim 7\,(\lambda_e H)^{1/2}}$$

thermal conduction important & anisotropic

Outline

- Linear Physics of MTI and HBI
- Nonlinear Saturation
- Interaction with External Sources of Turbulence

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- **Linear Physics of MTI and HBI**
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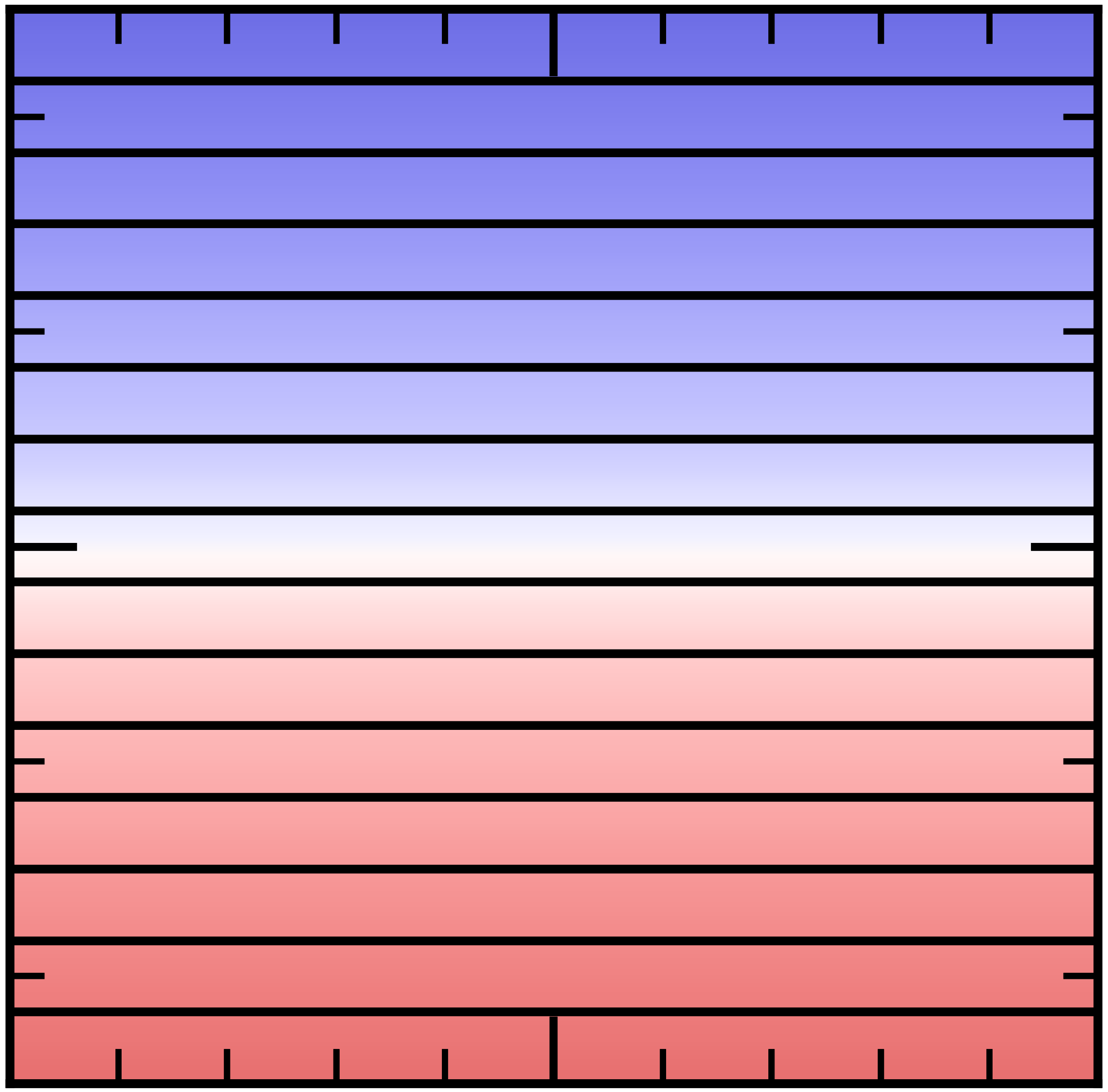
Linear Evolution of the MTI

Temperature (t = 0)



- Would be stable if adiabatic. $\partial s/\partial z > 0$
- Efficient conduction along magnetic field lines ⇒ field lines isothermal.
- New stability criterion:

$$\frac{\partial s}{\partial z} > 0 \rightarrow \frac{\partial T}{\partial z} > 0$$

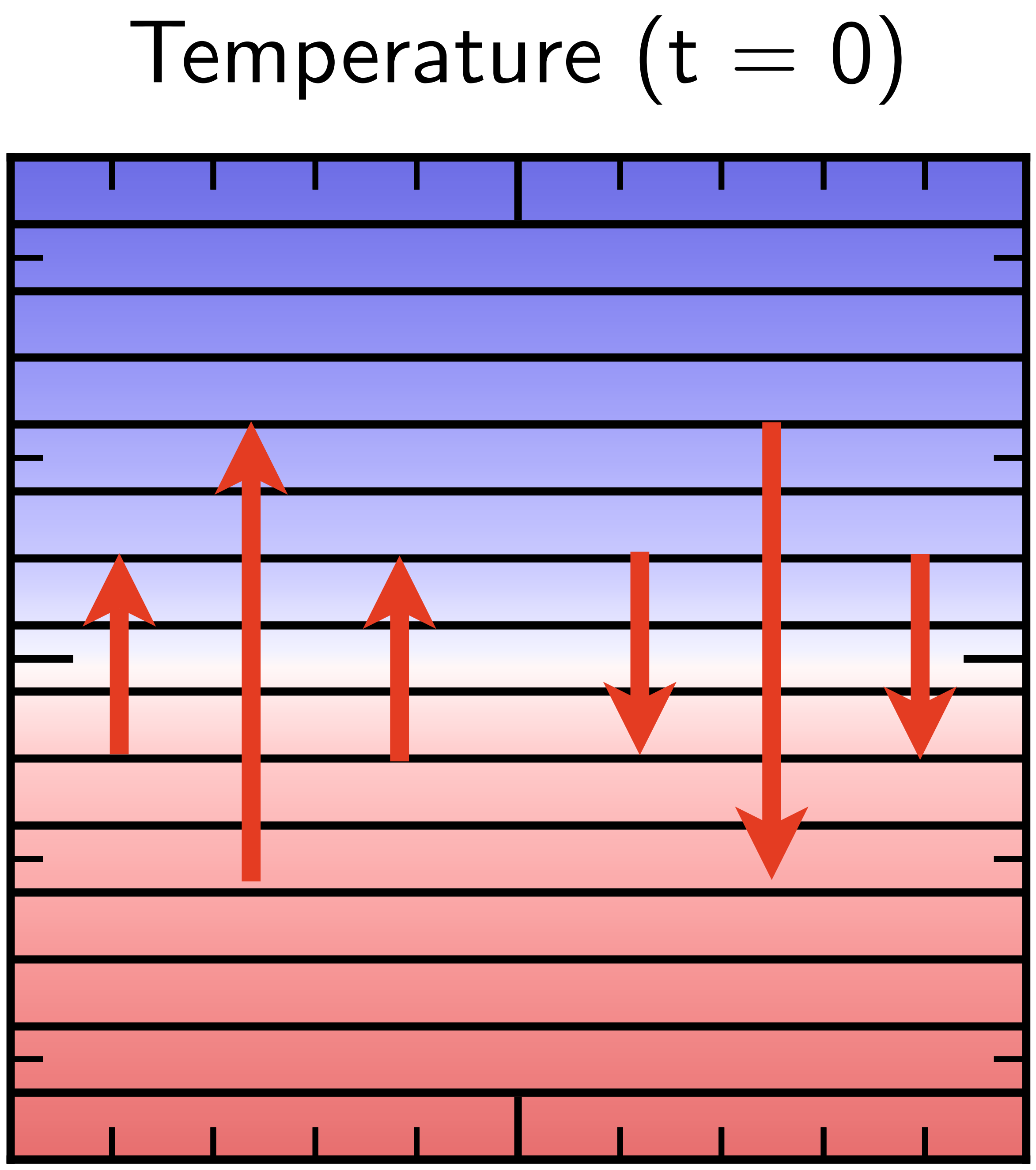
(displacements isothermal rather than adiabatic)

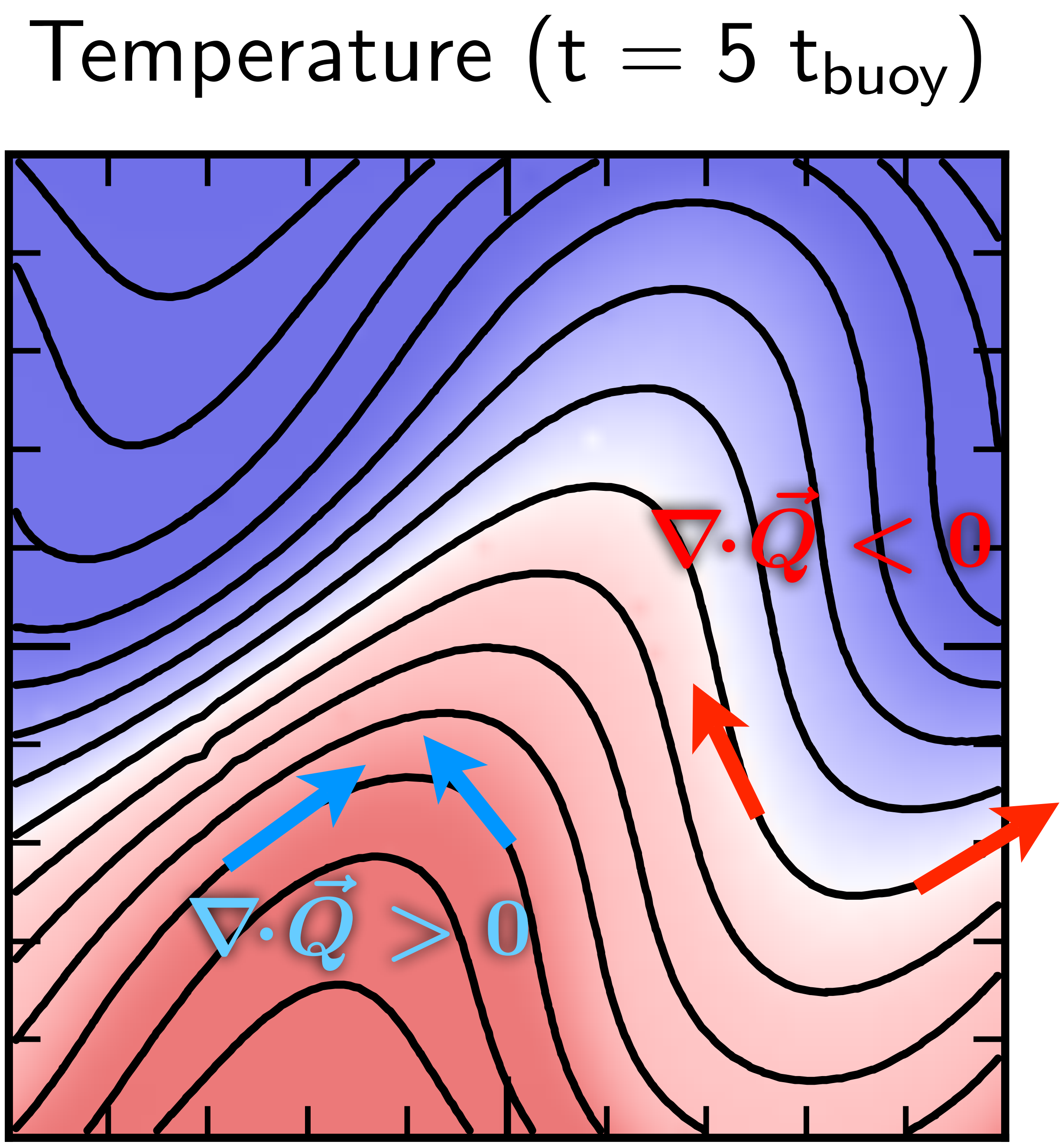
- Timescale for perturbations grow:

$$t_{\rm buoy} = \left(g \frac{\partial \ln T}{\partial z} \right)^{-1/2}$$

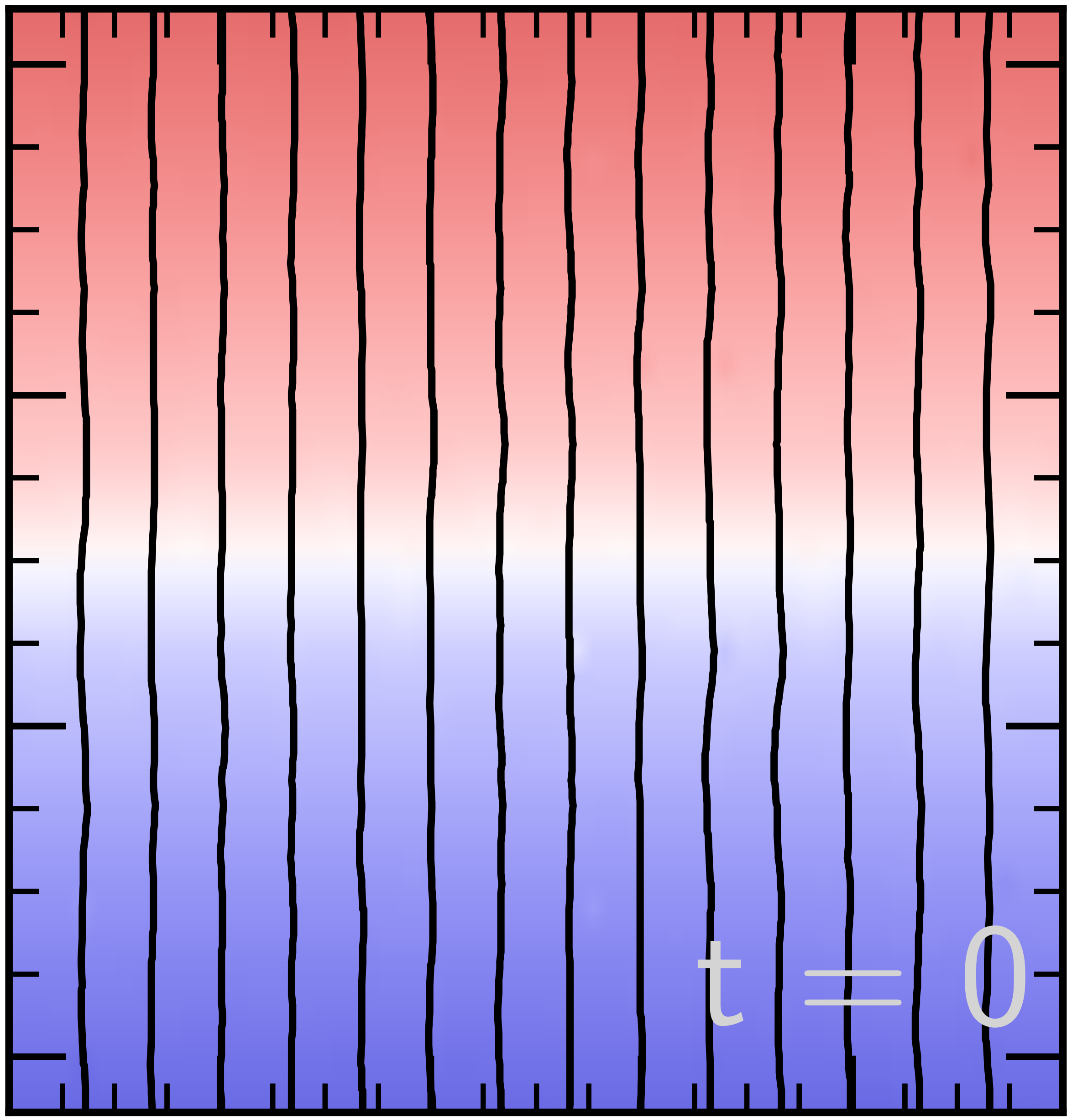
(Balbus, 2000; Parrish & Stone, 2005)

Linear Evolution of the MTI





Linear Evolution of the HBI



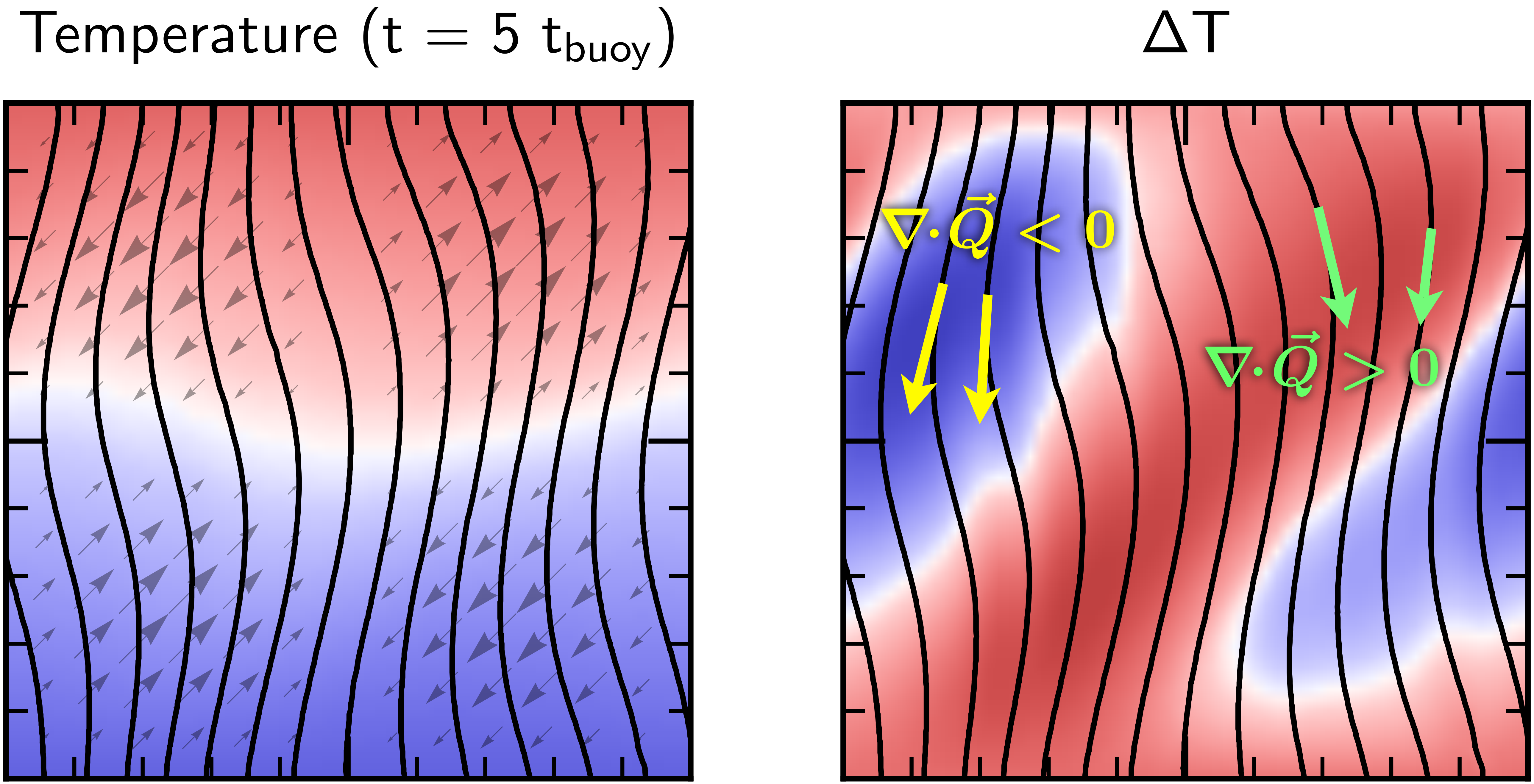
- Would be stable if adiabatic
- Stable according to MTI criterion
- Constant heat flux $\vec{Q}$
  $\Rightarrow \boldsymbol{\nabla} \cdot \vec{Q} = \mathbf{0}$
- Perturbations to $\vec{Q}$ can cause instability
- Timescale for perturbations grow:

$$t_{\rm buoy} = \left(g \frac{\partial \ln T}{\partial z} \right)^{-1/2}$$

(same as MTI)

(Quataert, 2008; Parrish & Quataert, 2008)

Linear Evolution of the HBI



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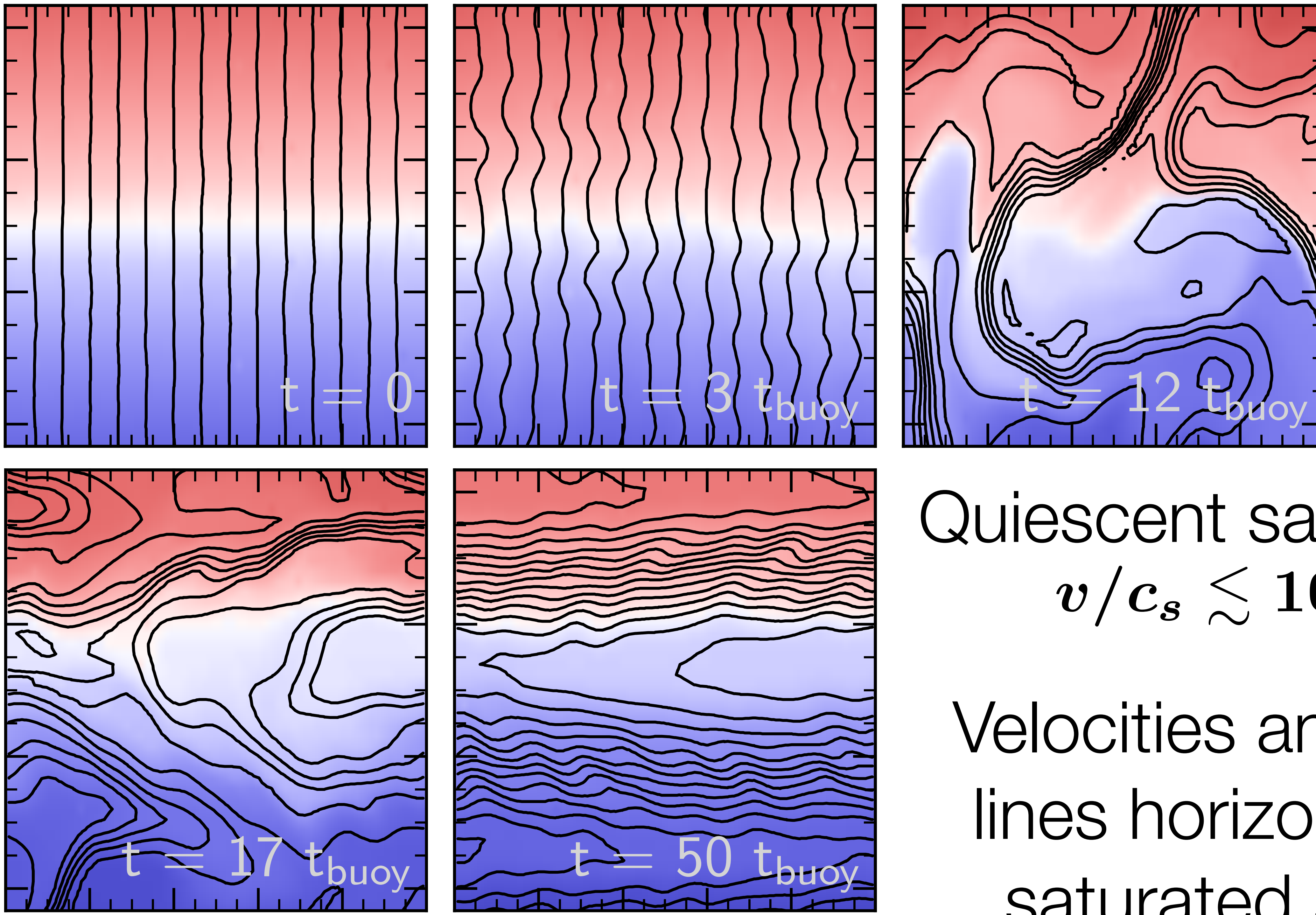
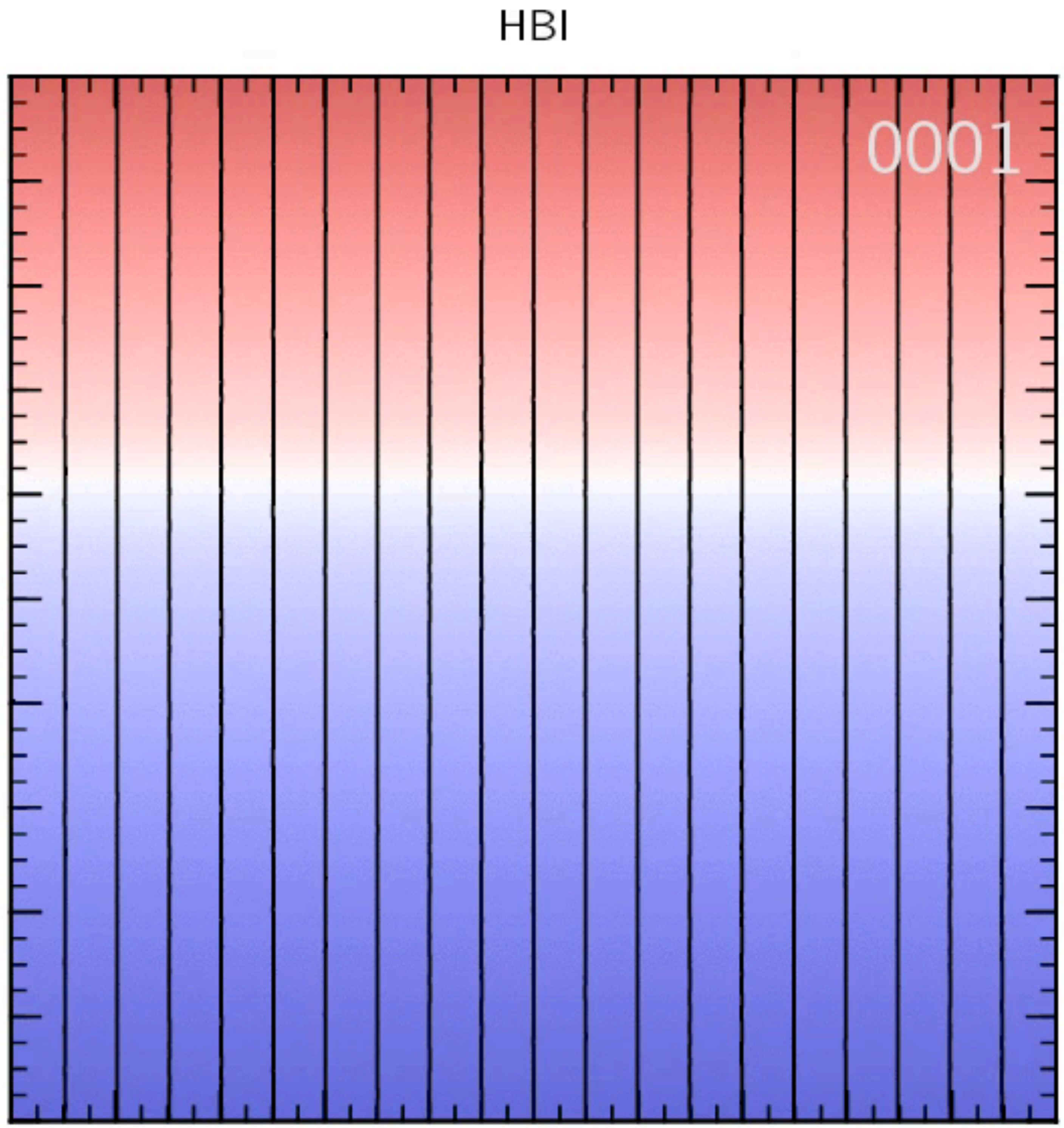
Saturation of the HBI



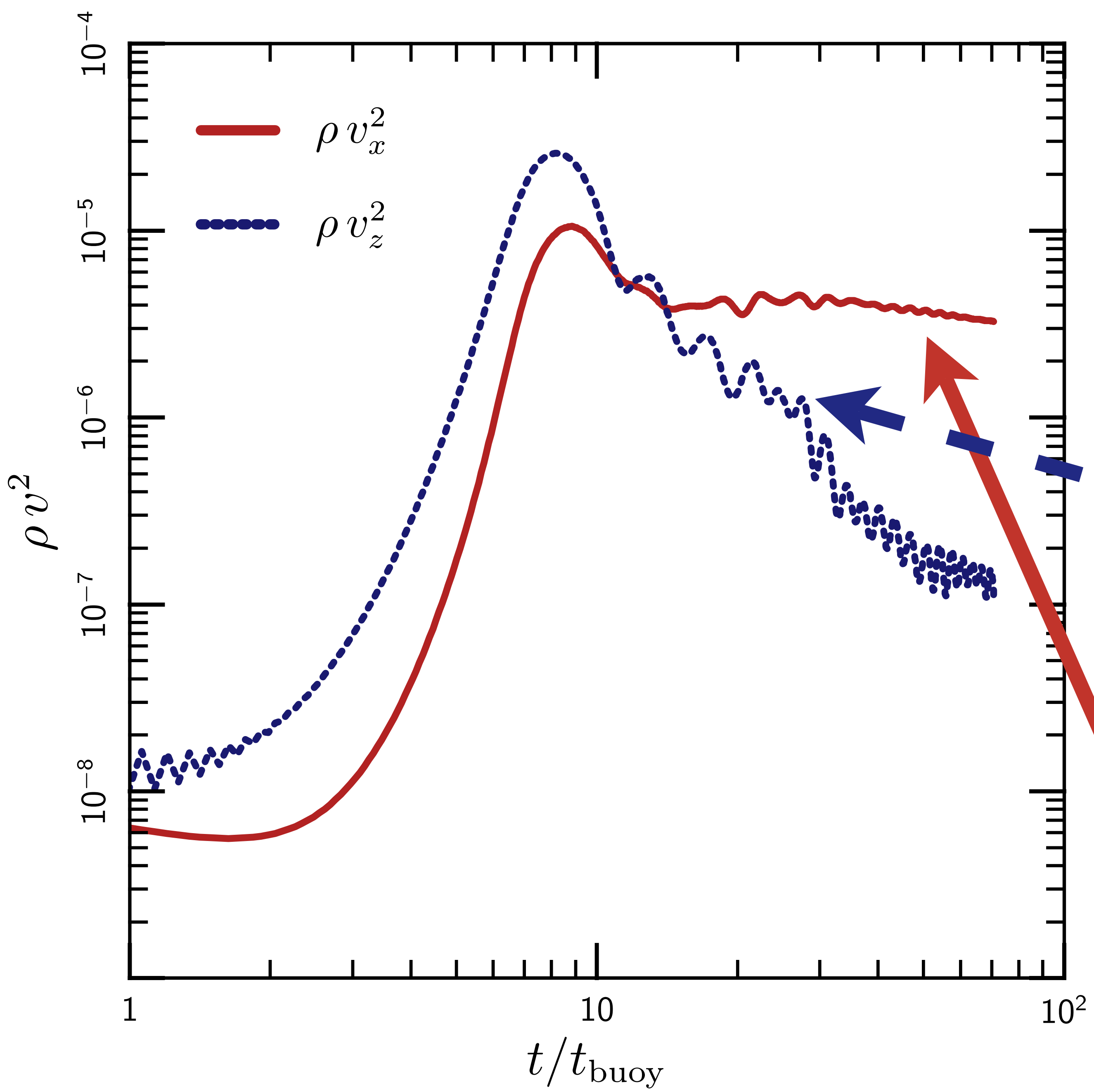
Quiescent saturation
$v/c_s \lesssim 10^{-2}$

Velocities and field lines horizontal in saturated state

Saturation of the HBI



Velocities in the Saturated State of the HBI



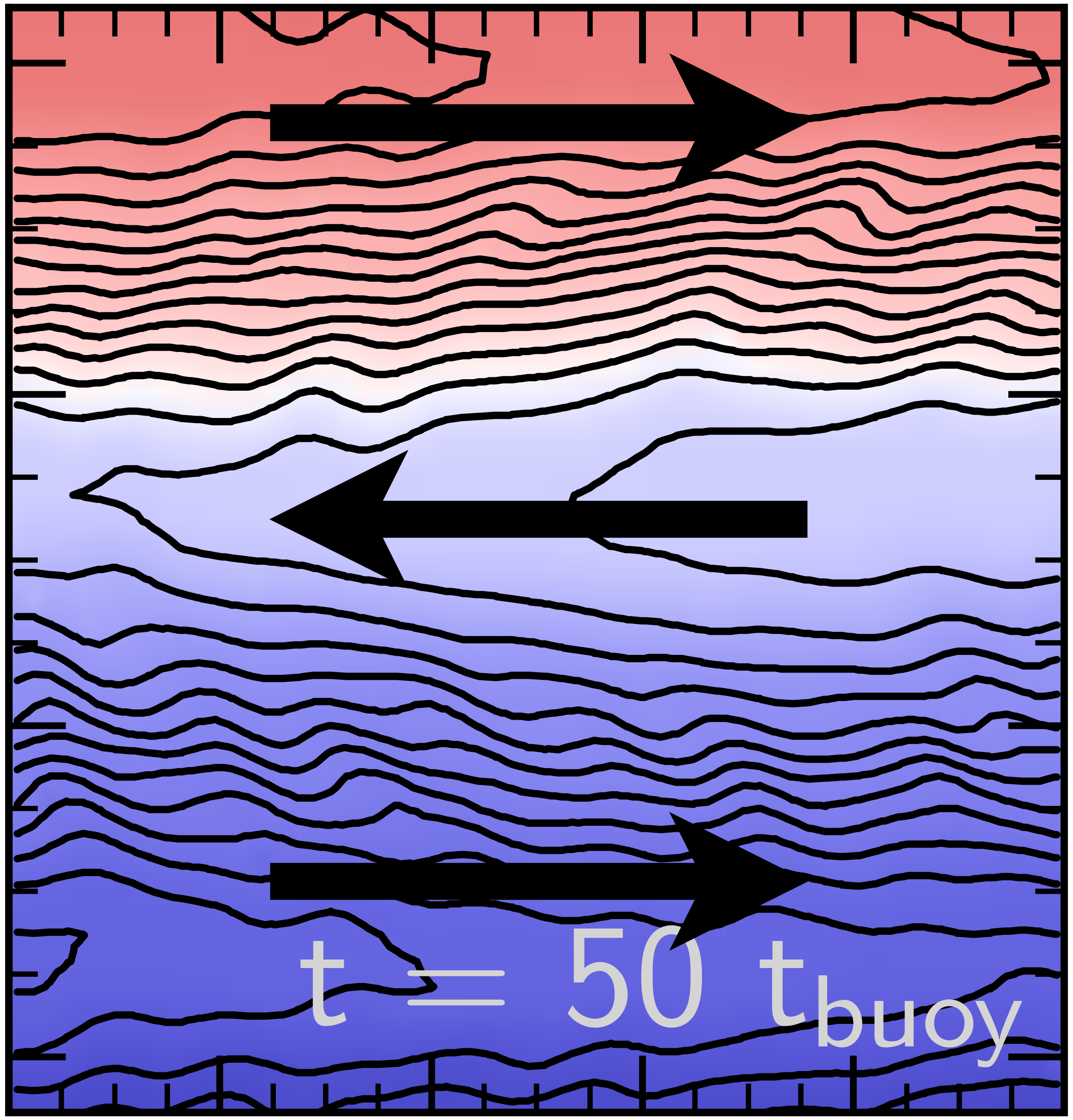
Limit that $b_z \rightarrow 0$:

$$\omega^2 = \omega^2_{\rm buoy}\left(1 - \hat{k}_z^2\right)$$

Vertical displacements oscillate (and decay)

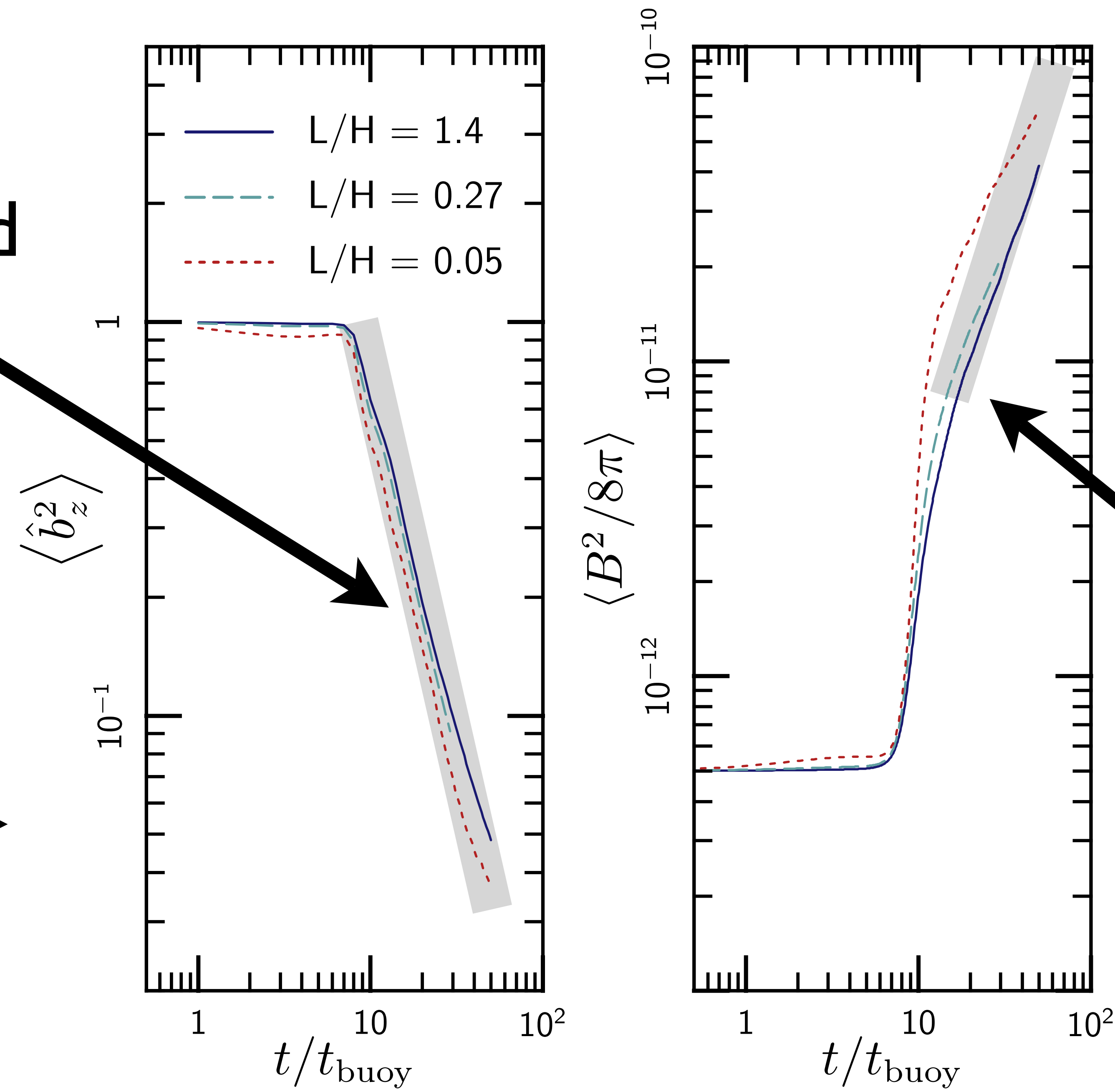
Horizontal displacements propagate

Velocities in the Saturated State of the HBI

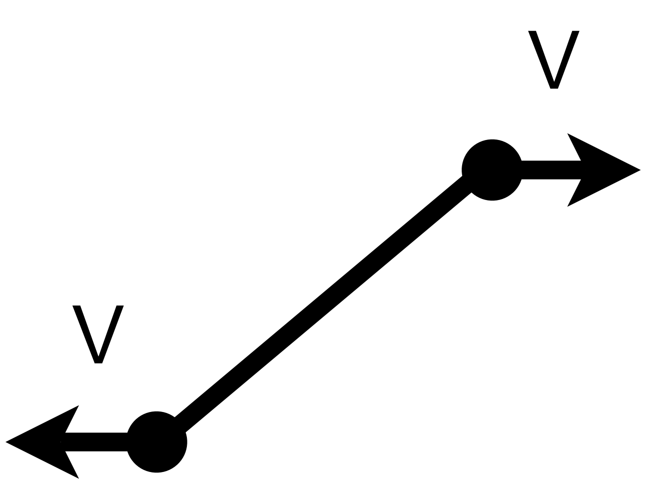


Magnetic Field in the Saturated State of the HBI

Horizontal displacements stretch out field lines ...

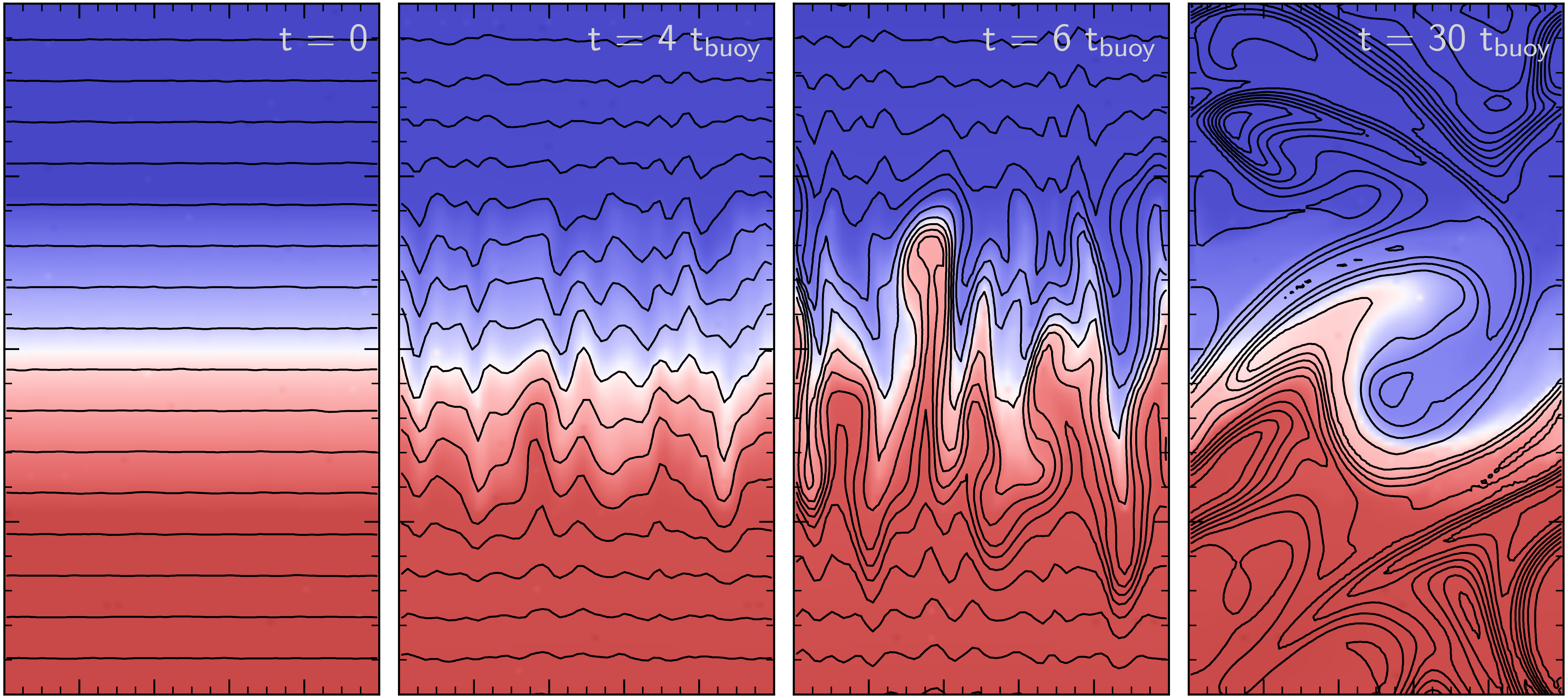


... and amplify them



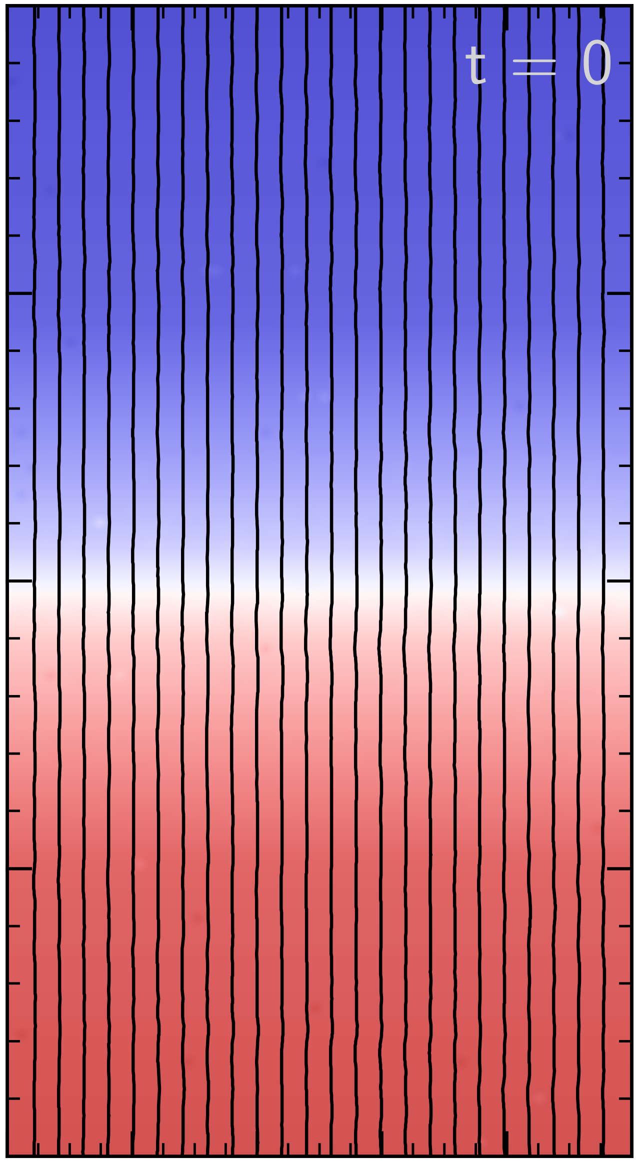
Constant velocity: $\hat{b}_z \propto t^{-1},\ B \propto t$

Nonlinear Evolution of the MTI



Saturation is **not** quiescent.

Equilibrium State of the MTI

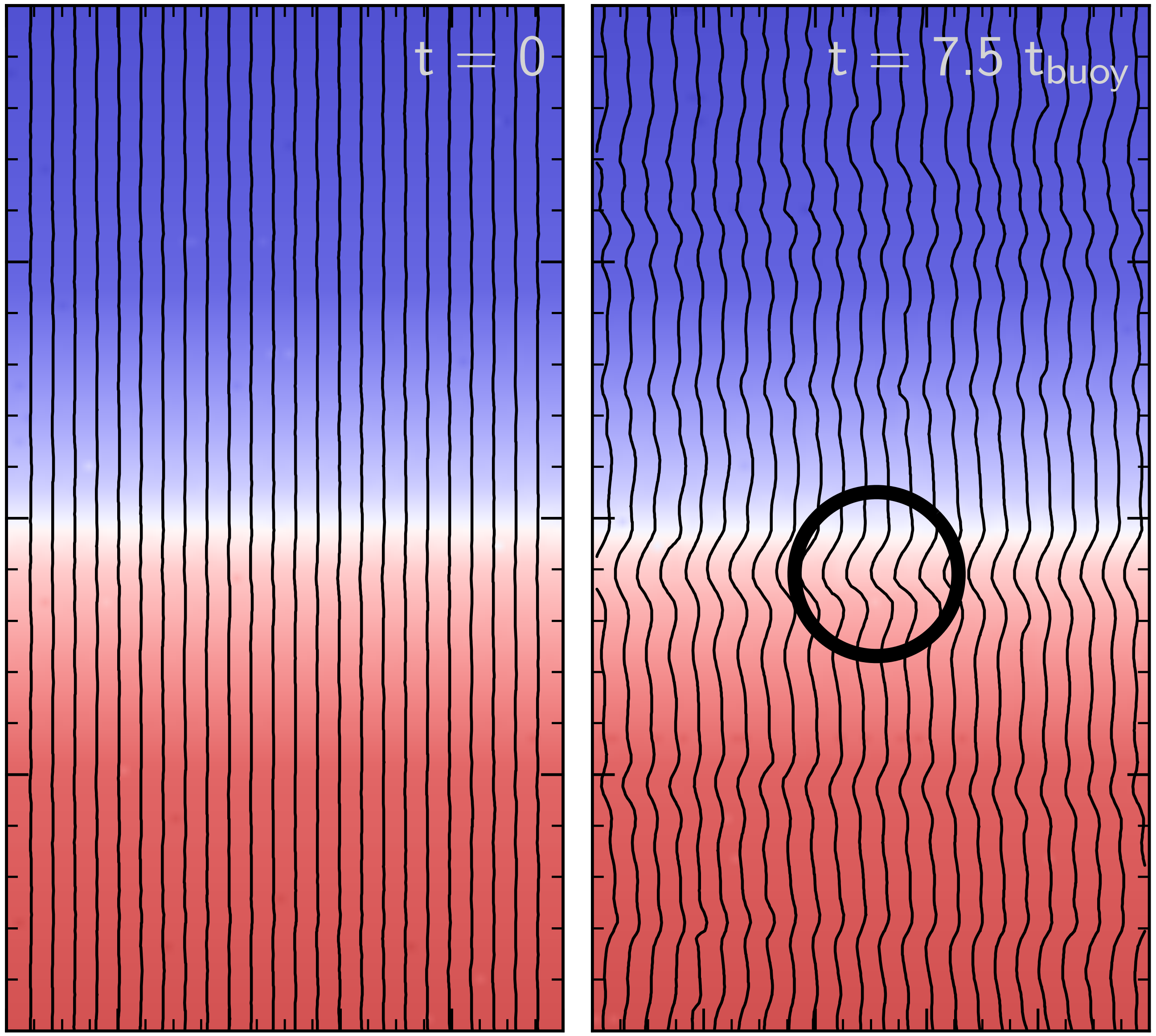


Limit that $b_z \rightarrow 1$:

$$\omega^2 = \omega^2_{\rm buoy}\left(1 - \hat{k}_z^2\right)$$

- Same dispersion relation as the HBI
- Stable stratification ⇒ horizontal displacements unaffected.

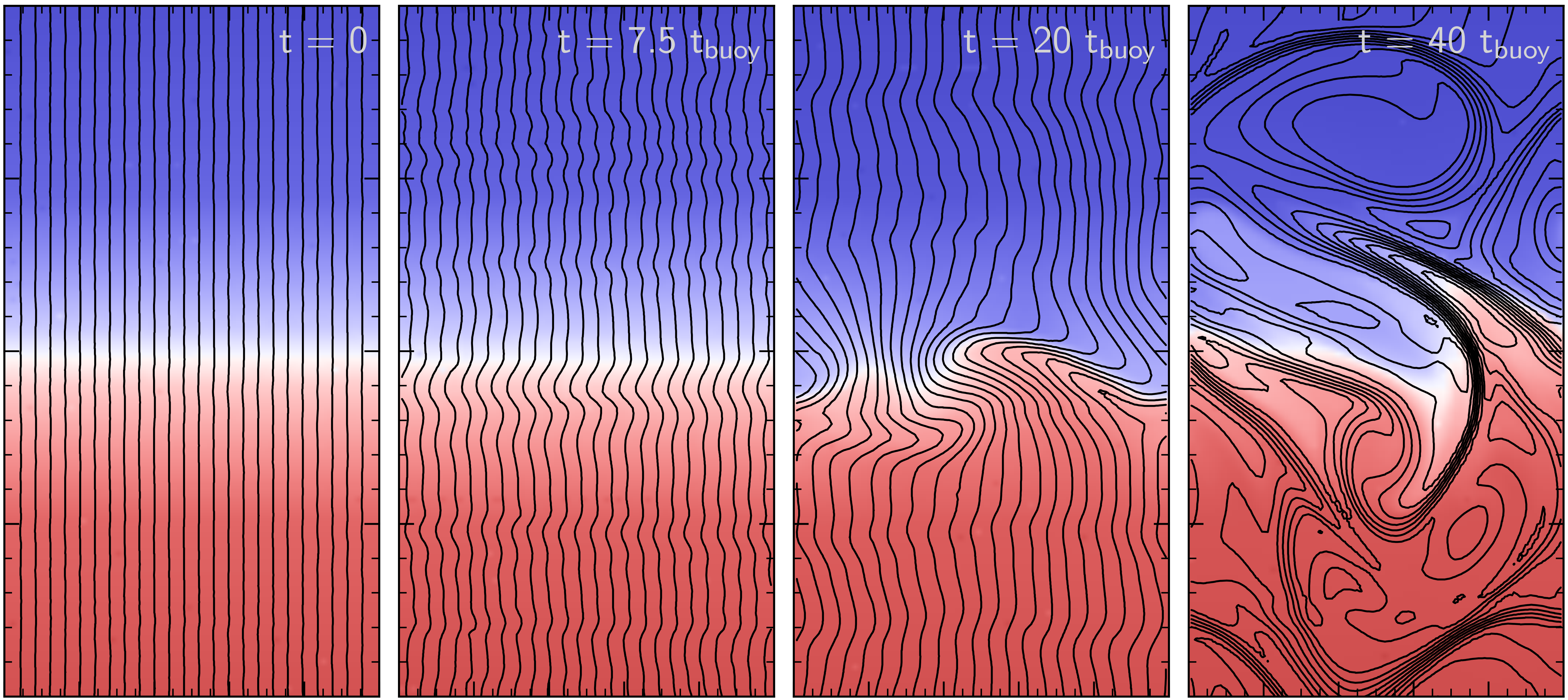
Equilibrium State of the MTI



$$\omega^2 \neq \omega^2_{\mathbf{buoy}} \left(1 - \hat{k}_z^2\right)$$

- No longer in equilibrium state
- Nonlinearly unstable
- Closes dynamo loop

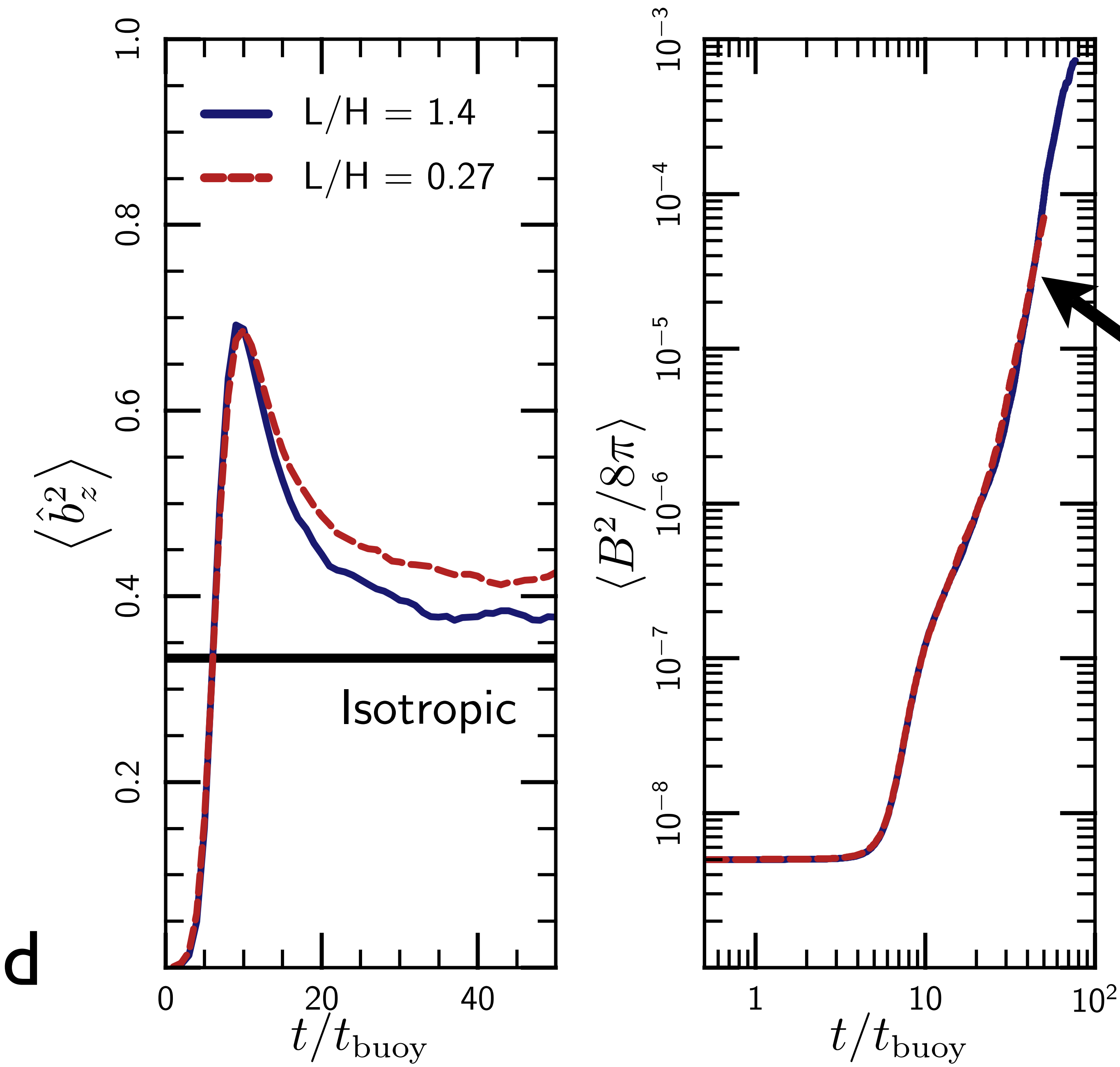
Equilibrium State of the MTI



At late times, can't tell whether or not the plasma was initially stable.

Magnetic Field in the Saturated State of the MTI

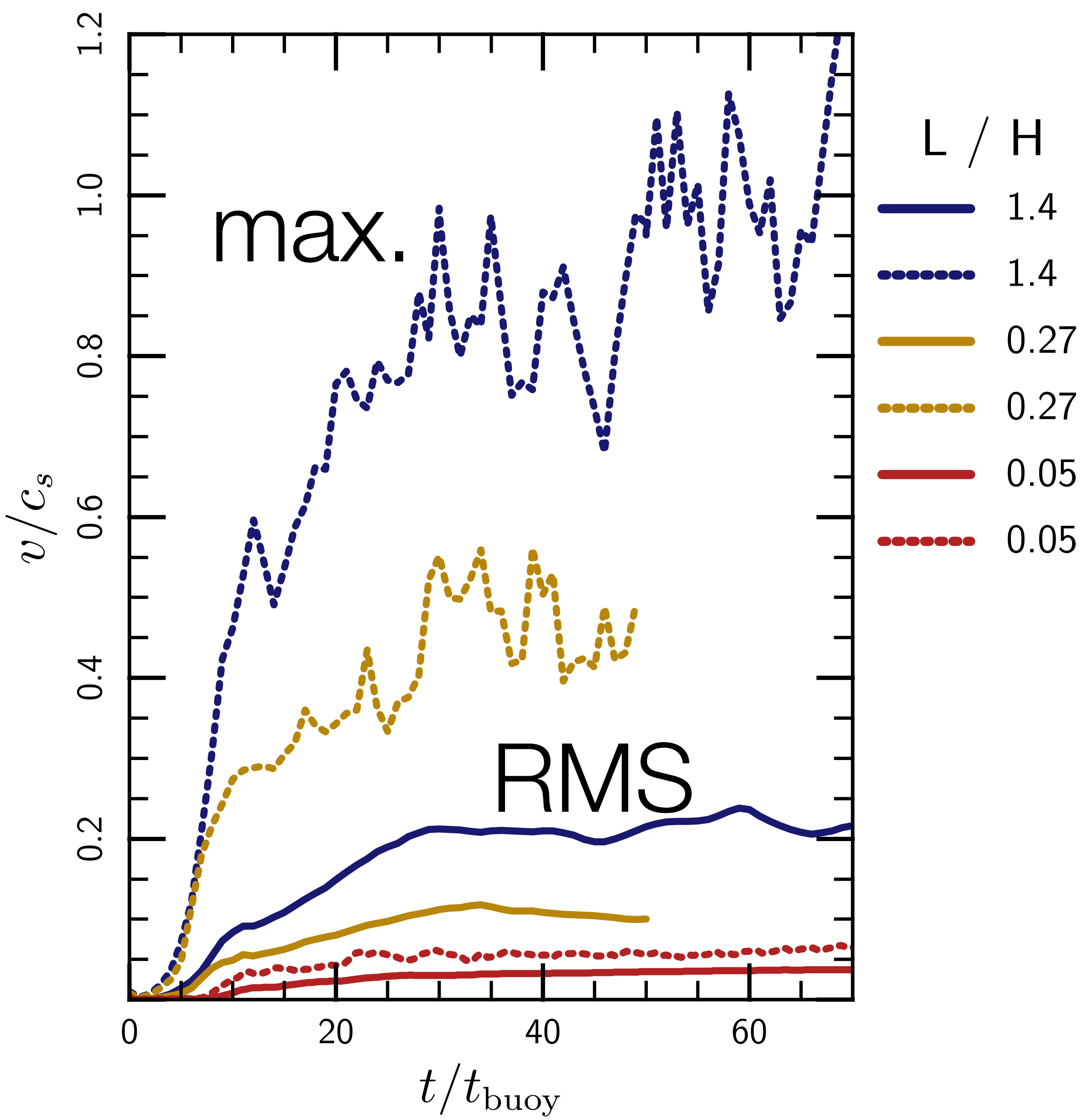
Vertical Field



Horizontal Field

Magnetic field in equipartition with kinetic energy

Turbulence Generated by the MTI



- MTI can drive ~ sonic turbulence
- Answer depends on size of simulation domain; need boxes of order H to get the right answer.
- Strong turbulence + magnetic fields $\Rightarrow$ 10s of % non-thermal pressure support

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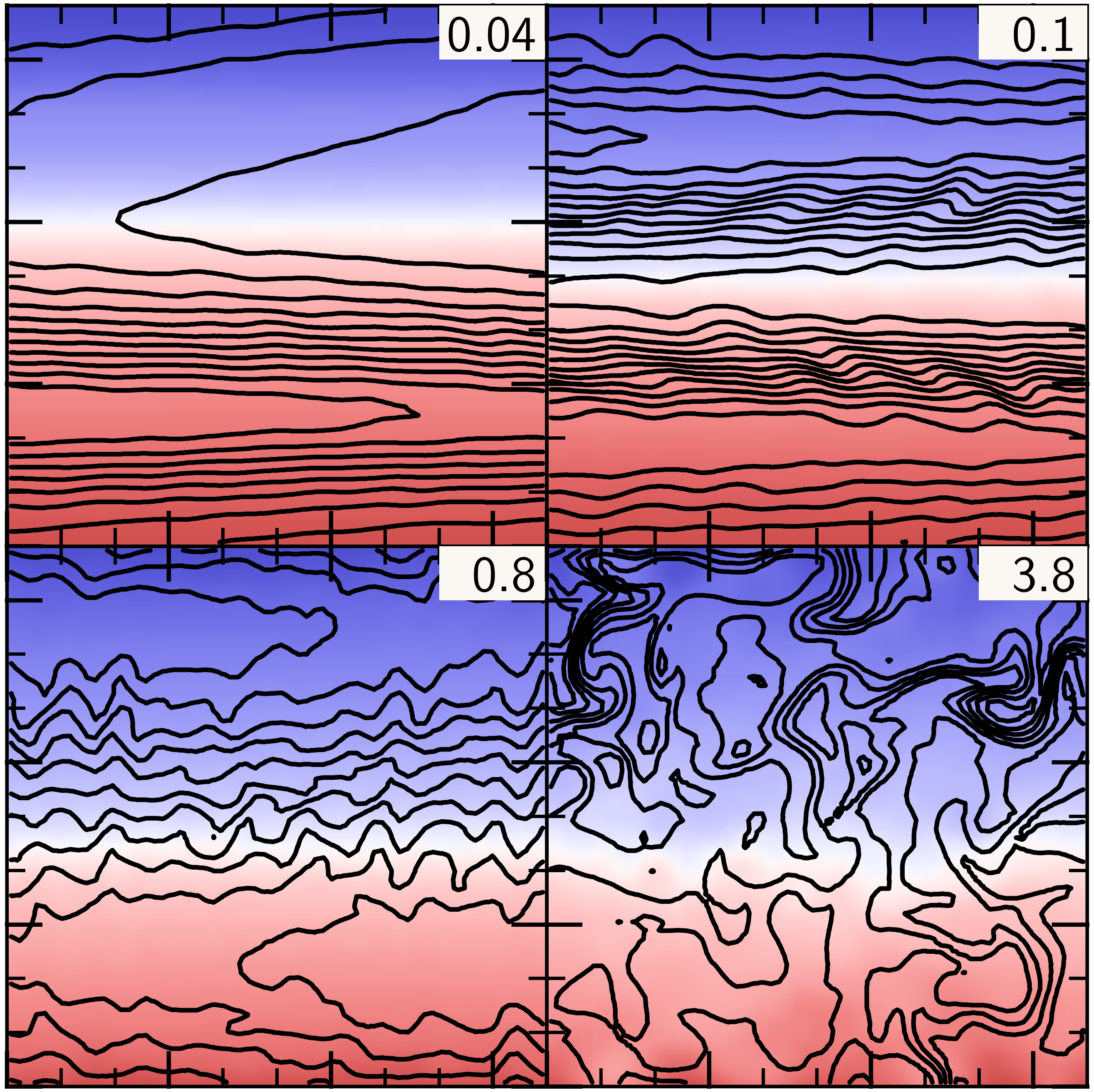
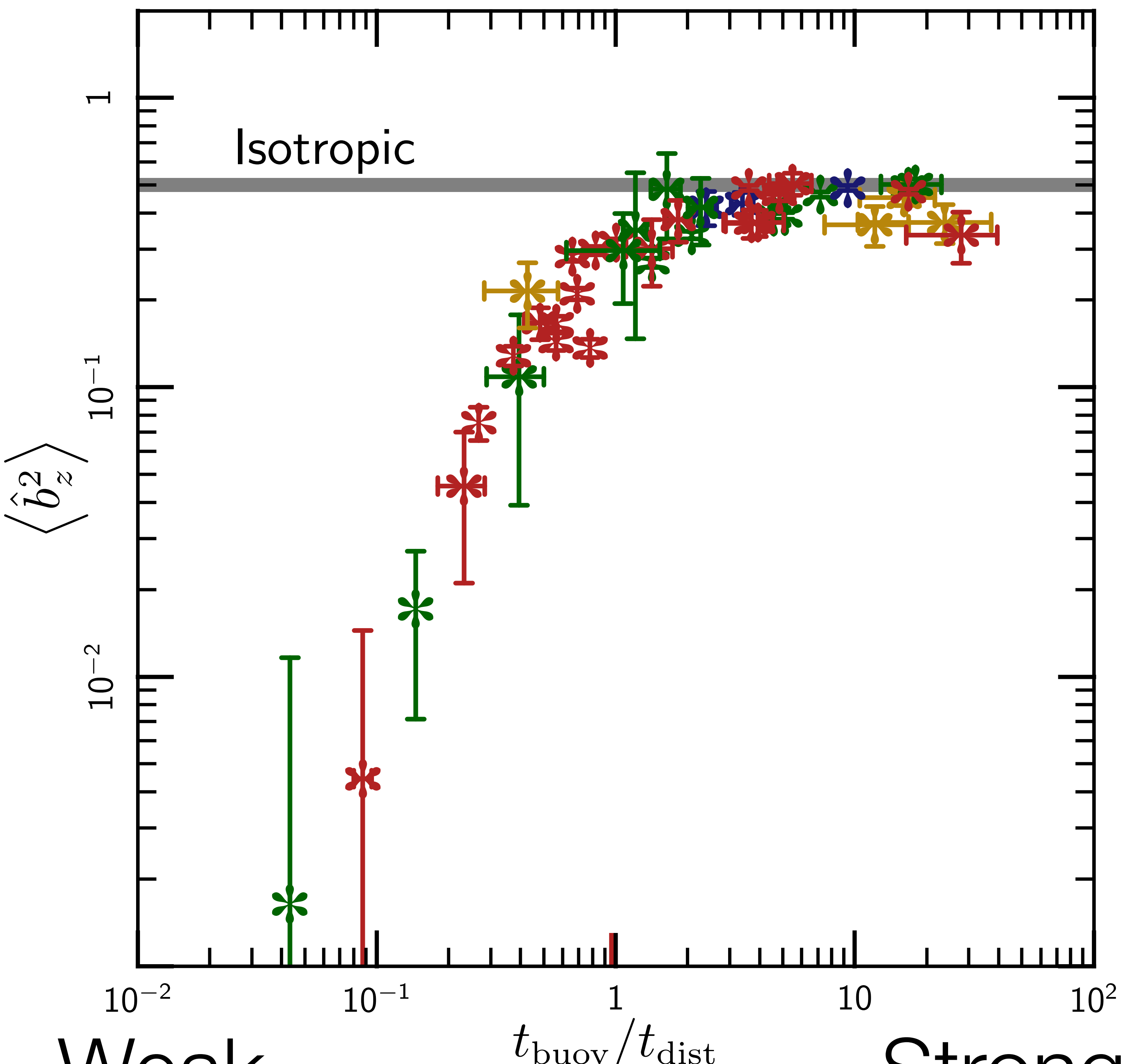
HBI + Turbulence



Statistical balance specifies b_z

Saturated Field Angles



Weak Turbulence

Strong Turbulence

Strong turbulence isotropizes the field

HBI dominates when turbulence is weak

Transition when $t_{\rm buoy} \sim t_{\rm eddy}$

Conclusions

- HBI and MTI both operate in clusters
- HBI saturates by reorienting the magnetic field, but the MTI does not
- MTI is a powerful dynamo and drives strong turbulence
- Interaction between HBI and turbulence determines suppression of the conductive flux