Minimalist Approach to Kahler Moduli Stabilization in Type IIB orientifolds and constraints on the String Axiverse

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KB, V. Braun, P. Kumar, S. Raby: hep-th 1003.1982
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Outline

- Introduction and motivation
- Moduli fixing in Type IIB CY orientifolds
- Axiverse and strong CP
- Conclusions

Introduction and motivation

- By compactifying to 4D, we obtain a multitude of scalar fields – moduli, parameterizing internal metric deformations, brane positions, etc.

- In String/M-theory all masses and couplings are functions of the moduli vevs

- Once the moduli are stabilized, all couplings are completely fixed and are computable in principle

- Can we stabilize all moduli so that we can make predictions?

• Moduli masses must be large enough to be compatible with observations (BBN => $M_{mod} > O(10)TeV$)

• Addressing the strong CP problem

$$f_{QCD} = \frac{4\pi}{g_3^2} + i\,\frac{\theta_{QCD}}{2\pi}$$

In SUSY Minkowski vacua θ_{QCD} is too heavy!
Conlon hep-th/0602233

Very heavy, the vev should be fixed at some high scale

Very light, the vev should be fixed by the QCD instanton effects

• CC should be tiny but positive (discrete fine tuning)

• SUSY must be broken such that new CP violating phases are compatible with current exp. limits

• FCNCs are suppressed, no rapid proton decay, etc.

Moduli fixing in Type IIB Calabi-Yau orientifolds

- Comlex structure moduli and the axio-dilaton can be stabilized at tree-level by turning on fluxes Dasgupta, Rajesh, Sethi; Giddings, Kachru, Polchinski; Larsen, O'Connell, Robbins

- Kahler moduli can be stabilized by including non-perturbative effects (instantons, gaugino condensation) Kachru, Kallosh, Linde, Trivedi (KKLT)

- Can obtain de Sitter vacua once supersymmetry breaking effects are included: alpha prime corrections (Balasubramanian, Berglund, Conlon, Quevedo), matter F-terms plus D-terms (Lebedev, Nilles, Ratz), anti D3-branes (KKLT)

- Here we concentrate on Kahler moduli

$$T_i = \tau_i + i\chi_i \,; \quad i = 1, \ldots, h_+^{1,1}$$

Volumes of 4-cycles
(fluctuations of the metric)

Axions
(periods of the 4-form, transform under a shift symmetry)

$$K_{Kahler} = -2\ln V_{CY}(\tau_i) \qquad W = W_{flux} + \sum_k A_k e^{-\frac{2\pi}{c_k}\vec{T}\cdot\vec{n}^{(k)}}$$

$$\tau^{(k)} = \vec{\tau}\cdot\vec{n}^{(k)} = \mathrm{Re}\,\vec{T}\cdot\vec{n}^{(k)} = \sum_{i=1}^{h_+^{1,1}} \tau_i n_i^{(k)}$$

Volume of the k-th divisor
(supersymmetric 4-cycle)

Integers, specifying the homology of the *k*-th divisor

• Common practice: assume as many divisors contributing to the superpotential as there are Kahler moduli and go to a basis where

$$W = W_{flux} + \sum_{k=1}^{h_{+}^{1,1}} A_k e^{-\frac{2\pi}{c_k} T_k}$$

• For a k-th divisor *D* to contribute the instanton must possess precisely two fermionic zero modes:

Witten hep-th/9604030

Necessary condition: $\chi_{+}(D,\mathcal{O}_D)-\chi_{-}(D,\mathcal{O}_D) = 1$

Sufficient: $h^{02}(D)=0$, $h^{01}(D)=0$

• Naively, one needs at least as many rigid divisors to contribute to the superpotential as there are Kahler moduli. If $h^{1,1}{}_{+}(X)=O(100)$ this is a rather daunting task

• Furthermore, visible sector divisor $D_{visible}$ supports chiral matter => extra zero modes Blumenhagen et. al. hep-th/0711:3389

Visible sector charged chiral matter

$$\delta W_{visible} = \prod_i \Phi_i e^{-2\pi T_{visible}}$$

• We already know that $\langle \Phi_i \rangle = 0$ in the visible sector

• Hence, despite the success of the KKLT approach the problem seems to comes back

• Is it possible to stabilize all Kahler moduli with fewer than $h^{1,1}{}_{+}(X)$?

• Consider an extreme case with a single divisor D

$$W = W_{flux} + Ae^{-\frac{2\pi}{c}T_D}, \text{ where } \quad \tau_D = \mathrm{Re}\, T_D = \sum_{i=1}^{h_+^{1,1}(X)} n_i \tau_i$$

• A divisor D corresponds to a zero locus of a section of a holomorphic line bundle $\mathcal{O}(D)$. If D is ample, its Poincare dual [ω] is inside the Kahler cone.

$$\{[\omega] \,|\, \omega = g_{i\bar{j}} dz^i \wedge dz^{\bar{j}}\} = Kc(X) \subset H^{1,1}(X)$$

$$c_1(\mathcal{O}(D)) = \lambda[\omega], \text{ where } \; 0 < \lambda \in \mathrm{R}$$

• The Kahler cone is given by a set of linear inequalities in the two-cycle volumes t_i. After a linear change of coordinates

$$Kc(X) = \{\vec{t} \mid t_i > 0\} \subset H^{1,1}(X)$$

• An ample divisor is transverse (has a positive intersection with) to all holomorphic curves

$$D = \sum_{i=1}^{h_+^{1,1}} n_i D_i \quad \text{is ample} \;\; => n_i > 0, i = 1, \ldots, h_+^{1,1}$$

• Hence, if a single ample divisor contributes to the superpotential, the vevs of all moduli are automatically inside the Kahler cone

- Recall: $K = -2\ln V_{CY}$, where $V_{CY} = \sum d_{ijk} t_i t_j t_k = \frac{1}{3}\sum \tau_i t_i$
- SUSY extremum (pick for definiteness $W_{flux} < 0$, $A > 0$)

$$D_i W = 0 \quad \Rightarrow \quad \vec{n}\cdot\vec{\chi} = 0 \bmod \frac{N}{2\pi}$$

$$t_i = \frac{n_i \tau_D^{1/2}}{\sqrt{3\sum d_{ijk} n_i n_j n_k}},\ \tau_i = \frac{\tau_D}{3}\frac{\partial}{\partial n_i}\ln\sum d_{ljk} n_l n_j n_k,\ V_{CY} = \frac{\tau_D^{3/2}}{3\sqrt{3\sum d_{ijk} n_i n_j n_k}}$$

where $\tau_D = \frac{N}{2\pi}\left(\ln\left|\frac{2A}{3W_{flux}}\right| + \ln\left(\frac{3}{2} + \ln\left|\frac{2A}{3W_{flux}}\right|\right) + \ldots\right)$

- All two- and four-cycle volumes as well as the volume of the Calabi-Yau are paremeterized by τ_D!

- Introduce anti D3-branes to break SUSY

Suppressed by the warp factor

$$V_{total} = e^K \left(G^{i\bar{j}} D_i W D_{\bar{j}} \overline{W} - 3|W|^2 \right) + \frac{D}{V_{CY}^{4/3}}$$

$$t_i = \frac{n_i \tau_D^{1/2}}{\sqrt{3 \sum d_{ijk} n_i n_j n_k}}, \ \tau_i = \frac{\tau_D}{3} \frac{\partial}{\partial n_i} \ln \sum d_{ljk} n_l n_j n_k, \ V_{CY} = \frac{\tau_D^{3/2}}{3\sqrt{3 \sum d_{ijk} n_i n_j n_k}}$$

$$\tau_D = \tau_D^{SUSY} + \frac{N^2}{48\pi^2} \frac{\left(\frac{4\pi^2}{N^2} \tau_D^{SUSY} + 3 \right)^2}{\left(\frac{\pi}{N} \tau_D^{SUSY} + 1 \right)\left(\frac{4\pi}{N} \tau_D^{SUSY} + 1 \right) \tau_D^{SUSY} W_{flux}^2 V_{CY}^{-\frac{2}{3}}} D + O\left(D^2\right)$$

- A three-parameter example CICY: $\begin{matrix} \mathbb{P}^1 \\ \mathbb{P}^2 \\ \mathbb{P}^3 \end{matrix}\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 3 \\ 3 & 1 & 0 \end{bmatrix}$

Define the orientifold action:

$$\Omega([x_0 : x_1 \mid y_0 : y_1 : y_2 \mid z_0 : z_1 : z_2 : z_3]) = [x_0 : -x_1 \mid y_0 : -y_1 : -y_2 \mid z_0 : -z_1 : -z_2 : -z_3]$$

Demand that the polynomials transform as

$$p_{(0,0,3)} \circ \Omega = -p_{(0,0,3)}, \quad p_{(1,0,1)} \circ \Omega = p_{(1,0,1)}, \quad p_{(1,3,0)} \circ \Omega = p_{(1,3,0)}$$

Kahler potential:

$$K = -2\ln\left(\frac{3}{2}t_2 t_3 (6t_1 + t_2 + t_3)\right) = -2\ln\left(\frac{1}{6\sqrt{3}}\sqrt{\tau_1(6\tau_2 - \tau_1)(2\tau_3 - \tau_1)}\right)$$

Ample divisor with $\chi_+ - \chi_- = 1$: $\tau_D = \tau_1 + \tau_2 + \tau_3$

- Recall the general expressions for the moduli vevs

$$t_i = \frac{n_i \tau_D^{1/2}}{\sqrt{3\sum d_{ijk} n_i n_j n_k}},\ \tau_i = \frac{\tau_D}{3}\frac{\partial}{\partial n_i}\ln\sum d_{ljk} n_l n_j n_k,\ V_{CY} = \frac{\tau_D^{3/2}}{3\sqrt{3\sum d_{ijk} n_i n_j n_k}}$$

- For the specific example we get:

$$\tau_1 = \frac{6}{30}\tau_D,\ \tau_2 = \frac{11}{30}\tau_D,\ \tau_3 = \frac{13}{30}\tau_D,\ t_1 = t_2 = t_3 = \frac{\tau_D^{1/2}}{3\sqrt{5}},\ V_{CY} = \frac{\tau_D^{3/2}}{9\sqrt{5}}$$

- All Kahler moduli are parameterized by a SINGLE parameter! This approach to Kahler moduli fixing is highly constraining and therefore potentially predictive

$$\alpha_{GUT}^{-1} \approx 25$$

$$\alpha_{GUT}^{-1} = \sum_{i=1}^{h_+^{1,1}} n_i^{vis}\tau_i \quad \Rightarrow \quad \tau_D = 3\left(\sum_{i=1}^{h_+^{1,1}} n_i^{vis}\frac{\partial}{\partial n_i}\ln\sum d_{ljk} n_l n_j n_k\right)^{-1} \times \alpha_{GUT}^{-1}$$

• In fluxless G_2 compactifications of M-theory we find a qualitatively similar result when the non-perturbative superpotential receives a contribution from a supersymmetric three-cycle Q, which is Poincare dual to the co-associative four-form $*\Phi$

B. Acharya, KB, hep-th/0810.3285
B. Acharya, KB, G. Kane, P. Kumar, J. Shao, hep-th/0701034
B. Acharya, KB, G. Kane, P. Kumar, D. Vaman, hep-th/0606262

Integers specifying the homology of Q

$$\tau_i = N_i \frac{7V_7}{3V_Q} > 0, \quad \Leftrightarrow \quad PD_X\left(Q\right) = \alpha * \Phi, \quad 0 < \alpha \in R$$

$$\tau_i \equiv \frac{\partial V_7}{\partial s_i} = \frac{1}{3}\int_X \phi_i \wedge *\Phi = \frac{1}{3}\int_{\beta_i} *\Phi, \text{where } \Phi = \sum_{i=1}^{b_3(X)} s_i \phi_i; \quad \phi_i \in H^2(X,Z)$$

The four-form is dynamically fixed by the homology of Q

$$G_2: \; s_i = \frac{4V_Q}{7}\frac{\partial}{\partial \tau_i} \ln V_7 \Big|_{\tau_k = N_k} \quad ; \; \text{TypeIIB}: \tau_i = \frac{\tau_D}{3}\frac{\partial}{\partial n_i} \ln \sum d_{ljk} n_l n_j n_k$$

• There may exist additional non-perturbative contributions. How robust are the vacua we found?

$$W_{total} = W_{flux} + Ae^{-\frac{2\pi}{N}\tau_D} + Be^{-\frac{2\pi}{M}\vec{\tau}\cdot\vec{m}} \approx W_{flux} - \frac{3}{2}W_{flux} + B\left|\frac{3W_{flux}}{2A}\right|^{\frac{N}{M}\times O(1)}$$

$$\tau_D \approx \frac{N}{2\pi}\ln\left|\frac{2A}{W_{flux}}\right|, \qquad \tau_i = \frac{\tau_D}{3}\frac{\partial}{\partial n_i}\ln\sum d_{ljk}n_l n_j n_k$$

• Clearly, when $N >> M$, the extra contributions are exponentially suppressed.

• This is especially true if the leading term is due to a gaugino condensate with *N~O(10)*, while the remaining terms come from instantons, i.e. *M=1*.

Moduli masses:

one is heavy $M_1 \approx O(60) \times m_{3/2}$

N-1 are light $m_i \approx O(1) \times m_{3/2} < 2 \times m_{3/2}$

- $m_{3/2} \sim O(10) TeV \Rightarrow$ moduli are probably heavy enough to decay before BBN

- For the explicit toy example we numerically obtain $m_i < 1.3 \times m_{3/2} \Rightarrow$ the light moduli cannot decay into the gravitinos. Large entropy production at late times, but before BBN, helps to avoid the gravitino problem

Acharya, et.al, hep-ph/0804.0863

String Axiverse and Strong CP

- Until now we only discussed fixing the volumes τ_i of four-cycles (and one linear combination of the axions). What about the remaining axions? Problem or virtue?
- Recall:

$$\chi_{\mathrm{i}} = \int_{[\tau_i]} C_4; \quad [\tau_i] \in H_4(X, Z);\ i = 1, \ldots, h_+^{1,1}$$

- Axions (arising from PQ symmetry) – the most elegant dynamical solution to the strong CP problem (Peccei, Quinn; Weinberg, Wilczek)
- May provide a significant fraction of Dark Matter
- Typically very light, so could have important consequences for cosmology and astrophysics

• Solving Strong CP via axions requires that the QCD axion acquires most of its mass from QCD instantons

$$m_{QCD} \sim \frac{\Lambda^2_{QCD}}{f_a} \sim 10^{-14} eV, \text{ for } f_a \approx 10^{16} GeV$$

• Typical moduli stabilization mechanisms give large masses to the axions, same as their saxion partners

• In Type IIB flux compactifications the dilaton and its axionic partner – the RR scalar are stabilized by fluxes near the string scale

• In a generic KKLT scenario as well as the LARGE volume scenario, the axionic partners of Kahler moduli get masses $m_\chi \sim O(m_{3/2}) >> m_{QCD}$ (Conlon hep-th/0602233)

- In the new scenario, a single non-perturbative term freezes the volumes τ_i and a single linear combination of axions

$$\vec{n}\cdot\vec{\chi} = 0 \ \text{mod}\ \frac{N}{2\pi}$$

- To fix all axions we must include the truncated non-perturbative contributions B. Acharya, KB, P. Kumar: hep-th/1004.5138

$$W = W_{flux} + A_1 e^{ib_1 f} + \sum_{k>1} A_k e^{ib_k f_k}$$

- Effective scalar potential after including the remaining non-perturbative terms and freezing all τ_i

$$V \approx V_0 - m_{3/2} m_{pl}^3 e^{K/2} \sum_{j>2} D_k e^{-b_j \tau^{(j)}} \cos \chi_j \ ; \ \chi_j = b_j \vec{n}_j \cdot \vec{\chi}$$

- The axion mass spectrum before the QCD effects

$$m_1 \approx O(60)\times m_{3/2}, \quad m_i^2 \approx O(1)\frac{m_{pl}^3 m_{3/2}}{f_i^2} e^{K/2} e^{-b_i\tau^{(i)}}, \; i = 2,\ldots, h_+^{1,1}$$

where

$$\tau^{(i)} = \vec{n}^{(i)}\cdot\vec{\tau} \sim O(1)\times\tau_D$$

- Consider $b_i = 2\pi$, $m_{3/2} \sim 10\ TeV$, $V_{CY} = 1000$, $f_i = 10^{16} GeV$

$$15 < \tau^{(j)} < 40 \Rightarrow 10^{-33} eV < m_i < 1 eV$$

- In generic compactifications $h^{1,1}{}_{+}(X) \sim O(100\text{-}1000)$

$\Rightarrow$ String Axiverse with a multitude of light axions

Arvanitaki et. al hep-th/0905.4720, hep-th/1004.3558

String Axiverse

Arvanitaki et. al hep-th/0905.4720, hep-th/1004.3558

- For masses below 10^{-33} eV, the axion mass is below the current Hubble scale, so they are essentially irrelevant although may contribute to dark energy
- Between 10^{-33} eV and 10^{-28} eV axions start oscillating between recombination and today. If they couple to E.B they can induce the rotation in the CMB polarization
- For m > 10^{-28} eV , axions can be a significant fraction of DM and can suppress power in small scale density perturbations
- Between 10^{-22} eV and 10^{-10} eV axions affect dynamics of rotating black holes. Superradiance.

What about the QCD axion?

• From QCD instantons $\delta V_{QCD} = \Lambda^4_{QCD}\left(1 - \cos\theta_{QCD}\right)$

$$\theta_{QCD} = 2\pi \sum_{i=1}^{h_+^{1,1}} n_i^{vis} \chi_i + \arg\det m_q = \theta_0 + \sum_{k=1}^{h_+^{1,1}} \frac{\psi_k}{\tilde{f}_k}; \quad \frac{1}{\tilde{f}_l} \equiv \sum_{i=1}^{h_+^{1,1}} \sum_{k=1}^{h_+^{1,1}} n_i^{vis} U_{ik} \frac{2\pi}{f_k} \tilde{U}_{kl}$$

Fixed by the vevs of the C.S. moduli

• The QCD instanton mass matrix is rank one =>

$$(m_k^2)_{QCD} = 0, \forall k = 1,\ldots, h_+^{1,1} - 1; \quad (m^2_{h_+^{1,1}})_{QCD} \approx \Lambda^4_{QCD} \sum_{l=1}^{h_+^{1,1}} \frac{1}{\tilde{f}_l^2}$$

• QCD effects give mass to the lightest mass eigenstate ψ_κ inside the linear combination representing θ_{QCD}

• Can easily achieve $|\theta_{QCD}| << 10^{-10}$, as long as at least one of the mass eigenstates inside θ_{QCD} is lighter than

$$m^2_{\exp} \approx 10^{-10}\left(m^2_{h_+^{1,1}}\right)_{QCD} \approx \left(10^{-14} eV\right)^2$$

• Pre BBN cosmological history is non-thermal so the standard estimates on the axion relic density do not apply. The mass of the QCD axion is such that it starts coherent oscillations during moduli dominated era. For all axions whose masses are greater then

Turner, 1983, Scherrer, Turner 1985, Dine Fischler 1983

$$\Gamma_{X_0} \approx O(1)\frac{m_{X_0}^3}{m_{pl}^2} \sim 10^{-14} eV; \quad m_{X_0} \approx 50\, TeV$$

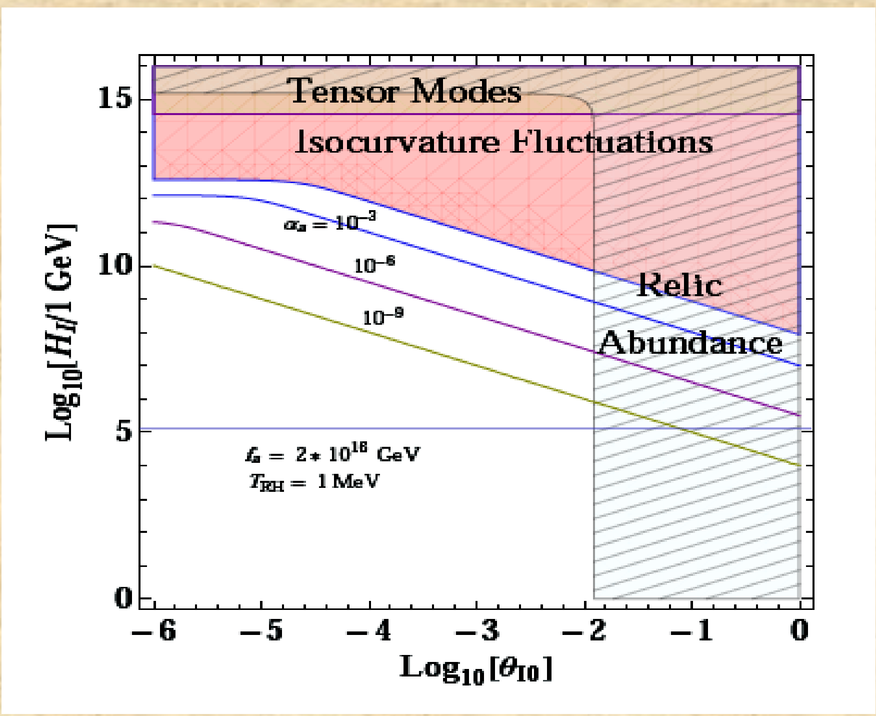
the relic abundance is independent of their mass!

$$\Omega_k h^2 = O(10)\left(\frac{f_k}{2\times 10^{16} GeV}\right)^2 \left(\frac{T_{RH}^{X_0}}{1MeV}\right)\left\langle \theta_{I_k}^2 \right\rangle \chi$$

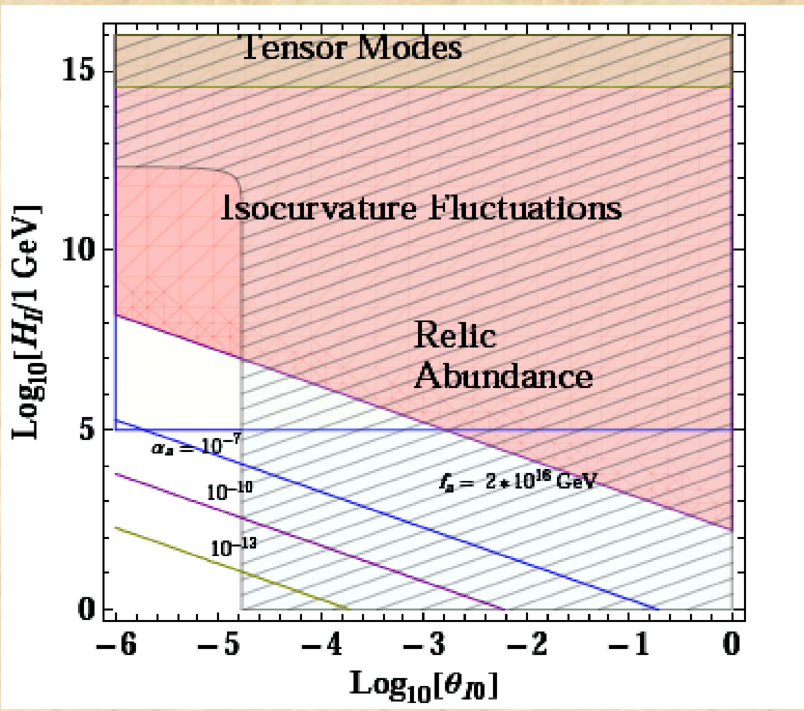
Fox, Pierce, Thomas hep-th:0409059,
Visinelli, Gondolo astro-ph:0912.0015

$$\Omega_{DM} h^2 \leq 0.11 \Rightarrow \left\langle \theta_{I_k}^2 \right\rangle \leq 10^{-2}$$

• Modest fine tuning of the misalignment angle. The entropy dilution due to late time moduli decays allows decay constants f_k much closer to the GUT scale



a) Non-thermal cosmology ($H_{inf} > M_{moduli}$)



b) Standard cosmology ($H_{inf} < M_{moduli}$)

- Consider one axion/e-folding between 10^{-33}eV and 1 eV
- Constraints from the relic abundance require a much smaller fine tuning of $<\theta_I>$ for case a) compared to case b)
- Isocurvature fluctuations can easily distinguish between the two scenarios (recall also Paolo Gondolo’s talk)

Falsifiable predictions

- Discovery of tensor modes in the near future would rule out the entire Axiverse (isocurvature fluctuations produced during inflation would be too large)
- Discovery of isocurvature componenent of in the CMB would strongly favor the non-thermal scenario
- Expect O(1) fraction of DM to come from axions
- Couplings of axions with masses $m_i << m_{QCD} \sim \Lambda^2{}_{QCD}/f$ to E.B are highly suppressed by a factor $(m_i/m_{QCD})^2$ essentially due to grand unification => axion decays to photons and the rotation of the CMB polarization are suppressed for light non-QCD axions

Conclusions

- Constructed a new class of compactifications in Type IIB on CY orientifolds, completely analogous to the M-theory models found earlier.
- All Kahler moduli can be stabilized inside the Kahler cone by a single non-perturbative contribution. Divisor supporting the visible sector is then fixed automatically
- SUSY CP violation is suppressed since the superpotential as well as the F-terms have the same overall phase.
- Strong CP problem can be solved with all the moduli stabilized. Moreover, obtain a multitude of light axions!