Dark Matter
of
Dark Energy

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in collaboration with

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What do we know about DM?

- It is dark
- It behaves like dust:

  - no pressure - energy flows along the time-like geodesics;

  - sound speed is vanishing

What do we know about DE?

- It is dark
- It mimics a cosmological constant equation of state

$$P \simeq -\mathcal{E}$$

New Class of Models for the Dark Sector of the Universe

- One scalar degree of freedom describing both DE and DM in linear regime
- No wave-like propagating degrees of freedom; sound speed is identically zero for all backgrounds
- Energy flows along time-like geodesics for all backgrounds
- No ghosts for $w_X \geq -1$

Action for two fields φ and λ

$$S=\int \mathrm{d}^4x\sqrt{-g}\left(K\left(\varphi,X\right)+\lambda\left(X-\frac{1}{2}\mu^2\left(\varphi\right)\right)\right)$$

where $$X\equiv\frac{1}{2}g^{\alpha\beta}\nabla_\alpha\varphi\nabla_\beta\varphi$$

$K\left(\varphi,X\right)$ **and** $\mu\left(\varphi\right)$ **are arbitrary functions**

λ **introduces a non-holonomic constraint:**

$$X=\frac{1}{2}\mu^2\left(\varphi\right)$$

Scalar version of the Einstein-Aether theory, Jacobson, Mattingly (2001)

Energy Momentum Tensor has the form of a Perfect Fluid:

$$\lambda\phi - \text{fluid}$$

$$T_{\alpha\beta} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\alpha\beta}} = (\varepsilon + p)\, u_\alpha u_\beta - p g_{\alpha\beta}$$

energy density: $\varepsilon(\lambda, \varphi) = \mu^2 (K_X + \lambda) - K$

pressure: $p(\varphi) = K\left(\varphi, \mu^2(\varphi)/2\right)$

4-velocity: $u_\alpha = \dfrac{\nabla_\alpha \varphi}{\sqrt{2X}} = \mu^{-1} \nabla_\alpha \varphi$

scalar field φ is an internal clock of this fluid

constraint: $X = \frac{1}{2}\mu^2(\varphi)$

NO acceleration :

$$a_\beta \equiv \dot{u}_\beta \equiv u^\lambda \nabla_\lambda u_\beta = \left(\frac{\nabla^\lambda \varphi}{\mu(\varphi)}\right) \nabla_\lambda \left(\frac{\nabla_\beta \varphi}{\mu(\varphi)}\right) = 0\,.$$

Energy flows along time-like geodesics--- "dust" with the pressure $p(\varphi)$ which depends on the internal clock only

arbitrary equation of state parameter

$$w_X \equiv p/\varepsilon$$

Equations of motion

$$\dot{\varphi} = u^{\alpha}\nabla_{\alpha}\varphi = \mu(\varphi) ,$$

$$\dot{\lambda} = u^{\alpha}\nabla_{\alpha}\lambda = -\mu^{-2}\left(\varepsilon_{\varphi}\mu + (\varepsilon + p)\,\theta\right) ,$$

where $\theta \equiv \nabla_{\alpha}u^{\alpha}$ **is expansion (in FRW** $\theta = 3H$**)**

two ordinary differential equations along the time-like geodesics

- **single degree of freedom (DoF)**
- **sound speed is identically zero; no wave-like propagating DoF**

Cosmological perturbations I

- evolution of the velocity potential is **identical to that for the dust case**

$$\frac{d}{dt}\left(\mu^{-1}\delta\varphi\right)=\Phi$$

- evolution of the energy perturbations

$$\delta\dot{\varepsilon}-\left(3\dot{\Phi}+\frac{\Delta}{a^2}\left(\frac{\delta\varphi}{\mu}\right)\right)(\varepsilon+p)+3H\left(\delta\varepsilon+\delta p\right)=0$$

- if $\delta p=p_{\varphi}\delta\varphi$ is negligible the perturbations behave as perturbations of dust

Cosmological perturbations II

- if $\lambda\phi - \mathrm{fluid}$ dominates, then the standard variable $u \propto \frac{\Phi}{\sqrt{\varepsilon + p}}$ evolves as

$$\frac{\partial^2 u}{\partial \eta^2} - \frac{1}{\theta}\frac{\partial^2 \theta}{\partial \eta^2} u = 0$$

where $\theta \propto \frac{1}{a\sqrt{1 + w_X}}$

sound speed is identically zero: $c_{\mathrm{s}} = 0$

- evolution of the Newtonian potential does not depend on scale:

$$\Phi = C_1(\mathbf{x}) + \frac{H}{a} C_2(\mathbf{x}) - C_1(\mathbf{x}) \frac{H}{a} \int^a \frac{\mathrm{d}a}{H}$$

$$\Phi = C_1(\mathbf{x}) + \frac{H}{a} C_2(\mathbf{x}) - C_1(\mathbf{x}) \frac{H}{a} \int^a \frac{\mathrm{d}a}{H}$$

If expansion history is *approximately* **that of** $\Lambda\mathrm{CDM}$

Perturbations are *approximately* **that of** $\Lambda\mathrm{CDM}$

Action for Cosmological Perturbations in terms of

$$u \propto \frac{\Phi}{\sqrt{\varepsilon + p}}$$

is not "ghosty" provided

$$\varepsilon + p > 0$$

smooth evolution across the "Phantom Divide"

$$w_X = -1$$

Indeed the classical solution for perturbations is smooth even during the crossing:

$$\Phi = C_1(\mathbf{x}) + \frac{H}{a} C_2(\mathbf{x}) - C_1(\mathbf{x}) \frac{H}{a} \int^a \frac{\mathrm{d}a}{H}$$

Simplest Example: Dust

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\lambda\left(X - \frac{1}{2}\mu^2\left(\varphi\right)\right)\right).$$

$$X \equiv \frac{1}{2}g^{\alpha\beta}\nabla_\alpha\varphi\nabla_\beta\varphi$$

Simple Example

ΛCDM

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(K\left(\varphi, X\right) + \lambda\left(X - \frac{1}{2}\mu^2\left(\varphi\right)\right)\right)$$

with

$$K\left(\varphi, X\right) = K\left(X\right) < 0$$
$$\mu\left(\varphi\right) = \mu_0 = \mathrm{const}$$

$$p = K\left(\mu_0\right) \quad \text{and} \quad \varepsilon = \varepsilon_0 a^{-3} - p$$

where ε_0 **is a constant of integration**

Less Simple Example: Dusty DE

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(K\left(\varphi,X\right)+\lambda\left(X-\frac{1}{2}\mu^2\left(\varphi\right)\right)\right)$$

$$K = -X \qquad \text{where}$$

$$\mu = \mu_0 \exp\left(-\frac{\varphi}{m}\right)$$

$$m = \sqrt{\frac{8}{3}}\,\frac{\sqrt{-w_{\mathrm{fin}}}}{1+w_{\mathrm{fin}}}M_{\mathrm{Pl}}$$

where the free parameter $w_{\mathrm{fin}} < 0$

for this model the equation of state is

$$w_X = \frac{1}{1-2\lambda}$$

if we start during radiation dominated époque with

$$\lambda \gg 1$$

$\Downarrow$

$\lambda\phi - \text{fluid}$ **behaves as dust**

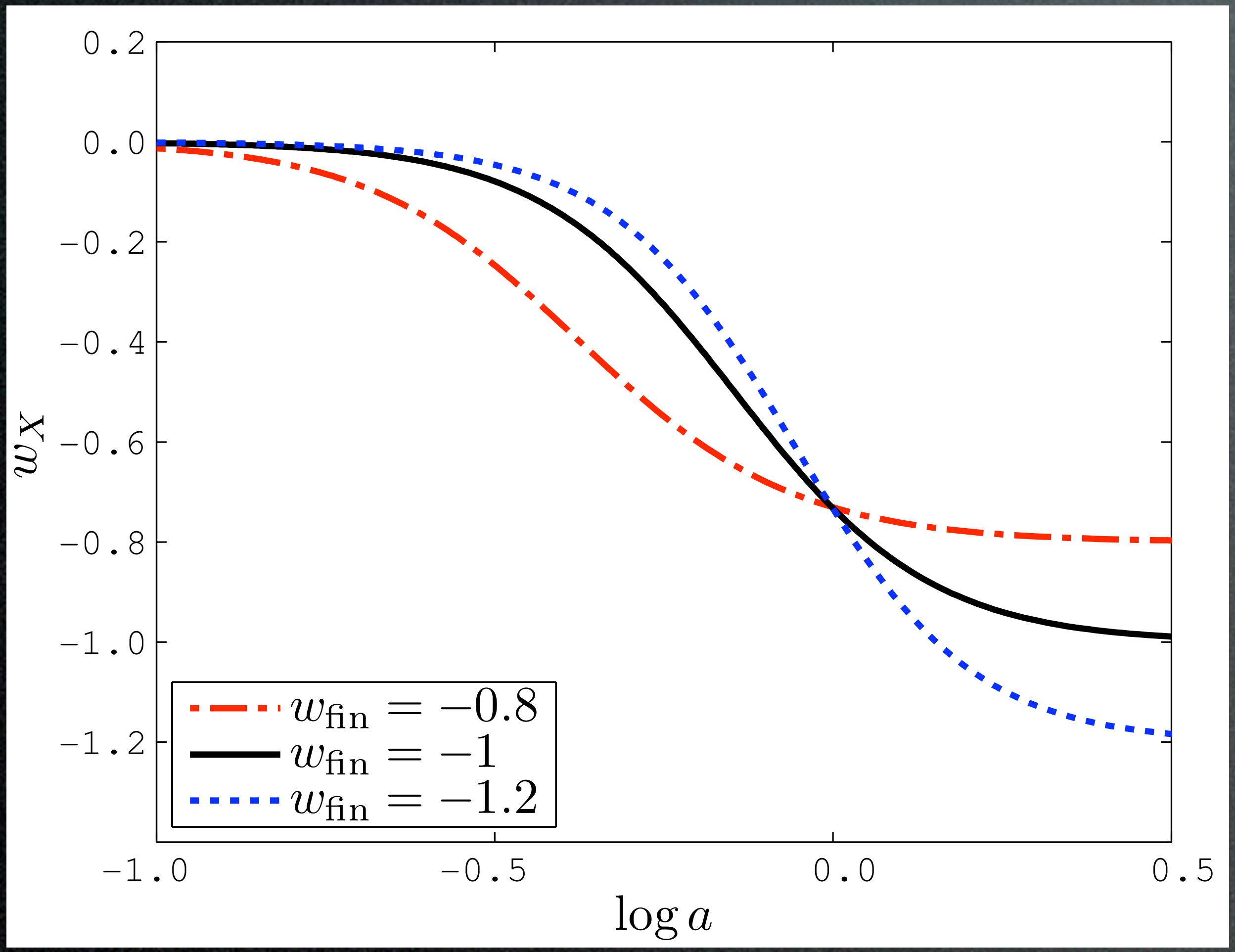
When $\lambda\phi - \text{fluid}$ dominates, equation of motion can be written in terms of the equation of state w_X :

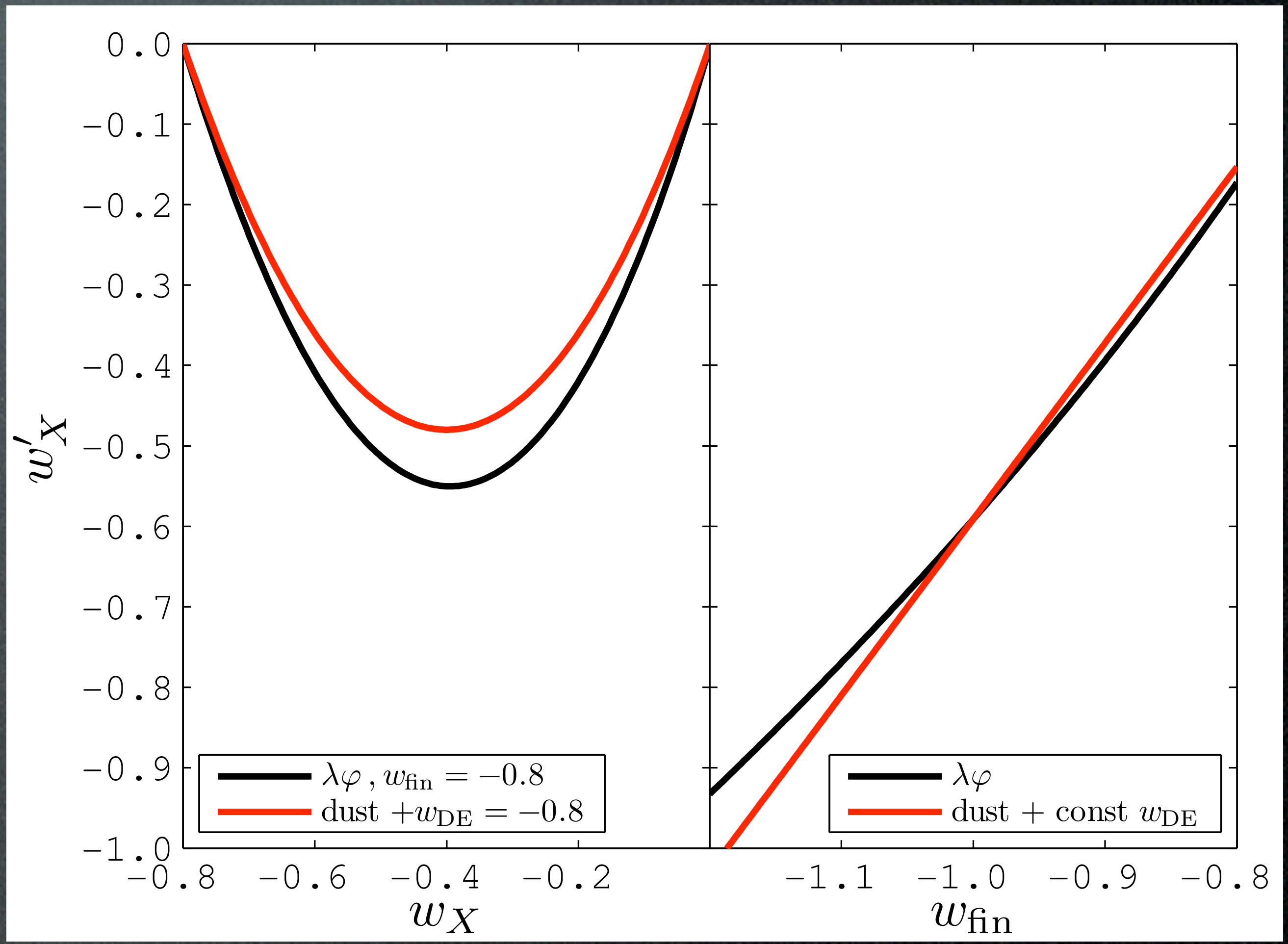
$$\frac{dw_X}{d\ln a} = 3w_X\left(1 + w_X - \sqrt{\frac{w_X}{w_{\text{fin}}}}\,(1 + w_{\text{fin}})\right),$$

two fixed points: repeller $w_X = 0$

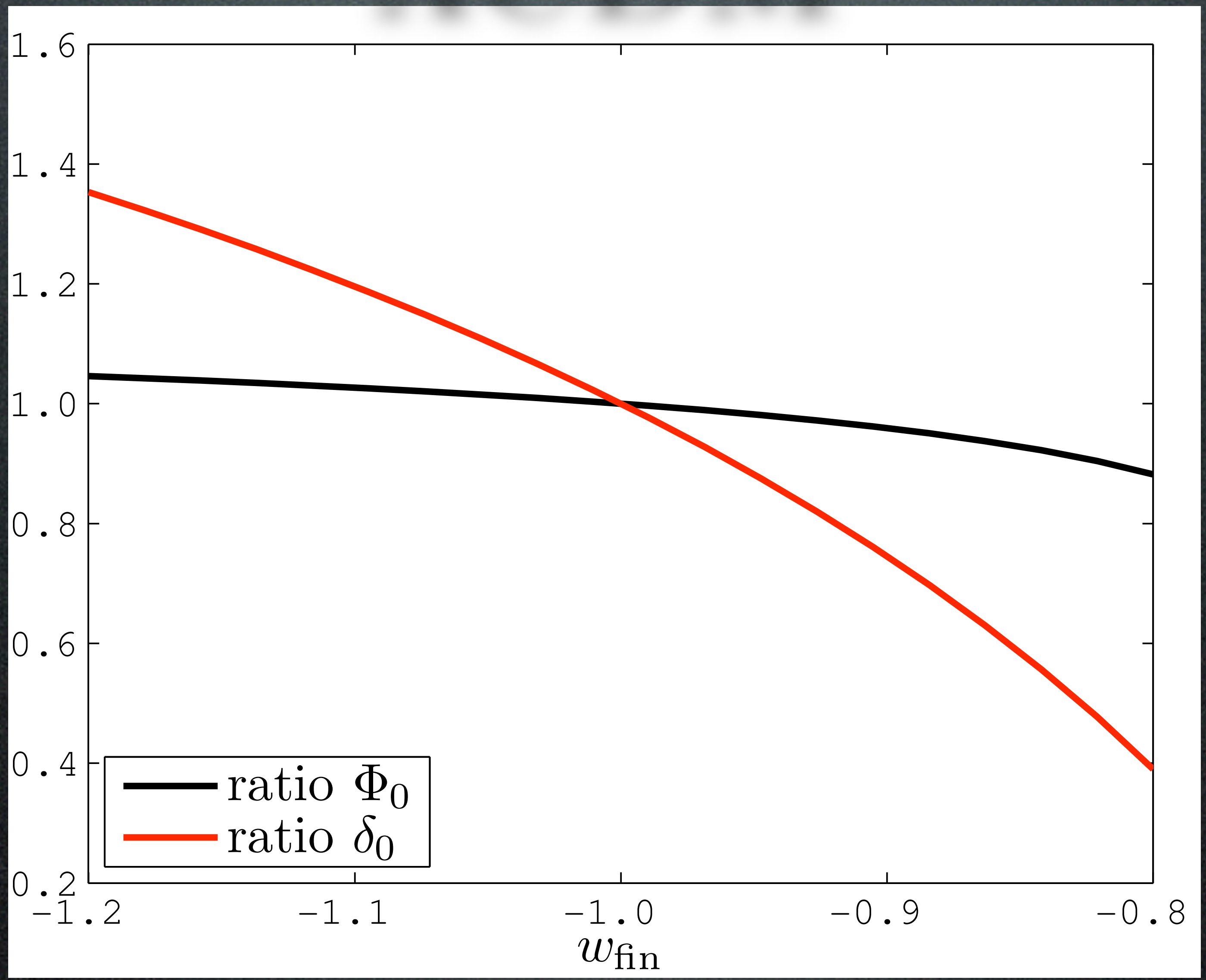
and attractor $w_X = w_{\text{fin}}$

expansion history is *approximately* that of ΛCDM





Perturbations are *approximately* **that of** ΛCDM



Open problems

- Caustics, how to interpret or avoid the multivalued regions?
- Can this “Dusty DE” virialize and form nonlinear structure?
- Can one obtain $\lambda\phi - \mathrm{fluid}$ as a limit of something less exotic?
- Quantization? What is the strong coupling scale for the cosmological perturbations?
- What is the origin of the initial conditions?

Thanks a lot for your attention !