dynamics of broken symmetry

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far-from equilibrium dynamics

- The challenge: how to characterise quantum dynamics far from equilibrium?

- Lack of broadly applicable principles and techniques

- Progress has been achieved in integrable models:
  - quench to CFT: Cardy & Calabrese PRL 96 2006
  - quench in transverse Ising chain: Calabrese, Essler & Fagottini PRL 106 (2011)

- Holography provides non-integrable yet solvable examples
  - plasma quench: Chesler & Yaffe PRL 102 (2009)
  - plasma thermalisation: de Boer et al. PRL 106 (2011)
holography in condensed matter

• We will endeavour to model QCPs using holography. **Scaling** symmetries encoded as **isometries** of `dual’ spacetime

• Continuum theory near QCP is encoded in dynamics of dual string theory

![Temperature and coupling phase diagram near a quantum critical point](image)

The two low temperature phases are separated by a region described by a scale-invariant theory at finite temperature. The solid line denotes a possible Kosterlitz-Thouless transition. Figure taken from reference [1].

Examples of systems that display quantum criticality. These will include both lattice models and experimental setups. Our discussion will be little more than an overview—the reader is encouraged to follow up the references for details. We shall focus on one-dimensional cases as we often will throughout these lectures. In several cases we will explicitly write down an action for the quantum critical theory. Typically the critical theory is strongly coupled and so the action is not directly useful for the analytic computation of many quantities of interest. Even in a large $N$ or $k$ for instance $d = w$, effects make the fixed point perturbatively accessible. Time-dependent processes, such as charge transport, are not easy to compute. This will be one important motivation for turning to the AdS/RNCFT correspondence. The correspondence will give model theories that share features of the quantum critical theories of physical interest, but which are amenable to analytic computations while remaining strongly coupled.

1.2.1 Example: The Wilson-Fisher fixed point

Let $\mathbf{r}$ be an $N$-dimensional vector. The theory described by the action

$$S = \sum_{d=3} \sum_{x} \sum_{\kappa_{\nu}} \frac{1}{2} n_{\nu}^{2} \kappa_{\nu} \sum_{l}^{2},$$

becomes quantum critical as $r \rightarrow r_{c}$ in mean field theory $r_{c} = s$, but the value gets renormalised and is known as the Wilson-Fisher fixed point. At finite $N$, the relevant state is realized in cold atoms.
1. non-equilibrium background
   “holography provides solvable examples”

2. model and background
   “holographic superconductors, time-dependent BCS”

3. a holographic setup for dynamical symmetry breaking
   “Numerical relativity, structure of quasi-normal modes”

4. conclusions and outlook
   “generic dynamical consequences of symmetry breaking”
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holography in a nutshell

- the effect of strong correlations (coupling) is recast into geometry: RG flow is incorporated into curved space and correlation functions are computed using (quantum) gravity
Superconductivity is a manifestation of symmetry breaking. New results here in a dynamical context are very general and extend beyond holography.

Specific example: minimal model of holographic superconductor

\[
S = \int d^4x \sqrt{-g} \left[ R + \frac{6}{\ell^2} - \frac{1}{4} F^2 - |D\psi|^2 - m^2 |\psi|^2 \right]
\]

Complex scalar $\Psi$ is dual to symmetry-breaking order parameter

1) RN: un-condensed normal phase, new hairy BH: s.c. phase

[Gubser; Hartnoll, Herzog, Horowitz]  
2) leading near-boundary term of $\Psi = $ source; subleading term = vev  
3) M-theory superconducting solutions exist!

[Gaunlett, Sonner, Wiseman]
an old dog and a new trick

• BCS theory is the celebrated microscopic explanation of conventional superconductivity. An old story!

BCS hamiltonian: \[ \mathcal{H} = \sum_{p,\sigma} \epsilon_p a_{p,\sigma}^\dagger a_{p\sigma} - \frac{\lambda(t)}{2} \sum_{q,p} a_{p\uparrow}^\dagger a_{-p\downarrow}^\dagger a_{-q\downarrow} a_{q\uparrow} \]

BCS groundstate: \[ |\Psi(t)\rangle = \prod_p \left[ u_p(t) + v_p(t) a_{p\uparrow}^\dagger a_{-p\downarrow}^\dagger \right] |0\rangle \]

pairing gap function: \[ \Delta(t) = \lambda \sum_p u_p(t) v_p^*(t) \]

• Recent (2004 - ...) new developments: the resulting (non-adiabatic) dynamics can be mapped onto a non-linear integrable system! [Barankov, Levitov & Spivak; Yuzbashyan, Altshuler, Kuznetsov & Enolskii]
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time-dependent BCS pairing problem

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The dynamics of this quench give rise to three distinct regimes:

I. Oscillation

II. Decay to finite gap

III. Decay to zero gap

Our achievement is twofold: 1) we exhibit analogous phenomena in a strongly-coupled system, with thermal and collisional damping
2) we identify a new and generic mechanism within dynamical symmetry breaking leading to this behaviour.
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• We wish to model a quench **holographically**: prescribe a sudden change in some physical parameter of the theory on the **boundary** and then evolve the non-linear PDEs numerically to ‘**fill in the bulk**’
more details of the setup [related work: Murata, Kinoshita & Tanahashi, 2010]

• for simplicity: take \textbf{homogeneous} quench

\[ ds^2 = \frac{1}{z^2} \left( -T(v, z) \, dv^2 - 2 \, dv \, dz + S(v, z)^2 \, dx_i^2 \right) \]

• the complex scalar can be expressed as

\[ \psi(v, z) = z \left( \psi_1(v) + \hat{\psi}(v, z) \right) \]

• and \( \Psi_1(t) \) is the \textbf{source} at the boundary. Use a spike in the source to quench the system (can think of different systems and different quenches)

• solve system of \((1+1)\) \textbf{non-linear PDE} by a pseudo-spectral method in spatial directions and ‘Crank-Nicholson’ finite differences in time direction (subtle issue about gauges. trial and error leads to stable choice)
The resulting dynamics give rise to three distinct regimes:

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2. Decay to finite gap

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\[ \psi_1(t) = \delta e^{-10(t-t_0)^2} \]
the resulting dynamics II  

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- we can dress the results up as a dynamical phase diagram

- BL-type analysis extended to include strong correlations, thermal damping,... we find similar behaviour: great! but why? and how?
clues from quasinormal mode structure

- let us study the structure of **quasi-normal modes** about the final state

\[
\begin{align*}
\psi(v, z) &= \psi_0(z) + \delta\psi(v, z) \\
g_{ab}(v, z) &= g_{ab,0}(z) + \delta g_{ab}(v, z) \\
A(v, z) &= A_0(z) + \delta A(v, z)
\end{align*}
\]

- deal with diffeo and U(1) gauge symmetry by defining **gauge-invariant variables** (c.f. cosmological perturbation theory)

\[
\delta \Phi_I(v, z) = e^{-i\omega v} \Phi_I^\omega(z)
\]

- The **analytic structure** of the $\Phi$ tells us about a) late-time behaviour of observables b) poles in two-point functions of dual operators
quasi-normal mode structure

\[ |\langle \mathcal{O}(t) \rangle| = |\langle \mathcal{O}_f \rangle + ce^{-i\omega_L t}| \]

- Off-axis poles lead to oscillations in broken phase
- Dynamics very well approximated by leading QNM
- Very good quantitative agreement with non-linear PDE code
quasi-normal pole dance
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dynamics of symmetry breaking

• T-reversal invariance means collective mode spectrum (manifested in our example as QNMs) must be \textbf{symmetric} under

\[ \omega \rightarrow -\omega^* \]

• Poles in spectral function (and other observables) come in two varieties:
  a) pairs of poles \textbf{off} imaginary axis
  b) single poles \textbf{on} imaginary axis

1. S.c. phase transition: coalescence of two poles at TC at \( \omega = 0 \)

2. Broken U(1) \( \Rightarrow \) Single pole (i.e. mode) at \( \omega = 0 \) (Goldstone mode)

3. At \( T=0 \) no source of dissipation \( \Rightarrow \) leading poles are oscillatory in nature

\[ 1 + 2 + 3 = \text{BL dynamical phase diagram!} \]
conclusions

• very interesting **far-from-equilibrium** problems are accessible at the intersection of numerical relativity and AdS/CFT. speculative comment: exact non-linear PDE methods may well be brought to bear on non-equilibrium field theory!

• simulated a **quantum quench** in ads/cmt: persistence of **BL phenomena** to strong coupling and in systems that thermalise makes it more likely to be observed in actual experiments

• in fact: our analysis shows that BL-type behaviour is **generic** for dynamical breaking of a continuous symmetry. This makes the **experimental point** even more emphatically.

• Are there different contexts? Higgs mechanism, early universe, you name it... [e.g. see this week’s “the QNM of quantum criticality” by Sachdev]
thanks for your attention!