Quantum Quenches

Alex Buchel

(Perimeter Institute & University of Western Ontario)

Work in collaboration with: Luis Lehner, Rob Myers, and Anton van Niekerk arXiv:1206.6785, 1209.xxxx (to appear)
⇒ Consider quantum mechanics with Hamiltonian dependent on an external parameter $\lambda$,

$$H_\lambda = H(\hat{p}, \hat{x}; \lambda).$$

The dynamics of the system induced by variation in $\lambda$ is well-understood:

- For a stationary state $|n\rangle$ with energy $E_n = \hbar \omega_n$, the slow changes in $\lambda$, i.e., $\frac{d \ln \lambda(t)}{\omega_n dt} \ll 1$, are adiabatic: the system continues to be in the state $|n\rangle$ with time-dependent energy $E_n = E_n(\lambda(t))$ tracing the change in $\lambda$.

- A fast (abrupt) change in $\lambda$, i.e., $\frac{d \ln \lambda(t)}{dt} = C \cdot \delta(t)$ results in the evolution of the wave-function $\psi_n$ of $|n\rangle$ for $t > 0$ as a mixed state of quenched Hamiltonian

$$H_\lambda \rightarrow H_{e^{C \cdot \lambda}}.$$
What about QFT?
The behavior of quantum quenches in QFT is a much more difficult question, i.e., the dynamics of the four dimensional quantum field theory under time-dependent variation of one of its coupling constants,

\[ \mathcal{L}_0 \rightarrow \mathcal{L}_\lambda = \mathcal{L}_0 + \lambda(t) \mathcal{O}. \]

Here, \( \mathcal{L}_0 \) is the undeformed Lagrangian of the theory, and \( \lambda(t) \) is a time-dependent coupling constant of a relevant operator \( \mathcal{O} \) in the theory. A textbook example in QFT — an interaction picture — is when \( \mathcal{L}_0 \) is a Lagrangian of a free theory, and the (small) coupling constant \( \lambda \) is turned-on adiabatically so that

\[
\lim_{t \to -\infty} \lambda(t) = 0, \quad \lim_{t \to +\infty} \frac{d \ln \lambda}{dt} = 0.
\]

Description of quantum quenches in strongly interactive systems, or with non-adiabatic profile of a coupling constant, has been studied to a lesser extent.
Some questions one can be interested in:

- How transition between the adiabatic and non-adiabatic regimes occur?
- What are the observables of a non-stationary QFTs?
- Are instantaneous quenches in QFT well-defined?
- How does a system relaxes as a result of a quench?
- Is there a difference in relaxation of one-point and many-point correlation functions?
- How does non-local observables (Wilson lines) relax?
- …
Outline of the talk:

- Thermal Quantum Quenches in non-interactive models
  - SHO
  - Bosonic free field theory
- Description of the interactive model: mass-deformed $\mathcal{N} = 4$ SYM
- Main tool: gauge/gravity correspondence
  - Holographic renormalization and ambiguities
- Results:
  - typical response of the system to a quench;
  - non-abiabaticity of the quench;
  - no instantaneous quenches;
  - renormalization scheme-dependence and divergences in $\langle T_{\mu\nu} \rangle$ and $\mathcal{O}_\Delta$;
  - renormalization scheme-dependence and the relaxation time;
  - constructing renormalization scheme-independent observables.
- Future directions
Follow Sotiriadis, Calabrese & Cardy (arXiv:0903.0895)

Consider SHO:

\[ H_0 = \frac{1}{2} \pi^2 + \frac{1}{2} \omega_0^2 \phi^2 \]

quenched abruptly at \( t = 0 \) as \( \omega_0 \to \omega \). Basic assumption:

\[ \langle \cdots \rangle|_{t=0_-} = \langle \cdots \rangle|_{t=0_+} \]

all physical observables are continuous across the quench.

In the thermal state \( \beta_0 = \frac{1}{T_0} \),

\[ \langle \phi^2 \rangle_{\beta_0} = \frac{1}{2\omega_0} \coth \frac{\beta_0 \omega_0}{2}, \quad \langle \pi^2 \rangle_{\beta_0} = \frac{\omega_0}{2} \coth \frac{\beta_0 \omega_0}{2} \]

\[ \langle H_0 \rangle_{\beta_0} = \frac{1}{2} \omega_0 \coth \frac{\beta_0 \omega_0}{2} \implies \langle H \rangle_{\beta_0} = \frac{\omega^2 + \omega_0^2}{4\omega_0} \coth \frac{\beta_0 \omega_0}{2} \]

\[ \frac{E - E_0}{E_0} = \frac{1}{2} \left( \left( \frac{\omega}{\omega_0} \right)^2 - 1 \right) \]

\[ \implies \text{In abrupt changes the work done on a system be positive/negative} \]
Bosonic free field theory:

\[ H_0 = \int d^3k \left( \frac{1}{2} \pi_k^2 + \frac{1}{2} \omega_{0,k}^2 \phi_k^2 \right), \quad \omega_{0,k} = \sqrt{m_0^2 + k^2} \]

Again, abrupt quench at \( t = 0 \) with \( m_0 \rightarrow m \) (same assumptions as in SHO).

It is straightforward to compute mixed momentum-time correlator in a quench:

\[
C_{\beta_0}(k; t_1, t_2) \equiv \langle T \{ \phi_k(t_1) \phi_k(t_2) \} \rangle_{\beta_0}
= \frac{1}{2\omega_k} e^{-i\omega_k|t_1-t_2|} + \left[ \frac{\omega_{0,k}}{4} \left( \frac{1}{\omega_{0,k}^2} + \frac{1}{\omega_k^2} \right) \coth \frac{\beta_0 \omega_{0,k}}{2} - \frac{1}{2\omega_k} \right] \cos \omega_k(t_1 - t_2)
+ \frac{\omega_{0,k}}{4} \left( \frac{1}{\omega_{0,k}^2} - \frac{1}{\omega_k^2} \right) \coth \frac{\beta_0 \omega_{0,k}}{2} \cos \omega_k(t_1 + t_2)
\]

It can be argued that at late times \( \{t_1, t_2\} \rightarrow \infty \) the last (time-translationary non-invariant) term can be neglected.
Thus, the quenched propagator at late times takes form

\[
C_{\beta_0}(k; t_1, t_2) = \frac{1}{2\omega_k} e^{-i\omega_k|t_1-t_2|} + \mathcal{A}(\beta_0, \omega_0, k, \omega_k) \cos \omega_k(t_1 - t_2)
\]

\[
C_\beta(k; t_1, t_2) = \frac{1}{2\omega_k} e^{-i\omega_k|t_1-t_2|} + \frac{1}{\omega_k(e^{\beta\omega_k} - 1)} \cos \omega_k(t_1 - t_2)
\]

Comparing the second terms we obtain the effective temperature \(\beta_{eff}(k)\) after the quench:

\[
\beta_{eff}(k) = \frac{1}{\omega_k} \ln \left(1 + \frac{1}{\omega_k \mathcal{A}}\right)
\]

In the "hot quench" limit, \(\frac{1}{\beta_0} \gg \omega_0, k\)

\[
\beta_{eff}(k) = \frac{2\beta_0}{1 + \omega_k^2/\omega_0^2}
\]

Further in the limit \(m_0 \gg m\)

\[
\beta_{eff} = 2\beta_0
\]

i.e., the final temperature is one-half the initial.
Along these lines, one can treat abrupt quenches in weakly interactive models.

**Rest of the talk:**
- quenches in interactive theory (true thermalization);
- we consider quenches *smoothed* over a time-scale $\tau \propto \alpha \beta_0$ (for some constant $\alpha$), and take a limit of “abrupt quench” as $\alpha \to 0$
Basic AdS/CFT correspondence:

\[
\begin{align*}
\text{gauge theory} & \quad \iff \quad \text{string theory} \\
\mathcal{N} = 4 \text{ } SU(N) \text{ } \text{SYM} & \iff N\text{-units of 5-form flux in type IIB string theory} \\
g_{YM}^2 & \iff g_s
\end{align*}
\]

\[
\Rightarrow \text{ Each of the duality frames are valid in complimentary regimes. In the 't Hooft limit (planar limit), } N \to \infty, g_{YM}^2 \to 0 \text{ with } Ng_{YM}^2 \text{ kept fixed:}
\]

- for \( g_{YM}^2 N \ll 1 \) we can use a standard perturbation theory
- for \( g_{YM}^2 N \gg 1 \) we can use effective supergravity description of type IIB string theory on \( AdS_5 \times S^5 \)

\[
\Rightarrow \text{ In the above regime we can incorporate corrections:}
\]

\[
\begin{align*}
\frac{1}{N}\text{-corrections} & \iff g_s\text{-corrections} \\
\frac{1}{Ng_{YM}^2}\text{-corrections} & \iff \alpha'\text{-corrections}
\end{align*}
\]
The ’basic’ holographic correspondence was extended to:

- non-conformal examples of gauge/string correspondence
- gauge theories in various dimensions
- beyond correspondence in vacuum — thermal states, near-equilibrium, etc

There are 2-ways to discuss gauge/gravity correspondence

- (a) Assume it is valid, and extract predictions
- (b) Do strong coupling computations and test the correspondence

I will go with (a)
Consider quenching the coupling $\lambda_\Delta$ in the deformation of large-$N$ $SU(N)$ $\mathcal{N} = 4$ supersymmetric Yang-Mills by a (gauge invariant) relevant operator $\mathcal{O}_\Delta$

$$\mathcal{L}_{SYM} \quad \rightarrow \quad \mathcal{L}_{SYM} + \lambda_\Delta \mathcal{O}_\Delta.$$ 

- We focus on two cases when $\Delta = 2, 3$
- The initial state is a thermal state of the gauge theory plasma.
- We discussed perturbative quenches, i.e., during the quench the coupling constant $\lambda_\Delta$ is always small compare to the temperature of the initial state $T_i$:

$$\frac{|\lambda_\Delta|}{T_i^{4-\Delta}} \ll 1.$$ 

- We allow for non-perturbative rates of change of $\lambda_\Delta = \lambda_\Delta(t)$:

$$\lambda_\Delta(t) = \lambda^0_\Delta \left( \frac{1}{2} \pm \frac{1}{2} \tanh \frac{t}{\mathcal{T}} \right), \quad \mathcal{T} = \frac{\alpha}{T_i},$$ 

i.e., we do not restrict values of $\alpha$.
- We are interested in the basic gauge invariant observables of the theory undergoing the quantum quench: the stress-energy tensor $T_{ij}$ and the VEV
The gravitational dual to the above quench:

\[ S_5 = \frac{1}{16\pi G_5} \int d^5 \xi \sqrt{-g} \left( R + 12 - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} m^2 \phi^2 + O(\phi^4) \right) , \]

with

\[ m^2 = \begin{cases} 
-3 , & \iff \text{corresponding operator } O_3 , \\
-4 , & \iff \text{corresponding operator } O_2 .
\end{cases} \]

Since our quenches are homogeneous and isotropic in the boundary spatial directions, we assume that both the background metric and the scalar field depend only on a radial coordinate \( r \) and a time \( v \). With the background ansatz

\[ ds_5^2 = -A(v, r) \, dv^2 + \Sigma(v, r)^2 \, (d\vec{x})^2 + 2dr dv , \quad \phi = \phi(v, r) , \]
From the effective gravitational action we obtain the following:

- evolution equations:

\[ 0 = \Sigma (\dot{\Sigma})' + 2\Sigma'\dot{\Sigma} - 2\Sigma^2 + \frac{1}{12} m^2 \phi^2 \Sigma^2 \]
\[ 0 = A'' - \frac{12}{\Sigma^2} \Sigma'\dot{\Sigma} + 4 + \phi'\phi - \frac{1}{6} m^2 \phi^2 \]
\[ 0 = \frac{2}{A} (\dot{\phi})' + \frac{3\Sigma'}{\Sigma A} \phi + \frac{3\phi'}{\Sigma A} \dot{\Sigma} - \frac{m^2}{A} \phi \]

- the constraint equations:

\[ 0 = \ddot{\Sigma} - \frac{1}{2} A'\dot{\Sigma} + \frac{1}{6} \Sigma (\dot{\phi})^2 \]
\[ 0 = \Sigma'' + \frac{1}{6} \Sigma (\phi')^2 \]

In above, for any function \( h(r, v) \),

\[ h' \equiv \partial_r h, \quad \dot{h} \equiv \partial_v h + \frac{1}{2} A \partial_r h. \]
When $m^2 = -3$,

$$\phi = \frac{1}{r} p_0 + \frac{1}{r^2} (p_0') + \frac{1}{r^3} \left( p_2 - \left( \frac{1}{2} p_0'' + \frac{1}{6} p_0^3 \right) \ln r \right) + {\mathcal{O}}(r^{-4} \ln r)$$

$$\Sigma = r + {\mathcal{O}}(r^{-1})$$

$$A = r^2 - \frac{1}{6} p_0^2 + \frac{1}{r^2} \left( a_4 + \left( \frac{1}{6} p_0 p_0'' + \frac{1}{36} p_0^4 - \frac{1}{6} (p_0')^2 \right) \ln r \right) + {\mathcal{O}}(r^{-3} \ln r)$$

where $\{p_0, p_2, a_4\}$ are functions of $v$.

In addition, a constraint equation implies:

$$0 = -2a'_4 + \frac{5}{27} p_0^3 p_0' + \frac{2}{3} p_0 p_2' - \frac{2}{3} p_0 p_2 - \frac{1}{9} p_0 p_0'' + \frac{4}{9} p_0 p_0'''$$
Physical meaning of \( \{p_0, p_2, a_4\} \):
- a 'source' [non-normalizable component],
  \[
p_0 \propto \lambda_3
\]
- a 'response' [normalizable component]
  \[
p_2 \sim O_3
\]
- Note that in the absence of the source/response the constraint implies
  \[
a'_4 = 0 \quad \Rightarrow \quad \text{energy density} = \text{constant}
\]
In general, the constraint equation can be integrated to quantify the change of \( \mathcal{E} \) during the quench:
\[
a_4 = \mathcal{C} + \frac{5}{216} p_0(v)^4 - \frac{5}{36} (p_0(v)')^2 + \frac{2}{9} p_0(v) p_0(v)'' - \frac{1}{3} p_0(v) p_2(v) + \frac{2}{3} \int_{-\infty}^{v} ds \ p_0(s)'
\]
where \( \mathcal{C} \) is a constant, related to the energy density in the infinite past.
Comment on numerical procedure (all to quadratic order in the source inclusive):

- Numerically solve the PDE for the scalar $\phi(v, r)$ for a given profile of the non-normalizable component
  \[ p_0 = p_0(v) \]

- Numerical solution determines normalizable component
  \[ p_2 = p_2(v) \]

- Given $\{p_0, p_2\}$ we can integrate the constraint equation to obtain
  \[ a_4 = a_4(v) \]

- Once $\{p_0, p_2, a_4\}$ are determined, we translate them in QFT observables:
  \[ E = E(v), \quad P = P(v), \quad O_3 = O_3(v) \]
To compute correlation functions of gauge-invariant observables, the theory has to be regularized and renormalized:

\[ S_{ct} = S_{ct}^{\text{divergent}} + S_{ct}^{\text{finite}} \]

\[ S_{ct}^{\text{divergent}} = \frac{1}{16\pi G_5} \int_{\partial M_5, \frac{1}{r} = \epsilon} d^4 x \sqrt{-\gamma} \left( 6 + \frac{1}{2} \phi^2 + \frac{1}{12} \phi^4 \ln \epsilon + \frac{1}{2} \gamma^{ij} \partial_i \phi \partial_j \phi \ln \epsilon + \frac{1}{12} R^\gamma \phi^2 \ln \epsilon \right) \]

\[ S_{ct}^{\text{finite}} = \frac{1}{16\pi G_5} \int_{\partial M_5, \frac{1}{r} = \epsilon} d^4 x \sqrt{-\gamma} \left( \delta_1 \phi^4 + \delta_2 \gamma^{ij} \partial_i \phi \partial_j \phi + \delta_3 R^\gamma \phi^2 \right) \]

where we have separated the counterterms which diverges in the limit \( \epsilon = \frac{1}{r} \to 0 \) from the finite counterterms.

The finite counterterms are parametrized by:

\[ \delta_1, \quad \delta_2, \quad \delta_3 \]
Once the theory is renormalized, we can compute 1-point correlation functions:

\[ 8\pi G_5 \mathcal{E} = -\frac{3}{2} a_4 - \frac{1}{12} (p'_0)^2 + \frac{1}{8} p_0^2 a_1^2 - \frac{1}{2} p_0 p_2 + \frac{1}{3} p_0 p'' + \frac{7}{288} p_0^4 + \mathcal{E}^{\text{ambiguity}} \]

\[ 8\pi G_5 \mathcal{P} = -\frac{1}{2} a_4 - \frac{1}{36} (p'_0)^2 + \frac{1}{6} p_0 p_2 - \frac{1}{18} p_0 p'' + \frac{7}{864} p_0^4 + \mathcal{P}^{\text{ambiguity}} \]

\[ 16\pi G_5 \langle \mathcal{O}_3 \rangle = \frac{1}{2} p''_0 - \frac{1}{12} p_0^3 - 2 a_1 p'_0 + \frac{1}{2} p_0 a_1^2 - 2 p_2 + \mathcal{O}_3^{\text{ambiguity}} \]

where we employ the label \( \text{ambiguity} \) to denote renormalization scheme ambiguities:

\[ \mathcal{E}^{\text{ambiguity}} = \frac{1}{2} \delta_1 p_0^4 + \frac{1}{2} \delta_2 (p'_0)^2, \]

\[ \mathcal{P}^{\text{ambiguity}} = -2 \delta_3 (p'_0)^2 - 2 \delta_3 p_0 (p''_0) - \frac{1}{2} \delta_1 p_0^4 + \frac{1}{2} \delta_2 (p'_0)^2, \]

\[ \mathcal{O}_3^{\text{ambiguity}} = 4 \delta_1 p_0^3 + 2 \delta_2 p''_0. \]
Note that for arbitrary $\delta_i$, the following (diffeomorphism) Ward identity,

$$\partial_i \langle T_{ij} \rangle = -\langle \mathcal{O}_3 \rangle \partial_j p_0,$$

is equivalent to the constraint

$$0 = -2a'_4 + \frac{5}{27} p_0^3 p'_0 + \frac{2}{3} p'_0 p_2 - \frac{2}{3} p_0 p'_2 - \frac{1}{9} p'_0 p'' + \frac{4}{9} p_0 p'''

⇒ We focus on the quenches of the type

$$\lim_{\tau \to \pm \infty} p_0(\tau) = constant$$

so, provided that the same is true for $p_2(\tau)$, i.e.,

$$\lim_{\tau \to \pm \infty} p_2(\tau) = constant$$

(numerically we verified that this is indeed the case), we have a thermal equilibrium state in the infinite past, and a thermal equilibrium state in the infinite future.
For example, if
\[
\lim_{\tau \to -\infty} p_0 = 0, \quad \lim_{\tau \to +\infty} p_0 = 1
\]
i.e., we quench from a thermal state of a CFT to a thermal state of a massive gauge theory,

\[
\mathcal{E} = \frac{3}{8} \pi^2 N^2 T_i^4 \left( 1 - \left( 2a_4 + \frac{1}{3} (p')^2 \ln \frac{\pi T_i}{\Lambda_2} + \frac{1}{9} (p')^2 + \frac{2}{3} p_0 p_2 \right) \frac{(m_f^0)^2}{\pi^2 T_i^2} + \mathcal{O} \left( \frac{(m_f^0)^4}{T_i^4} \right) \right)
\]

\[
\mathcal{P} = \frac{1}{8} \pi^2 N^2 T_i^4 \left( 1 - \left( 2a_4 + \frac{1}{9} (p')^2 - \frac{2}{3} p_0 p_2 + \frac{2}{9} p_0 p'' - \frac{2}{3} (p_0 p'' + (p')^2) \ln \frac{\pi T_i}{\Lambda_3} \right) \right)
\]

\[
O_3 = -\frac{\sqrt{2}}{2} N^2 T_i^2 m_f^0 \left( p_2 - \frac{1}{4} p'' + \frac{1}{2} \ln \frac{\pi T_i}{\Lambda_2} p'' + \mathcal{O} \left( \frac{(m_f^0)^2}{T_i^2} \right) \right)
\]

where

\[
\delta_2 = \frac{1}{2} \ln \Lambda_2, \quad \delta_3 = \frac{1}{12} \ln \Lambda_3
\]

Note that the number of ambiguities in renormalization scheme is precisely what is needed to make sense of \(\ln(T)\) terms once the gravity data is translated into gauge theory data.
Similarly, we can analyze the quenches

$$\lim_{\tau \to -\infty} p_0 = 1, \quad \lim_{\tau \to +\infty} p_0 = 0$$

i.e., we quench from a thermal state of a massive gauge theory to a thermal state of a CFT.

Another interesting observables are:

$$\frac{T_f}{T_i} = \left( 1 + \left( \pm \frac{\Gamma \left( \frac{3}{4} \right)^{\frac{4}{3}}}{3\pi^2} - \frac{1}{2} a_4^\infty \right) \frac{(m_f^0)^2}{\pi^2 T_i^2} + O \left( \frac{(m_f^0)^4}{T_i^4} \right) \right)$$

$$\frac{\mathcal{E}_f}{\mathcal{E}_i} = \left( 1 + \left( \pm \frac{2\Gamma \left( \frac{3}{4} \right)^{\frac{4}{3}}}{3\pi^2} - 2a_4^\infty \right) \frac{(m_f^0)^2}{\pi^2 T_i^2} + O \left( \frac{(m_f^0)^4}{T_i^4} \right) \right)$$

$$\frac{\mathcal{P}_f}{\mathcal{P}_i} = \left( 1 - \left( \pm \frac{2\Gamma \left( \frac{3}{4} \right)^{\frac{4}{3}}}{3\pi^2} + 2a_4^\infty \right) \frac{(m_f^0)^2}{\pi^2 T_i^2} + O \left( \frac{(m_f^0)^4}{T_i^4} \right) \right).$$

where

$$a_4^\infty = \lim_{\tau \to +\infty} a_4(\tau)$$
Consider quenches of the type

\[ p_0 = \frac{1}{2} + \frac{1}{2} \tanh \frac{\pi T_i \tau}{\alpha} \]

where \( T_i \) is the initial temperature.

⇒ For \( \alpha \gg 1 \) the quenches are slow compared to a characteristic thermal scale \( \propto \frac{1}{T_i} \), we expect an "adiabatic" response

\[ p_2(\tau) \bigg|_{\text{adiabatic}} = -\frac{\Gamma \left( \frac{3}{4} \right)^4}{\pi^2} p_0(\tau) \]

⇒ note that for the adiabatic response, from

\[ 0 = -2a_4' + \frac{2}{3} p_0' p_2 - \frac{2}{3} p_0 p_2' - \frac{1}{9} p_0' p'' + \frac{4}{9} p_0 p''' \implies a_4' \approx 0 + \mathcal{O}(\alpha^{-2}) \]

⇒

\[ \frac{T_f}{T_i} = \left( 1 + \left( \pm \frac{\Gamma \left( \frac{3}{4} \right)^4}{3\pi^2} + \mathcal{O}(\alpha^{-1}) \right) \frac{(m_f^0)^2}{\pi^2 T_i^2} + \mathcal{O} \left( \frac{(m_f^0)^4}{T_i^4} \right) \right) \]

and similarly for \( \mathcal{E}, \mathcal{P} \)
Typical response of the system:

\[ p_2(\tau) \]

Figure 1: Evolution of the normalizable component \( p_2 \) during the quench with \( \alpha = 1 \). The dashed red lines represent the adiabatic response.
More evolutions:
Recall:

\[
\frac{T_f}{T_i} = \left(1 + \left( \pm \frac{\Gamma \left(\frac{3}{4}\right)^4}{3\pi^2} - \frac{1}{2} a_4^\infty \right) \frac{(m_f^0)^2}{\pi^2 T_i^2} + O \left( \frac{(m_f^0)^4}{T_i^4} \right) \right)
\]

⇒ Note that quenches always results in pumping energy into the system
slow:  \[ \ln(-a_{2,4}^\infty) \bigg|_{\text{red,dashed}}^{fit} = -2.46(5) - 1.0(2) \ln \alpha, \quad \alpha \gg 1 \]

fast:  \[ \ln(-a_{2,4}^\infty) \bigg|_{\text{red,dashed}}^{fit} = -2.17(0) - 2.0(2) \ln \alpha, \quad \alpha \ll 1 \]

Above asymptotic behaviour translates into

\[ \frac{|\Delta T|}{T_i} = \frac{|T_f - T_i|}{T_i} = \begin{cases} \propto \frac{1}{\alpha} \left(\frac{m_0^f}{T_i}ight)^2, & \alpha \gg 1 \\ \propto \frac{1}{\alpha^2} \left(\frac{m_0^i}{T_i}ight)^2, & \alpha \ll 1 \end{cases} \]

and similarly for the relative change in the energy density \( \mathcal{E} \) and the pressure \( \mathcal{P} \).

\[ \Rightarrow \text{Note that infinitely sharp quenches} \]

\[ \alpha \rightarrow 0 \]

are not allowed
Entropy production and irreversibility of quenches

⇒ Forward/backwards quenches can be specified as

\[ p_{0}^{f/b} = \frac{1}{2} \pm \frac{1}{2} \tanh \frac{\pi T_{i}\tau}{\alpha} \]

Since

\[ p_{0}^{f}(\tau) + p_{0}^{b}(\tau) = 1 \quad \Rightarrow \quad p_{2}^{f}(\tau) + p_{2}^{b}(\tau) = p_{2}^{\text{equilibrium}} \bigg|_{p_{0} = 1} \]

⇒ Using

\[ 0 = -2a_{4}' + \frac{2}{3} p_{0}'p_{2} - \frac{2}{3} p_{0}p_{2}' - \frac{1}{9} p_{0}'p_{0}'' + \frac{4}{9} p_{0}p_{0}''' \]

above it enough to establish

\[ a_{4}^{f}(\tau) = a_{4}^{b}(\tau) \quad \Rightarrow \quad a_{4}^{\infty} = a_{4}^{\infty} \]
During the evolution the entropy is not well-defined. However, as system is at equilibrium at $\tau \pm \infty$ we can unambiguously compute $S_i$ and $S_f$. We find:

$$\frac{S_f}{S_i} = 1 - \frac{3}{2}a_{4,\infty}^f \frac{(m_f^0)^2}{\pi^2 T_i^2} + O\left(\frac{(m_f^0)^4}{T_i^4}\right)$$

- Since

$$a_{4,\infty}^f = a_{4,\infty}^b$$

the equilibration process is non-reversible;

- In all simulations

$$a_{4,\infty}^b < 0$$

Thus, quenches/equilibration are associated with the entropy production.

- Finally, recall

$$a_{4,\infty}^b \to 0 \quad \text{as} \quad \alpha \to \infty$$

Thus, there is no entropy production in adiabatic quenches
Quenching the coupling $\lambda_\Delta$ of $\mathcal{O}_\Delta$ depends on $\Delta$!

Figure 3: Left plot: $\Delta = 3$, right plot $\Delta = 2$

$$|T_f - T_i| = \frac{T_i}{T_i} \begin{cases} \propto \frac{1}{\alpha^2} \left(\frac{m_f^0}{T_i}\right)^2, & \alpha \ll 1, \quad \text{when} \quad \Delta = 3 \\ \propto (-\ln \alpha) \left(\frac{m_b^0}{T_i^4}\right), & \alpha \ll 1, \quad \text{when} \quad \Delta = 2 \end{cases}$$
Comment on scheme-independent observables:

- while the following observables are scheme-dependent,

\[ E = \frac{3}{8} \pi^2 N^2 T_i^4 \left( 1 - \left( 2a_4 + \frac{1}{3} (p'_0)^2 \right) \ln \frac{\pi T_i}{\Lambda_2} + \frac{1}{9} (p'_0)^2 + \frac{2}{3} p_0 p_2 \right) \frac{(m_f^0)^2}{\pi^2 T_i^2} + O \left( \frac{(m_f^0)^4}{T_i^4} \right) \]

\[ O_3 = -\frac{\sqrt{2}}{2} N^2 T_i^2 m_f^0 \left( p_2 - \frac{1}{4} p''_0 + \frac{1}{2} \ln \frac{\pi T_i}{\Lambda_2} p''_0 + O \left( \frac{(m_f^0)^2}{T_i^2} \right) \right) \]

- the following combination is renormalization-scheme independent:

\[ \left( E(\tau) - \frac{m_f^0}{\sqrt{2}} \int_{-\infty}^{\tau} ds \, p'_0(s) O_3(s) \right) \]

\[ \Rightarrow \text{Ideally, one would like to extract the relaxation time from renormalization-scheme independent observables.} \]
Universality of slow quenches:

\[ p_{1,2}(\tau) \bigg|_{\text{adiabatic}} = -\frac{\Gamma \left(\frac{3}{4}\right)^4}{\pi^2} p_{1,0}(\tau) \]

\[ p_{1,0}(\tau) \bigg|_{\text{adiabatic}} = -\ln 2 \ p_{1,0}^l(\tau) \]
⇒ Universality of fast quenches for $\Delta = 3$:

Introduce

$$\| p_{1,2} - \mathcal{F}(\alpha)\, p''_{1,0} \| \equiv \int_0^1 d(p_{1,0}) \, (p_{1,2} - \mathcal{F}(\alpha)\, p''_{1,0})^2$$

$$= \int_{-\infty}^{\infty} d\tau \, p'_{1,0} \, (p_{1,2} - \mathcal{F}(\alpha)\, p''_{1,0})^2$$
From the fit,

\[ \mathcal{F}_{\text{fit}} \simeq 0.43 - 0.50 \ln \alpha \]

define a 'universal' profile

\[ \hat{p}_{1,2} \equiv \alpha^2 \left( p_{1,2} - \mathcal{F}(\alpha) p^{''}_{1,0} \right) , \]
Open questions:

- Fully-nonlinear quenches, not necessarily of thermal states
- Sound waves in quenches
- Quenches in various dimensions
- Non-local observables during quenches
- Quenches of SUSY couplings
- Quenches across the phase transitions
- ...
Fast, fully-nonlinear quenches are **universal**!

Consider fully-backreacted quench of $\Delta = 3$ near the boundary:

$$\phi = p_0 \rho + \rho^2 p'_0 + \rho^3 \left( p_{2,0} + \left( \frac{1}{6} p_0^3 + \frac{1}{2} p_0'' \right) \ln \rho \right)$$

$$+ \rho^4 \left( p'_{2,0} - \frac{1}{3} p'''_0 + \left( \frac{1}{2} p_0^2 p'_0 + \frac{1}{2} p_0''' \right) \ln \rho \right) + \mathcal{O}(\rho^5 \ln \rho)$$

$$\Sigma = \frac{1}{\rho} \left( 1 - \frac{1}{12} \rho^2 p_0^2 + \mathcal{O}(\rho^3) \right)$$

$$A = \frac{1}{\rho^2} \left( 1 - \frac{1}{6} \rho^2 p_0^2 + \mathcal{O}(\rho^4 \ln \rho) \right)$$

where

$$p_0 = p_0 \left( \frac{v}{\alpha} \right)$$
Consider a scaling symmetry $\alpha \to 0$, i.e., introduce

$$t \equiv \frac{v}{\alpha}$$

Then:

$$' \equiv \frac{1}{\alpha} \partial_t$$

If above scaling is accompanied by

$$\rho \to r\alpha$$

Above asymptotic expansions have a homogeneous scaling in the limit $\alpha \to 0$

$$\phi \to \alpha \phi, \quad \Sigma \to \alpha^{-1}\Sigma, \quad A \to \alpha^{-2}A$$

provided

$$\hat{p}_{2,0} \equiv p_{2,0} + \frac{1}{2} \ln \alpha \, p_0'' \quad \to \quad \alpha^{-2}\hat{p}_{2,0}$$

Such rescaling can be understood at the level of non-linear equations, not just the asymptotic solution; it:
- effectively linearizes equations of motion
- neglects the scalar backreaction on the geometry
- it has a simple physical interpretation...
Note: the scaling explains the universal curve for the fast quench of $\Delta = 3$ operator! Previously:

$$\hat{p}_{1,2} \equiv \alpha^2 \left( p_{1,2} - \mathcal{F}(\alpha) \ p''_{1,0} \right) ,$$

$$\mathcal{F}|_{fit} \simeq 0.43 - 0.50 \ \ln \alpha$$

while the scaling predict that

$$\hat{p}_{1,2} \equiv \alpha^2 \left( p_{1,2} + \frac{1}{2} \ln \alpha \ p''_{1,0} \right) ,$$

is a universal curve.
Similar arguments can be made for any $\Delta$. For example, for a marginal operator, we have a prediction

$$\hat{p}_{4,0} \equiv p_{4,0} - \frac{1}{16} \ln \alpha \frac{1}{\alpha^4} \partial_t^4 p_0 \quad \rightarrow \quad \alpha^{-4} \hat{p}_{4,0}$$

or in other words:

$$\alpha^4 p_{4,0} = \frac{1}{16} \ln \alpha \partial_t^4 p_0 + \text{const}$$

Results of the quench:
The red line is

$$-0.029 + \frac{1}{16} \ln \alpha$$

$$\implies$$ A Fit of the first 20 blue points gives $\ln \alpha$ coefficient:

0.0624986, note: $\frac{1}{16} = 0.0625$
Time-ordered correlator of SHO for abrupt quench

\[ C_{\beta_0}(t_1, t_2) = \left\langle T \{ \phi(t_1) \phi(t_2) \} \right\rangle |_{\beta_0} \]

From EOM

\[ \dddot{\phi} + \omega^2 \phi = 0 \]

we have

\[ \phi(t) = \phi(0) \cos \omega t + \pi(0) \frac{\sin \omega t}{\omega} \]

For \( t_1 > t_2 \),

\[ \left\langle \phi(t_1) \phi(t_2) \right\rangle |_{\beta_0} = \left\langle \phi(0)^2 \right\rangle_{\beta_0} \cos \omega t_1 \cos \omega t_2 + \left\langle \pi(0)^2 \right\rangle_{\beta_0} \frac{\sin \omega t_1 \sin \omega t_2}{\omega^2} \]

\[ + \left\langle \phi(0) \pi(0) + \pi(0) \phi(0) \right\rangle_{\beta_0} \frac{\sin \omega (t_1 + t_2)}{2\omega} - i \frac{\sin \omega (t_1 - t_2)}{2\omega} \]
Using

\[ \langle \phi(0)^2 \rangle_{\beta_0} = \frac{1}{2\omega_0} \coth \frac{\beta_0 \omega_0}{2}, \quad \langle \pi(0)^2 \rangle_{\beta_0} = \frac{\omega_0}{2} \coth \frac{\beta_0 \omega_0}{2} \]

\[ \langle \phi(0)\pi(0) + \pi(0)\phi(0) \rangle_{\beta_0} = 0 \]

We find

\[ C_{\beta_0}(t_1, t_2) = \frac{1}{2\omega} e^{-i\omega|t_1-t_2|} + \left[ \frac{\omega_0}{4} \left( \frac{1}{\omega_0^2} + \frac{1}{\omega^2} \right) \coth \frac{\beta_0 \omega_0}{2} - \frac{1}{2\omega} \right] \cos \omega(t_1 - t_2) \]

\[ + \frac{\omega_0}{4} \left( \frac{1}{\omega_0^2} - \frac{1}{\omega^2} \right) \coth \frac{\beta_0 \omega_0}{2} \cos \omega(t_1 + t_2) \]