Holographic Spacetimes From Entanglement Renormalization

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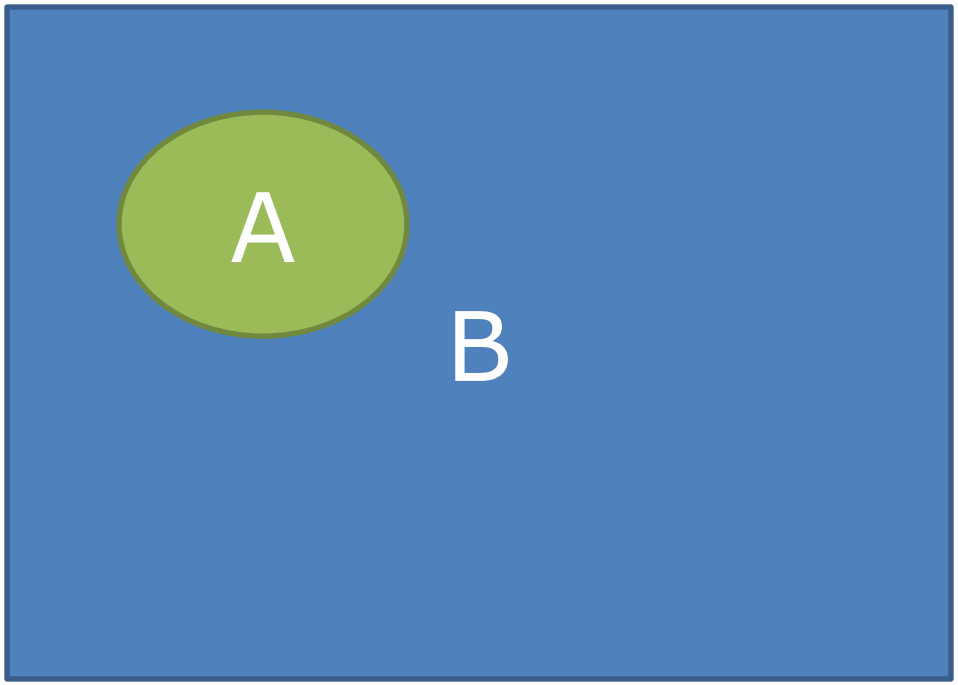
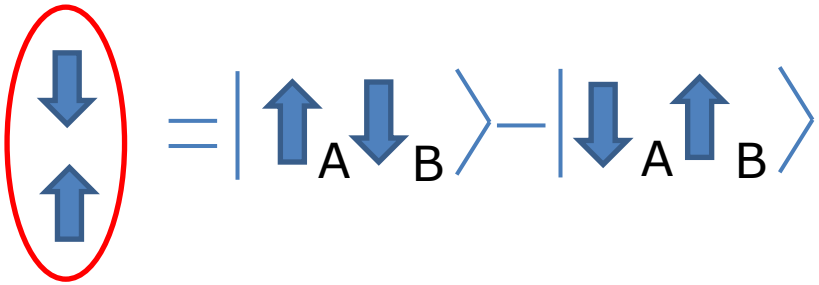
- Thanks to the organizers Matt, Henriette, Jim for inviting me

1. Entanglement
2. Motivations
3. Related results
4. Preview

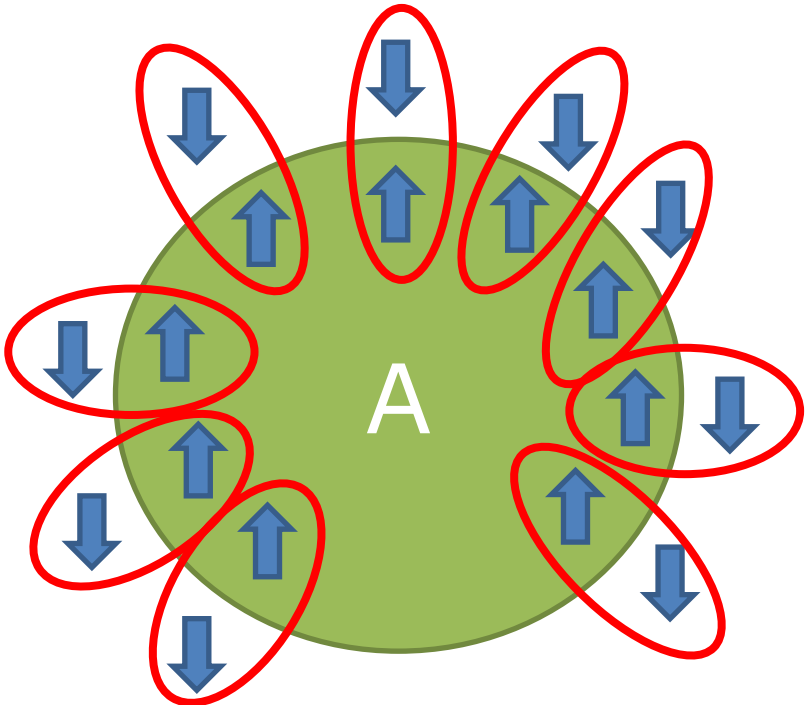
MOTIVATION AND BACKGROUND

Entanglement





$$S_n(A) = S_n(B) = \ln 2$$



Renyi entropy:

$$S_n = \frac{1}{1-n} \ln\left(\mathrm{tr}(\rho_A^n)\right)$$

$$S(A) = \lim_{n\to 1} S_n(A)$$

Why do we care?

Resource — Quantum computation and communication

- State teleportation
- Measurement-based quantum computation
- Topological quantum computation (long-range entanglement)

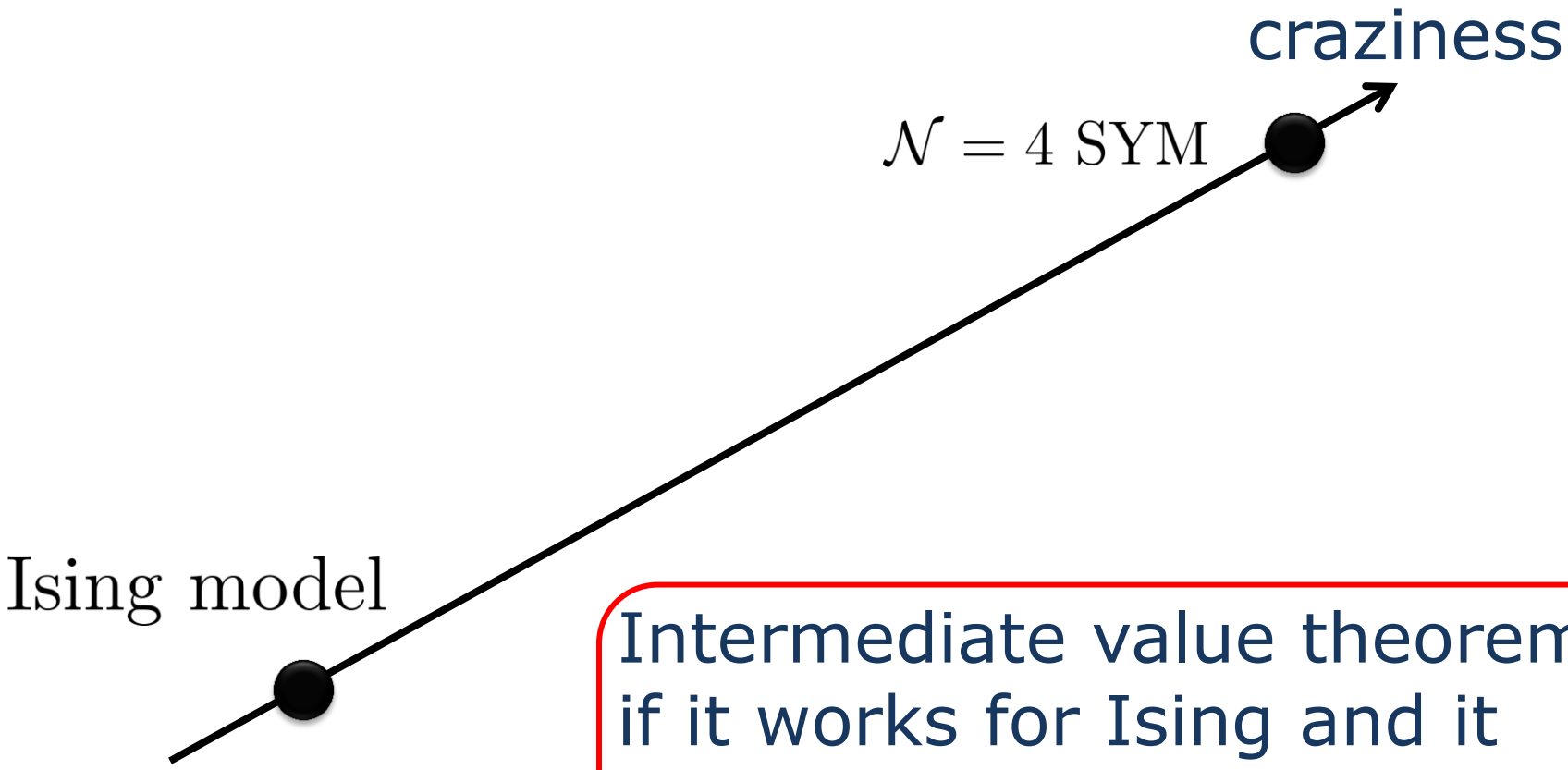
Diagnostic — Identifying and probing quantum matter

- Topological entanglement entropy (spin liquids/fractionalized phases)
- Fermi surfaces (FS), hidden or otherwise
- Critical points

Foundational — Understanding structure of quantum matter

- Entanglement-based simulation methods (DMRG, MERA, PEPS, …)
- Thermalization
- Holographic duality

My dream: entanglement renormalization everywhere!



Intermediate value theorem: if it works for Ising and it works for SYM then it works for everything!

Renormalization

UV

$r = \epsilon$

Microscopic model

Intermediate scale physics

Low energy physics

IR

$r \to \infty$

An example:

$$H = -w \sum_{<rr'>\sigma} c^\dagger_{r\sigma} c_{r'\sigma} + h.c.$$
$$+ \sum_r U n_{r\uparrow} n_{r\downarrow} - \sum_r \mu (n_{r\uparrow} + n_{r\downarrow})$$

$$\mathcal{S} = \sum \int d\hat{n} \int \frac{dk}{2\pi} \psi^\dagger_{\hat{n}\sigma} (i\partial_t - v_F k) \psi_{\hat{n}\sigma}$$

$$\langle \psi_{\hat{n}\uparrow} \psi_{-\hat{n}\downarrow} \rangle \neq 0$$

… but rarely so clear-cut in experiment

Recent results: Fermi surfaces

- Widom formula: *(Klich-Gioev '06, BGS '09)*

$$S_n(A) = \left(1 + \frac{1}{n}\right) \frac{1}{24} \frac{1}{(2\pi)^d} \int_{\partial A} \int_{FS} |n_x \cdot n_k| \ln(L)$$

- Fermi liquids: $U(1)^\infty$ symmetry

$$H_{Landau} = \sum_k \epsilon_k n_k + \frac{1}{2} \sum_{kk'} f_{kk'} n_k n_{k'} \quad (n_k = c_k^\dagger c_k)$$

(BGS '10, Ding-Seidel-Yang '12)

- Exactly solvable models!

1. Variable quasiparticle residue
2. Quantum phase transition
3. Non-Fermi liquid metal

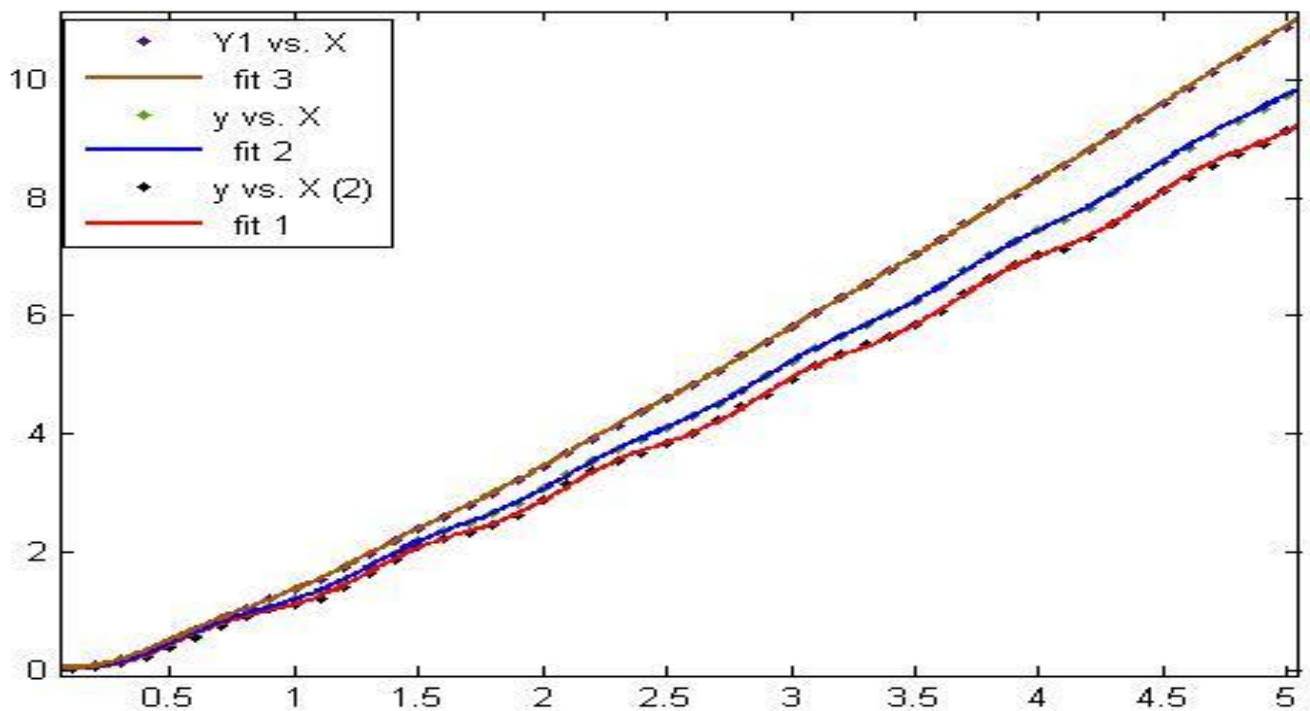
(BGS '12)

Recent results: Fermi surfaces

(BGS-McMinis-Tubman coming soon)

- Sub-leading oscillating terms

$$\delta S_n = f_n \frac{1}{4} \int_{\partial A} \int_{FS} |n_x \cdot n_k| \frac{\cos\left(2k_F L_{eff}(x,k)\right)}{\left(2k_F L_{eff}(x,k)\right)^{2/n}}$$



Recent results: topological phases

- Topological entanglement entropy:

$$S(A) = \alpha L - \boxed{\beta}$$

(Levin-Wen, Kitaev-Preskill '05)

- Entanglement spectrum: *(Li-Haldane '09, …)*

$$\rho_A = e^{-K_A}, \ \mathrm{spec}(K_A)$$

- Bulk-edge correspondence: K_A same universal features as physical edge Hamiltonian! *(Senthil-BGS '11)*

$$K_{\text{half space}} \rightarrow \text{Rindler space}$$

(Unruh-Davies-Fulling)

$$K_{\text{ball}} \rightarrow \text{hyperbolic space}$$

(Casini-Myers-Huerta '11)

Thermal-entanglement crossover

$$\rho_A(T) = \mathrm{tr}_B \left(e^{-H/T} / Z \right)$$

$S(A,T) =$ universal cross-over function
+ non-universal area law terms

(early 1d results: Cardy-Calabrese '04, Korepin '03)

$$S(A,T) = \frac{1}{12} \frac{1}{(2\pi)^d} \int_{\partial A} \int_{FS} |n_x \cdot n_k| \ln \left(\frac{v_F}{\pi T \epsilon} \sinh \left(\frac{\pi T L_{eff}(x,k)}{v_F} \right) \right)$$

(BGS '10)

$S(A,T) \sim T^{\phi} f(T L^{1/z})$ S~L ln L for NFL

(Senthil-BGS '11)

RG monotonicity?

- Wonderful results for CFTs!

(Zamolodchikov, Schwimmer-Komagordski '11, Casini-Huerta '12)

- Nothing(?) for anything more general …

1. Fermi points, 1d $\epsilon_k = vk + v'k^3/k_0^2$
2. Limit cycles (other talks!)
3. Lifshitz points *(Hsu et al. '09, Oshikawa '10, Stephan et al. '09)*
4. Disorder *(Refael-Moore '04)*
5. CFT -> FS -> "CFT" e.g. d-wave SC

Is there a general framework?

Preview

Large N

- Operator dimensions
- Entropy/area
- Mutual information

cMERA

- Scaling transformations
- Entanglement per scale

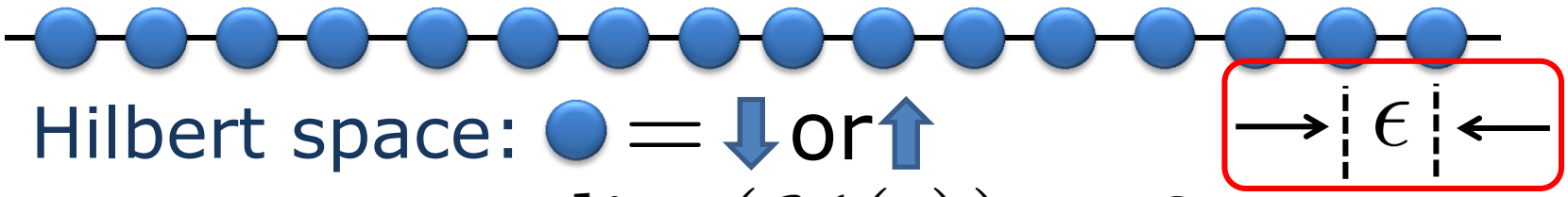
Fermi surfaces

- Emergent dimensions/branching MERA
- Field theory route to hyperscaling spacetimes?

1. Entanglement and locality
2. Disentanglers and coarse-grainers
3. Operators and correlation functions

ENTANGLEMENT RENORMALIZATION

Spin chains



- Hilbert space: ● = ⬇ or ⬆

$$\dim(\mathcal{H}(\bullet)) = 2$$

- Hamiltonian:

$$H = \sum_r h_r^{(1)} + \sum_{rr'} h_{rr'}^{(2)} + \sum_{rr'r''} h_{rr'r''}^{(3)} + \ldots$$

Locality: $h_{rr'}^{(2)} \to 0 \text{ as } |r - r'| \to \infty, \text{ etc.}$

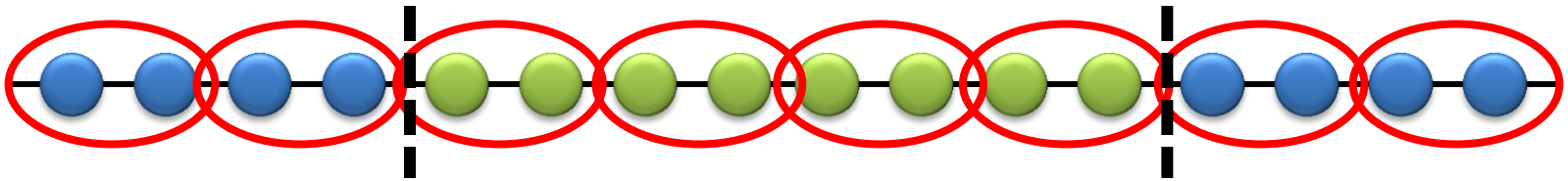
- Later, large N limit:

$$\dim(\mathcal{H}(\bullet)) = e^N$$

Short Range Entanglement

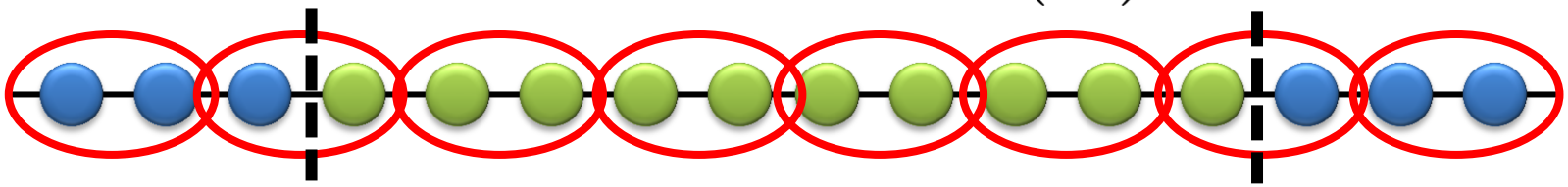
- Example 1:

A $S(A) = 0$

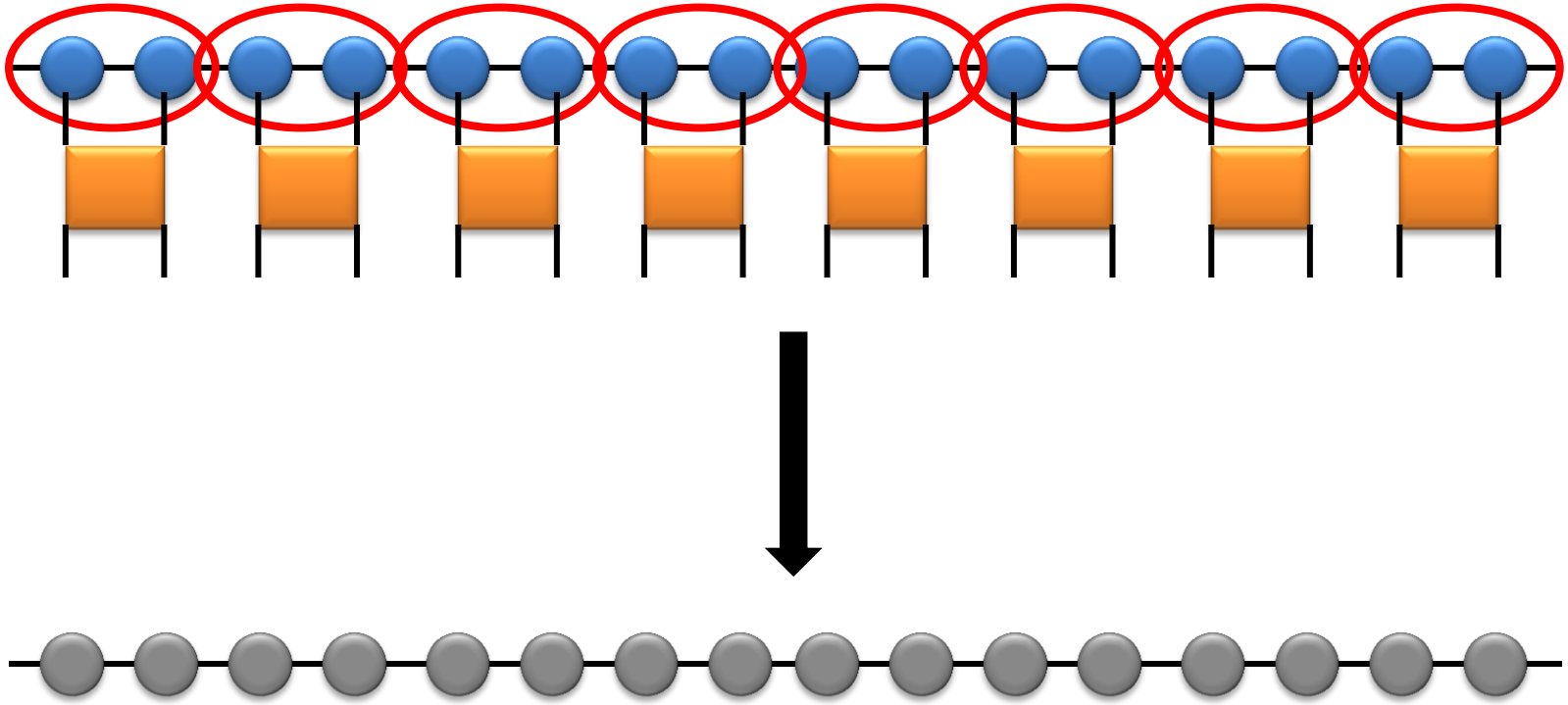


- Example 2:

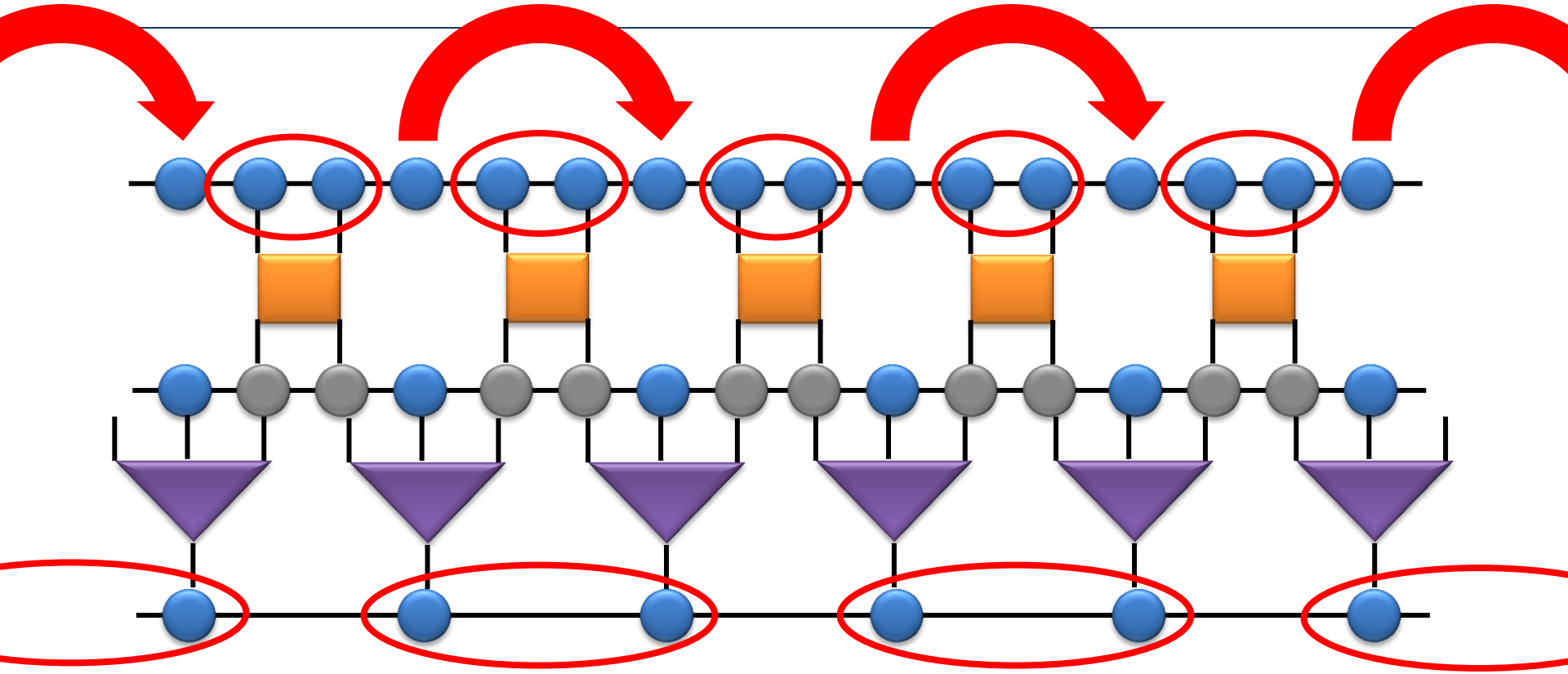
A $S(A) = 2\ln 2$

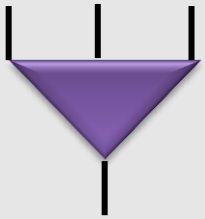


Building up entanglement

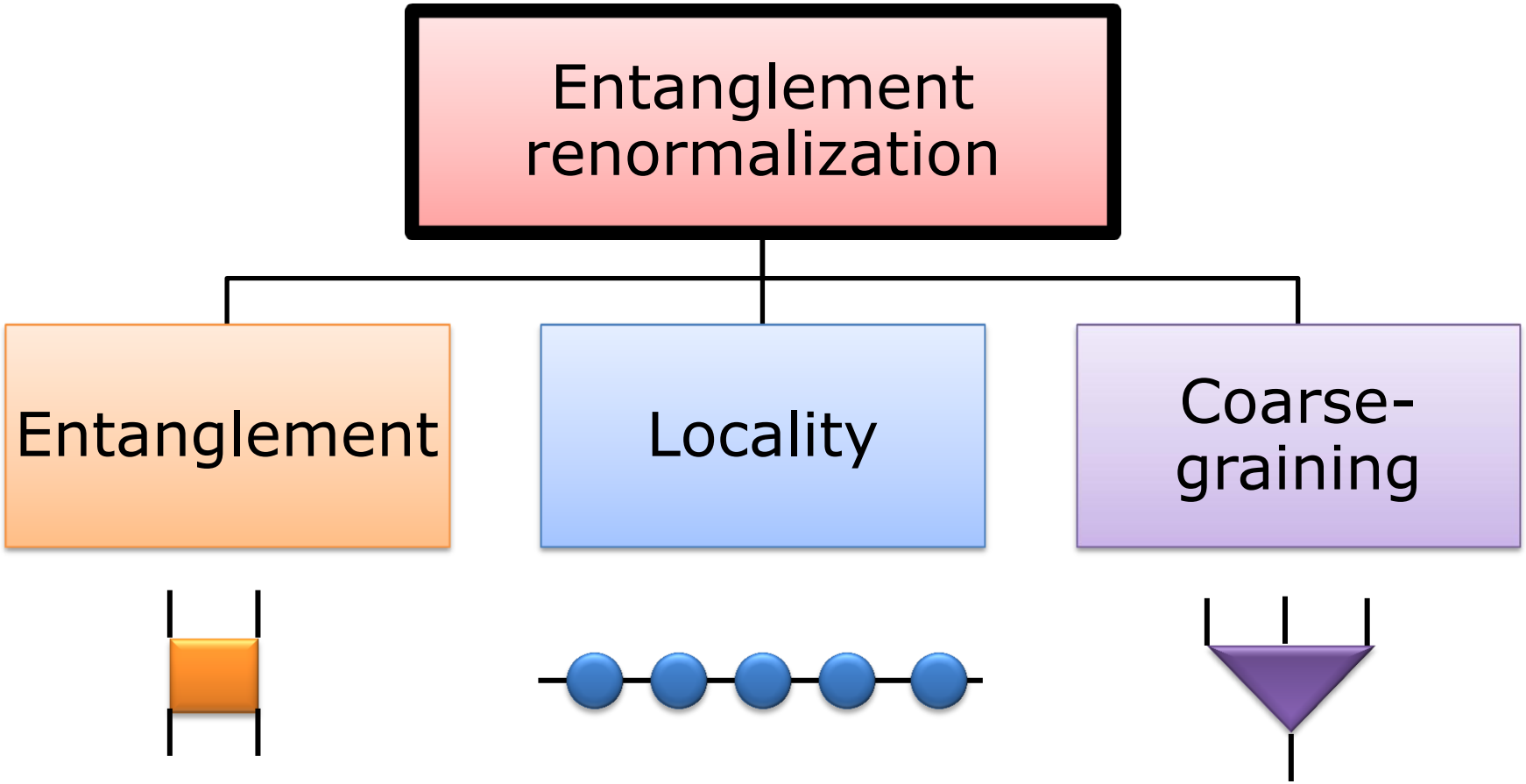


Longer range entanglement

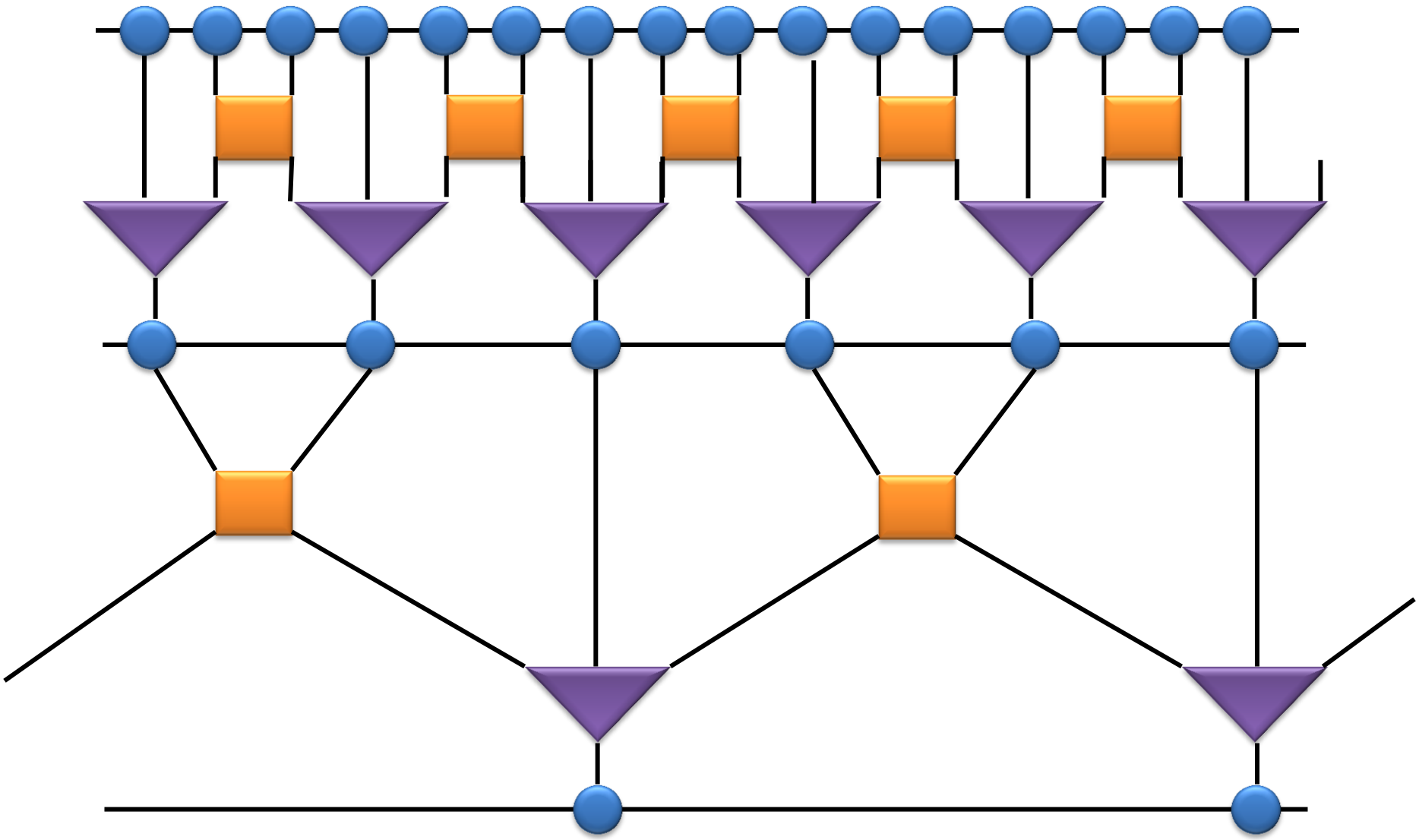




= coarse-grainer



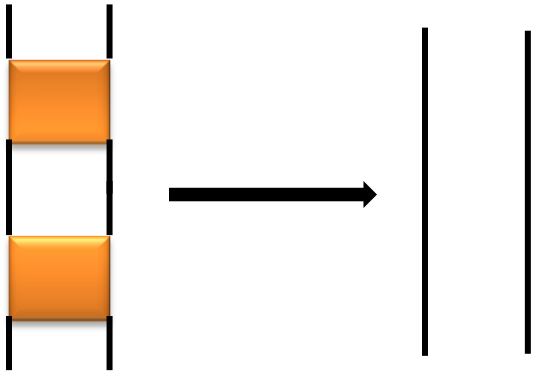
multi-scale entanglement *(Vidal '05)*
renormalization ansatz (MERA)



$k = 3,\ 3 \to 1$ scheme

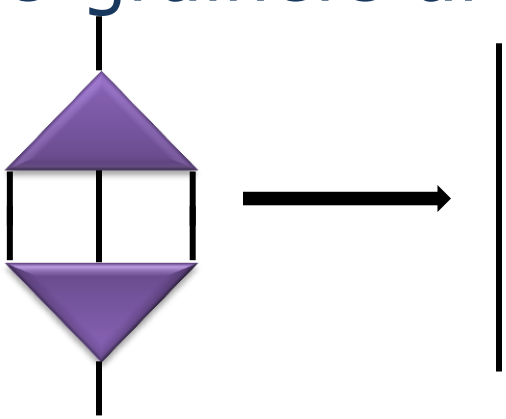
Unitaries and isometries

- Disentanglers are <u>unitaries</u>



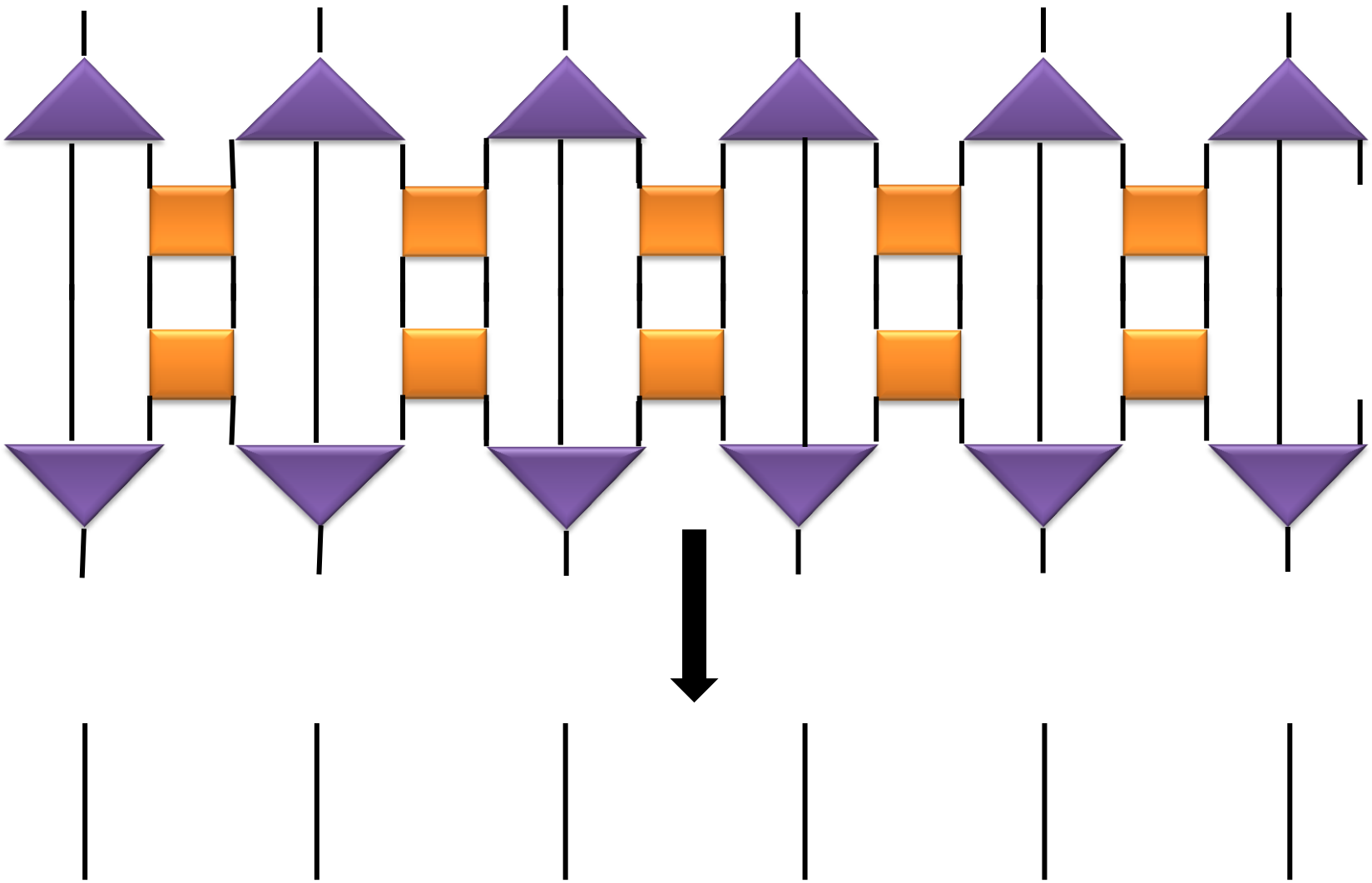
$U =$ 

- Coarse-grainers are <u>isometries</u>

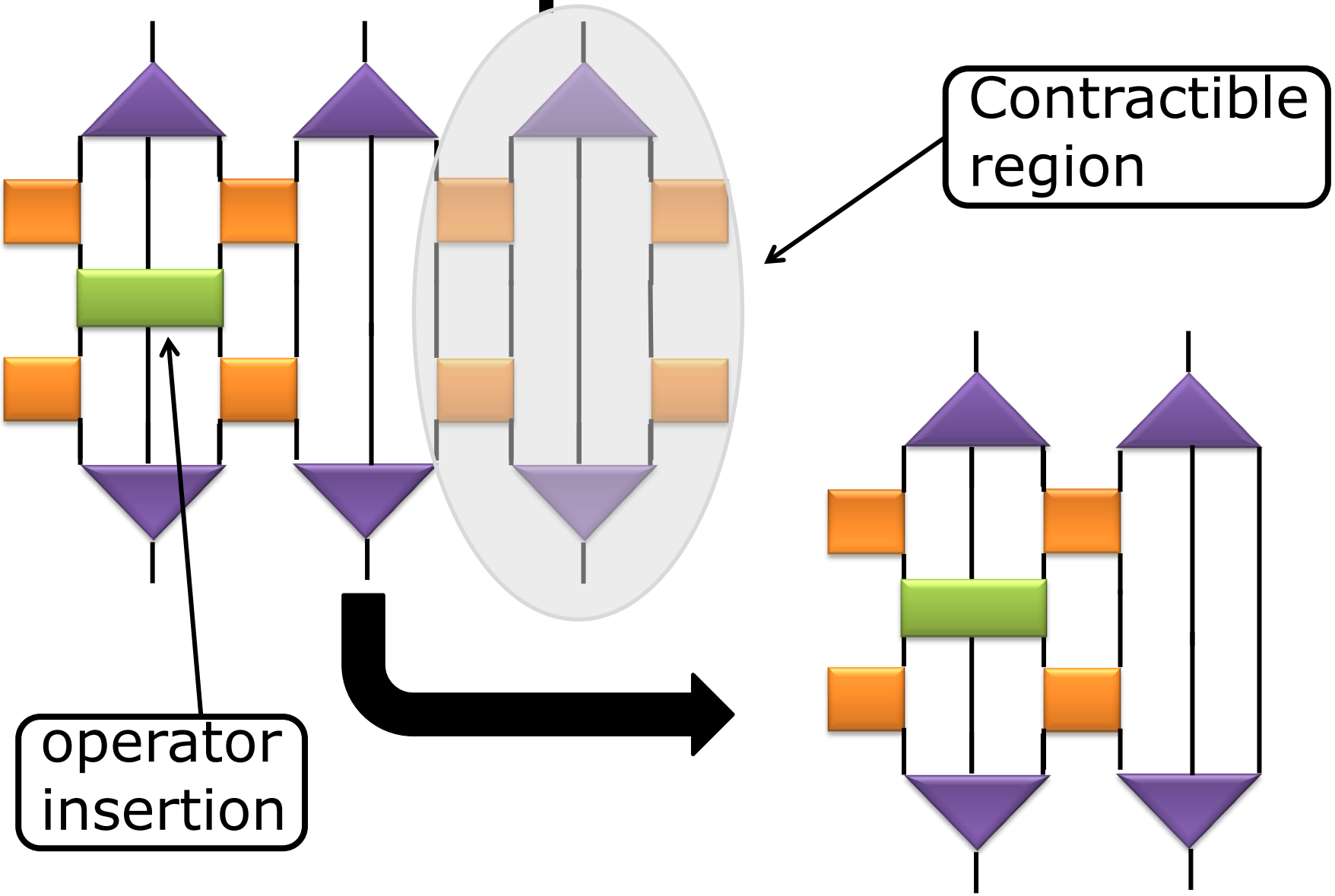


$W =$ 

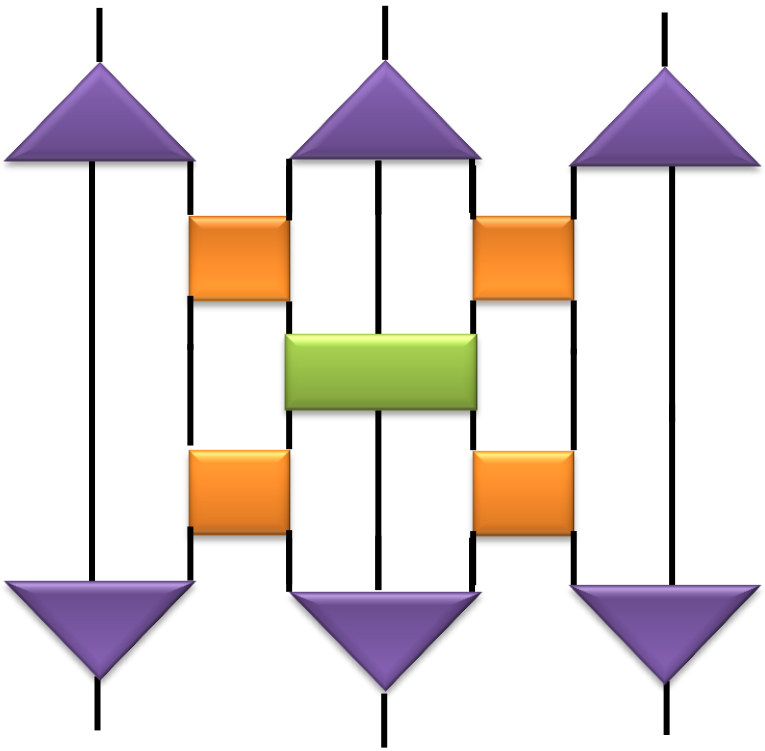
Contracting the network



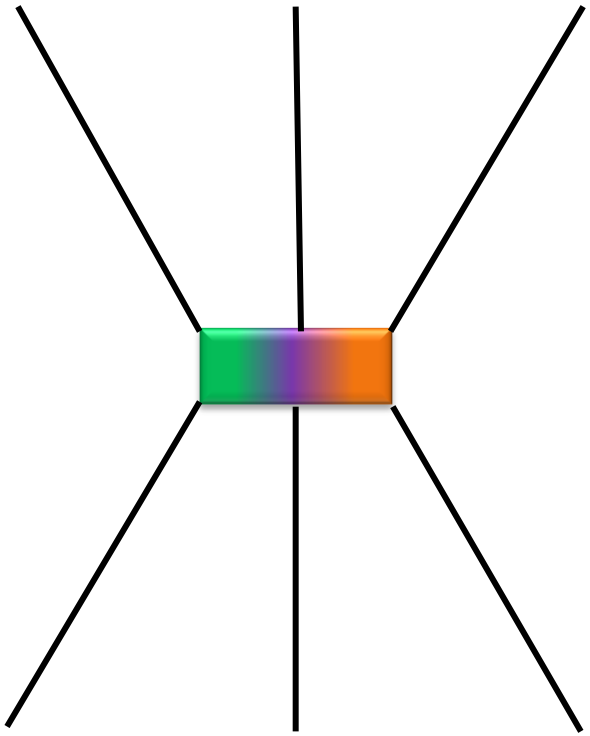
Operators



Renormalized operators



(Montangero et al. '08, Pfeifer et al. '09)

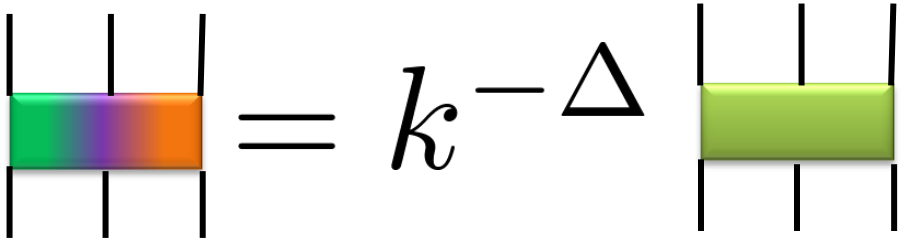


$$O_{ren} = \mathcal{C}(O) = V^{\dagger} O V$$

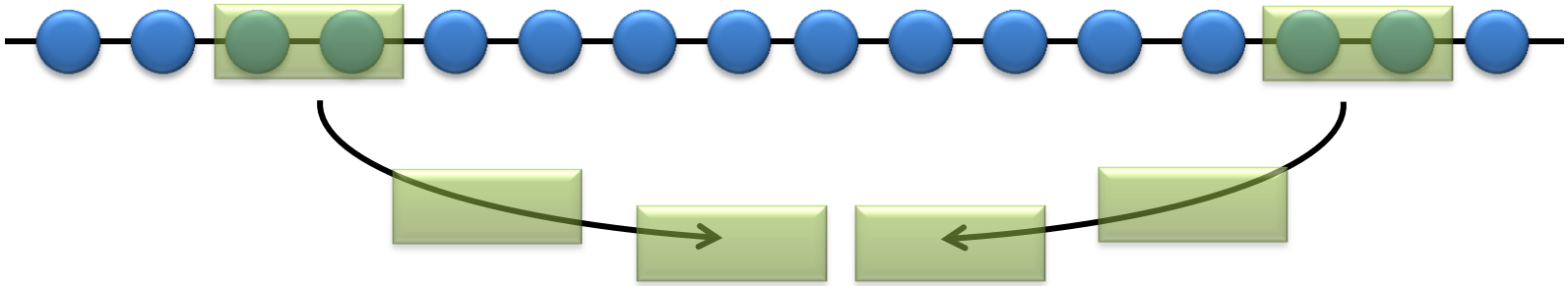
$$V = (\otimes U)(\otimes W)$$

Scaling operators

$$\mathcal{C}(O(x)) = k^{-\Delta} O(x/k)$$

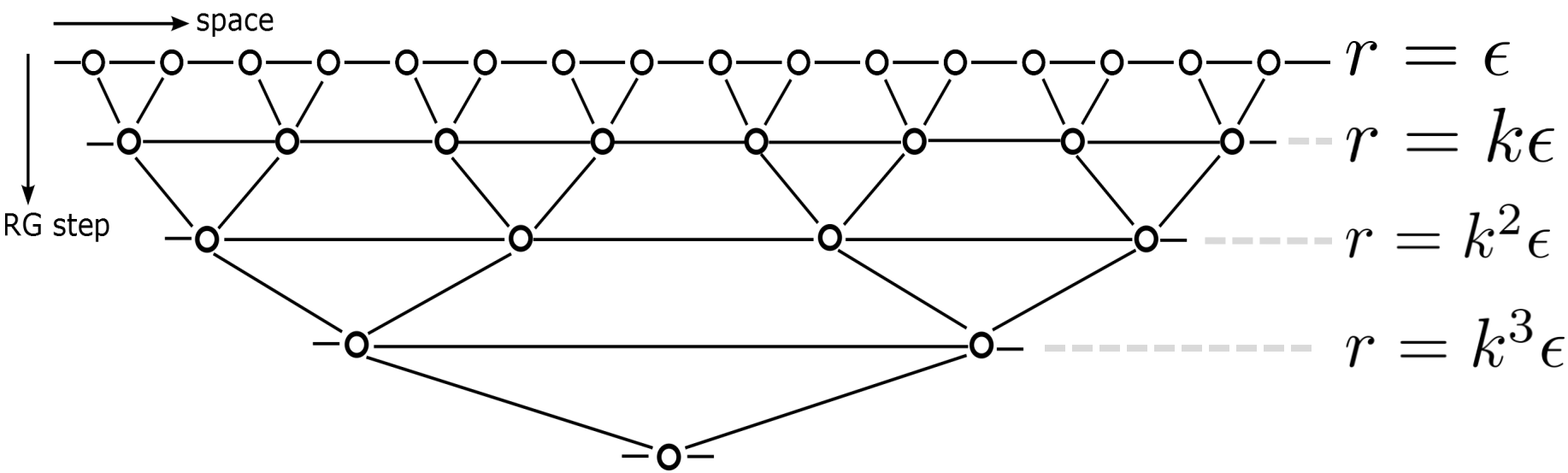


$$\mathcal{C}(O_1(0)O_2(x)) = k^{-\Delta_1} O_1(0) k^{-\Delta_2} O_2(x/k)$$



Collision after $\log_k(x/\epsilon)$ steps, $k^{-2\Delta \log_k(x/\epsilon)} = \left(\frac{\epsilon}{x}\right)^{2\Delta}$

Entanglement renormalization and holography



- Laplacian: $\mathrm{spec}(\triangle_G) \sim \mathrm{spec}(\triangle_{AdS})$
- Geodesics and correlations
- Entropy bounds

(Ryu-Takayanagi '06)

(BGS '09)

(see also van Raamsdonk '09)

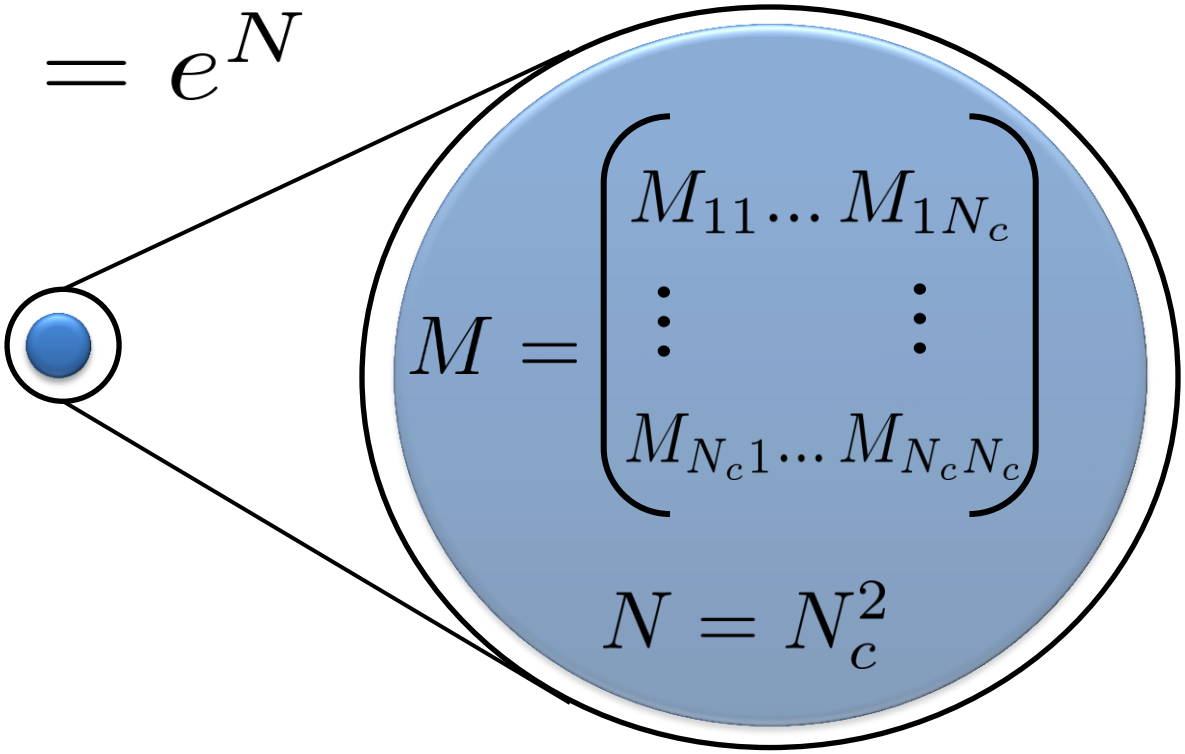
1. Large N limit
2. Operator dimensions
3. Geometry from entanglement
4. Mutual information
5. Comparing to holography

LARGE N AND STRONG COUPLING

(BGS '12 monday night)

Large N

$$\dim(\mathcal{H}(\bullet)) = e^N$$



$$\mathcal{S} = \frac{1}{g^2} \int dt d^d x \, \mathrm{tr} \left(\frac{1}{2} F^2 + (DM_1)^2 + (DM_2)^2 + [M_1, M_2]^2 \dots \right)$$

Operator dimensions

-  and  are strongly mixing at large N and strong coupling

$$O_{ren} = \mathcal{C}(O) = V^\dagger O V$$

- Operators reduced to identity

$$U \text{ is } \chi^2 \times \chi^2 \text{ generic unitary,}$$

$$\int dU_{Haar} U^\dagger (O \otimes 1) U = \frac{tr(O)}{\chi}(1 \otimes 1)$$

- Corrections $\sim 1/\chi^p$ typical dimension:

$$1/\chi^p \sim k^{-\Delta} \rightarrow \Delta \sim N \text{ or greater}$$

Low dimension operators

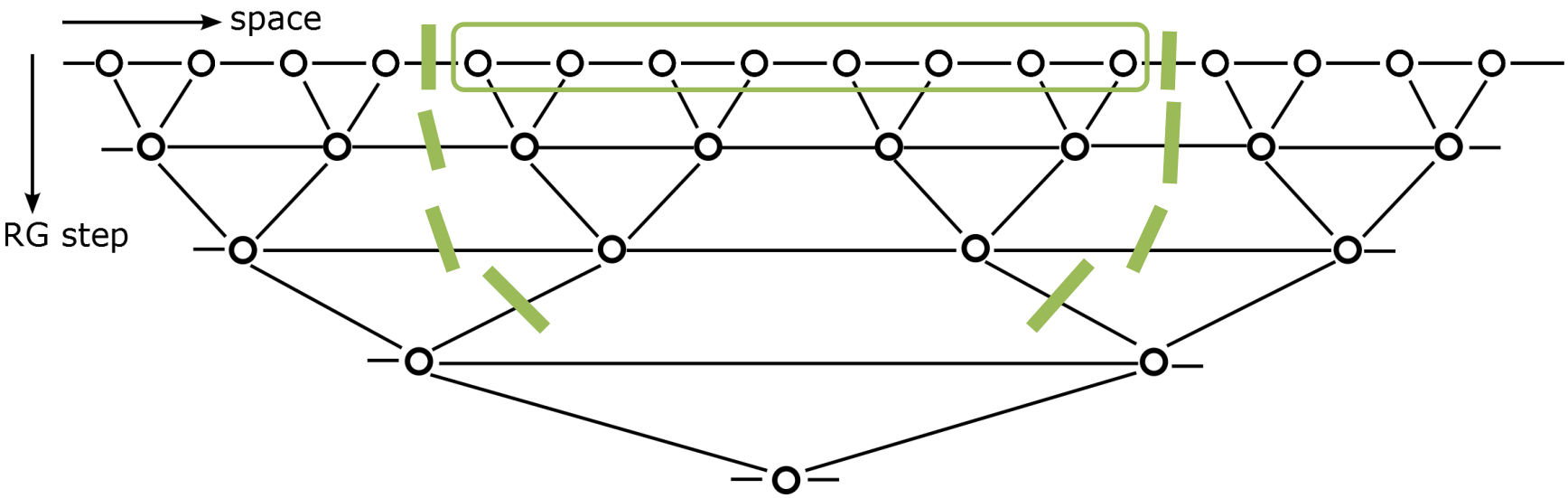
- Symmetry protects existence of a few low dimension operators, e.g.
  - Translation invariance $\mathcal{T}V = V\mathcal{T}$

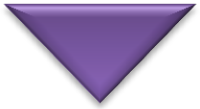
$$\partial O = \frac{O(x) - O(0)}{x}$$

$$\mathcal{C}(\partial O) = k^{-\Delta}\frac{O(x/k) - O(0)}{x} = k^{-\Delta-1}\frac{O(x/k) - O(0)}{x/k}$$

  - Hamiltonian density $O = h,\ \Delta_h = d + z$
  - Global symmetries, SUSY, …

Additive entanglement entropy



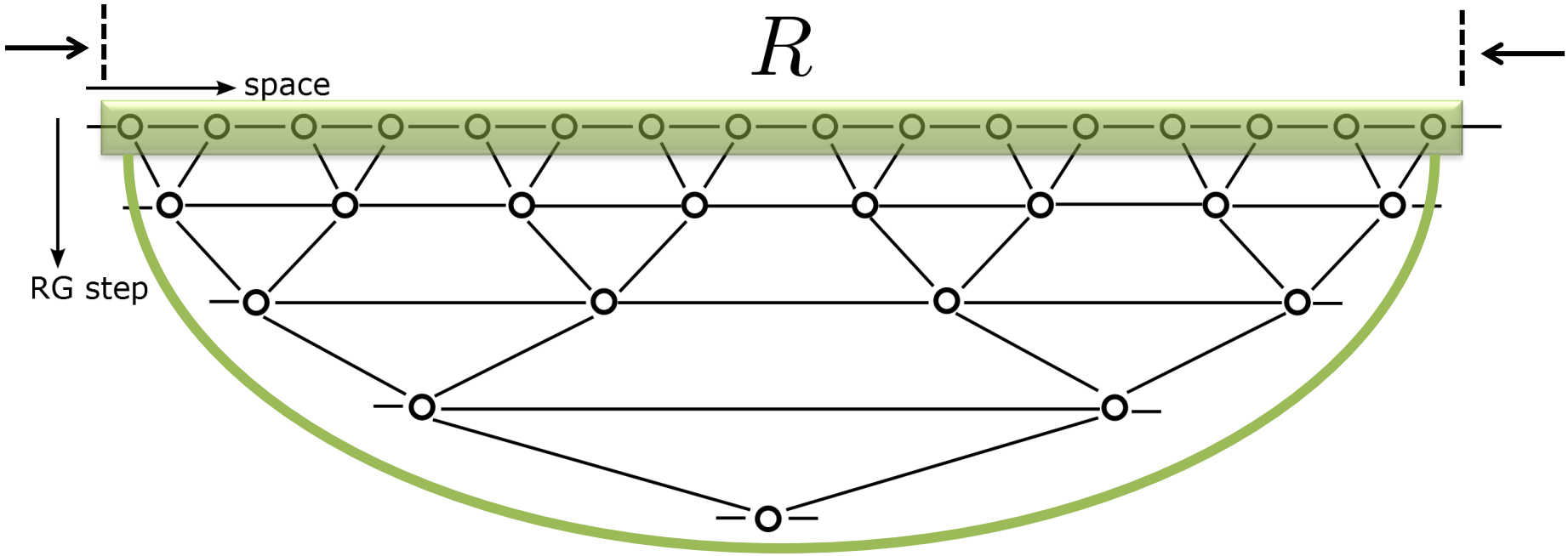
 and  are strongly mixing:

 every bond cut gives a definite entropy $\delta S \sim N$

(c.f. quantum expanders – ask me later)

(Hastings '07, Ben-Aroya Ta-Shma '07)

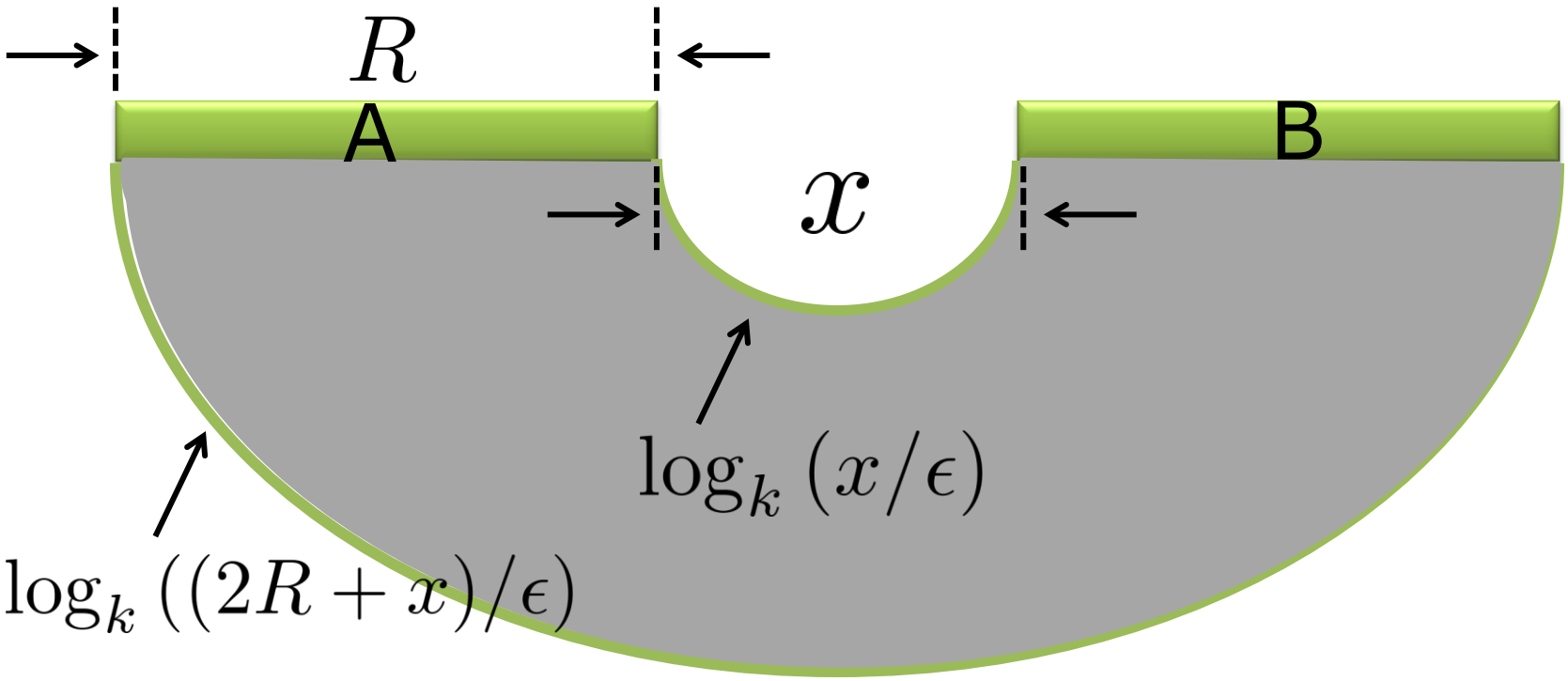
Geometry and entropy



Number of bonds cut = $2\log_k(R/\epsilon)$

Entropy = $S = \delta S(2\log_k(R/\epsilon))$

Mutual information

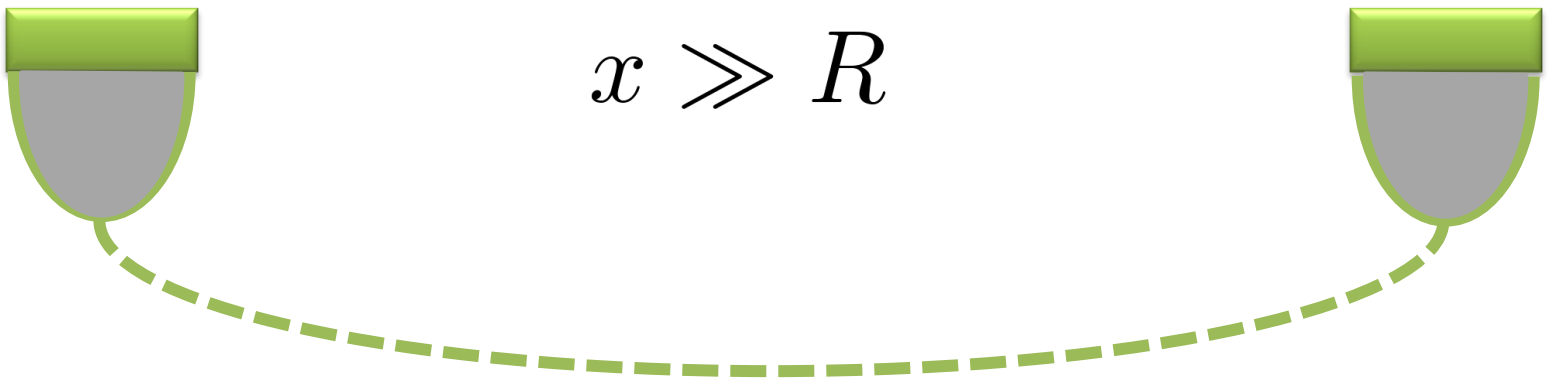


$$\mathcal{I}(A,B) = S(A) + S(B) - S(AB)$$

$$\mathcal{I} = \delta S(4\log_k(R/\epsilon) - 2\log_k(x/\epsilon) - 2\log_k((2R+x)/\epsilon))$$

$$\mathcal{I} = 2\delta S\ \log_k\left(\frac{R^2}{(2R+x)x}\right),\ x \ll R$$

Mutual information



$$\mathcal{I}(A,B) = S(A) + S(B) - S(AB)$$
$$\rightarrow \mathcal{I}(A,B) \approx 0$$

Corrections from low dim. operators:

$$\rho_{AB} = \rho_A \otimes \rho_B + \delta\rho,\ \mathcal{I}(A,B) \sim \delta\rho^2$$
$$\delta\rho \sim \langle O_A O_B \rangle_c,\ \mathcal{I} \sim \langle O_A O_B \rangle_c^2$$

(consistent with Wolf et al. '07)

(information locking, data hiding states?)

(Hayden et al.)

Holography proper

e.g. Einstein gravity *(Maldacena, Witten, Gubser-Polyakov-Klebanov)*

- Most operators have large dimension

$$\Delta \sim L_{AdS}/\ell_s \sim \lambda^{1/4} \ (\mathcal{N}=4,\ 3{+}1\mathrm{d}),\ \text{D-branes, etc.}$$

- Entropy and area, RT formula: *(Ryu-Takayanagi '06)*

$$S(A) = \frac{|\Sigma(A)|}{4G_N},\ \Sigma(A) = \text{bulk surface anchored at } \partial A$$

- Phase transitions in MI *(Headrick ...)*

CONJECTURE: basic MERA network ~ coarse holography; strongly coupled disentanglers and coarse-grainers ~ sharp holography

(Heemskerk-Penedones-Polchinski-Sully '09)

1. Continuous MERA
2. Entanglement per scale
3. RG causality
4. Fermi surfaces and branching MERA

ADDITIONAL TOPICS

(BGS '12 monday night)

Continuous MERA

- Replace discrete lattice with a continuous theory with cutoff
- UV state $|UV\rangle$, IR state $|IR\rangle$ (unentangled)

$$|UV\rangle = V|IR\rangle$$

$$V = e^{-iu_{IR}D} \qquad r = \epsilon e^{u}$$

$$D = \text{dilatation generator (regulated)}$$

(Haegeman-Osborne-Verschelde-Verstraete '11)

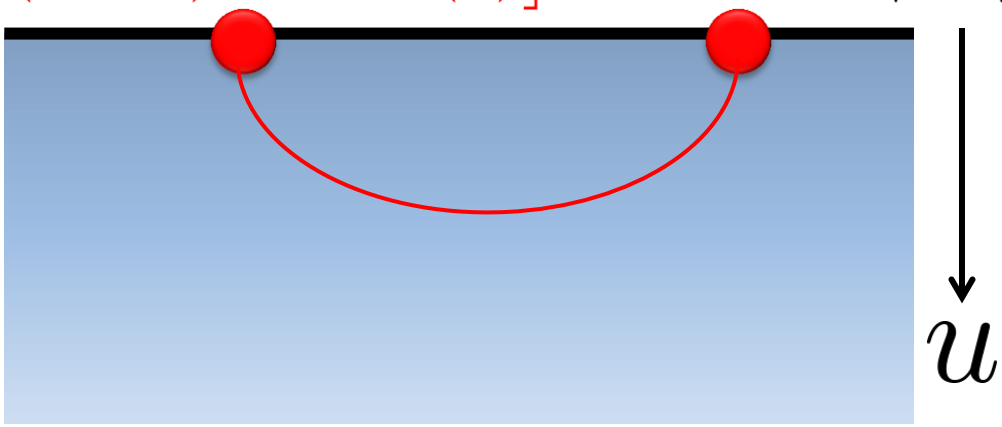
Correlations

$$e^{iuD}O(x)e^{-iuD} = e^{-\Delta u}O(xe^{-u})$$

Compare: $\mathcal{C}(O) = V^\dagger O V = k^{-\Delta} O$

$$\langle UV|O(x)O(0)|UV\rangle$$
$$= \langle IR|e^{i(u_{IR}-u)D}\left[e^{-u\Delta}O(xe^{-u})e^{-u\Delta}O(0)\right]e^{i(u_{IR}-u)D}|IR\rangle$$
$$= e^{-2\Delta\ln(x/\epsilon)}\langle O(\epsilon)O(0)\rangle$$

(Haegeman et al. '11)



Geometry

- State at scale u: $|u\rangle = e^{-iuD}|IR\rangle$
- DEFINE bulk "area element" =

$$dS(A) = S(A, u + du) - S(A, u)$$

- CHALLENGE: show that this satisfies the axioms of an area function

$$A \to |\Sigma(A)| \propto S(A)$$

(BGS '09, see also nice work in Nozaki-Ryu-Takayanagi '12)

- Use twist fields (focus on d=1): Φ_n

$$\langle \Phi_n(L)\Phi_n(0)\rangle_n = \mathrm{tr}(\rho^n_{[0,L]})$$

(Cardy-Calabrese, Caraglio-Gliozzi '08)

RG quantum quench

- Twist fields are non-local $(|IR\rangle \text{ is unentangled})$

$$\langle IR|\Phi_n(L)\Phi_n(0)|IR\rangle_n = 1$$

- Twist fields are scaling fields $\Delta_n = \frac{c}{12}\left(n - \frac{1}{n}\right)$

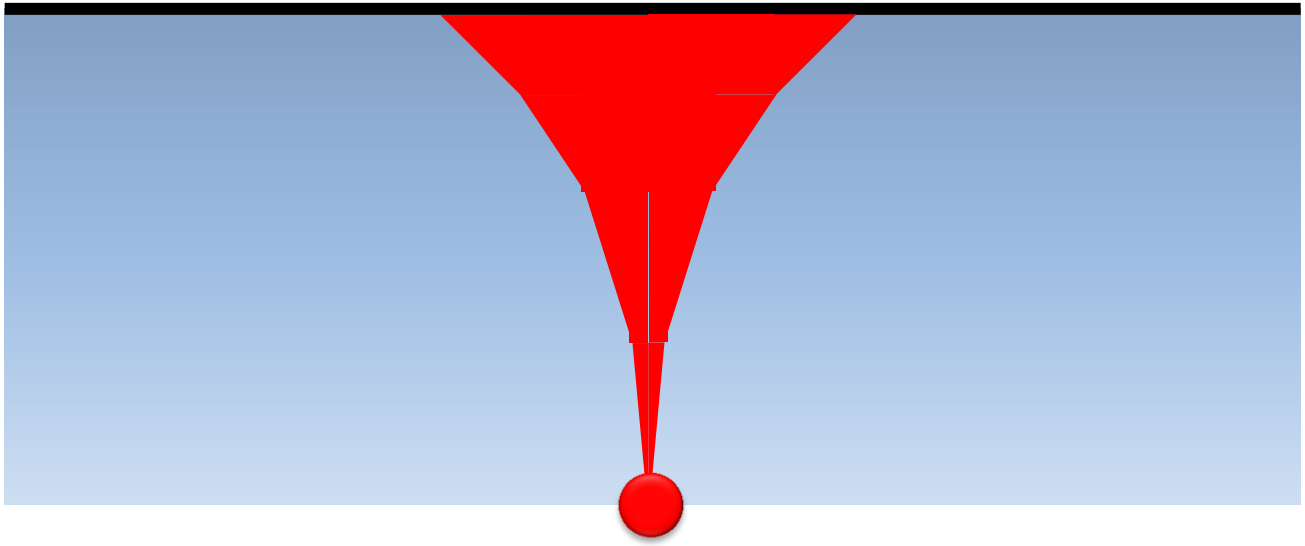
- State $|u\rangle = e^{-iuD}|IR\rangle$ like a quantum quench! Compute entropy added …

$$\langle u|\Phi_n(L)\Phi_n(0)|u\rangle_n$$
$$= e^{-2u\Delta_n}\langle IR|\Phi_n(Le^{-u})\Phi_n(0)|IR\rangle_n$$

Provided u < ln(L)

$$\frac{dS(L)}{du} = \frac{c}{3}, \text{ saturates at } u = \ln\left(L/\epsilon\right)$$

RG causality



- Exponential "RG light cone":

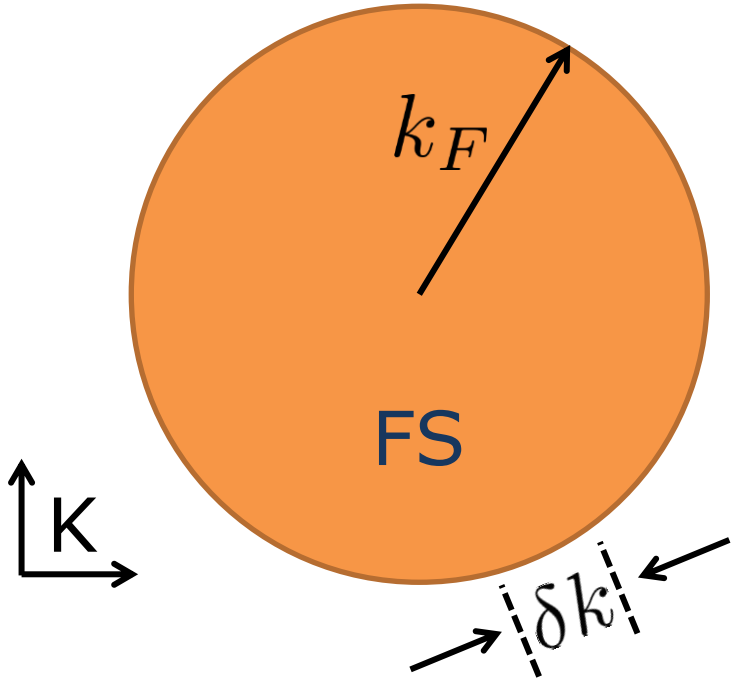
$$e^{iuD}O(x)e^{-iuD} = e^{-\Delta u}O(xe^{-u})$$

- UV perturbations don't affect much of the RG circuit

(see also Evenbly and Vidal)

Fermi surface

- How should we understand the Fermi surface? Emergent dimensions!?



- UV-IR duality $\delta k \sim 1/r$
- Growing number of low energy modes: $(1{+}1\mathrm{d\ CFTs})$

$$dS(A) \sim \left(\frac{R}{r}\right)^{d-1} (k_F r)^{d-1} \frac{dr}{r}$$

like branching MERA!

- Locality? Fermi-liquid – no! FS+gauge field – maybe?

(Evenbly-Vidal `12)

Hyperscaling violation geometry?

- Different kind of dilatation generator

$$D \sim \int_{FS} D_{1d}$$

$$V = e^{-iu_{IR}D}$$

$$|UV\rangle = V|IR\rangle$$

- Entanglement per scale

$$\frac{dS(A)}{du} \sim (k_F R)^{d-1} du$$

- Consistent with a geometrical calculation!

(Ogawa-Takayanagi-Ugajin '11, Huijse-Sachdev-BGS '12)

Summary

- Geometrical picture of entropy in cMERA
- Fermi surfaces as emergent dimensions
- RG causality
- Sparse spectrum of scaling dimensions emerges with definite entropy at large N
- Phase transitions in mutual information
- General (holographic?) framework for combining entanglement and renormalization?

Future

Dynamics

- More covariant form of ER/holo relationship
- Entanglement in time dependent states

Holography and numerics

- Look for CFTs by imposing entanglement + symmetry
- Suggestions from holography to improve tensor network algorithms

Many other directions,
so get involved! Thanks!