

Asymptotics of 4D Quantum Field Theory



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Outline

Results are controversial...

J. Fortin, B. Grinstein, A. Stergiou, arXiv:1106.2540,
1107.3840, 1110.1634, 1202.4657, 1206.2921, 1208.3674

4D example consistency

- New version of perturbative argument that addresses criticisms of FGS
- Remarks about discrepancy
- Non-perturbative scale without conformal

Introduction

Old idea: QFT simplifies in UV and IR

Masses unimportant

$$E \gg m \Leftrightarrow m \rightarrow 0$$

$$E \ll m \Leftrightarrow m \rightarrow \infty \quad \text{integrate out massive states}$$

Asymptotic UV/IR dynamics has scale symmetry?

This talk:

- Only possible perturbative UV/IR asymptotics is conformal invariance
- Non-perturbative scale without conformal implausible

Dilaton Effective Action

(Z. Komargodski, A. Schwimmer arXiv:1107.3987)

$g_{\mu\nu}(x)$ = source to define $T^{\mu\nu}$

$$e^{iW[g_{\mu\nu}]} = \int d[\Phi] e^{iS[\Phi, g_{\mu\nu}]}$$

$$\langle T^{\mu\nu}(x) \rangle = \frac{2}{\sqrt{-g(x)}} \frac{\delta W}{\delta g_{\mu\nu}(x)}$$

Sufficient to work with conformally flat metric

$$g_{\mu\nu}(x) = \Omega^2(x) \eta_{\mu\nu} \quad \Omega(x) = 1 + \frac{\varphi(x)}{f}$$

$$\langle T^\mu{}_\mu(x) \rangle = f \frac{\delta W}{\delta \varphi(x)}$$

Dilaton Amplitude

$$(2\pi)^4 \delta^4(p_1 + \cdots + p_4) \mathcal{M}(\varphi(p_1)\varphi(p_2) \rightarrow \varphi(-p_3)\varphi(-p_4))$$

$$\equiv \frac{\delta}{\delta\varphi(p_1)} \cdots \frac{\delta}{\delta\varphi(p_4)} W$$

$$\begin{aligned} \mathcal{M} = \frac{1}{f^4} & \left[\langle T(p_1) \cdots T(p_4) \rangle \right. \\ & + \langle T(p_1 + p_2)T(p_3 + p_4) \rangle + \text{permutations} \\ & \left. + \langle T(p_1 + p_2 + p_3)T(p_4) \rangle + \text{permutations} \right] \end{aligned}$$

$$T = T^\mu{}_\mu \quad p_1^2 = \cdots = p_4^2 = 0$$

Think of φ as particle

$\Rightarrow \mathcal{M} = \text{leading contribution to } \varphi \text{ scattering}$
as $f \rightarrow \infty$

Counterterms

$\mathcal{M} \sim p^4 \Rightarrow$ possible counterterms are $O(\partial^4)$

$O(\partial^0) : \sqrt{-g}$ tune away (cosmological const)

$O(\partial^2) : \sqrt{-g} R(g) \propto \Omega \square \Omega$

$O(\partial^4) : \sqrt{-g} R^2(g) \propto (\Omega^{-1} \square \Omega)^2$

} vanish on shell

$$\sqrt{-g} W^2 = 0$$

$$\sqrt{-g} E_4(g) = \text{total derivative}$$

No counterterm!

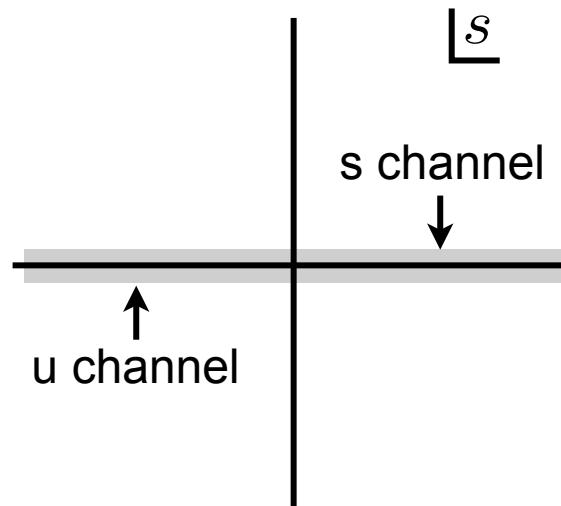
Analyticity & Unitarity

$$A(s) = \lim_{t \rightarrow 0} \mathcal{M}(s, t, u)$$

$$s + t + u = 0$$

$$s \leftrightarrow u \text{ crossing} \Rightarrow A(s) = A(-s)$$

Analytic structure:

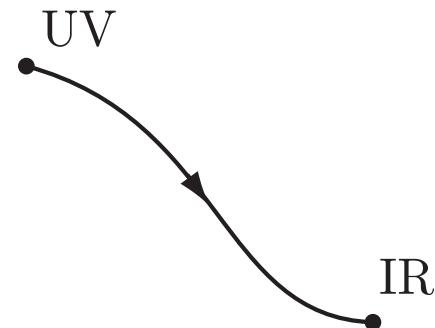


$$A(s) = \alpha(s) \frac{s^2}{f^4}$$

Unitarity: $\text{Im } \alpha(s) \geq 0$

a-theorem

Flow from UV CFT to IR CFT



UV: $W[e^{-2\tau} \eta_{\mu\nu}] \rightarrow W_{\text{UV}}[e^{-2\tau} \eta_{\mu\nu}] + \text{relevant}$

IR: $W[e^{-2\tau} \eta_{\mu\nu}] \rightarrow W_{\text{IR}}[e^{-2\tau} \eta_{\mu\nu}] + \underbrace{\text{irrelevant}}_{\text{no } \partial^4 \text{ counterterm}}$

UV or IR CFT:

$$W_{\text{CFT}}[e^{-2\tau} g_{\mu\nu}] = W_{\text{CFT}}[g_{\mu\nu}] + \underbrace{S_{\text{WZ}}[\tau; g_{\mu\nu}]}_{\text{local dilaton interaction}}$$

a-theorem

$$A(s) = \frac{\alpha(s)s^2}{f^4} \quad \alpha(s) \rightarrow -8a_{\text{IR,UV}}$$

$$\tilde{\alpha}(s) = \alpha(s) + 8a_{\text{UV}}$$

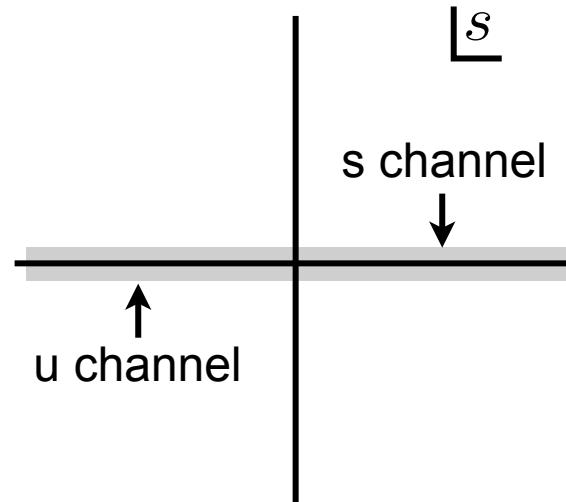
Satisfies unsubtracted dispersion relation

$$\tilde{\alpha}(0) = 8(a_{\text{UV}} - a_{\text{IR}}) = \frac{2}{\pi} \int_0^\infty ds \frac{\text{Im } \alpha(s)}{s} \geq 0$$

QED

Perturbative Flows

$$\alpha(s) \sim s^0$$



⇒ write subtracted dispersion relation

$$\alpha(s) = \alpha(s_0) + \frac{s - s_0}{\pi} \int_{-\infty}^{\infty} dx \frac{\text{Im } \alpha(x)}{(x - s)(x - s_0)}$$

$s_0 \gg s \Rightarrow s_0 = \text{UV cutoff}$

$s = \text{IR scale}$

IR Limit

As $s \rightarrow 0$

$$\alpha(s) \rightarrow \frac{2}{\pi} \int_0^{\infty} \frac{dx}{x} \operatorname{Im} \alpha(x) + \text{finite}$$

No counterterm for $\alpha(s)$

$\Rightarrow \alpha(s)$ calculable in terms of renormalized couplings

$\Rightarrow \alpha(s)$ finite as $s \rightarrow 0$

\Rightarrow integral must converge

$\Rightarrow \operatorname{Im} \alpha(s) \rightarrow 0$ as $s \rightarrow 0$

Analogous argument for UV limit...

Unitarity

$$\text{Im } \alpha(s) \geq 0$$

$$\text{Im } \alpha(s) \propto \sum_{n=2}^{\infty} \sum_{\Psi_n} \int d\Phi_n \underbrace{|\mathcal{M}(\varphi\varphi \rightarrow \Psi_n)|^2}_{\geq 0}$$

$n = \text{number of particles}$

$$\begin{aligned} \mathcal{M}(\varphi\varphi \rightarrow \Psi_n) &= \langle \Psi_n | T(p_1)T(p_2) + T(p_1 + p_2) | 0 \rangle \\ &\rightarrow 0 \quad \text{for all states } \Psi_n \end{aligned}$$

Strongly suggests $T \rightarrow 0$ as an operator

$$(\langle \Psi | \mathcal{O} | 0 \rangle = 0 \text{ for all } \Psi \Rightarrow \mathcal{O} \equiv 0)$$

Trace Anomaly

$$T = \sum_A \beta_A \mathcal{O}^A + \mathcal{E} + R^2 \text{ terms} \quad \mathcal{L}_{\text{int}} = \sum_A \lambda_A \mathcal{O}^A$$

definition!

\mathcal{E} = equation of motion operator
vanishes except for contact terms

$$\langle \mathcal{E}(x) \mathcal{O}(y) \rangle = \delta^4(x - y) \langle \delta \mathcal{O}(x) \rangle$$

R^2 terms: additional contact terms
for multiple T insertions

Trace Anomaly

Contact don't matter for states with $J \neq 0$

$$\begin{aligned}\mathcal{M}(\varphi\varphi \rightarrow \Psi_n) &= \langle \Psi_n | T(p_1) T(p_2) | 0 \rangle \\ &= \sum_{A,B} \beta_A \beta_B \langle \Psi_n | \mathcal{O}_A(p_1) \mathcal{O}_B(p_2) | 0 \rangle\end{aligned}$$

$$\sum_{J \neq 0} \int d\Phi |\langle \Psi | T(p_1) T(p_2) | 0 \rangle|^2 = \sum_{A,B} \beta_A^2(s) \beta_B^2(s) \underbrace{[C_{AB} + O(\hbar)]}_{\geq 0}$$

$A, B =$ largest term

$$\Rightarrow \int_0 \frac{ds}{s} \beta_A^2(s) \beta_B^2(s) < \infty$$

$$\Rightarrow \beta_A \rightarrow 0 \text{ for all } A \quad \Rightarrow \text{conformal in IR}$$

Controversy

- “Running a ” vs. $\alpha(s)$

CFT asymptotics: $\alpha(s) \rightarrow -8 a_{\text{IR,UV}}$

$\alpha(s)$ = non-local amplitude away from CFT

- Neglect of counterterm?

$$\Delta\mathcal{L} = \partial^\mu \lambda_A J_\mu^A \quad J_\mu^A = \Phi S^A \overleftrightarrow{\partial}_\mu \Phi + \Psi F^A \sigma_\mu \Psi$$

Relevant for definition of composite operators

$$T = \beta_A \mathcal{O}^A \quad \mathcal{O}^A(x) = \frac{\delta}{\delta \lambda_A(x)}$$

definition!

Trace Anomaly Redux

Structure of trace anomaly and perturbative RG flows
greatly clarified in the 90's

I. Jack, H. Osborn, NPB 343 (1990); H. Osborn, NPB 363 (1991).

$$T = \sum_A \beta_A \mathcal{O}^A + \partial_\mu K^\mu + \mathcal{E} + R^2 \text{ terms}$$

$$K_\mu = s_{ab}(\lambda) \Phi^a \overleftrightarrow{\partial}_\mu \Phi^b + f_{ij}(\lambda) \Psi^i \sigma_\mu \Psi^j$$

= flavor current

Equations of motion (field redefinition)

$$\Rightarrow \partial_\mu K^\mu = \delta_K \mathcal{L} = (\delta_K \lambda_A) \mathcal{O}^A$$

$$T = \sum_A B_A \mathcal{O}^A + \tilde{\mathcal{E}} + R^2 \text{ terms} \quad \underbrace{B_A}_{\text{invariant under field redefinitions}} = \beta_A + \delta_K \lambda_A$$

Field Redefinitions

$$\Phi^a \rightarrow F_{ab}(\lambda)\Phi^b$$

$$\frac{1}{2}\partial^\mu\Phi^a\partial_\mu\Phi^a \rightarrow \underbrace{\frac{1}{2}F_{ab}F_{ac}\partial^\mu\Phi^b\partial_\mu\Phi^c}_{\begin{array}{l} \text{wavefunction} \\ \text{renormalization} \end{array}} + \underbrace{\frac{1}{2}\partial^\mu F_{ab}\partial_\mu F_{ac}\Phi^b\Phi^c}_{\begin{array}{l} \text{relevant only for} \\ [\Phi^4][\Phi^4] \text{ contact term} \end{array}}$$
$$+ \underbrace{F_{ac}\partial^\mu F_{ab}\Phi^b\partial_\mu\Phi^c}_{\text{origin of } \partial_\mu K^\mu \text{ term}}$$

- $\alpha(s)$ invariant under field redefinitions
- β_A, K^μ not invariant

Beyond Perturbation Theory

Assume scale without conformal symmetry

$$S^\mu = T^\mu{}_\nu x^\nu + V^\mu \quad C_\mu = \text{source for } V^\mu$$

Scale Ward identity

$$W[e^{2\sigma}g_{\mu\nu}, C_\mu + \partial_\mu\sigma] = W[g_{\mu\nu}, C_\mu] + \underbrace{S_{\text{WZ}}[\sigma; g_{\mu\nu}, C_\mu]}_{\text{local}}$$
$$\text{Im } \alpha(s) = Cs^2 \quad \text{exactly}$$

$$\alpha(s) = \text{finite} \Rightarrow C = 0 \quad \Rightarrow \alpha(s) = \text{constant}$$

$$\mathcal{M}(\varphi\varphi \rightarrow \Psi_n) = \langle \Psi_n | T(p_1)T(p_2) + T(p_1 + p_2) | 0 \rangle$$

$\rightarrow 0$ for all states Ψ_n

$\Rightarrow T \equiv 0 ?$

Factorization?

$$\langle T(x_1)T(x_2)T(x_3)T(x_4) \rangle = \frac{\text{const}}{(x_1 - x_2)^8(x_3 - x_4)^8} + \text{perms}$$
$$+ \text{local}$$

$\Rightarrow \text{Im } \alpha(s) = \text{ local, may vanish}$

Factorized form holds at leading order in $1/N \dots$

OPE: $TT \rightarrow 1 + \underbrace{\dim}_{\text{no } T} \geq 8$

Looks implausible, but not ruled out

Riva-Cardy Model

Non-unitary, scale but not conformal

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\xi}{2}(\partial^\mu A_\mu)^2$$

Allow gauge non-invariant operators, states

$$V_\mu = (\partial^\nu A_\nu)A_\mu$$

Checks:

- $\alpha(s) = \text{constant}$
- Relax on-shell condition

$$\Rightarrow \mathcal{M} \sim \ln s$$

Corresponds to $\sqrt{-g} R^2(g)$ counterterm

Conclusions

- $T \rightarrow 0$ in perturbation theory
- Scale without conformal invariance looks implausible beyond perturbation theory