RG Limit Cycles

(Part I)

Andy Stergiou
UC San Diego

based on work with
Jean-François Fortin and Benjamín Grinstein

Outline

- ➡ The physics:
  - Background and motivation
  - New improved SE tensor and scale invariance
  - Associated RG flows
  - Stability and scheme changes
- ➡ The examples:
  - Method of search
  - Scalars; scalars and spinors in $d = 4 - \epsilon$
  - A QFT in $d = 4$
- ➡ Future work and conclusion

The physics

Why care about scale without conformal?

Phases of QFTs:

- IR free
  - With mass gap: Exponentially decaying correlators (e.g. confinement)
  - Without mass gap: Trivial correlators (e.g. Coulomb phase)
- IR interacting
  - CFTs: Power-law correlators
  - SFTs???: Power-law correlators???

IR-limits of RG flows

- Strong coupling (e.g. QCD)
- Fixed points (CFT)
- Limit cycles (???)
- Ergodic trajectories (???)

Scale and conformal invariance

The symmetry group of QFTs usually consists of the Poincaré group and an internal symmetry group.

The Poincaré group can be extended to include supersymmetry, and/or conformal transformations.

But, in principle, only scale transformations could be allowed, without special conformal ones.

In a lot of examples if the theory is unitary and scale-invariant, then it is automatically conformally invariant.

Scale $\overset{?}{\Rightarrow}$ conformal invariance

In $d = 2$ the answer is yes! (Polchinski, 1988; Zamolodchikov, 1986)

Operating assumptions:

- Unitarity
- Finiteness of correlators of SE tensor

Relaxing these assumptions, leads to examples.

(Riva & Cardy, 2006; Hull & Townsend, 1986)

Many examples in $d > 2$ have suggested the same answer, but there has been no proof.

Counterexamples are classical:

- An unconventional field theory for a scalar (Jackiw & Pi, 2011)
- Free Maxwell theory in $d \neq 4$ (Pons, 2009; Jackiw & Pi, 2011; El-Showk, Nakayama & Rychkov, 2011)
- Holography: Kerr-AdS black holes in $d = 5, 7$ (Awad & Johnson, 1999)
- Extension of Liouville theory (Iorio, O'Raifeartaigh, Sachs & Wiesendanger, 1997)

Is there room for scale without conformal invariance?

Maybe there is a proof, since so many examples show that scale implies conformal invariance.

But it would be much more interesting if such a proof did not exist:

- CFTs are very tractable nontrivial QFTs, and we can hope that SFTs are also tractable
- SFTs are almost completely unexplored
- Since the symmetry is weaker, they must be richer than CFTs
- Possibly, there's a whole new class of QFTs!

Conformal invariance

Object of interest: Stress-energy tensor, $T_{\mu\nu}$ (symmetric)

A theory has conformal invariance iff

$$T_\mu^{\ \mu} = \partial_\mu \partial_\nu L^{\mu\nu}$$

Equivalently, there is then a new improved SE tensor, $\Theta_{\mu\nu}$, with

(Callan, Coleman & Jackiw, 1970)

$$\boxed{\Theta_\mu^{\ \mu} = 0}$$

Conformal implies scale, but algebra doesn't imply the converse:

$$[M_{\mu\nu}, P_\rho] = i(\eta_{\mu\rho}P_\nu - \eta_{\nu\rho}P_\mu), \qquad [M_{\mu\nu}, K_\rho] = i(\eta_{\mu\rho}K_\nu - \eta_{\nu\rho}K_\mu),$$
$$[M_{\mu\nu}, M_{\rho\sigma}] = i(\eta_{\mu\rho}M_{\nu\sigma} - \eta_{\nu\rho}M_{\mu\sigma} + \eta_{\nu\sigma}M_{\mu\rho} - \eta_{\mu\sigma}M_{\nu\rho}),$$
$$[D, P_\mu] = -iP_\mu, \qquad [D, K_\mu] = iK_\mu, \qquad [K_\mu, P_\nu] = 2i(\eta_{\mu\nu}D - M_{\mu\nu})$$

Scale invariance

If we ask for just scale invariance, then the dilatation current

$$\mathcal{D}^\mu = x_\nu \Theta^{\mu\nu} - V^\mu$$

Virial current

is conserved if

$$\boxed{\Theta_\mu^{\ \mu} = \partial_\mu V^\mu}$$

with $V^\mu \neq J^\mu + \partial_\nu L^{\mu\nu}$ where $\partial_\mu J^\mu = 0$. (Polchinski, 1988)

In an SFT the trace of the SE tensor is nonzero, but just a total derivative.

Scale without conformal invariance

The question of scale without conformal invariance can thus be asked as follows: (Polchinski, 1988)

Are there nontrivial candidates for V^μ ?

Constraints on virial:

- Gauge-invariant
- Scaling dimension $d-1$ in d spacetime dimensions

Attempt: ϕ^4 theory in $d=4-\epsilon$

Answer: No nontrivial candidate for V^μ

→ **No possibility for scale without conformal invariance**

Same conclusion for ϕ^6 in $d=3-\epsilon$ and ϕ^3 in $d=6-\epsilon$.

Most general QFT in $d = 4$

But in more complicated theories there are nontrivial candidates for V^μ.

Consider the most general renormalizable QFT:

$$\begin{aligned}
\mathscr{L} = &-\mu^{-\epsilon} Z_A \frac{1}{4g_A^2} F^A_{\mu\nu} F^{A\mu\nu} + \tfrac{1}{2} Z^{\frac{1}{2}}_{ab} Z^{\frac{1}{2}}_{ac} D_\mu \phi_b D^\mu \phi_c \\
&+ \tfrac{1}{2} Z^{\frac{1}{2}*}_{ij} Z^{\frac{1}{2}}_{ik} \bar{\psi}_j i \bar{\sigma}^\mu D_\mu \psi_k - \tfrac{1}{2} Z^{\frac{1}{2}*}_{ij} Z^{\frac{1}{2}}_{ik} D_\mu \bar{\psi}_j i \bar{\sigma}^\mu \psi_k \\
&- \tfrac{1}{4!} \mu^\epsilon (\lambda Z^\lambda)_{abcd} \phi_a \phi_b \phi_c \phi_d \\
&- \tfrac{1}{2} \mu^{\frac{\epsilon}{2}} (y Z^y)_{a|ij} \phi_a \psi_i \psi_j - \tfrac{1}{2} \mu^{\frac{\epsilon}{2}} (y Z^y)^*_{a|ij} \phi_a \bar{\psi}_i \bar{\psi}_j
\end{aligned}$$

Nontrivial virial

The virial is

$$V^\mu = Q_{ab}\phi_a D^\mu \phi_b - P_{ij}\bar{\psi}_i i\bar{\sigma}^\mu \psi_j$$

with

$$Q_{ab} = -Q_{ba} \qquad\qquad P_{ij} = -P^*_{ji}$$

Q and P also satisfy conditions for gauge invariance of V^μ.

The virial generates an internal transformation of the fields.

It cannot be improved away from $\mathcal{D}^\mu = x_\nu \Theta^{\mu\nu} - V^\mu$.

Scale vs conformal

Assume that we have a theory in 4 dimensions with scale but without conformal invariance.

Then the virial

- Has no anomalous dimension (scaling dimension exactly 3)
- Is gauge invariant
- Is not conserved

This is impossible in a unitary CFT
(Mack, 1977 (also Intriligator, Grinstein & Rothstein, 2008))

It is however within bounds on operator dimensions in non-conformal scale-invariant unitary theories. (Intriligator, Grinstein & Rothstein, 2008)

Algebraic condition for SI

The new improved SE tensor is

$$\Theta_\mu{}^\mu(x) = \frac{\beta_A}{2g_A^3} F^A_{\mu\nu} F^{A\mu\nu} + \gamma_{aa'} D^2\phi_a\phi_{a'} - \gamma^*_{i'i}\bar\psi_i i\bar\sigma^\mu D_\mu\psi_{i'} + \gamma_{ii'} D_\mu\bar\psi_i i\bar\sigma^\mu\psi_{i'}$$
$$- \frac{1}{4!}(\beta_{abcd} - \gamma_{a'a}\lambda_{a'bcd} - \gamma_{b'b}\lambda_{ab'cd} - \gamma_{c'c}\lambda_{abc'd} - \gamma_{d'd}\lambda_{abcd'})\phi_a\phi_b\phi_c\phi_d$$
$$- \frac{1}{2}(\beta_{a|ij} - \gamma_{a'a}y_{a'|ij} - \gamma_{i'i}y_{a|i'j} - \gamma_{j'j}y_{a|ij'})\phi_a\psi_i\psi_j + \text{h.c.}$$

The divergence of the dilatation current is

$$\partial_\mu\mathcal{D}^\mu(x) = \frac{\beta_A}{2g_A^3} F^A_{\mu\nu} F^{A\mu\nu} + (\gamma_{aa'} + Q_{aa'}) D^2\phi_a\phi_{a'} - (\gamma^*_{i'i} + P^*_{i'i})\bar\psi_i i\bar\sigma^\mu D_\mu\psi_{i'} + (\gamma_{ii'} + P_{ii'}) D_\mu\bar\psi_i i\bar\sigma^\mu\psi_{i'}$$
$$- \frac{1}{4!}(\beta_{abcd} - \gamma_{a'a}\lambda_{a'bcd} - \gamma_{b'b}\lambda_{ab'cd} - \gamma_{c'c}\lambda_{abc'd} - \gamma_{d'd}\lambda_{abcd'})\phi_a\phi_b\phi_c\phi_d$$
$$- \frac{1}{2}(\beta_{a|ij} - \gamma_{a'a}y_{a'|ij} - \gamma_{i'i}y_{a|i'j} - \gamma_{j'j}y_{a|ij'})\phi_a\psi_i\psi_j + \text{h.c.}$$

Algebraic condition for SI

We can now use the equations of motion to find that

$$\partial_\mu \mathcal{D}^\mu(x) = 0$$

when

$$\begin{aligned}
\beta_A &= 0 \\
\beta_{abcd} &= -Q_{a'a}\lambda_{a'bcd} - Q_{b'b}\lambda_{ab'cd} - Q_{c'c}\lambda_{abc'd} - Q_{d'd}\lambda_{abcd'} \\
\beta_{a|ij} &= -Q_{a'a}y_{a'|ij} - P_{i'i}y_{a|i'j} - P_{j'j}y_{a|ij'}
\end{aligned}$$

A solution to these equations with nonzero beta functions defines a theory with scale but without conformal invariance.

These equations are true to all orders in perturbation theory.

How do generators of dilatations generate dilatations?

The naive Ward identity for scale invariance becomes the Callan–Symanzik equation in the quantum theory:

$$\left[\mu\frac{\partial}{\partial\mu}+\beta_i\frac{\partial}{\partial g_i}+\gamma_j^{\,i}\int d^4x\,\varphi_i(x)\frac{\delta}{\delta\varphi_j(x)}\right]\Gamma[\varphi(x),g,\mu]=0$$

The dilatation generator can be redefined to account for anomalous dimensions, but not for beta functions. (Coleman & Jackiw, 1971)

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It seems like the only case where we can have scale invariance is in CFTs.

How do generators of dilatations generate dilatations?

$$\left[\mu\frac{\partial}{\partial\mu}+\beta_i\frac{\partial}{\partial g_i}+\gamma_j^{\ i}\int d^4x\,\varphi_i(x)\frac{\delta}{\delta\varphi_j(x)}\right]\Gamma[\varphi(x),g,\mu]=0$$

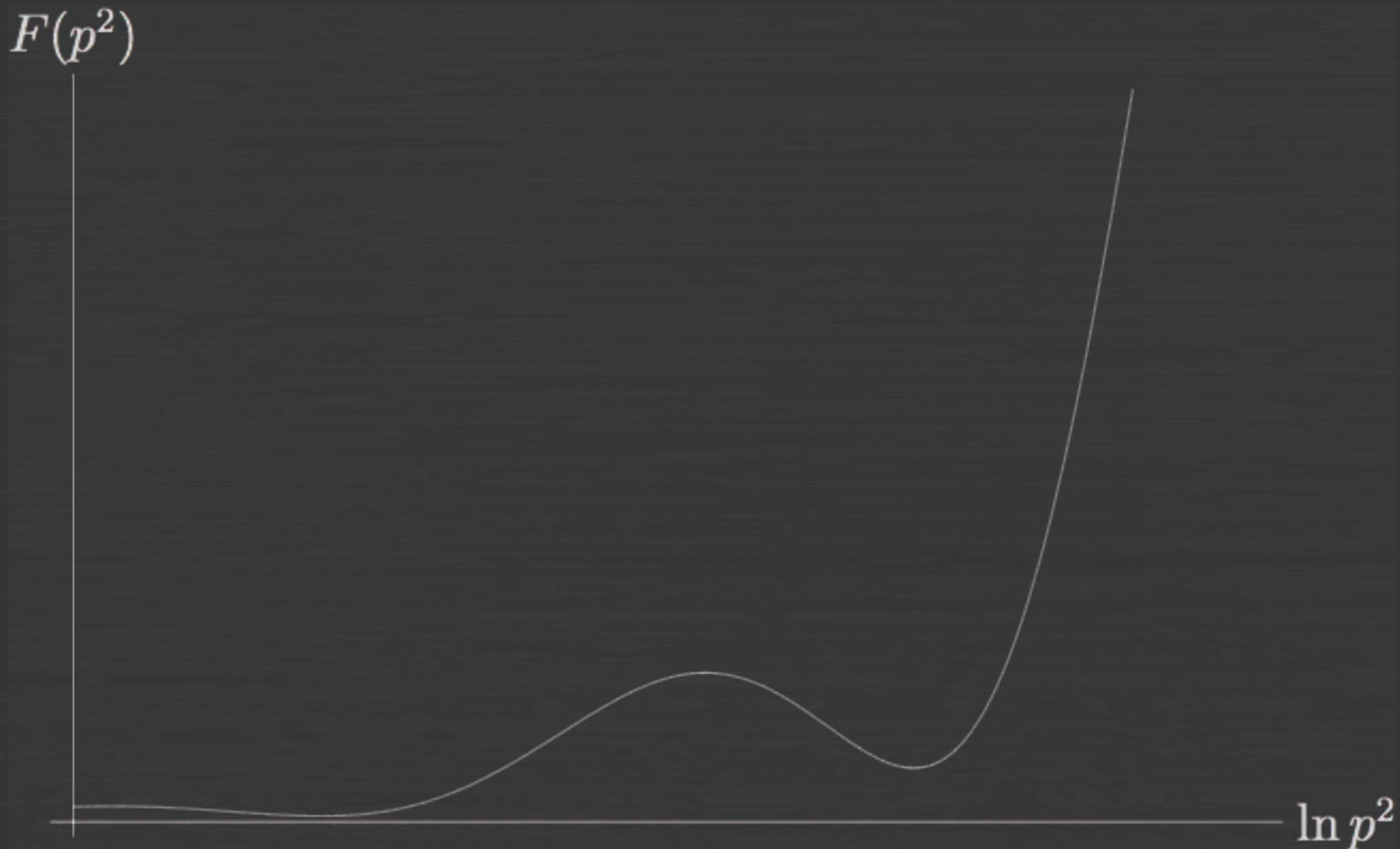
$$\left[\mu\frac{\partial}{\partial\mu}+(\gamma_j^{\ i}+Q_j^{\ i})\int d^4x\,\varphi_i(x)\frac{\delta}{\delta\varphi_j(x)}\right]\Gamma[\varphi(x),g,\mu]=0$$

The dilatation generator can be redefined to account for these very special beta functions too! (Fortin, Grinstein & AS, 2011)

Effects on correlators

The correlators of an SFT are not those of a CFT.

They can be studied, also with applications to unparticle physics in mind. (Fortin, Grinstein & AS, 2011)



How do generators of dilatations generate dilatations?

$$D = \int d^3x\, \mathcal{D}^0 = \int d^3x\, (x_\mu \Theta^{0\mu} - V^0)$$

E.g. in a theory with scalars and fermions

$$[D, \phi_a(x)] = -i(x \cdot \partial + 1)\phi_a(x) - iQ_{ab}\phi_b(x)$$

$$[D, \psi_i(x)] = -i(x \cdot \partial + \tfrac{3}{2})\psi_i(x) - iP_{ij}\psi_j(x)$$

There are contributions to scaling dimensions from the beta functions:

$$\Delta_{ab} = \delta_{ab} + Q_{ab} + \gamma_{ab}$$

$$\Delta_{ij} = \tfrac{3}{2}\delta_{ij} + P_{ij} + \gamma_{ij}$$

RG trajectories in SFTs

If we find a point where the theory is scale-invariant, then there must be an RG trajectory through that point.

What is the nature of such a trajectory?

Schematically:

$$-\frac{dg^i}{dt} = Q^i{}_j g^j$$

The system can be solved, and if Q is antisymmetric or anti-Hermitian we'll get a periodic or quasi-periodic solution.

Running of the couplings

If we have a scale-invariant point, then all points

$$\bar{\lambda}_{abcd}(t) = \widehat{Z}_{a'a}(t)\widehat{Z}_{b'b}(t)\widehat{Z}_{c'c}(t)\widehat{Z}_{d'd}(t)\lambda_{a'b'c'd'}(0)$$

$$\bar{y}_{a|ij}(t) = \widehat{Z}_{a'a}(t)\widehat{Z}_{i'i}(t)\widehat{Z}_{j'j}(t)y_{a'|i'j'}(0)$$

where $\widehat{Z}_{ab}(t) = (e^{Qt})_{ab}$ and $\widehat{Z}_{ij}(t) = (e^{Pt})_{ij}$ are scale-invariant.

Q and P are constant and so $\widehat{Z}_{ab}$ is orthogonal and $\widehat{Z}_{ij}$ unitary.

Scale invariance $\Rightarrow$ Recurrence

Trajectories that go through scale-invariant points are periodic or quasi-periodic!

We can get both limit cycles and ergodic behavior. This is behavior first speculated to exist in QFTs by Wilson in 1971!

Such behavior in a field theory appears to disagree with expectations derived from the c-theorem.

The usual intuition is that massless degrees of freedom are lost as one coarse-grains.

This intuition is violated on scale-invariant trajectories.

If scale-invariant trajectories exist, then RG flows are not gradient flows.

Stability in CFTs

To study stability we linearize around the fixed point:

$$\beta(t) = [g(t) - g_*] \left.\frac{\partial \beta}{\partial g}\right|_{g=g_*} + \cdots$$

Very easy to solve:

$$g(t) = g_* + (g_0 - g_*) \exp\left(- \left.\frac{\partial \beta}{\partial g}\right|_{g=g_*} t \right)$$

The eigenvalues of $\partial\beta/\partial g|_{g=g_*}$ determine the nature of the approach to the fixed point.

Stability in SFTs

To study stability we linearize around the trajectory:

$$\beta(t) = \beta\big|_{g=g_*(t)} + [g(t) - g_*(t)]\,\frac{\partial\beta}{\partial g}\bigg|_{g=g_*(t)} + \cdots$$

Non-trivial to solve because everything is RG-time-dependent.

But there must be a variable that takes the RG-time-dependence out of $\partial\beta/\partial g\big|_{g=g_*(t)}$.

Using the "comoving" variable $\delta g(t) = [g(t) - g_*(t)]e^{-Qt}$

$$-\frac{d\,\delta g(t)}{dt} = \delta g(t) S + \cdots\,, \qquad S = \left(\frac{\partial\beta}{\partial g}\bigg|_{g=g_*(0)} + Q\right)$$

Stability in SFTs

The eigenvalues of the stability matrix

$$S = \left(\left. \frac{\partial \beta}{\partial g} \right|_{g=g_*(0)} + Q \right)$$

tell us if a deformation is attractive or repulsive.

S has scheme-independent eigenvalues.

S always has a zero eigenvalue with corresponding eigenvector the beta function on the trajectory.

Scheme changes

Q and P are scheme-independent.

CFTs	SFTs
Existence of fixed point	Existence of SI trajectory
Eigenvalues of γ	Eigenvalues of $\gamma + Q$
Eigenvalues of $\dfrac{\partial\beta}{\partial g}$	Eigenvalues of $\dfrac{\partial\beta}{\partial g} + Q$
First coefficient in β	(Same)
First coefficient in γ	(Same)

The examples

Scale $\overset{?}{\Rightarrow}$ conformal invariance
(Scalars in $d = 4 - \epsilon$)

In multi-flavor ϕ^4 theory the condition for scale invariance becomes

$$\beta_{abcd} \overset{(*)}{=} \mathcal{Q}_{abcd}$$

from $T^\mu_{\ \mu}$

from virial
$V_\mu = Q_{ab}\phi_a\partial_\mu\phi_b$

where $\mathcal{Q}_{abcd} = Q_{ae}\lambda_{ebcd} +$ permutations, with Q_{ab} antisymmetric.

In $d = 4 - \epsilon$, a solution to $(*)$ is automatically a solution that sets both sides to zero. This can be shown at one (Polchinski, 1988) and two loops. (Fortin, Grinstein & AS, 2011)

No possibility for scale without conformal invariance!

Scale $\overset{?}{\Rightarrow}$ conformal invariance
(Scalars and spinors in $d = 4 - \epsilon$)

The condition for scale invariance is

$$\beta_{abcd} \overset{(*)}{=} \mathcal{Q}_{abcd} \qquad \text{and} \qquad \beta_{a|ij} \overset{(**)}{=} \mathcal{P}_{a|ij}$$

where $\mathcal{P}_{a|ij} = Q_{ab} y_{b|ij} - (P_{ki} y_{a|kj} + i \leftrightarrow j)$ with P_{ij} anti-Hermitian.

$(*)$ and $(**)$ are solved at fixed points at one loop: (Dorigoni & Rychkov, 2009)

$$\mathcal{P}^*_{a|ij}\mathcal{P}_{a|ij} = \mathrm{Re}(\mathcal{P}^*_{a|ij}\beta^{(1\text{-loop})}_{a|ij}) = 0 \Rightarrow \mathcal{P}_{a|ij} = 0$$

$$\mathcal{Q}_{abcd}\mathcal{Q}_{abcd} = \mathcal{Q}_{abcd}\beta^{(1\text{-loop})}_{abcd} = 0 \Rightarrow \mathcal{Q}_{abcd} = 0$$

Scale $\overset{?}{\Rightarrow}$ conformal invariance
(Scalars and spinors in $d = 4 - \epsilon$)

At two loops, however, (Fortin, Grinstein & AS, 2011)

$$\mathcal{P}^*_{a|ij}\mathcal{P}_{a|ij} = \mathrm{Re}(\mathcal{P}^*_{a|ij}\beta^{(\text{2-loop})}_{a|ij}) \neq 0$$

and there's a chance that

$$\beta_{abcd} \overset{(*)}{=} \mathcal{Q}_{abcd} \qquad \text{and} \qquad \beta_{a|ij} \overset{(**)}{=} \mathcal{P}_{a|ij}$$

have solutions that are not fixed points.

Necessary (but not sufficient) condition for the existence of scale without conformal invariance is satisfied.

Specific models

With one scalar and any number of Weyl spinors, scale implies conformal invariance to all orders in perturbation theory.

(Fortin, Grinstein & AS, 2011)

We need at least two scalars and at least one Weyl spinor.

In $d = 4 - \epsilon$ we want to find well-defined examples:

- Unitary
- With bounded tree-level scalar potential

Scale $\overset{?}{\Rightarrow}$ conformal invariance
(Scalars and spinors in $d = 4 - \epsilon$)

Is there really a solution to $(*)$ and $(**)$ with Q and/or P nonzero?

Search à la Wilson–Fisher:

$$\lambda_{abcd} = \sum_{n\geq 1} \lambda^{(n)}_{abcd}\epsilon^n \qquad y_{a|ij} = \sum_{n\geq 1} y^{(n)}_{a|ij}\epsilon^{n-\frac{1}{2}}$$

$$Q_{ab} = \sum_{n\geq 2} Q^{(n)}_{ab}\epsilon^n \qquad P_{ij} = \sum_{n\geq 2} P^{(n)}_{ij}\epsilon^n$$

Plug into

$$\beta_{abcd} \overset{(*)}{=} \mathcal{Q}_{abcd} \qquad \text{and} \qquad \beta_{a|ij} \overset{(**)}{=} \mathcal{P}_{a|ij}$$

and solve order by order in ϵ ($\epsilon^{3/2}, \epsilon^2, \ldots$).

Scale $\overset{?}{\Rightarrow}$ conformal invariance
(Scalars and spinors in $d = 4 - \epsilon$)

$$\beta_{abcd} \overset{(*)}{=} \mathcal{Q}_{abcd} \qquad\qquad \beta_{a|ij} \overset{(**)}{=} \mathcal{P}_{a|ij}$$

First order:

- System of coupled nonlinear equations
- Many solutions
- Throw away "bad" ones

Beyond first order:

- Use solutions of first order
- System of linear equations
- Unique solution

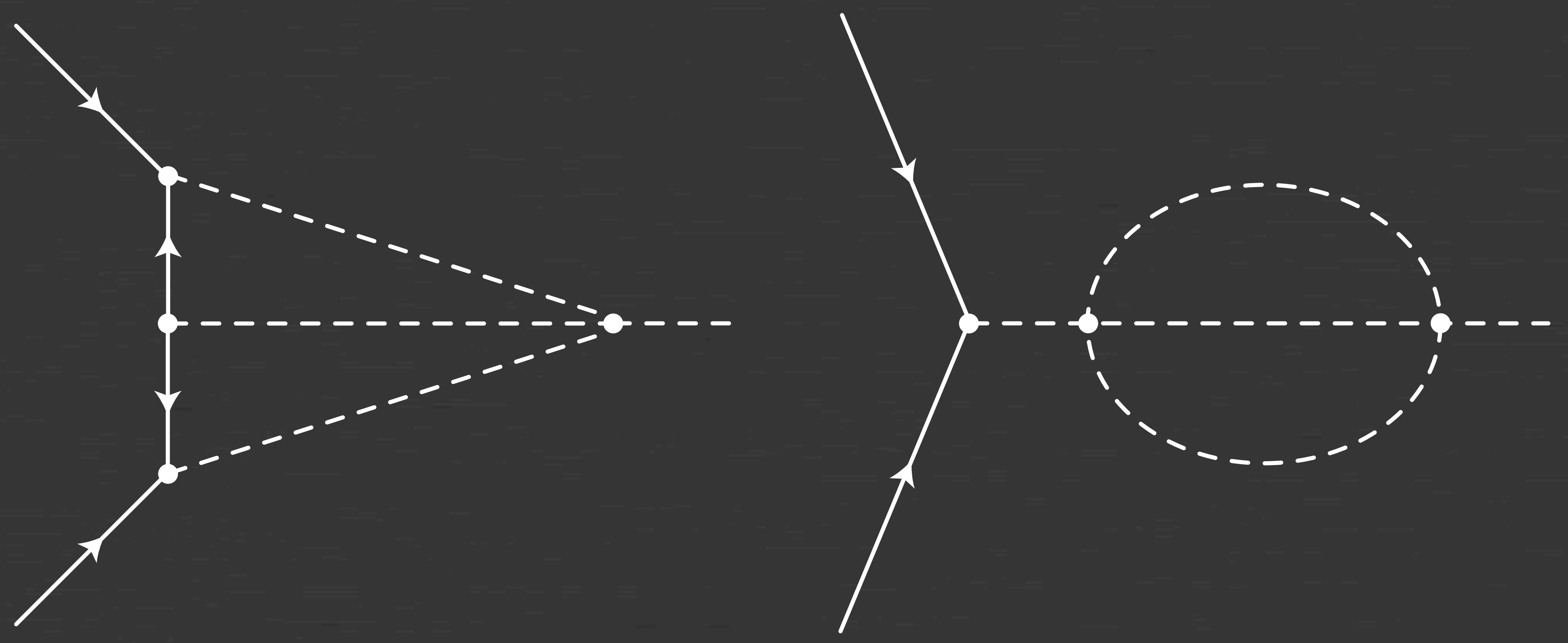
Second order in ϵ

Specific model: 2 scalars and 2 Weyl spinors (17 couplings)

$Q_{12} = q$ and $\mathrm{Im}\, P_{12} =$ undetermined

7 independent couplings

Do we get a nonzero q?



$$q^{(2)} \propto b_1 + 24 b_2$$

Second order in ϵ

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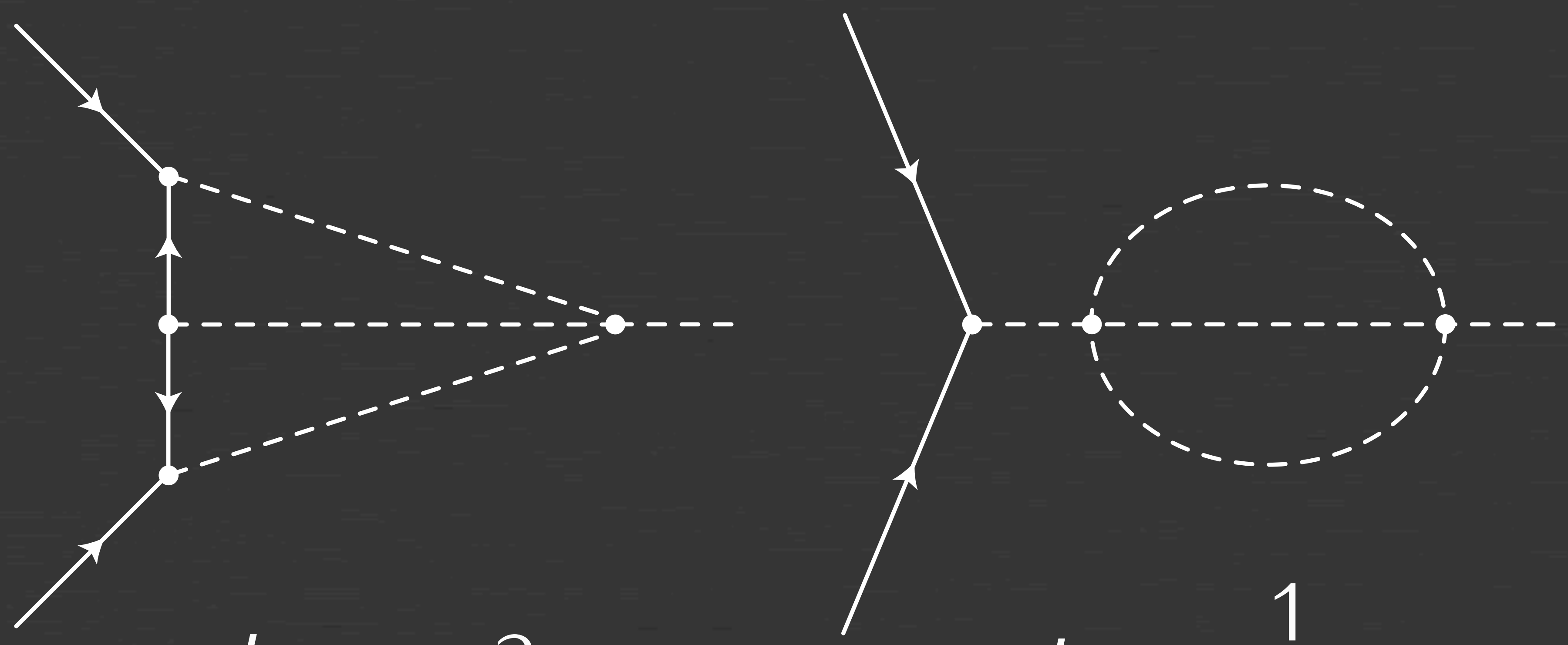
7 independent couplings

Not at two loops...



$$b_1 = -2 \qquad\qquad b_2 = \frac{1}{12}$$

$$\boxed{q^{(2)} \propto b_1 + 24 b_2 = 0} \qquad \text{(but } q^{(3)} \neq 0\text{)}$$

Chance for scale without conformal invariance is not utilized???

Second order in ϵ

Specific model: 2 scalars and 2 Weyl spinors (17 couplings)

$$Q_{12} = q \ \text{ and } \ \text{Im}\, P_{12} = \ \text{undetermined}$$

7 independent couplings

Not at two loops...

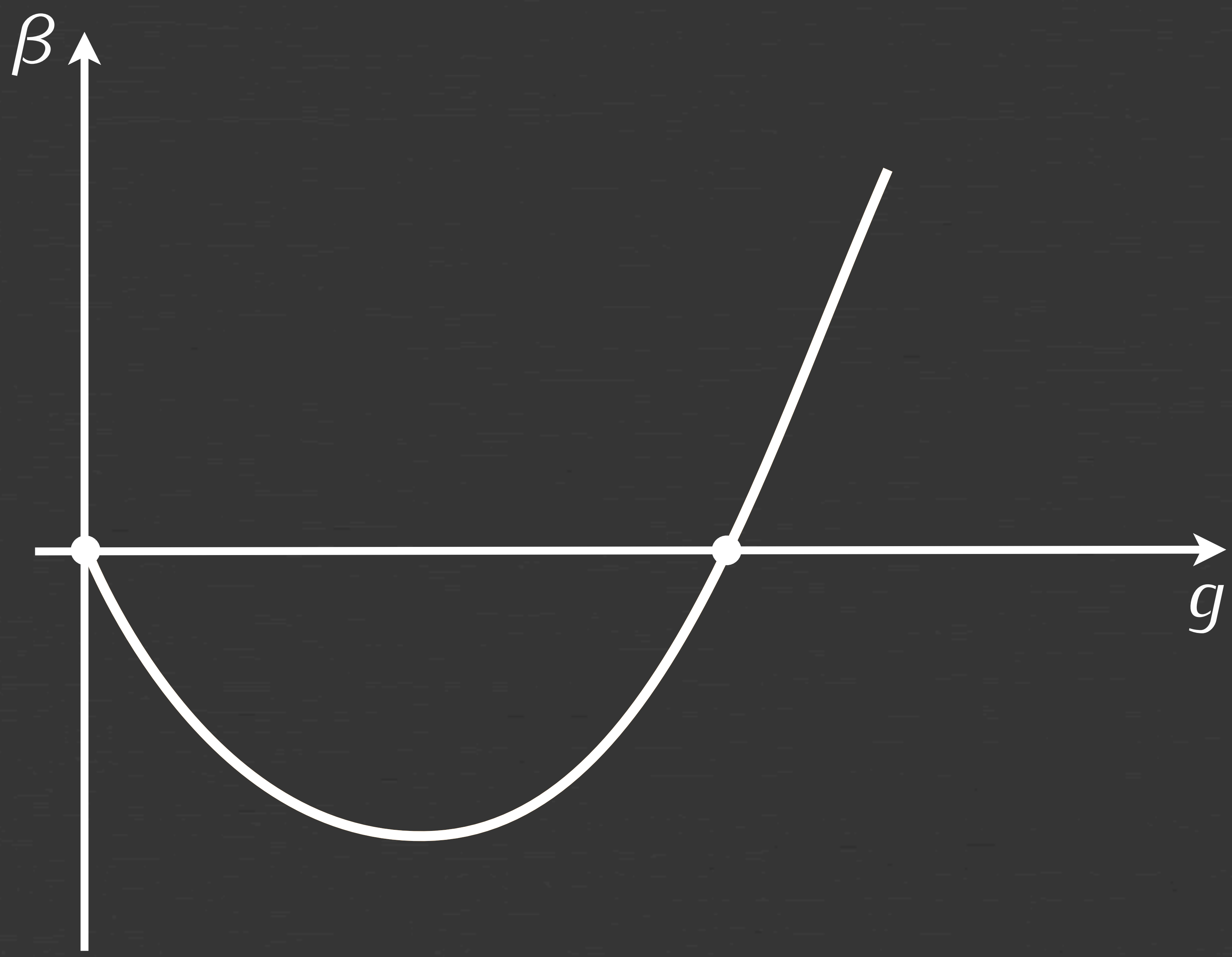
$b_1 = -2$ $\qquad$ $b_2 = \dfrac{1}{12}$

$$q^{(2)} \propto b_1 + 24 b_2 = 0$$

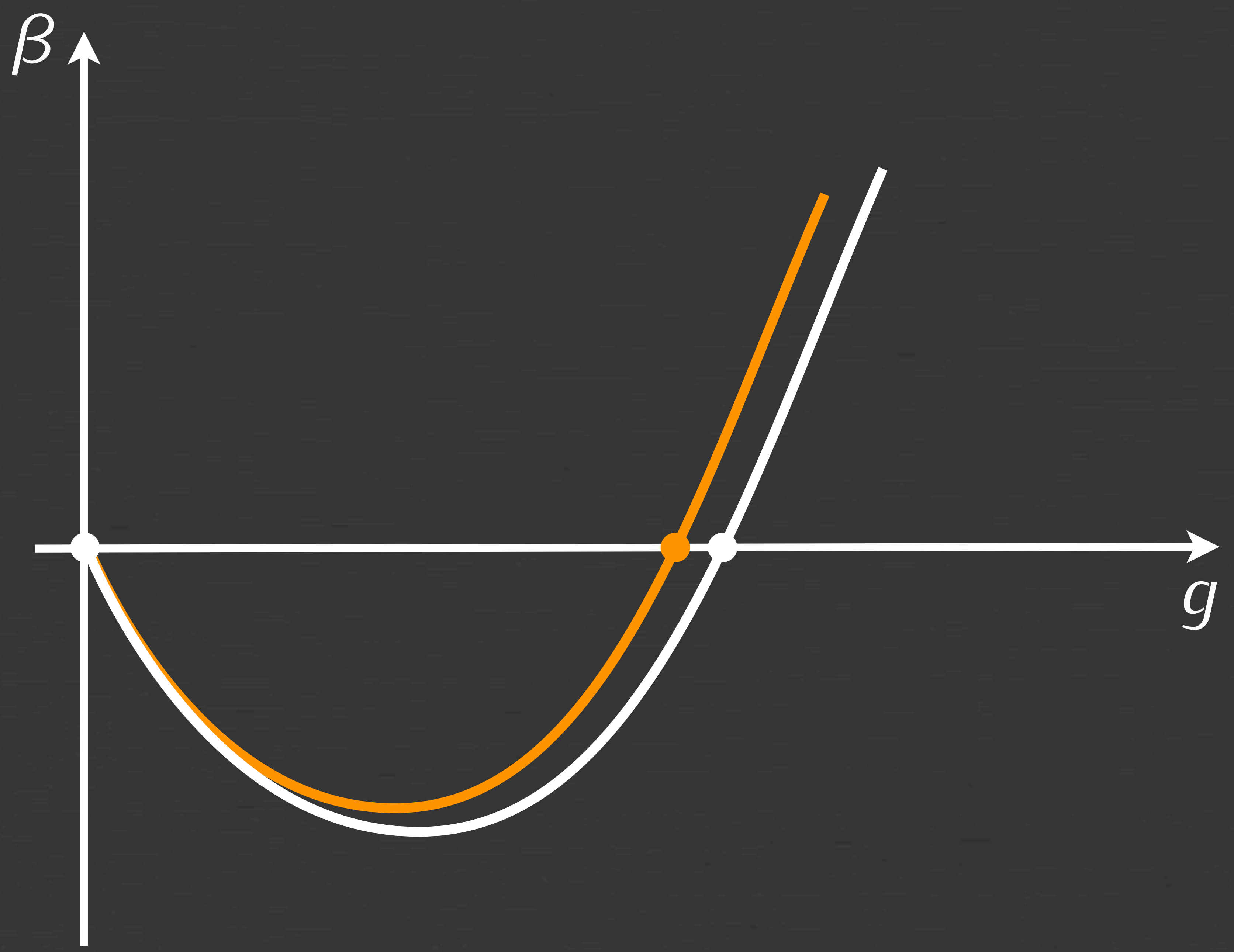
(but $q^{(3)} \neq 0$)

Chance for scale without conformal invariance is not utilized???

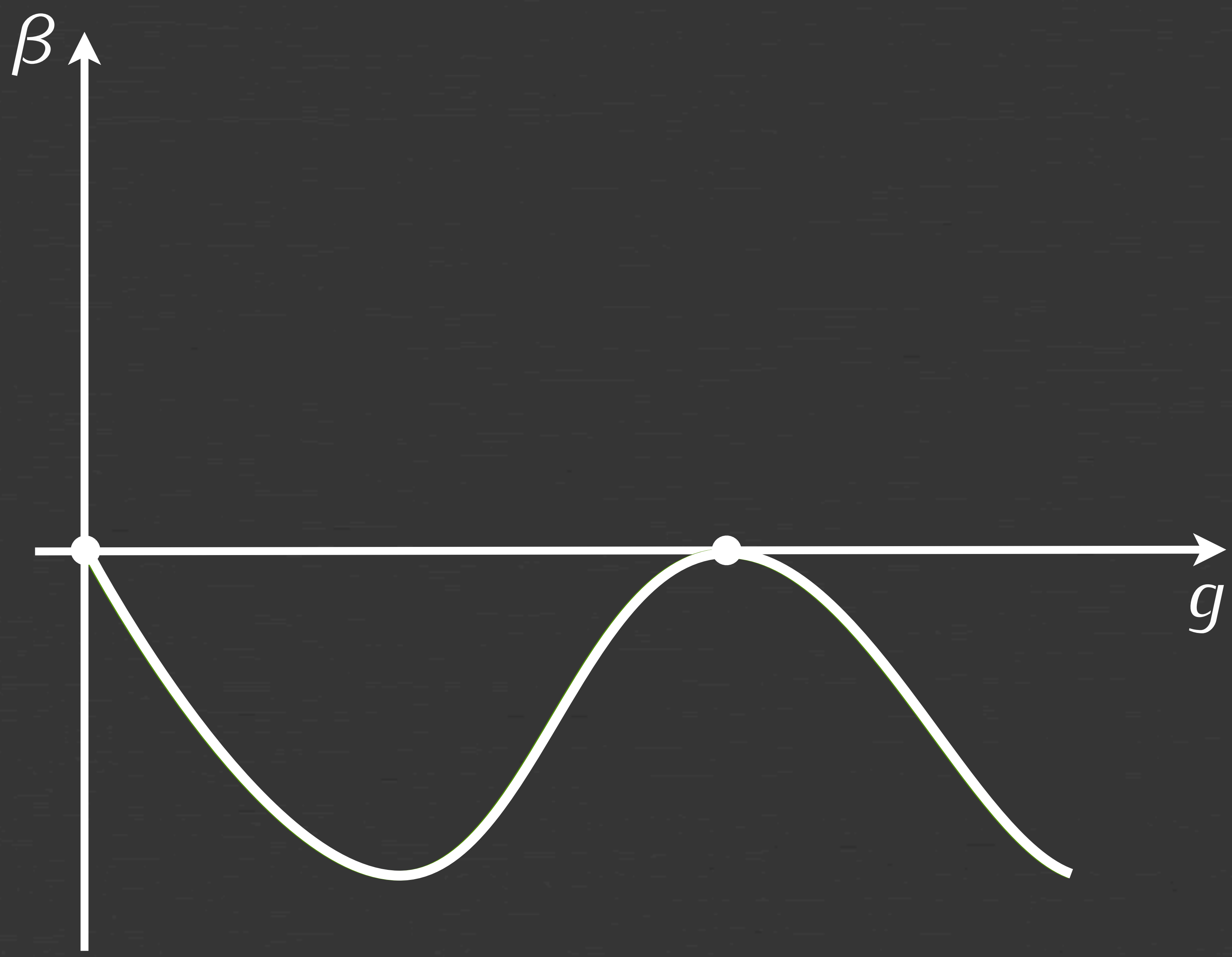
More loops in CFTs



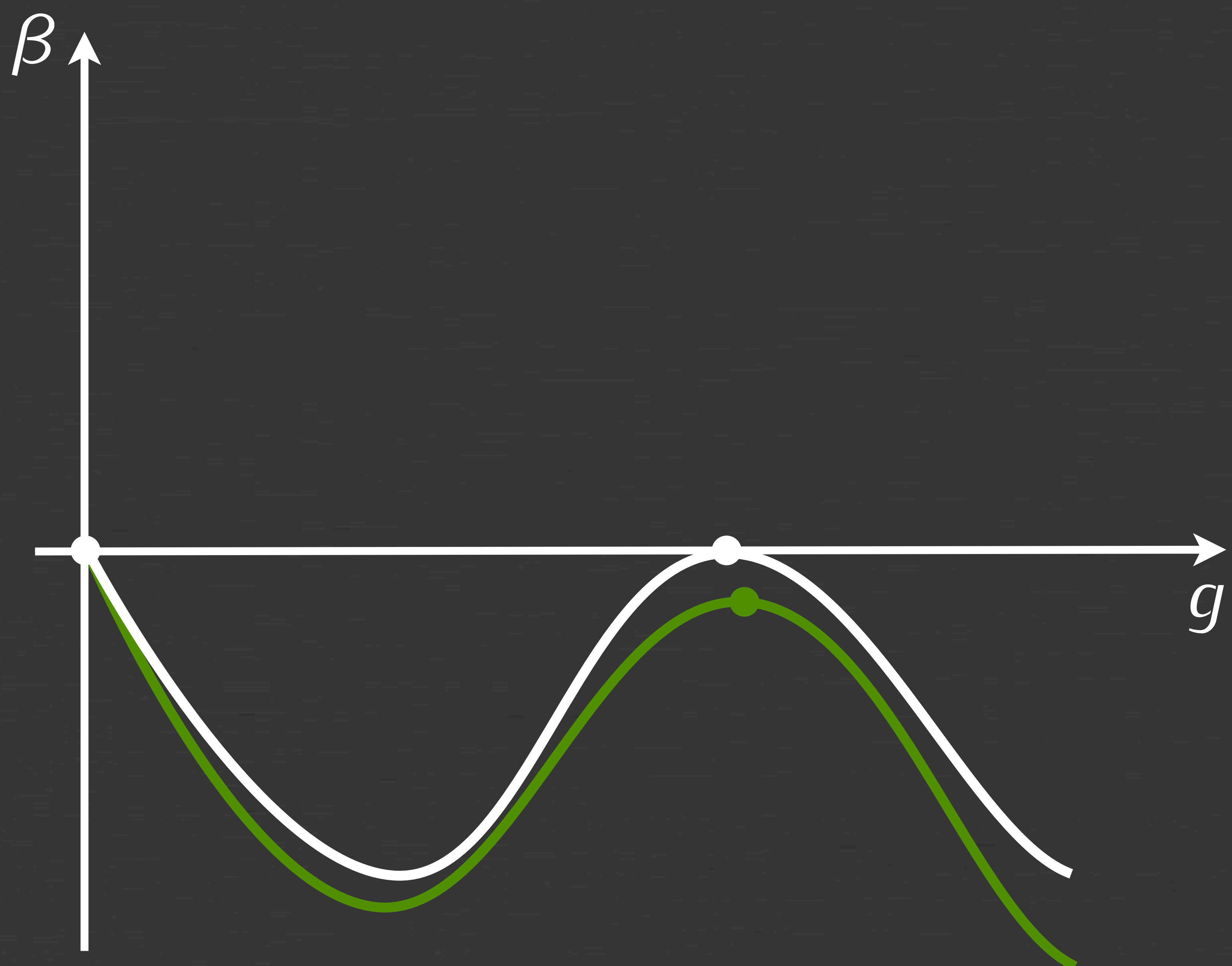
More loops in CFTs



More loops in SFTs

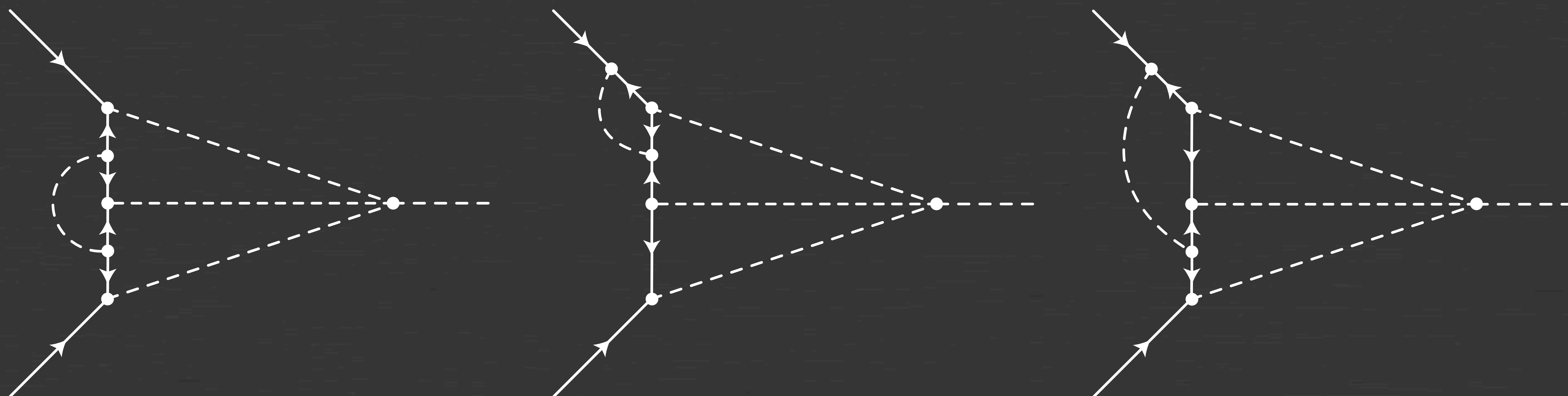


More loops in SFTs

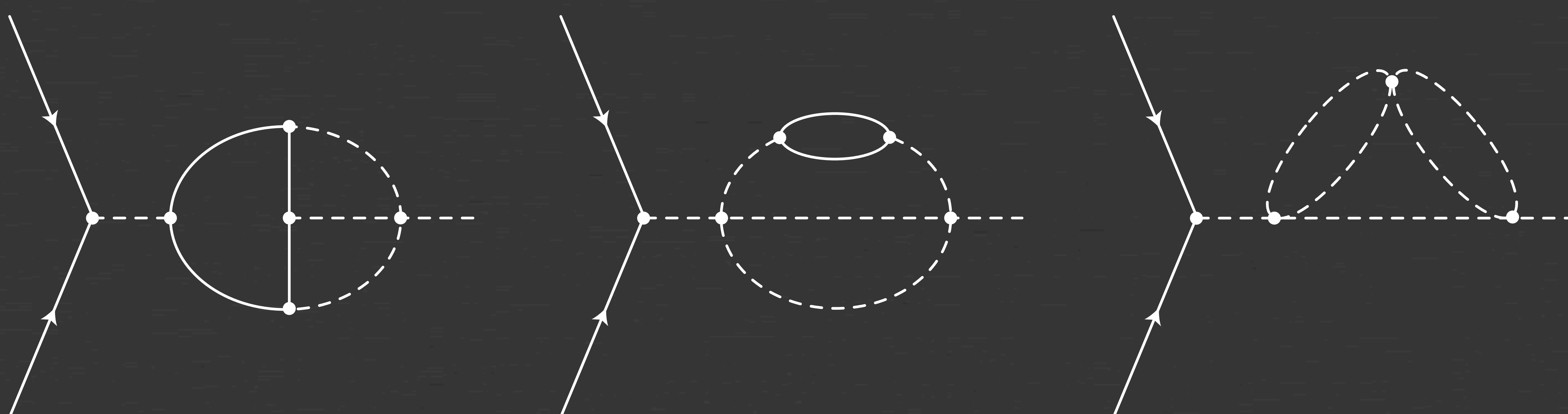


Third order in ϵ

New diagrams contribute to q, e.g.



$\cdots$



Third order in ϵ

12 diagrams in total contribute to q:

$$q^{(3)} \propto -71 + 3(c_1 + 2c_2 + 2c_3 + c_4 + 2c_5 + 4c_6 + 8c_7) + 4(c_8 + 2c_9 + 3c_{10} + 4c_{11} + 58c_{12})$$

We computed these diagrams and

$$\boxed{q^{(3)} \neq 0}$$

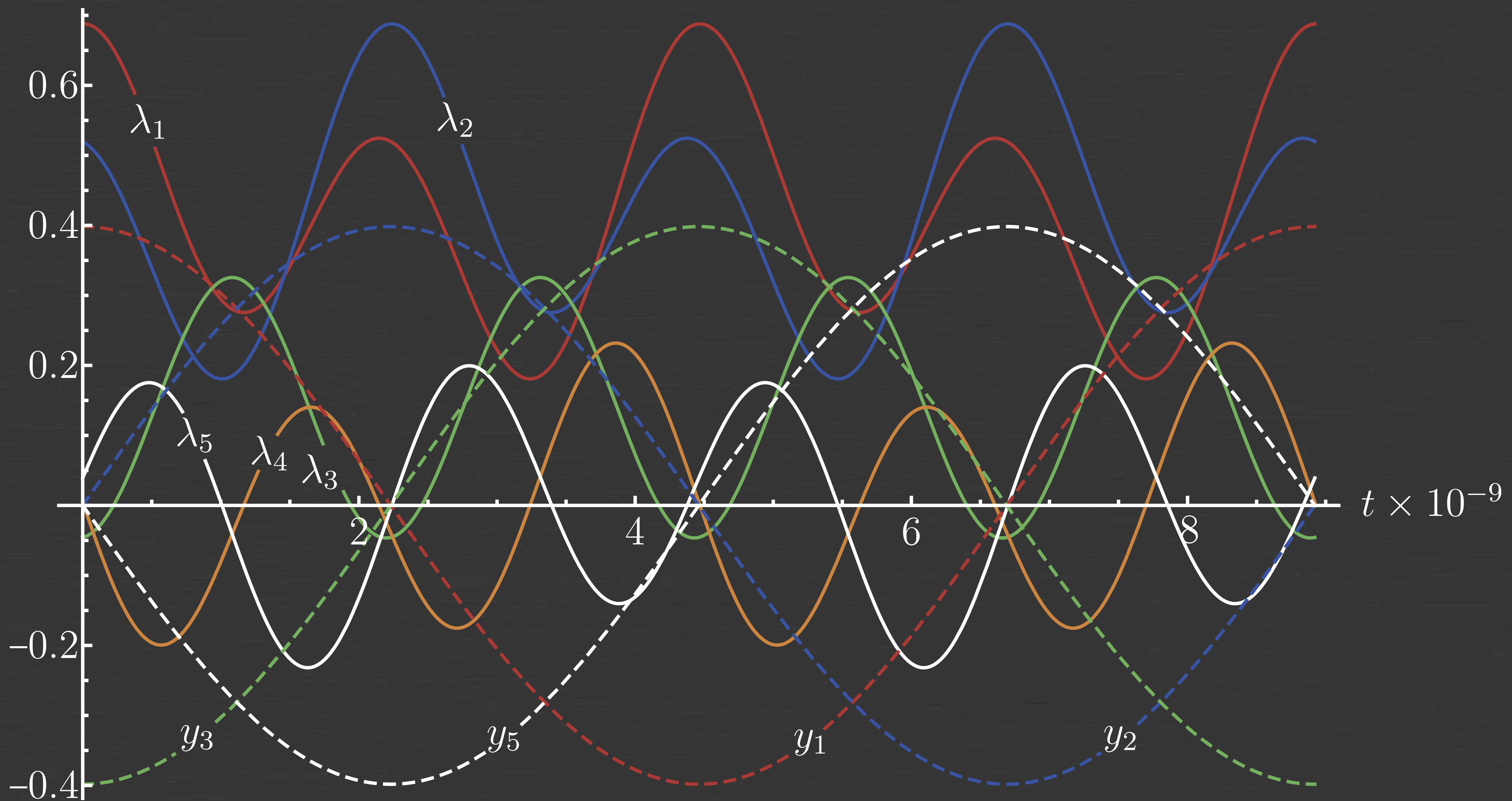
We thus have the first example of scale without conformal invariance!

This theory is unitary with bounded-from-below scalar potential.

We have a limit cycle with “frequency” $q^{(3)}$.

The eigenvalues of the stability matrix tell us that there are 5 attractive and 1 repulsive direction.

Oscillating couplings



Scale $\not\Rightarrow$ conformal invariance

Limit cycle is established!

For ergodic behavior we need at least 4 scalars and 2 spinors (at most 59 couplings).

The solutions have been found in an expansion in ϵ, and they disappear when $\epsilon \to 0$ (like, e.g., the Wilson–Fisher fixed point).

The $\epsilon \to 1$ limit leads to strong coupling, and so we cannot claim a result in $d = 3$.

Scale $\overset{?}{\nRightarrow}$ conformal invariance

$d = 4 - \epsilon$ has always been useful in the study of properties of the renormalization group.

But we want to study theories in integer spacetime dimensions.

In $d = 4$ we can add gauge fields and go to a Banks–Zaks fixed point for the gauge coupling.

Are SFTs possible in $d = 4$?

$$\beta_g = \frac{c_1}{16\pi^2} g^3 + \frac{c_2}{(16\pi^2)^2} g^5 + \cdots$$

$$\beta_y = -\frac{1}{16\pi^2} g^2 y + \cdots$$

$$\beta_\lambda = -\frac{1}{16\pi^2} g^2 \lambda + \cdots$$

Examples in $d = 4$

Take

- SU(3) gauge theory
- Two singlet scalars
- Two fundamental and two antifundamental Weyl spinors
- $(29 - 3\epsilon)/2$ sterile Weyl spinors

In the end we'll take the limit $\epsilon \to \frac{1}{3}$.

Checks on calculation:

- Gauge invariance
- No ABJ-like anomaly for the fermionic part of the virial:

$$\mathrm{Tr}\, P = 0$$

Examples in $d = 4$

$$
\begin{aligned}
q^{(3)} \propto\ & 2061664 + 143986c_1 + 127268c_2 - 735868c_3 + 63634c_4 \\
& - 735868c_5 - 1117968c_6 - 1593120c_7 + 654696c_8 \\
& + 1309392c_9 + 1726320c_{10} + 2146752c_{11} - 25316928c_{12} \\
& + 24431904c_{13} - 863136c_{14} + 4779648c_{15} + 106491c_{16} \\
& - 212982c_{17} + 212982c_{18} + 106491c_{19} - 212982c_{20}
\end{aligned}
$$

$$
\begin{aligned}
p_1^{(3)} + p_3^{(3)} \propto\ & 389632 + 4300c_1 + 50720c_2 - 105124c_3 + 25360c_4 \\
& - 105124c_5 - 94632c_6 - 357744c_7 + 93528c_8 \\
& + 187056c_9 + 276648c_{10} + 276648c_{11} - 3616704c_{12} \\
& + 3490272c_{13} - 155844c_{14} + 862992c_{15} + 15213c_{16} \\
& - 30426c_{17} + 30426c_{18} + 15213c_{19} - 30426c_{20}
\end{aligned}
$$

We find gauge invariance of the answer and absence of anomalous dimension of the virial.

Conclusion

- Scale $\not\Rightarrow$ conformal invariance
- Scale-invariant theories are less constrained than conformal theories with novel unexplored features
- Scale invariance $\Rightarrow$ recurrent behaviors in the RG running
- Phenomenological applications: Cyclic unparticle physics

Future work:

- BSM phenomenology
- Supersymmetry
- Holographic description
- Condensed matter

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Thank you!