What has Bell measurement have to do with (the lack of) infall

Work in progress with Steve Avery

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Motivation: Information loss and infall

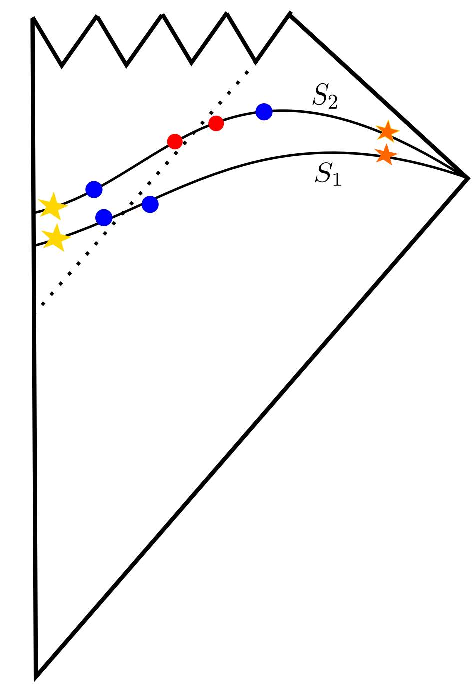
General Relativity mandates black holes

Black holes have unique state at the horizon - Unruh vacuum

Evaporation of Unruh vacuum from horizon leads to violation of unitarity - information loss

Small deviations from Unruh vacuum do not restore unitarity [Mathur, Avery, Avery-BDC-Puhm]

Inverting Mathur's argument and putting it in infalling frame argued against black hole complementarity [AMPS]



Motivation: Requirements for free infall

The state at the horizon has to be the Unruh vacuum

Near horizon geometry Rindler. Falling through requires another Rindler maximally entangled with first *and* together in Minkowski vacuum

Evaporation maintaining this leads to mixed state [Hawking, Mathur]

Some non-local models claim inside is made from radiation degrees of freedom thus restoring entanglement [Papadodimas-Raju]

Even so the full state one gets from above models is not the Unruh vacuum [Bousso, BDC]

Counter claim no matter what the entangled state, it will be defined to be the vacuum [Papadodimas-Raju]

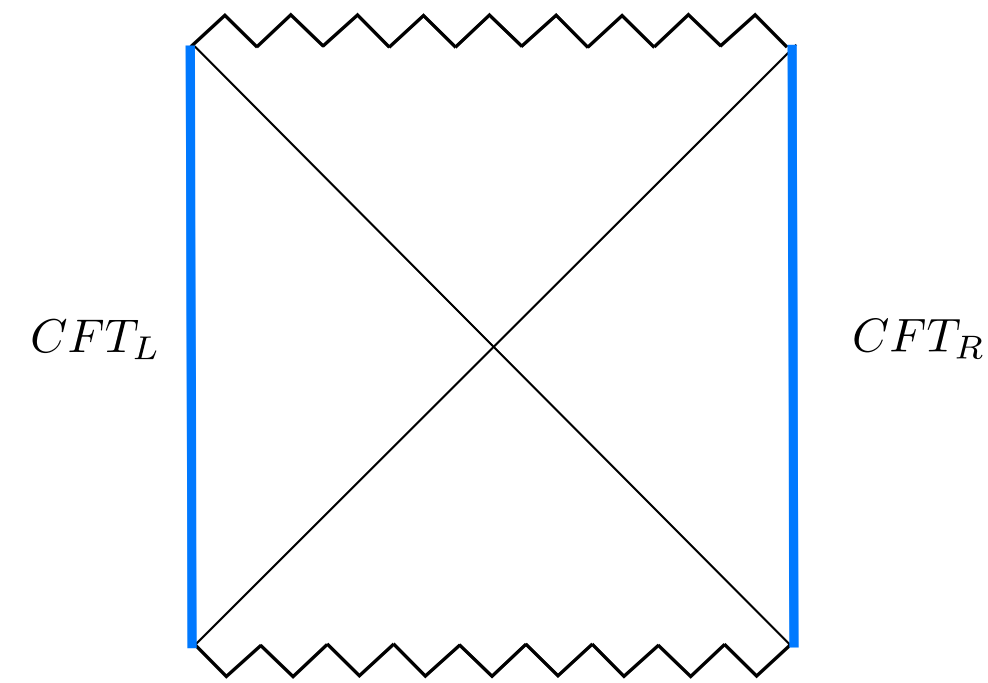
ER=EPR models say the state does not matter as long as there is entanglement. [Maldacena-Susskind]

Motivation: Eternal AdS Black Holes

Many of these ideas based on Maldacena's proposal

Eternal AdS is dual to two *non-interacting* CFTs in TFD state

$$|BH\rangle = \frac{1}{\sqrt{Z}} \sum e^{-E/2T} |E\rangle_L \otimes |E\rangle_R$$



This has led to two kinds of claims

1. Radiation is like the other CFT but on a system with arbitrary dynamics and not TFD state - no free fall. [Avery-BDC]

2. Radiation is like the other CFT and irrespective of the full state gives free infall. [Papadodimas-Raju, Maldacena-Susskind]

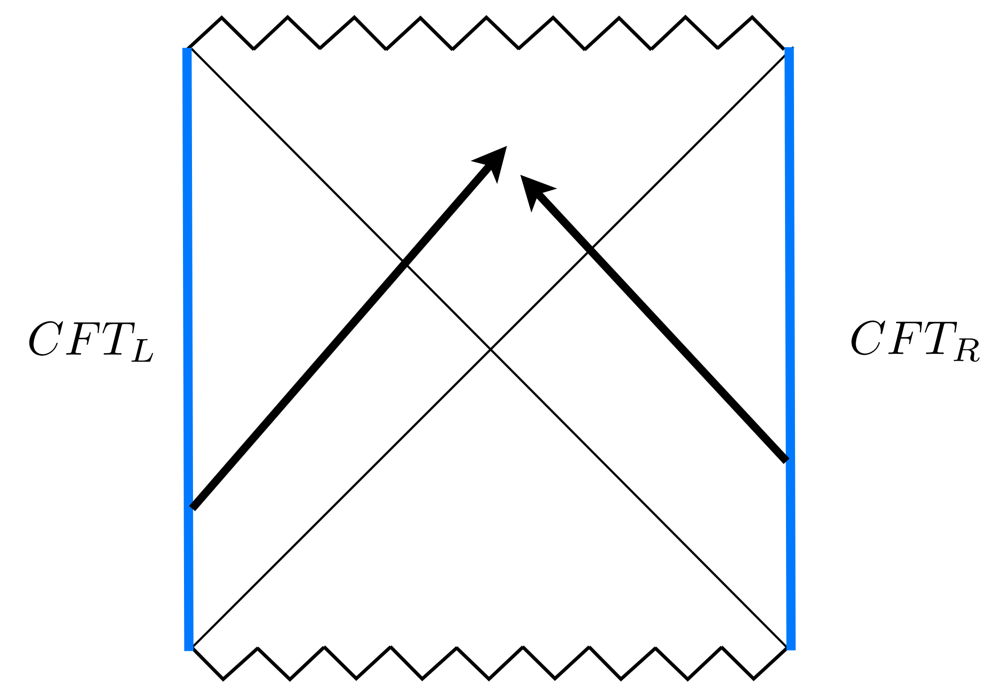
Motivation: Putting in the observer

Observer and observation should be included in the theory using decoherence ideas. [BDC-Puhm]

We will put the observers in the eternal AdS and the dual CFT

What do we want to test?

Two decoupled CFTs on spheres <u>have a forward wedge</u> where Alice, a creature on one CFT, can go. [Marolf-Wall, Maldacena-Susskind]



The <u>forward wedge is common</u> so she can meet her cousin, a creature on the other CFT there. Do both CFTs capture the amplitude for this process?

Plan

1. Bell measurements
2. Infall as a Bell measurement
3. Infall into topological black holes/ dual process in CFT
4. Infall into eternal AdS black holes/ dual process in CFT

Bell Measurements

Claim: Infalling observer does Bell measurements, accelerating one does non-Bell measurements

Take a basis of maximally entangled two qubit system

$$|\varphi_1\rangle := \frac{1}{\sqrt{2}}\big(|\hat{0}\rangle|0\rangle + |\hat{1}\rangle|1\rangle\big)\,,$$
$$|\varphi_2\rangle := \frac{1}{\sqrt{2}}\big(|\hat{0}\rangle|0\rangle - |\hat{1}\rangle|1\rangle\big)\,,$$
$$|\varphi_3\rangle := \frac{1}{\sqrt{2}}\big(|\hat{0}\rangle|1\rangle + |\hat{1}\rangle|0\rangle\big)\,,$$
$$|\varphi_4\rangle := \frac{1}{\sqrt{2}}\big(|\hat{0}\rangle|1\rangle - |\hat{1}\rangle|0\rangle\big)\,,$$

Reduced density matrix of each is the same

$$\hat{\rho} = \frac{1}{2}(|\hat{0}\rangle\langle\hat{0}| + |\hat{1}\rangle\langle\hat{1}|), \qquad \rho = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|)$$

Bob with access to only one system $\hat{I}\otimes\sigma_x,\ \hat{I}\otimes\sigma_y$ and $\hat{I}\otimes\sigma_z$

will get identical results on all 4 states

Bell Measurements

Bell states are eigenstates of $\hat{\sigma}_x \otimes \sigma_x,\ \hat{\sigma}_y \otimes \sigma_y \text{ and } \hat{\sigma}_z \otimes \sigma_z.$

state	$\hat{\sigma}_x \otimes \sigma_x$	$\hat{\sigma}_y \otimes \sigma_y$	$\hat{\sigma}_z \otimes \sigma_z$
$\|\varphi_1\rangle$	+1	-1	+1
$\|\varphi_2\rangle$	-1	+1	+1
$\|\varphi_3\rangle$	+1	+1	-1
$\|\varphi_4\rangle$	-1	-1	-1

Alice with access to both systems can distinguish between the four states

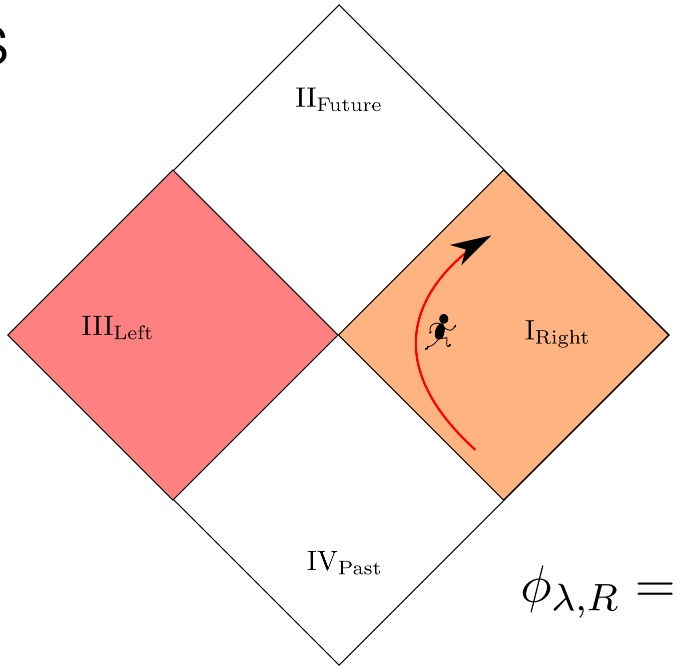
This is called a Bell measurement and is useful in quantum teleportation and quantum dense coding.

What has this got to do with infall?

Consider massless scalar fields in Minkowski spacetime. [Parentani]

For lightlike coordinate u,v eqn. of motion gives $\phi(u) + \phi(v)$.

Consider only left movers



$$U = t - x,$$

$$u_R = -\theta(-U)\frac{1}{a}\ln(-aU),$$
$$u_L = \theta(U)\frac{1}{a}\ln(aU)$$

$$\phi_k(U) = \frac{1}{\sqrt{4\pi k}}e^{-ikU}.$$

$$\phi_{\lambda,R} = \frac{1}{\sqrt{4\pi\lambda}}e^{-i\lambda u_R}\theta(-U) = \frac{1}{\sqrt{4\pi\lambda}}\theta(-U)(-aU)^{i\lambda/a}$$
$$\phi_{\lambda,L} = \frac{1}{\sqrt{4\pi\lambda}}e^{-i\lambda u_L}\theta(U) = \frac{1}{\sqrt{4\pi\lambda}}\theta(U)(aU)^{-i\lambda/a}.$$

$$\phi_{\lambda,M} = \int_0^\infty dk\frac{1}{\sqrt{2\pi k}}(k/a)^{-i\lambda/a}\phi_k(U).$$

$$= \begin{cases} \frac{\Gamma(-i\lambda/a)}{\sqrt{2\pi a/\lambda}}[e^{\pi\lambda/(2a)}\phi_{\lambda,R} + e^{-\pi\lambda/(2a)}\phi^*_{\lambda,L}] & \lambda > 0 \\ \frac{\Gamma(-i\lambda/a)}{\sqrt{-2\pi a/\lambda}}[e^{\pi\lambda/(2a)}\phi^*_{-\lambda,R} + e^{-\pi\lambda/(2a)}\phi_{-\lambda,L}] & \lambda < 0 \end{cases}$$

What has this got to do with infall?

Bogolubov coefficients

$$b_{\lambda,R} = \cosh\theta_\lambda a_\lambda + \sinh\theta_\lambda a^\dagger_{-\lambda}$$
$$b_{\lambda,L} = \cosh\theta_\lambda a_\lambda + \sinh\theta_\lambda a^\dagger_{-\lambda}$$

with $\lambda > 0$ and $\tanh\theta_\lambda = e^{-\pi\lambda/a}$.

Minkowski vacuum in terms of Rindler modes

$$|0_M\rangle = \frac{1}{\sqrt{Z}}\prod_\lambda e^{\tanh\theta_\lambda b^\dagger_{\lambda,R} b^\dagger_{\lambda,L}}|0_R\rangle|0_L\rangle.$$

Consider just one mode

$$|0_{M,\lambda}\rangle = \frac{1}{\sqrt{Z_\lambda}}\sum \tanh^n\theta_\lambda\ |n_{\lambda,R}\rangle|n_{\lambda,L}\rangle.$$

Make it fermionic and take high temperature limit - qubit Bell state

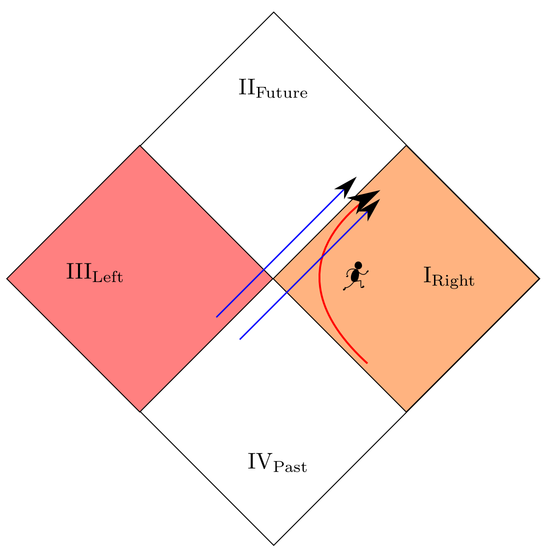
$$|0_{M,\lambda}\rangle = \frac{1}{\sqrt{2}}(|0_{\lambda,R}\rangle|0_{\lambda,L}\rangle + |1_{\lambda,R}\rangle|1_{\lambda,L}\rangle).$$

What has this got to do with infall?

Minkowski vacuum may be thought of as qubits mode by mode for our purpose

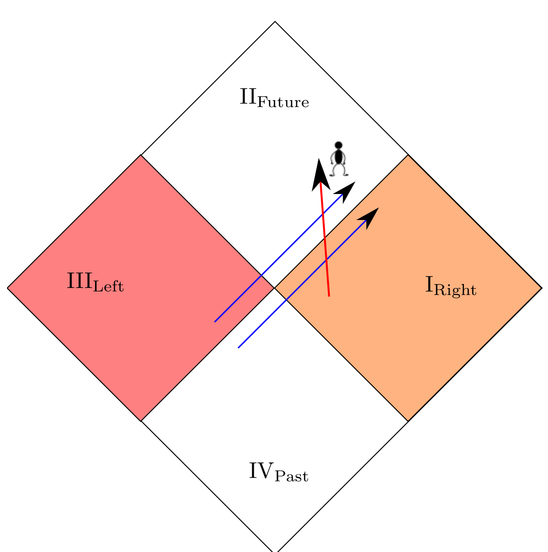
For a left moving wavepacket interaction with right moving modes

Accelerated wavepacket has access only to half the right moving modes (one qubit)



non-Bell measurement
can't identify full state

Inertial wavepacket has access to both right moving modes (both qubit)



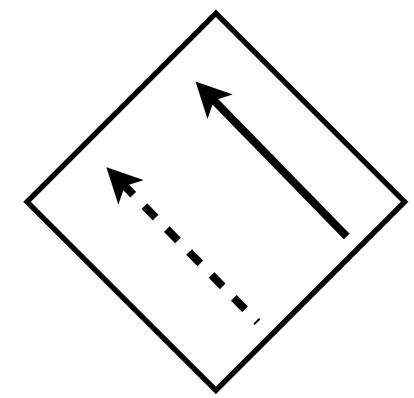
Bell measurement
can identify full state

A surprise

Alice is a right moving wavepacket trying to see if left movers in vacuum

Creating a null right moving Alice in the right wedge

$$a_\lambda^\dagger |0_M\rangle = \frac{1}{\cosh\theta_\lambda} b_{\lambda,R}^\dagger |0_M\rangle$$



Has a *state dependent* effect on the left wedge

$$\rho_{\lambda,L} = \sum_{n=0}^{\infty} \tanh^n \theta (n+1) |n_L\rangle\langle n_L|$$

In fact $$a_\lambda^\dagger |0_M\rangle = \frac{1}{\cosh\theta_\lambda} b_{\lambda,R}^\dagger |0_M\rangle = \frac{\sinh\theta_\lambda}{\cosh^2\theta_\lambda} b_{\lambda,L} |0_M\rangle$$ [Unruh-Wald, Unruh]

But this is no different from the *collapse of wavefunction* in EPR paradox

$$|\eta_{\theta,\phi,\chi}\rangle = \frac{1}{\sqrt{2}}\left[\left(\cos\left(\frac{\theta}{2}\right)|\hat{0}\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|\hat{1}\rangle\right))|0\rangle + e^{i\chi}\left(\sin\left(\frac{\theta}{2}\right)|\hat{0}\rangle - e^{i\phi}\cos\left(\frac{\theta}{2}\right)|\hat{1}\rangle\right)|1\rangle\right]$$

Unitary evolution does not change the left density matrix but projection does

Firewalls

Minkowski vacuum in terms of Rindler modes

$$|0_{M,\lambda}\rangle = \frac{1}{\sqrt{2}}(|0_{\lambda,R}\rangle|0_{\lambda,L}\rangle + |1_{\lambda,R}\rangle|1_{\lambda,L}\rangle).$$

permutations

$$\frac{1}{\sqrt{2}}\big(|\hat{0}\rangle|0\rangle - |\hat{1}\rangle|1\rangle\big)$$
$$\frac{1}{\sqrt{2}}\big(|\hat{0}\rangle|1\rangle + |\hat{1}\rangle|0\rangle\big)$$
$$\frac{1}{\sqrt{2}}\big(|\hat{0}\rangle|1\rangle - |\hat{1}\rangle|0\rangle\big)$$

leave the density matrix invariant

$$\hat{\rho} = \frac{1}{2}(|\hat{0}\rangle\langle\hat{0}| + |\hat{1}\rangle\langle\hat{1}|), \qquad \rho = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|)$$

so non-Bell measurements (staying outside horizon) show no difference

but Bell measurements (falling into horizon) shows difference

AdS Rindler spacetime/ topological black hole

Consider global AdS $$ds^2 = -(1+\rho^2)d\tau^2 + \frac{d\rho^2}{1+\rho^2} + \rho^2 d\Omega_{d-1}^2$$

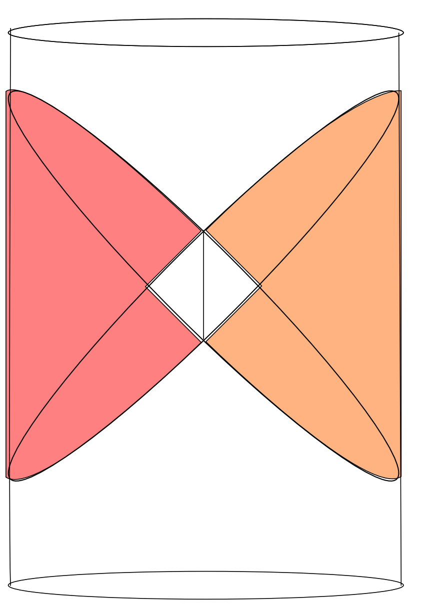
This can be Rindlerized by the transformation

$$\rho^2 = \xi^2 \,\frac{\cosh(2\chi) + \cosh(2t)}{2} + \sinh^2(\chi)$$
$$\tan\psi = \frac{\sqrt{\xi^2+1}\,\sinh(\chi)}{\xi\cosh(t)}$$
$$\tan\tau = \frac{\xi\sinh(t)}{\sqrt{\xi^2+1}\,\cosh(\chi)}$$

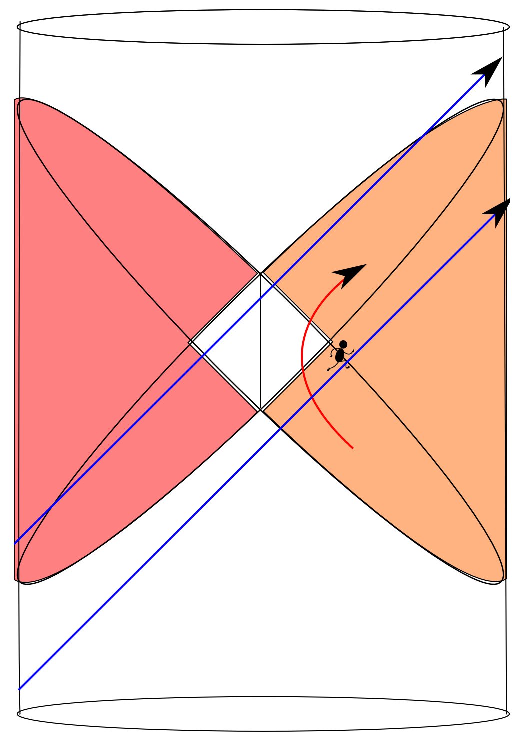
To give $$ds^2 = -\xi^2 dt^2 + \frac{d\xi^2}{1+\xi^2} + (1+\xi^2)\left[d\chi^2 + \sinh^2(\chi)d\Omega_{d-2}^2\right].$$

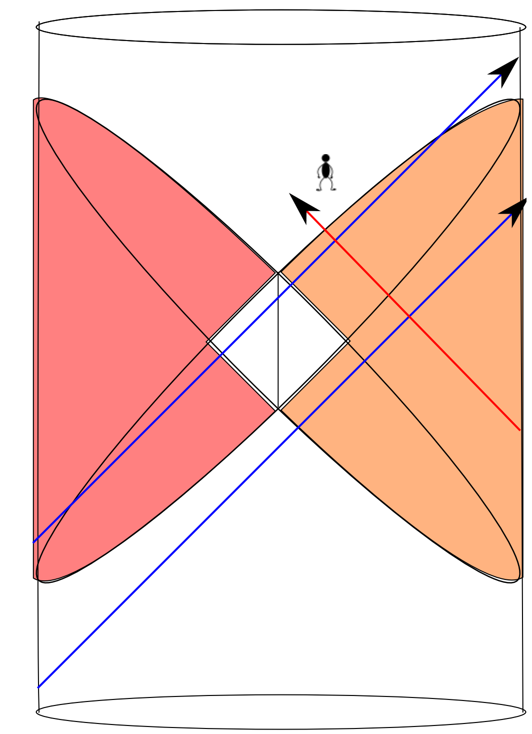
Global AdS is a cylinder

Rindler coordinates cover *the inside of a wedge*



(Non-) Bell measurements in bulk





There is a Rindler horizon in the bulk

Observers staying outside perform non-Bell measurement

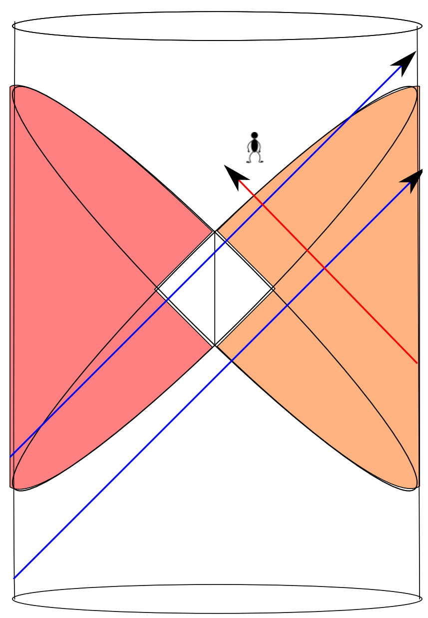
Observers falling in perform Bell measurement

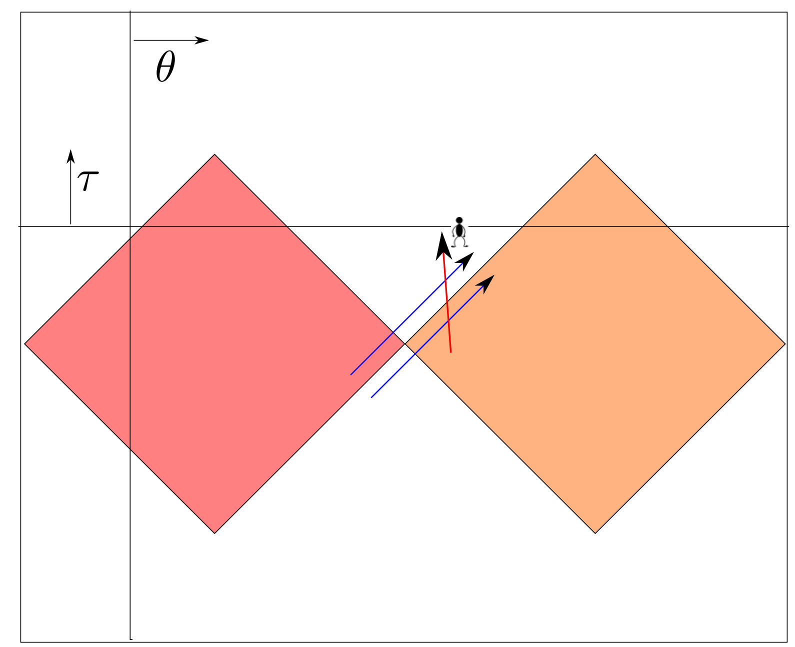
Since this is AdS/CFT what is the CFT version of this?

Bell measurements in CFT

The boundary of global AdS_{d+1} is R x S^{d-1} so CFT lives on a sphere x time

Bulk horizons intersect the boundary to form causal diamonds

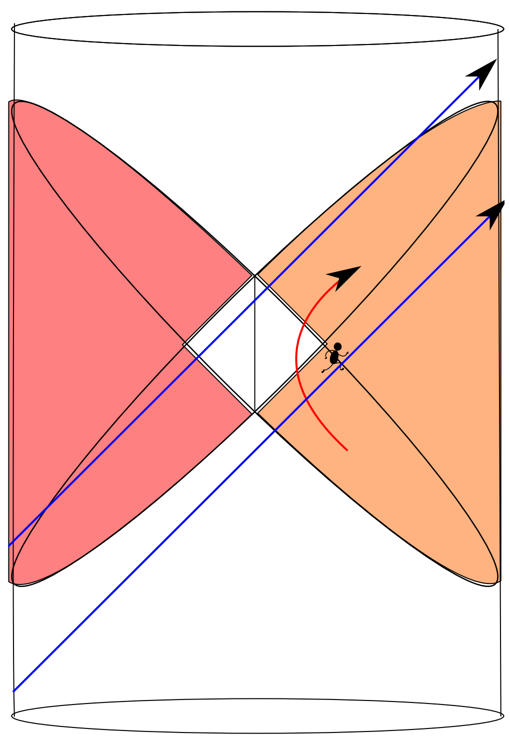




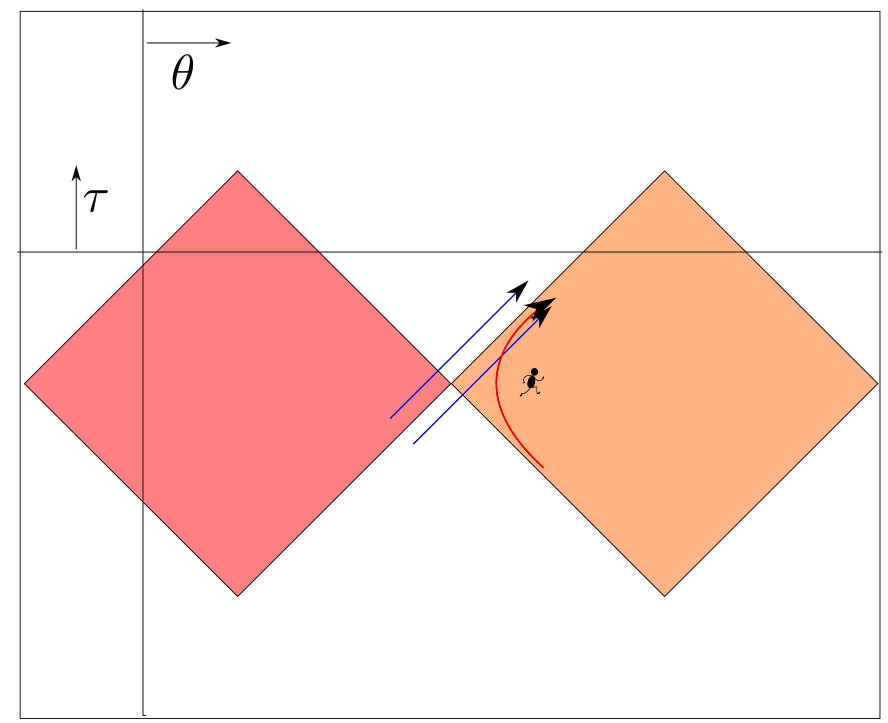
Bell measurement (infall) in bulk corresponds to two point function in CFT with one point in and one out of the relevant diamond [Lunin-Mathur]

Non-Bell measurements in CFT?

Observer should stay within the Rindler wedge



On boundary should correspond to the second point in a two point function not making it out



If only we could confine our theory to the open interval inside the right causal diamond......

Non-Bell measurements in CFT?

Metric of topological black hole $$ds^2 = -\xi^2 dt^2 + \frac{d\xi^2}{1+\xi^2} + (1+\xi^2)\left[d\chi^2 + \sinh^2(\chi) d\Omega^2_{d-2}\right].$$

Boundary is hyperbolic CFT R x H^{d-1}

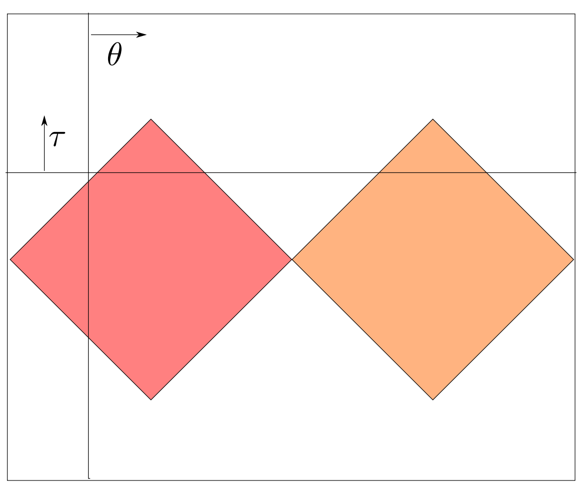
Bulk map induces boundary map

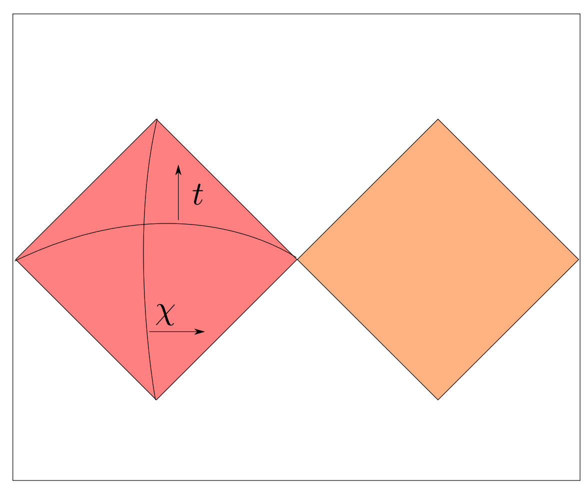
$$\rho^2 = \xi^2 \frac{\cosh(2\chi)+\cosh(2t)}{2} + \sinh^2(\chi)$$
$$\tan\psi = \frac{\sqrt{\xi^2+1}\sinh(\chi)}{\xi\cosh(t)}$$
$$\tan\tau = \frac{\xi\sinh(t)}{\sqrt{\xi^2+1}\cosh(\chi)}$$
$$\longrightarrow$$
$$\tan(\tau) = \frac{\sinh(t)}{\cosh(\chi)},$$
$$\tan(\psi) = \frac{\sinh(\chi)}{\cosh(t)}$$

Takes open interval inside sphere on boundary to hyperbolic space

$$ds^2 = -dt^2 + d\psi^2 + \sin^2\psi\, d\Omega^2 \longrightarrow ds^2 = \frac{2}{\cosh(2\chi)+\cosh(2t)}(-dt^2 + d\chi^2 + \sinh^2\chi\, d\Omega^2).$$

Takes vacuum state to thermal state (pretty much like Rindler-Minkowski)
[Casini-Huerta-Myers]



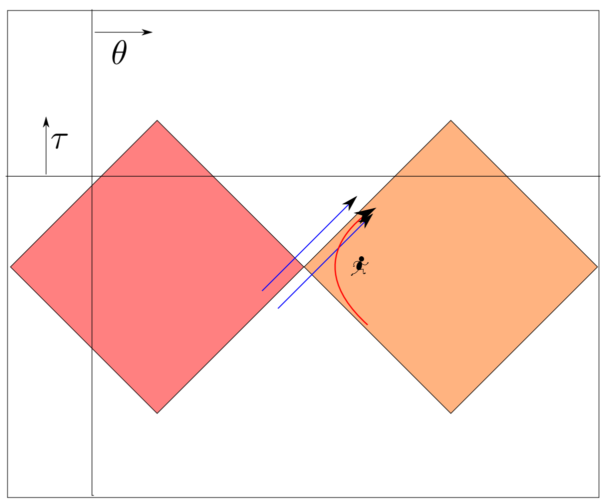


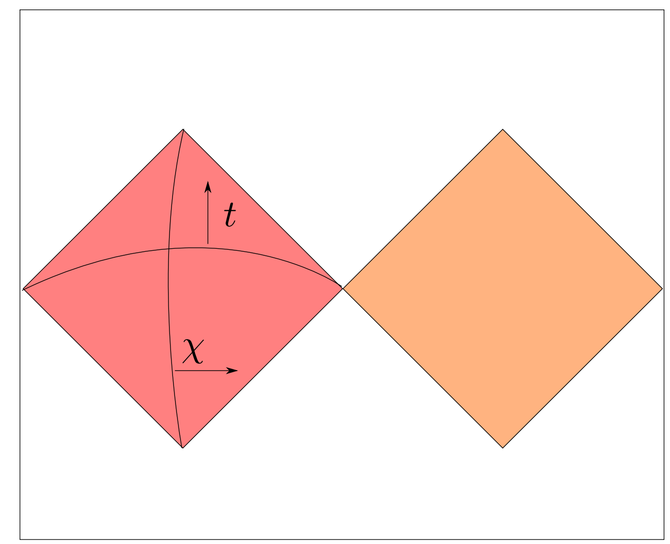
$t, \chi \in (-\infty, \infty)$

Non-Bell measurements in CFT?

On boundary should correspond to the second point in a two point function not making it out

Hyperbolic CFT naturally captures this





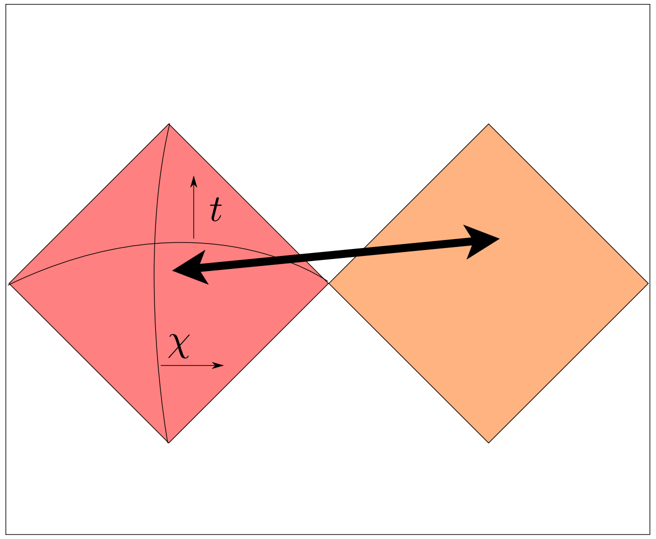
Calculations in hyperbolic CFT correspond to non-Bell measurements

<u>CFTs are non-interacting but by construction.</u>

The degrees of freedom keep decreasing with time as seen by entanglement entropy at any t>0.

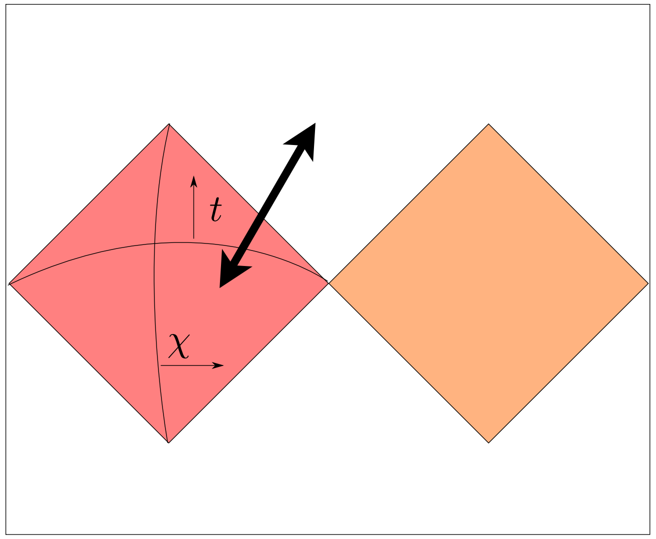
Alice can never make it out because time steps keep decreasing. Kind of like Zeno's paradox

Non-Bell measurements in CFT?



Can both hyperbolic CFTs do a Bell measurement

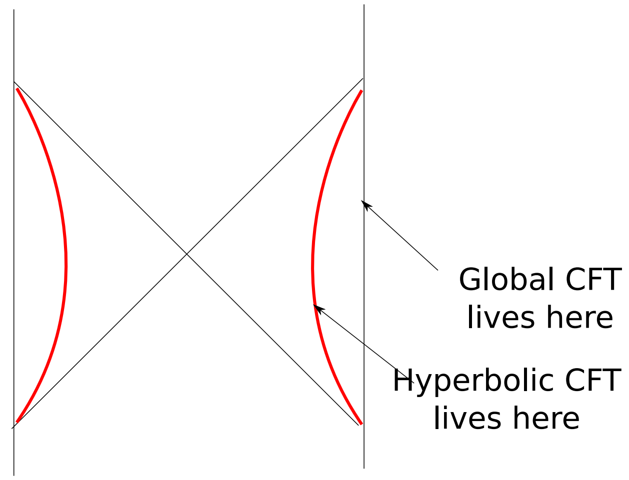
Assuming the other in TFD some 2-pt functions come for free. Correspond to both points <u>inside</u> the diamonds



For infall we need one point inside and one outside

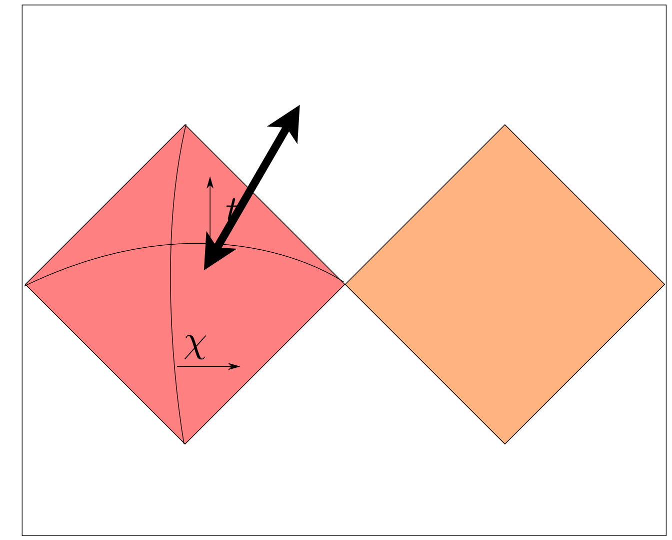
The diamond of hyperbolic CFT captures open intervals (0,π)

The two missing points and more general complementary closed interval [0,π] (dis-)allows "leakage".



Presence of global CFT contains that information, allows tilting the two point function

Non-Bell measurements in CFT?



Presence of global CFT contains that information, allows tilting the two point function

Interaction between the degrees of freedom of the two CFTs is what is responsible for producing the correct entangled state in the first place presumably.

The topological black hole is geodesically incomplete.

Dual statement? Open interval?

There is a smooth completion.

Dual statement?

Summary (only for topological BHs)

There is a **dual horizon** to the bulk horizon in the global CFT

Lore is that two hyperbolic CFTs are not individually enough to give the future and past wedge but together they do. Entanglement produces smoothness. [van Raamsdonk]

This is not true. You need the missing points at $0,\pi$

Similar for poincare patch

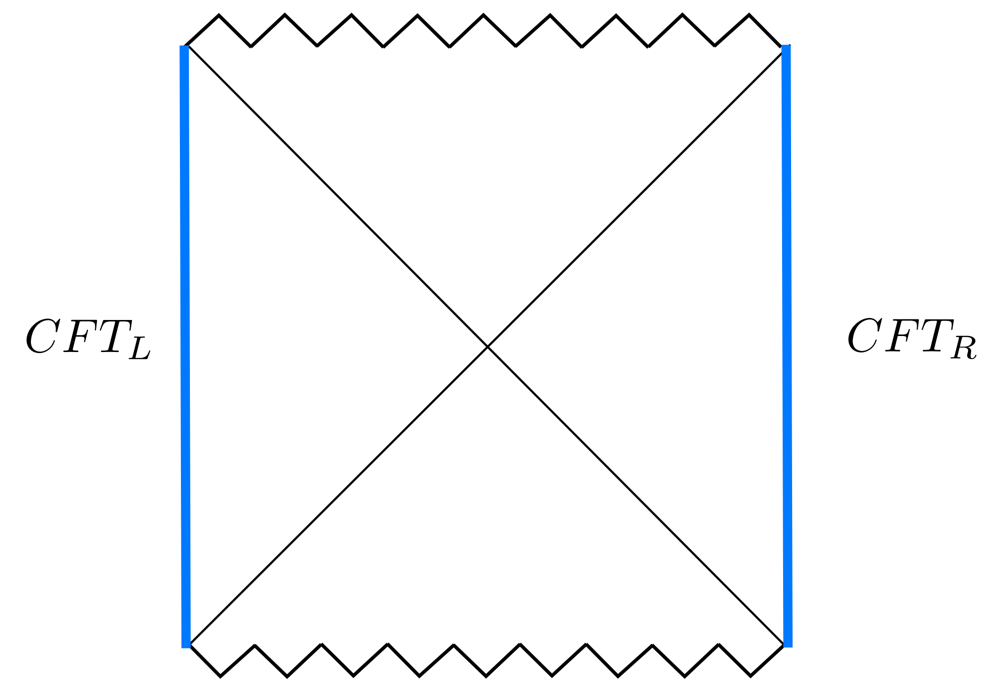
Screwy data gives firewalls

The amplitude for Alice to meet her cousin (or even fall in) is contained in the parent theory.

Eternal AdS black hole

Conjecture: Eternal AdS is dual to two *non-interacting* CFTs in TFD state [Maldacena]

$$|BH\rangle = \frac{1}{\sqrt{Z}} \sum e^{-E/2T} |E\rangle_L \otimes |E\rangle_R$$



The CFTs are spherical

The CFTs are non-interacting

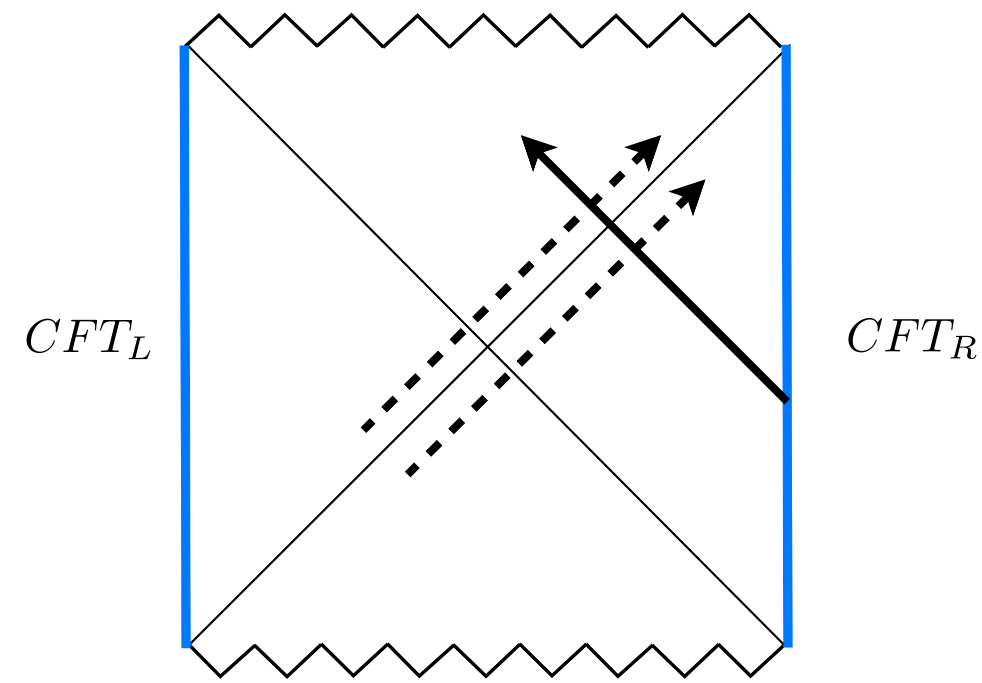
The claim is entanglement as the TFD state produces forward and backward wedge

Not below the deconfinement phase transition though!

Infall in the bulk

In the bulk, left moving Alice falling in performs a Bell measurement on right movers

It is widely believed that such an Alice is originally created in the right CFT and then falls through. [Marolf-Wall, Maldacena-Susskind]



Alice on one CFT

Can create Alice by hitting one CFT

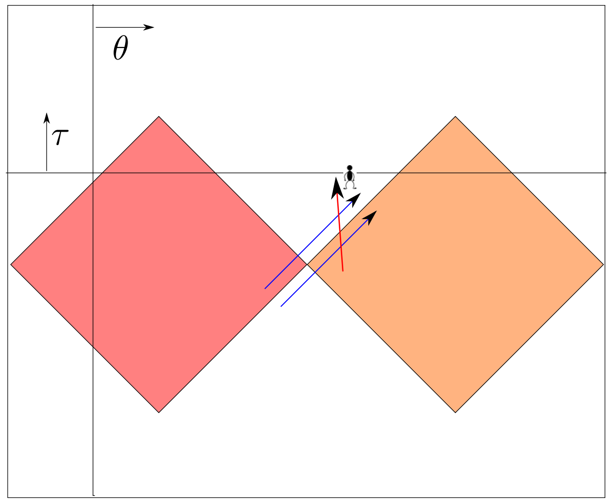
Such a “wavepacket” can only interact with one CFT

$$H_{int} = I_{Left} \otimes |+, E_1\rangle\langle -, E_2|$$

Such a “wavepacket” can not interact with the other sphere so cannot do a Bell measurement

Thus the bulk and boundary do not agree!

Difference from hyperbolic CFT case



CFT defined on open interval

QN mode in bulk - decaying correlators in CFT

CFT defined on closed interval

QN mode in bulk - quasiperiodic CFT [Maldacena]

Bulk horizon does not intersect hyperbolic CFT but induces “horizon” on global CFT

Bulk horizon does not intersect CFT. No parent theory.

Interactions which cause entanglement not turned off

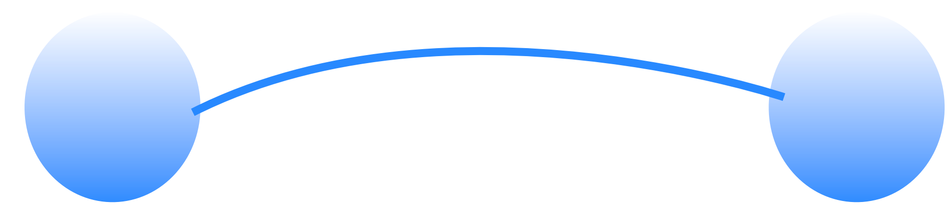
Entanglement causing interactions turned off

Bulk smooth = CFT smooth

Bulk smooth, CFT not smooth

Some general comments

If we try to keep the (small) interaction
which causes entanglement



we get a bigger sphere
O(1) modification of the proposal

Some hidden “parent” theory?

Not clear how - quasi periodic CFT so no
leakage + we think we know CFT

For hyperbolic CFT the interaction which entangled the CFT remains on and is responsible for Alice meeting her cousin. For spherical CFTs we turn that interaction off.

Some general comments

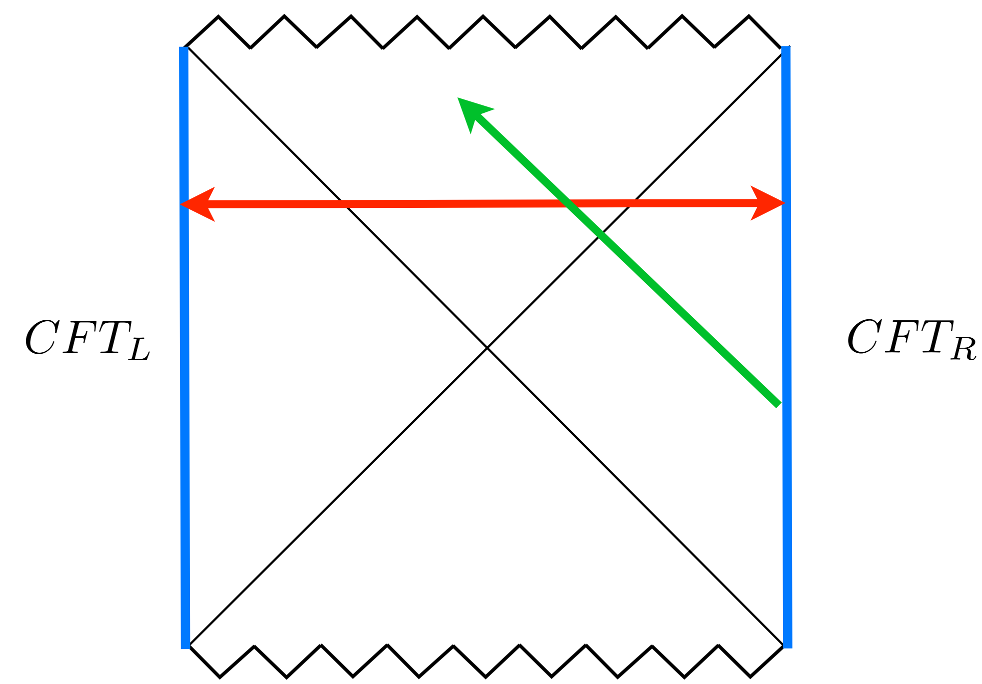
What about evidences in favor of Maldacena's proposal?

They work for insertion in both CFTs/ spatial geodesics

Our claim is inserting Alice on one CFT cannot lead to free infall as she cannot measure the combined state (no Bell measurement)

Inserting Alice on one CFT is important for ER=EPR and related ideas

Seems Maldacena's proposal useful for some questions but not for infall.



Conclusions

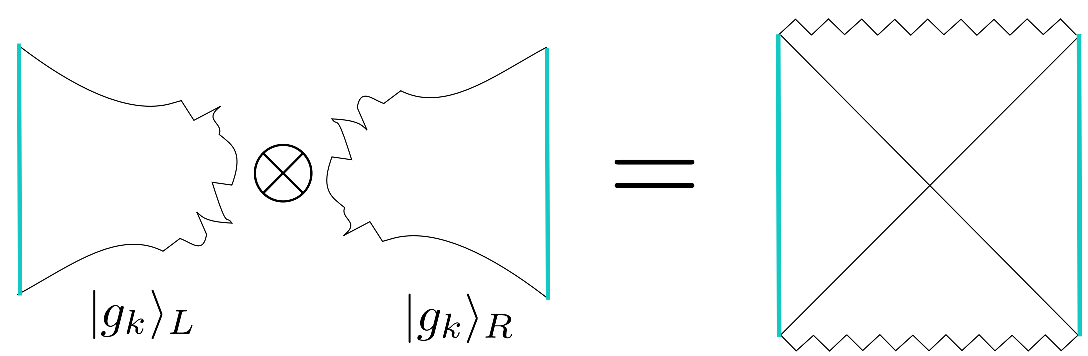
Motion across horizon is a Bell measurement

Bulk and Boundary Bell measurement for topological black holes - consistent

Bulk Bell measurement but not Boundary Bell measurement for eternal black holes

Entanglement of CFT does not imply smoothness

$$\sum_k e^{-\frac{E_k}{2kT}}$$



Entangled fuzzballs do not have common forward wedge.

What goes wrong?

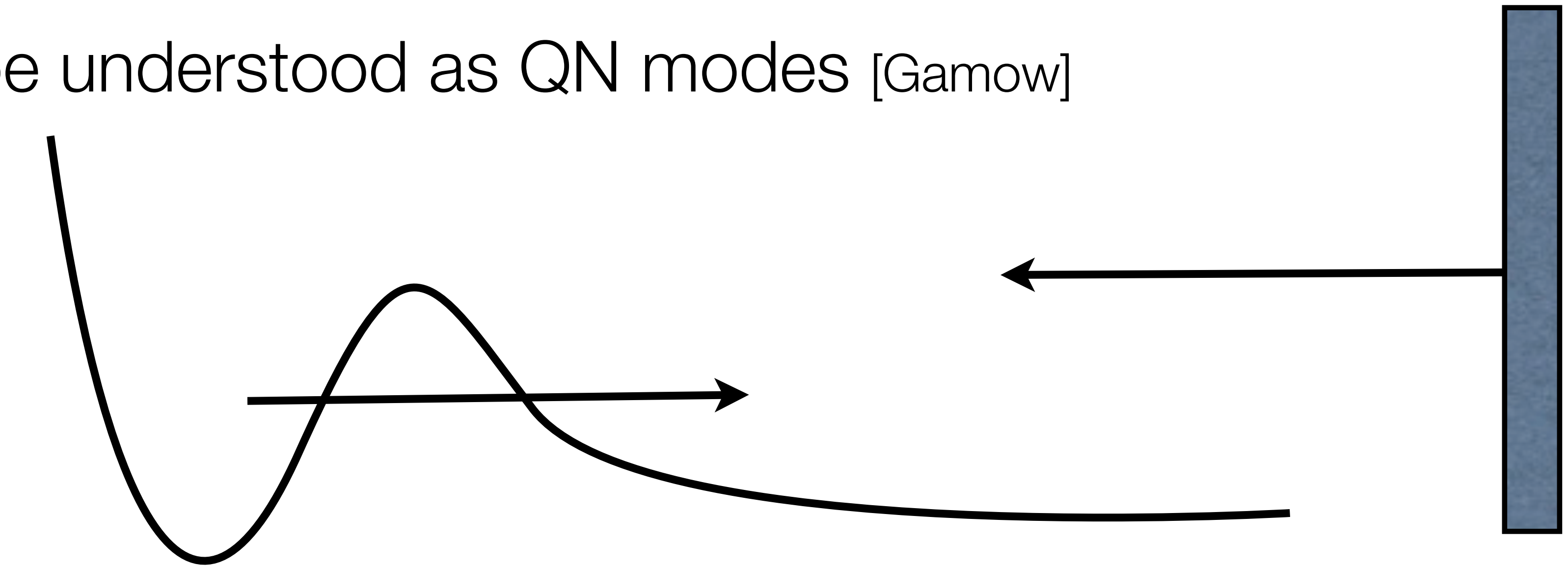
What is the dual to an eternal AdS BH ? **Nothing**

What is the dual to TFD state of two CFTs? Entangled fuzzballs, no forward wedge

More than a claim of what does not work.

Future directions

Tunneling can be understood as QN modes [Gamow]



Physically thats early time behavior.

Standing waves if we put a wall.

Seems related to black holes and fuzzballs. Purely ingoing BC correspond to pile up at the edge of open interval.

Topological black holes and dual unbounded CFTs have quasi-normal modes. However, global AdS/CFT do not.

What is geodesic incompleteness of bulk dual to?

What happens to periodicity of parent CFT upon Rindlerization? Some truncation of modes of hyperbolic CFT?