Blackfolds: effective worldvolume theory for black branes (part II)

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Niels Obers, NBI

0912.2352 (JHEP), 0910.1601 (JHEP), 0902.0427 (PRL) + 1106.4428 (JHEP)
(with R. Emparan, T. Harmark, V. Niarchos)

1012.5081 (PRD) (with J. Armas)

1012.1494 (JHEP), 1101.1297 (NPB), 1112.
(with G. Grignani, T. Harmark, A. Marini, M. Orselli)

0708.2181 (JHEP) (with R. Emparan, T. Harmark, V. Niarchos, M.J. Rodriguez)

1307.0504 & 1209.2127 (PRL) (with J. Armas, J. Gath)

1210.5197 (PRD) (with J. Armas)

1110.4835 (JHEP) (with J. Armas, J. Camps, T. Harmark)
Intro + overview

- long wave length perturbations of black branes
  - construction of new BH solutions in higher dimensions (ST)
  - properties of QFTs via holography

In long-wave length regime: black branes behave like any other type of continuous media with dynamics governed by some (specific) effective theory

- new insights into GR/geometry
- find BHs in higher dimensions and discover their properties
- effective theory that integrates out gravitational degrees of freedom
- AdS/CFT (fluid/gravity) inspired new way to look at gravity
- find universal features of black branes in long wave length regime described by “every day” physics
- reduce complicated gravitational physics to simple response coefficients
- cross-fertilization between classical elasticity/fluid theory and gravity (cf. rigorous development of fluid and superfluid dynamics using fluid/gravity correspondence)
Blackfold approach: a unified framework

two types of deformations:

- **intrinsic**:
  time (in)dependent fluctuations along worldvolume/boundary directions

- **extrinsic**: 
  stationary perturbations along directions transverse to worldvolume

**effective theory of viscous fluid flows**

- Bhattacharyya, Hubeny, Minwalla, Rangamani
- Erdmenger, Haack, Kaminski, Yarom/Nanerjee et al (fluid/gravity ....)
- Camps, Emparan, Haddad/Gath, Pedersen
- Emparan, Hubeny, Rangamani

**effective theory of thin elastic branes**

- Emparan, Harmark, Niarchos, NO
- Armas, Camps, Harmark, NO
- Camps, Emparan
- Armas, Gath, NO/Armas, NO/Armas

fluids living on dynamical surfaces ("fluid branes") = blackfold approach

(unified general framework of the two descriptions)

Reviews:

- Emparan, Harmark, Niarchos, NO
- Emparan/
- Harmark, NO (to appear)
Blackfolds: framework for dynamics of black branes

- based on bending/vibrating of (flat) black branes

very much like other extended solitonic objects:

- Nielsen-Olesen vortices and NG strings
- open strings and DBI action

difference: - short-distance d.o.f. = \textit{gravitational} short-wavelength modes
- extended objects possess black hole \textit{horizon}
  -> \textit{worldvolume} thermodynamics

blackfold = black brane wrapped on a compact submanifold of spacetime
Effective worldvolume theory – leading order

widely separated scales: perturbed black brane looks locally like a flat black brane

- effective stress tensor of black branes correspond to specific type of fluid to leading order: perfect fluid

\[ T^{ab} = (\varepsilon + P)u^a u^b + P \gamma^{ab} \]

for charged black branes of sugra: novel type of (an)isotropic charged fluids

notation: spacetime worldvolume

\[ X^\mu, \mu, \nu \ldots = 0, \ldots, D - 1 \]
\[ \sigma^a, a, b \ldots = 0, \ldots, p \]
\[ n = D - p - 3 \]
BF equations

blackfold equations

intrinsic (Euler equations of fluid + charge conservation)

extrinsic (generalized geodesic eqn. for brane embedding)

fluid excitations (+ charge waves)

elastic deformations

• gives novel stationary black holes (metric/thermo) + allows study of time evolution
• generalizes (for charged branes) DBI/NG to non-extremal solns. (thermal)
• possible in principle to incorporate higher-derivative corrections (self-gravitation + internal structure/multipole)
• BF equations have been derived from Einstein equations

general emerging picture (from hydro of non-extremal D3-branes)

Membrane paradigm ⊂ Fluid/gravity correspondence ⊂ Blackfolds
Main ingredients

- Identify collective coordinates of the brane

\[ X^\mu(\sigma), \varepsilon(\sigma), u^\mu(\sigma), q(\sigma), \ldots \]

positions transverse to worldvolume
energy density (horizon thickness)
local boost velocity
charge density

- Blackfold equations of motion follow from conservation laws (stress tensor, currents,..)

\[ \nabla_\mu T^\mu\nu = 0, \nabla_\mu J^\mu = 0, \ldots \]

effective (charged) fluid living on a dynamical worldvolume:

Extrinsic equations (D-p-1)
\[ T^\mu\nu K_{\mu\nu}^\rho = 0 \]
\[ D_a T^{ab} = 0 \]

Intrinsic equations (p+1)

\[ D_a J^{a_1 \ldots a_p+1} = 0 \]

Leading order BF equations
Stationary solutions

- equilibrium configurations stationary in time = stationary black holes

\[ u = \frac{k}{|k|}, \quad \nabla(\mu k_{\nu}) = 0, \quad \mathcal{T}(\sigma^a) = \frac{T}{|k|} \]

\[ k = \xi + \Omega \chi \]

can solve intrinsic blackfold equations explicitly (e.g. for thickness and velocity)

\[ r_0 = \frac{n\sqrt{1-V^2}}{2\kappa} \quad \quad V^2 = \sum_i \Omega_i^2 R_i^2(\sigma) \]

- blackfolds with boundaries: fluid approaches speed of light at bdry. (horizon closes off !)

extrinsic equations:

\[ K^\rho = \perp_{\rho\mu} \partial_\mu \ln(-P) \]

\[ \tilde{I} = \int_{\mathcal{W}_{p+1}} d^{p+1}\sigma \sqrt{-\gamma} P \]

- thermodynamics: all global quantities: mass, charge, entropy, chemical potentials by integrating suitable densities over the worldvolume
for any embedding (not nec. solution) the “mechanical” action is proportional to Gibbs free energy:

\[ \beta^{-1} I = G = M - \sum_i \Omega_i J_i - TS \]

varying \( G \) \( \rightarrow \) 1st law of thermodynamics

\[ dM = TdS + \Omega dJ \quad \text{(fixed } Q_p) \]

\[ (D - 3)M - (D - 2)(TS + \Omega J) - n\Phi_H^{(p)} Q_p = \mathcal{T}_{\text{tot}} \]

• can also use Smarr relation to show that:
  total tension vanishes for stationary blackfolds
Applications of BF

- new (approximate) stationary black hole solutions

- thermal probe brane method and connection to DB

- perturbations:
  - elasticity response coefficients
  - hydrodynamcis and GL instability
I. New stationary BHs

- Solve the BF equations for stationary solutions:
  - typically: zero tension condition
  - e.g. black ring: gravitational attraction balanced by centrifugal repulsion

- Kerr regime
- regime of mergers and connections
  i) new solution branches (via 0-modes)
  ii) topology changing transition
- BF regime (ultraspinning)

hierarchy of scales:
Example: Black ring

- **wrap black string** on a compact 1D space (topologically $S^1$)

  specify embedding: $S^1$ in $\mathbb{R}^2$ (times point in $\mathbb{R}^{D-3}$)

  \[
  \mathbb{R}^2 : (r, \phi) \quad r = R(\sigma), \quad \phi = \sigma
  \]

  action

  \[
  I_{\text{WW}} \propto \int \sqrt{-\gamma(1-\Omega^2)} \frac{\mathbb{R}}{\mathbb{R}} = \int d\sigma \sqrt{(R')^2 + R^2(1-\Omega^2R^2)} \frac{\mathbb{R}}{\mathbb{R}}
  \]

  full EOM is:

  \[
  (1-\Omega^2R^2)RR'' + ((n+2)\Omega^2R^2 - 2)R'^2 + ((n+1)\Omega^2R^2 - 1)R^2 = 0
  \]

- highly non-linear DE; simple solution with constant $R$. \[ R = \frac{1}{\sqrt{n+1}} \]

  or directly from Carter equation: \[ \frac{\tau_{11}}{R} = 0 \quad \text{(total tension vanishes)} \]

- **zero tension condition** is equivalent to balancing forces on ring

  - centrifugal repulsion balances gravitational tension
  - solution with horizon topology $S^1 \times S^{D-3}$
Neutral blackfolds

First applied to neutral black branes of higher dim gravity:

Quick overview of results:

- new helical black strings and rings

- odd-branes wrapped on odd-spheres (generalizes 5D black ring)

- even-branes wrapped on even-balls correctly reproduce MP BHs in ultraspining (pancaked) limit

- non-uniform black cylinders

- static minimal blackfolds (non-compact)
Blackfolds in supergravity and string theory

- BF method originally developed for neutral BHs, but even richer dynamics when considering charged branes

- extra equations: charge conservation
  consider dilatonic black branes that solve action (includes ST black branes)

\[
S = \frac{1}{16\pi G} \int d^D x \sqrt{-g} \left[ R - \frac{1}{2} (\nabla \phi)^2 - \frac{1}{2(q+2)!} e^{a\phi} H_{[q+2]}^2 \right]
\]

p-branes with q-charge: q=0, particle charge, q=1: string charge, etc.

anisotropic (charged) fluids

\[
T_{ab} = \varepsilon u_a u_b + P_\perp (\gamma_{ab} + u_a u_b - v_a v_b) + P_\parallel v_a v_b
\]

q=1

\[
J_{ab}^{(0)} = Qu_{[b} v_{a]}
\]

spacelike vector \( v \) along the directions of the 1-charge (string)
Effective fluid

for charged dilatonic p-branes:
use exact sugra solution to read off the properties of the fluid:

\[
\begin{align*}
\varepsilon &= \frac{\Omega_{n+1}}{16\pi G} r_0^n (n + 1 + nN \sinh^2 \alpha), \\
P &= -\frac{\Omega_{n+1}}{16\pi G} r_0^n (1 + nN \sinh^2 \alpha), \\
T &= \frac{n}{4\pi r_0 (\cosh \alpha)^N}, \\
s &= \frac{\Omega_{n+1}}{4G} r_0^{n+1} (\cosh \alpha)^N, \\
Q_p &= \frac{\Omega_{n+1}}{16\pi G} n\sqrt{N} r_0^n \sinh \alpha \cosh \alpha, \\
\Phi_p &= \sqrt{N} \tanh \alpha.
\end{align*}
\]

functions of \( r_0 \) and \( \alpha \), \( N = 1,2,3 \) (depending on case)

stress tensor takes form:

\[
T_{ab} = T s \left( u_a u_b - \frac{1}{n} \gamma_{ab} \right) - \Phi_p Q_p \gamma_{ab}
\]

near extremality:

\[
T_{ab}^{(exc)} \sim T s \left( u_a u_b + \left( \frac{N}{2} - \frac{1}{n} \right) \gamma_{ab} \right)
\]
Blackfolds with brane currents

- can perform general analysis of blackfolds with brane currents on them (q=0 and q=1 simplest): anistropic charged perfect fluids (entirely new type of fluid dynamics)
- able to capture thermal excitations of e.g. D-branes with lower D-brane or F-string currents

brane currents induce differences in pressures in directions parallel and transverse to them (due to effective tension $\Phi_q Q_q$ along the current)

- charge density conserved along the q-brane but can redistribute itself in transverse directions
- stability of charge waves governed by isothermal permittivity

$$\epsilon_q \equiv \left( \frac{\partial \Phi_q}{\partial Q_q} \right)_{Q_p,T}$$
ST/M: BF with dipole p-brane charge & brane currents

- find new odd-sphere (+products) stationary black hole solutions with dipole-like (local) charge (checks with exact 5D dipole rings (Emparan))

- new stationary black holes in string/M-theory, w. novel horizon topology
- new type of charge (generalizing dipole charge of ring) entering 1\textsuperscript{st} law of thermo (cf. Copsey, Horowitz)
- (presumably) stable for sufficiently high charge (positive specific heat)
- standard extremal limit gives Dirac: \[ T_{ab} = P \gamma_{ab} \]
- interesting new extremal limits with null waves (beyond DBI):
  \[ T_{ab} = \mathcal{K} l_a l_b - \sqrt{N} Q_p \gamma_{ab} \quad l_a l^a = 0 \]

- can perform general analysis of blackfolds with brane currents on them (q=0 and q=1 simplest): anistropic charged perfect fluids (entirely new type of fluid dynamics)
- able to capture thermal excitations of e.g. D-branes with lower D-brane or F-string currents

- 3-charge example: D1-D5-P (e.g. in D=6: horizon S\^1 x S\^3) with finite entropy in extremal limit could be first example of stable, asymptotically flat, extremal, non-supersymmetric brane in ST with non-spherical horizon topology in D > 5
Odd-sphere blackfolds in string theory

Table 1: A list of horizon topologies for stationary non-extremal black holes in type IIA/IIB string theory based on the singly-charged blackfolds of the theory with worldvolumes curved into products of odd-spheres. The $s^{n+1}$ denotes the ‘small’ sphere in horizon directions orthogonal to the worldvolume. The number $l$ of ‘large’ odd-spheres spanned by the worldvolume is limited by (3.11).

```
<table>
<thead>
<tr>
<th>Brane (IIA)</th>
<th>Worldvolume</th>
<th>⊥ Sphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>$S^1$</td>
<td>$s^7$</td>
</tr>
<tr>
<td>D2</td>
<td>$T^2$</td>
<td>$s^6$</td>
</tr>
<tr>
<td>D4</td>
<td>$S^3 \times S^1$, $T^4$</td>
<td>$s^4$</td>
</tr>
<tr>
<td>NS5</td>
<td>$S^5$, $S^3 \times T^2$</td>
<td>$s^3$</td>
</tr>
<tr>
<td>D6</td>
<td>$S^3 \times S^3$, $S^5 \times S^1$</td>
<td>$s^2$</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>Brane (IIB)</th>
<th>Worldvolume</th>
<th>⊥ Sphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>$S^1$</td>
<td>$s^7$</td>
</tr>
<tr>
<td>F1</td>
<td>$S^1$</td>
<td>$s^7$</td>
</tr>
<tr>
<td>D3</td>
<td>$S^3$, $T^3$</td>
<td>$s^5$</td>
</tr>
<tr>
<td>D5</td>
<td>$S^5$, $S^3 \times T^2$</td>
<td>$s^3$</td>
</tr>
<tr>
<td>NS5</td>
<td>$S^5$, $S^3 \times T^2$</td>
<td>$s^3$</td>
</tr>
</tbody>
</table>
```

Table 2: The analogue of Table 1 in M-theory for M2 and M5 black branes.

```
<table>
<thead>
<tr>
<th>Brane</th>
<th>Worldvolume</th>
<th>⊥ Sphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>M2</td>
<td>$T^2$</td>
<td>$s^7$</td>
</tr>
<tr>
<td>M5</td>
<td>$S^5$, $S^3 \times T^2$, $T^5$</td>
<td>$s^4$</td>
</tr>
</tbody>
</table>
```
Examples (2 charges)

blackfolds based on 2-charge brane systems:  
- D0-Dp (p=2,4,6)  
- F1-Dp (p >0)

<table>
<thead>
<tr>
<th></th>
<th>Worldvolume</th>
<th>⊥ Sphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>D0-D2</td>
<td>F1-D2</td>
<td>(T^2)</td>
</tr>
<tr>
<td></td>
<td>F1-D3</td>
<td>(S^3, T^3)</td>
</tr>
<tr>
<td>D0-D4</td>
<td>F1-D4</td>
<td>(S^3 \times S^1, T^4)</td>
</tr>
<tr>
<td></td>
<td>F1-D5</td>
<td>(S^5, S^3 \times T^2)</td>
</tr>
<tr>
<td>D0-D6</td>
<td>F1-D6</td>
<td>(S^3 \times S^3, S^5 \times S^1)</td>
</tr>
</tbody>
</table>
Examples (3 charges)

3-charge example: D1-D5-P  (e.g. in D=6: horizon $S^1 \times s^3$

with finite entropy in extremal limit

could be first example of stable, asymptotically flat, extremal, non-supersymmetric brane in ST with non-spherical horizon topology in $D > 5$

<table>
<thead>
<tr>
<th>Dimension (non-compact)</th>
<th>Worldvolume</th>
<th>$\perp$ Sphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D = 10$</td>
<td>$S^5, S^3 \times T^2$</td>
<td>$s^3$</td>
</tr>
<tr>
<td>$D = 9$</td>
<td>$S^3 \times S^1, T^4$</td>
<td>$s^3$</td>
</tr>
<tr>
<td>$D = 8$</td>
<td>$S^3, T^3$</td>
<td>$s^3$</td>
</tr>
<tr>
<td>$D = 7$</td>
<td>$T^2$</td>
<td>$s^3$</td>
</tr>
<tr>
<td>$D = 6$</td>
<td>$S^1$</td>
<td>$s^3$</td>
</tr>
</tbody>
</table>

Table 4: A list of horizon topologies for stationary extremal rotating black holes with D1-D5 dipoles in a spacetime with $D$ non-compact dimensions and $10 - D$ compact KK circles. We do not distinguish whether the D1-P current wraps some of the compact directions, which gives different kinds of black holes.

other new extremal black holes in ST ?
Thermal probe branes and BFs

- Brane/string probes widely used in ST, including AdS/CFT
  - uncover features of backgrounds, phase transitions, stringy observables, non-perturbative aspects of FT, dual operators in CFT, ads/CMT
  - learn new things about fundamentals of ST/M-theory by studying low energy theories on D/M-branes

Conventionally used (at weak coupling):

- **F-stings:** NG (Wilson loops, q-qbar potential, energy loss of quarks)
- **D-branes:** DBI (Wilson loops in large sym/antisym reps, flavors, meson spectroscopy, giant gravitons)
- **M-branes:** PST (giant gravitons, self-dual string)

What happens when we heat this up?
Open/closed perspectives

**worldvolume (DBI,NG)**
- microscopic, open
- weak coupling

**spacetime (SUGRA)**
- macroscopic, closed
- strong coupling

- for SUSY configs can interpolate between the two (exactly)
- for non-SUSY (finite T): qualitative matching (more control for near-extremal)

“shapes of branes/strings” are determined dynamically

most work on curved D-branes performed using open string picture:
- probe (N=1)
  \[ K_{ab}^\rho T^{ab} = J \cdot F^\rho \]

can use symmetries, ansätze consistency to construct the exact backgrounds in SUGRA (N >> 1)

extrinsic curvature (2\textsuperscript{nd} fundamental form)

(DBI) EM tensor of the brane

Note: any probe brane will have EOM of this form Carter
Example: Blon

DBI

D3-brane DBI with constant electric flux (D3->F1)

Callan, Maldacena

SUGRA

derive appropriate PDEs and prove existence of SUSY sol

Lunin

branes follow harmonic profiles (unique, given BCs)

match: open/closed duality beyond decoupling limit

Q: Can one extend this open/closed picture to finite T? (non-SUSY)

interesting since:
- develop horizon, learn about BH physics
- branes are used to probe spaces at finite T (hot flat or AdS space, AdS BH)
- thermal states in gauge theories (AdS/CFT)
Intermezzo: conventional method for probe branes in thermal background

conventionally used method: ‘Euclidean DBI probe’ method:
- Wick rotate background and classical DBI action
- find solns. of EOM
- identify the radii of thermal circle in background and DBI soln.

(see also: Kiritsis/Kiritsis,Taylor/Kiritsis,Kehagias)

boils down to: solving same (local) EOMs but different BCs

\[(T_E)^{ab}_{\text{DBI}}(K_E)_{ab} = (\perp E)^{\rho\lambda}_E \frac{1}{4!}(J_E)^{abcd}(F_E)_{\lambda abcd}\]

this global condition is not enough to ensure that probe is in thermal equilibrium with the background

reason: to ensure thermal equilibrium we need to also modify the EOMs (via the stress tensor) since the brane DOFs get thermally excited

Example: single D3-brane near extremality (at weak coupling)
- gas of photons (+ superpartners)

\[T_{ab} = -T_D3\eta_{ab} + T_{ab}^{(\text{NE})}, \quad T_{00}^{(\text{NE})} = \rho, \quad T_{ii}^{(\text{NE})} = p, \quad i = 1, 2, 3 \quad \rho = 3p = \pi^2 T^4 / 2.\]
Open/closed at finite T

open (weak coupling), \( N=1 \)

thermal DBI
(thermal SUSY gauge theory
+ string corrections)

Grignani, Harmark, Marini, Orselli (to appear)

thermal NG: quantize string in
finite T background

de Boer, Hubeny, Rangamani, Shigemori

closed (strong coupling), \( N >> 1 \)

black branes (solitons)
- curved black brane solutions
  in SUGRA

exact solutions already hard at \( T=0 \)

go to regime where brane is approximately locally flat:
-> can use probe approximation
= 0th order blackfold construction

gives the geometry to leading order in perturbative expansion governed \( r_0/R \)

like DBI/NG this is (to leading order) probe computation:
dynamics in both cases described by Carter equation:

- difference is EM tensor that you put
  + different regime! (match when \( T->0, N=1 \))
Applications

-> Heating up DBI/NG solutions using:
   * blackfolds as thermal probe branes/strings in string theory

  shows new qualitative & quantitative effects

  black probe          background
  D3-F1                hot flat 10D
  M5-M2                11D
  F1                   AdS BH
  D3                   hot..
  AdS5 x S5

  M2                   AdS7 x S4
  AdS4 x S7

  • Thermal **Bion** solutions (wormhole & spike)
    Grignani,Harmark,Marini,NO,Orsell

  • Thermal **string probes in AdS** &
    finite T Wilson loops
    Grignani,Harmark,Marini,NO,Orsell

  • Thermal (spinning) **giant gravitons**
    (also: new null-wave giant gravitons)
    Armas,Harmark,NO,Orselli,Vigand Pedersen
    Armas,NO,Vigand Pedersen
Black branes as fluids and elastic materials

Goal: show that asymptotically flat (charged) black branes have both elastic and fluid properties

Method: perturb -> consider derivative corrections

two ways:

- **intrinsic perturbations** parallel to the worldvolume (wiggle)
  
  viscosities (shear, bulk)  
  charge diffusion  
  gives connection to GL instability, fluid/gravity, …

  can be used in AdS/Ricci flat map

- **extrinsic perturbations** transverse to the worldvolume (bend)

  response coefficients are inputs to effective theory

* generalizes Polyakov QCD string + actions considered in theoretical biology

Camps, Emparan, Haddad  
Gath, Pedersen  
Emparan, Hubeny, Rangamani,  
Caldarelli, Camps, Gouteraux, Skenderis
Elasticity: Fine structure corrections to blackfolds

- can explore corrections in BF approach that probe the fine structure: go beyond approximation where they are approximately thin

\[
\hat{T}^{\mu\nu}(x^\alpha) = \int d^{p+1}\sigma \sqrt{-\gamma} \left[ T^{\mu\nu}_{(0)}(\sigma^a) \frac{\delta^D(x^\alpha - X^\alpha(\sigma^a))}{\sqrt{-g}} - \nabla_\rho \left( T^{\mu\nu\rho}_{(1)}(\sigma^a) \frac{\delta^D(x^\alpha - X^\alpha(\sigma^a))}{\sqrt{-g}} \right) \right] + \ldots
\]

Vasilic, Vojinovic

accounts for:

- dipole moment of wv stress energy
  
  = bending moment (density)

\[
D^{ab\rho} = \int_\Sigma d^{D-1}x \sqrt{-g} \hat{T}^{ab} x^\rho = \int_{B_p} d^p\sigma \sqrt{-\gamma} d^{ab\rho}
\]

- internal spin degrees of freedom
  
  (conserved angular momentum density)

\[
J^{\mu\nu}_\perp = \int_\Sigma d^{D-1}x \sqrt{-g} \left( \hat{T}^\mu_0 x^\nu - \hat{T}^\nu_0 x^\mu \right) = \int_{B_p} d^p\sigma \sqrt{-\gamma} j^{0\mu\nu}_\perp
\]
Fine structure: Charged branes

- branes charged under Maxwell fields: multipole expansion of current

\[
\hat{J}^\mu(x^\lambda) = \int_{W_{p+1}} dV \left[ \frac{J^\mu(0)(\sigma^a)}{\sqrt{-g}} \delta^{(D)}(x^\lambda - X^\lambda(\sigma^a)) - \nabla_\rho \left( \frac{J^{\mu \rho}(\sigma^a)}{\sqrt{-g}} \delta^{(D)}(x^\lambda - X^\lambda(\sigma^a)) \right) + \ldots \right]
\]

dipoles of charge

electric dipole moment:

\[
J^{\mu \nu}_{(1)} = m^{\mu \nu} + u^\mu p^{\alpha \nu} + J^{\mu \alpha}_{(1)} u^\nu
\]

\[
P^{a \rho} = \int_{\Sigma} d^{D-1}x \sqrt{-g} \hat{J}^\mu u^a_\mu x^\rho = \int_{B_p} dp \sigma \sqrt{-\gamma} p^{a \rho}
\]

can also write generalization for p-branes carrying q-charge (omit details)

corrected pole/dipole BF equations generalize those of general relativistic (charged) spinning point particle (p=0, q=0) to extended charged objects

Armas, Gath, NO
Relativistic Young modulus

bending moment a priori unconstrained -> assume classical Hookean elasticity theory:

\[ d^{a b \rho} = \tilde{Y}^{a b c d} K_{c d \rho} \]

extrinsic curvature like Lagrangian strain (measures variation of induced metric transverse to \( w v \)).

bending moment (not present for point particle)

relativistic Young modulus

general structure of \( Y \) can be classified using effective action approach (done for neutral isotropic fluids):

generalization to (isotropic) case with \( w v \) charge:

\[
\tilde{Y}^{a b c d} = -2 \left( \lambda_1 (k; T, \Phi_H) \gamma^{a b} \gamma^{c d} + \lambda_2 (k; T, \Phi_H) \gamma^{(a \gamma^b)(c \gamma^d)} + \lambda_3 (k; T, \Phi_H) k^{(a \gamma^b)(c \gamma^d)} \right) + \lambda_4 (k; T, \Phi_H) \frac{1}{2} (k^a k^b \gamma^{c d} + \gamma^{a b} k^c k^d) + \lambda_5 (k; T, \Phi_H) k^a k^b k^c k^d
\]

\( k = \) Killing vector, \( T = \) global temperature, \( \Phi = \) chemical potential

upshot: (charged) black branes are described by this effective theory + characterized by particular values of the response coefficients \( \lambda \).
piezo electric moduli

• for piezo electric materials: dipole moment proportional to strain ($q=0$)

\[ p^{a\rho} = \tilde{\kappa}^{abc} K_{c d \rho} \]

relativistic generalization of piezo-electric modulus found in electro-elasticity

structure of kappa not yet classified from effective action, but from symmetries/covariance

\[ \tilde{\kappa}^{abe} = -2 \left( \kappa_1(k; T, \Phi_H) \gamma^{a(b} k^{c)} + \kappa_2(k; T, \Phi_H) k^a k^b k^c + \kappa_3(k; T, \Phi_H) k^a \gamma^{be} \right) \]

similarly: for $p$-branes with $q$-charge:
  - possible anomalous terms in Young-modulus
  - piezo electric effect with new types of piezo electric moduli

(note: piezo electric effect also encountered in context of superfluids) Erdmenger,Fernandez,Zeller

upshot: (charged) black branes are described by this effective theory + characterized by particular values of the response coefficients kappa
Measuring Young/piezo electric moduli for charged BB

- can be measured in gravity by computing the first order correction to bent charged black branes

simplest example: charged black branes of EMD theory obtained by uplift-boost-reduce from neutral bent branes

more involved: charged black p-branes with q-charge of E[(q+1)-form]D theory can use again same procedure to charge up branes + use in string theory setting U-dualities to generate higher form charge

- bending of black string (or brane) induces dipole moments of stress can be measured from approximate analytic solution (obtained using MAE)

\[ g_{\mu\nu} = \eta_{\mu\nu} + h^{(M)}_{\mu\nu} + h^{(D)}_{\mu\nu} + \mathcal{O}(r^{-n-2}) \quad \nabla^2_h^{(D)} = 16\pi G d_{\mu\nu} r^\perp \partial r^\perp \delta^{(n+2)}(r) \]

- bending of charged black string (or brane) induces dipole moments of charge can be measured from approximate analytic solution (obtained using MAE)

\[ A_\mu = A^{(M)}_\mu + A^{(D)}_\mu + \mathcal{O}(r^{-n-2}) \quad \nabla^2 A^{(D)}_\nu = 16\pi G p_\nu r^\perp \partial r^\perp \delta^{(n+2)}(r) \]
Examples of results for new response coeffs

p-branes with 0-form (Maxwell) charge: 3+1 response coefficients

Young modulus

\[
\lambda_1(k; T, \Phi_H) = \frac{\Omega_{(n+1)}}{16\pi G} \xi_2(n) \left( \frac{n}{4\pi T} \right)^{n+2} |k|^{n+2} \left( 1 - \frac{\Phi_H^2}{|k|^2} \right)^{\frac{n}{2}} \left( \frac{3n+4}{2n^2(n+2)} - \bar{k} \left( 1 - \frac{\Phi_H^2}{|k|^2} \right) \right)
\]

\[
\lambda_2(k; T, \Phi_H) = \frac{\Omega_{(n+1)}}{16\pi G} \xi_2(n) \left( \frac{n}{4\pi T} \right)^{n+2} |k|^{n+2} \left( 1 - \frac{\Phi_H^2}{|k|^2} \right)^{\frac{n}{2}+1} \frac{1}{2(n+2)},
\]

\[
\lambda_3(k; T, \Phi_H) = \frac{\Omega_{(n+1)}}{16\pi G} \xi_2(n) \left( \frac{n}{4\pi T} \right)^{n+2} |k|^n \left( 1 - \frac{\Phi_H^2}{|k|^2} \right)^{\frac{n}{2}},
\]

piezo electric

\[
\kappa_1(k; T, \Phi_H) = \frac{\Omega_{(n+1)}}{16\pi G} \xi_2(n) \left( \frac{n}{4\pi T} \right)^{n+2} \frac{\Phi_H |k|^n \left( 1 - \frac{\Phi_H^2}{|k|^2} \right)^{\frac{n}{2}}}{2}
\]

similar expressions for p-branes with q-form charge: 3+1 response coefficients
Effective action for elastic expansion of branes

old physical problem: **fluids living on surfaces**: response to bending
(e.g. biconcave shape of red blood cells: cannot be described by standard soap bubble action, with minimal surface)

Helfrich-Canham bending energy: add
\( F[X^\mu] = \alpha \int dA K^2 \)

in physics: improved effective action for QCD string (Polyakov & Kleinert)

**general framework for higher order corrections** (stationary brane fluids)

**EM tensor**

\( T_{ab} = \frac{2}{\sqrt{-\gamma}} \frac{\delta \mathcal{L}}{\delta \gamma_{ab}} \)

**dipole moment**

\( D_{ab}^i = \frac{1}{\sqrt{-\gamma}} \frac{\delta \mathcal{L}}{\delta K_{ab}^i} \)

**spin current**

\( S_{ij}^a = \frac{1}{\sqrt{-\gamma}} \frac{\delta \mathcal{L}}{\delta \omega_{a ij}} \)

\( K_{ab}^\mu = \nabla_a u_b^\mu \)

\( \omega_{a ij} = -n^j_\mu \nabla_a n^{i\mu} \)
Leading order effective action

\[ I [X^\mu] = \int_{W_{p+1}} \mathcal{L} (\sqrt{-\gamma}, k) = \int_{W_{p+1}} dp+1 \sqrt{-\gamma} \lambda_0 (k) \]

gives perfect fluid

\[ T^{ab} = T^{ab}_{(0)} = \lambda_0 (k) \gamma^{ab} - \lambda'_0 (k) k u^a u^b \]

\[ P = \lambda_0 (k), \quad \epsilon + P = -\lambda'_0 (k) k \]

strain tensor

\[ U_{ab} = -\frac{1}{2} (\gamma_{ab} - \tilde{\gamma}_{ab}) \]

\[ dU_{ab} = N_\rho K_{a \beta}^\rho \]

can define elasticity tensor (measuring compression/stretching)

\[ E^{abcd} = 2 \left( \lambda_0 (k) \gamma^{(a} \gamma^{b} \gamma^{c} \gamma^{d)} - \left( \frac{\partial \lambda_0 (k)}{\partial \gamma_{ab}} \right) \gamma^{cd} - 2 \left( \frac{\partial^2 \lambda_0 (k)}{\partial \gamma_{ab} \partial \gamma_{cd}} \right) \right) \]

extrinsic dynamics in transverse directions to the surface correspond to that of elastic brane

Armas, NO
Second order corrections

first order:

\[ k^a \nabla_a k , \nabla_a k^a \]

are zero

in agreement with analysis of stationary & non-dissipative fluids (expansion and shear vanish)

2\textsuperscript{nd} order elastic

\[ \lambda_1(k) K^i K_i , \lambda_2(k) K^{abi} K_{abi} , \lambda_3(k) k^a k^b K_{ac} K^c_{bi} , \]
\[ \lambda_4(k) k^a k^b K_{ab} K_i , \lambda_5(k) k^a k^b k^c k^d K_{ab} K_{cdi} . \]

2\textsuperscript{nd} order spin

\[ \varpi_1(k) \omega_{ij} \omega^i_j , \varpi_2(k) k^a k^b \omega_{aij} \omega^i_j \]

2\textsuperscript{nd} order hydrodynamic

\[ \nu_1(k) \nabla_a \nabla^a k , \nu_2(k) \mathcal{R} , \nu_3(k) k^a k^b \mathcal{R}_{ab} , \]
\[ \nu_4(k) \nabla_{[a} k_{b]} \nabla^{[a} k^{b]} , \nu_5(k) \nabla_a k \nabla^a k , \nu_6(k) R^a_{ba} b , \nu_7(k) k^a k^b R^c_{acb} \]
coupling between elastic and hydrodynamic modes

using field redefinitions, ibp and other props:
one finds for codimension higher than 1 branes
- 3 elastic response coefficients
- 5 hydrodynamic response coefficients
- 1 spin response coefficient

but: coupling between elastic and hydro due to geometric constraints

Gauss-Codazzi

\[ R_{abcd} = \mathcal{R}_{abcd} - K_{ac}^i K_{bdi} + K_{ad}^i K_{bei} \]

new terms compared to stationary and non-dissipative space-filling fluids

cf. Banerjee, Bhattacharya, Bhattacharyya, Jain, Minwalla et al
Jensen, Kaminski, Kovtun, Meyer, Ritz et al
Bhattacharya, Bhattacharyya, Rangamani
Young modulus from effective action

using 2\textsuperscript{nd} order elastic corrections one finds from the action

\[ \mathcal{D}_{ab} = \mathcal{Y}_{abcd} K_{cd} \]

with

\[ \mathcal{Y}_{abcd} = 2 \left( \lambda_1 \gamma^{ab} \gamma^{cd} + \lambda_2 \gamma^{a(c} \gamma^{d)b} + \lambda_3 k^{(a} \gamma^{b)}(c \kappa^d) + \frac{\lambda_4}{2} \left( \gamma^{ab} k^c k^d + \gamma^{cd} k^a k^b \right) + \lambda_5 k^a k^b k^c k^d \right) \]

black branes in gravity are a particular case of this!
Intrinsic transport coefficients: viscosities

- to first order in derivative expansion

\[ T^{ab} = (\epsilon u^a u^b + PP^{ab} - 2\eta \sigma_{ab} - \zeta \theta P^{ab}) \delta^{n+2}(x^\rho - X^\rho) \]

\[ \eta = \frac{s}{4\pi}, \quad \zeta = 2\eta \left(\frac{1}{\rho} - c_s^2\right) \]

can apply e.g. to GL instability: include viscous damping of the soundmode perturbations in dispersion relation

\[ \Omega = \frac{k}{n+1} \left(1 - \frac{n+1}{n\sqrt{n+1}}kr_0\right) \]

fits very well with GL curve

recnetly: 2^{nd} order transport via AdS/Ricci flat map
Viscous corrections to charged black branes

charged branes:

simplest case is **Reisner-Nordstrom branes** (solution of higher-D EM theory)

extra contribution to charge current:

```
y^{(1)} = -\mathcal{D} \left( \frac{Q T}{w} \right)^2 \Delta^{ab} \partial_b \left( \frac{\Phi}{T} \right)
```

results:

- shear viscosity/entropy satisfies usual bound, bulk viscosity provides counterexamples to various (proposed) bounds in certain regimes of charge

analyzed effect on first order dispersion relation for effective fluid

- results are in agreement with thermo for smeared black D0-branes

interesting relations with AdS/fluid-gravity (using AdS/Ricci flat map)
Effective hydrodynamics of black D3-branes

Emparan, Hubeny, Rangamani,

relation between various fluid descriptions encountered:

Membrane paradigm $\subset$ Fluid/gravity correspondence $\subset$ Blackfolds.

study effective hydrodynamics on non-extremal black D3-brane
(geometry includes three regions, Rindler, AdS throat, flat Minkowski)

enclose D3-brane in a “box”; Dirichlet boundary conditions + allow long wavelength fluctuations along D3-brane worldvolume

$\rightarrow$ constitutive relations for the effective theory and transport coeffs can be read off (charge is fixed, only energy density fluctuates)
  gives neutral relativistic fluid capturing low wave length modes on surface outside horizon

two non-trivial scales (horizon & cutoff scale, in terms of charge radius scale)
$\rightarrow$ can interpolate between the different descriptions
Relevance of BF method

- **new stationary BH solutions:**
  - approximate analytic construction of BH metrics in higher D gravity/supergravities (cf. String Theory)
  - possible horizon topologies, thermodynamics, phase structure, …
  - new non-extremal and extremal BH solutions
  - useful for insights/checks on exact analytic/numeric solutions

- **BH instabilities and response coefficients:**
  - understand GL instabilities in long wavelength regime, dispersion relation, elastic (in) stabilities, new long wavelength response coefficients for BHs, Young modulus (hydro + material science)

- **Thermal probe branes/strings:**
  - new method to probe finite T backgrounds with probes that are in thermal equilibrium with the background (e.g. hot flat space, BHs)

- **AdS/CFT:**
  - many potential applications
    - (new black objects in AdS, connection with fluid/gravity, thermal probes thermal giant gravitons, BHs on branes, …)
  - interrelations between the four items above

References:
- EHONR/EHON/ Caldarelli,Emparan,Rodriguez
- Armas,NO/Camps,Emparan,Giusto,Saxena/..
Outlook

- systematic **effective actions** for elastic/hydrodynamic properties of charged fluid branes
  - obtained useful inputs/insights from gravity

Cf. development of fluids/superfluids inspired by gravity and holography

- elastic corrections for D3-branes and AdS/CFT !
- responses for **spinning charged branes**
- response coefficients in other backgrounds with **non-zero fluxes** (susceptibility, polarizability)
- **Chern-Simons** couplings
- **multi-charge** bound states
- **entropy current**
- effective hydrodynamics of **spinning D3-branes** (Dp,M)
- further explore **AdS/Ricci flat** connection
The end