Forecasting Multi-field Inflation

What halo bias can tell us

Simone Ferraro
(Princeton University)

In collaboration with K. Smith, D. Baumann and D. Green
[based on arXiv:1209.2173 and arXiv:1209.2175
+ work in progress]

University of Michigan, Ann Arbor
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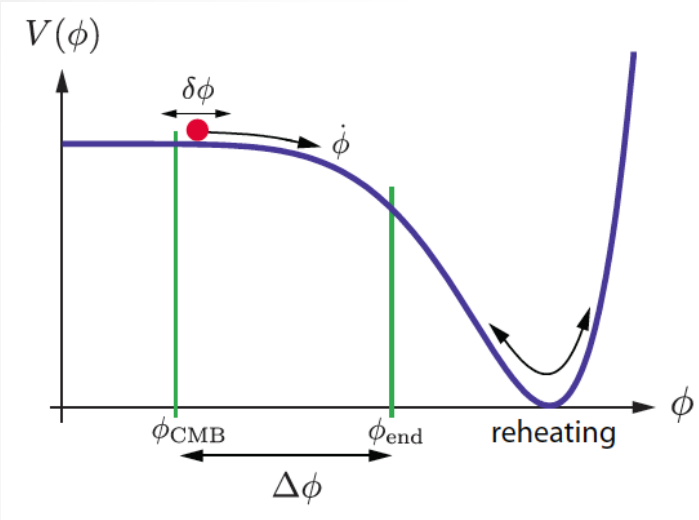
Outline

- Clustering in f_{NL} cosmology
- Clustering with general Non-Gaussian initial conditions
- Stochastic bias
- Examples: Curvaton and τ_{NL}
- Forecasts: single and multiple tracers
- Conclusions

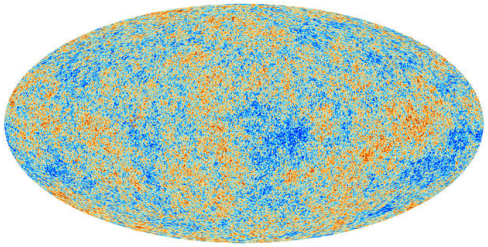
The big picture

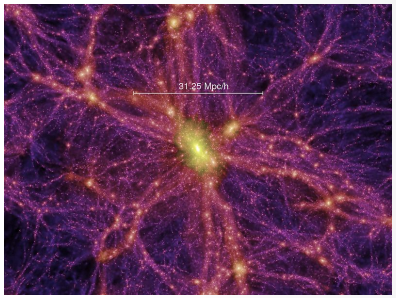
Inflationary perturbations

Late universe



gravity

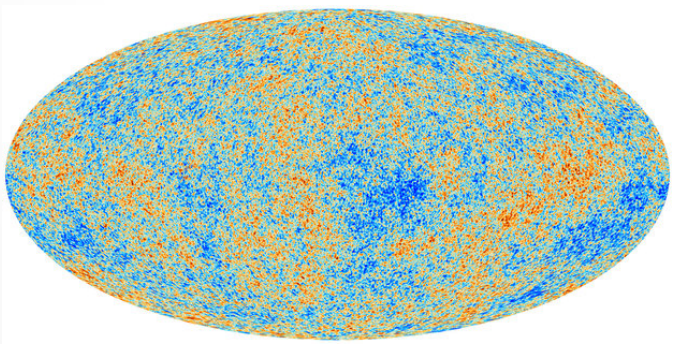




- Statistics of inflationary perturbations determine late time density field.
- Inflationary models can make different predictions for the initial conditions.
- Understanding the relationship between dark matter and galaxies is key for LSS to make constrain cosmology.

Constraining inflation

- So far perturbations are… adiabatic, scale invariant, superhorizon.
- How do we *distinguish* between models ?
- Primordial gravitational waves.
- (Non-)Gaussianity: from Planck



$$f_{\rm NL}^{\rm local} = 2.7 \pm 5.8$$
$$f_{\rm NL}^{\rm equil} = -42 \pm 75$$
$$f_{\rm NL}^{\rm ortho} = -25 \pm 39$$
$$\tau_{\rm NL} < 2800\ (95\%\ {\rm CL})$$

- Challenge for LSS to improve on this!

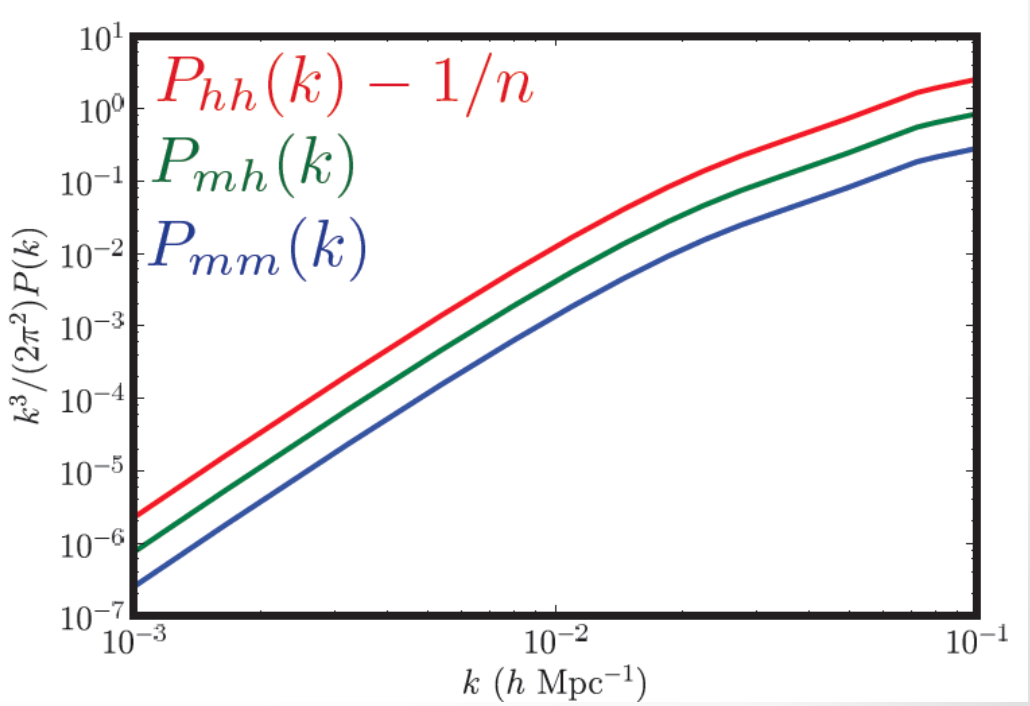
Clustering in Gaussian cosmology

- In a Gaussian cosmology, the halo density is proportional to the dark matter density on large scales

$$\delta_h = b_g \delta$$

$$P_{mh}(k) = b_g P_{mm}(k)$$

$$P_{hh}(k) = b_g^2 P_{mm}(k)$$



Clustering in $f_{\rm NL}$ cosmology

- Local model: $\Phi(\mathbf{x}) = \phi(\mathbf{x}) + f_{\rm NL}(\phi^2(\mathbf{x}) - \langle\phi^2(\mathbf{x})\rangle)$

Generates *scale dependent* halo bias:

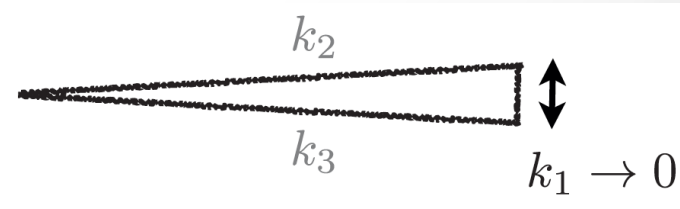
$$P_{\rm mh} = \overbrace{\left(b_g + \beta_f \frac{f_{\rm NL}}{\alpha(k)}\right)}^{\textbf{bias(k)}} P_{\rm mm}$$

$$\alpha(k) \propto k^2$$

$$P_{\rm hh} = \left(b_g + \beta_f \frac{f_{\rm NL}}{\alpha(k)}\right)^2 P_{\rm mm}$$

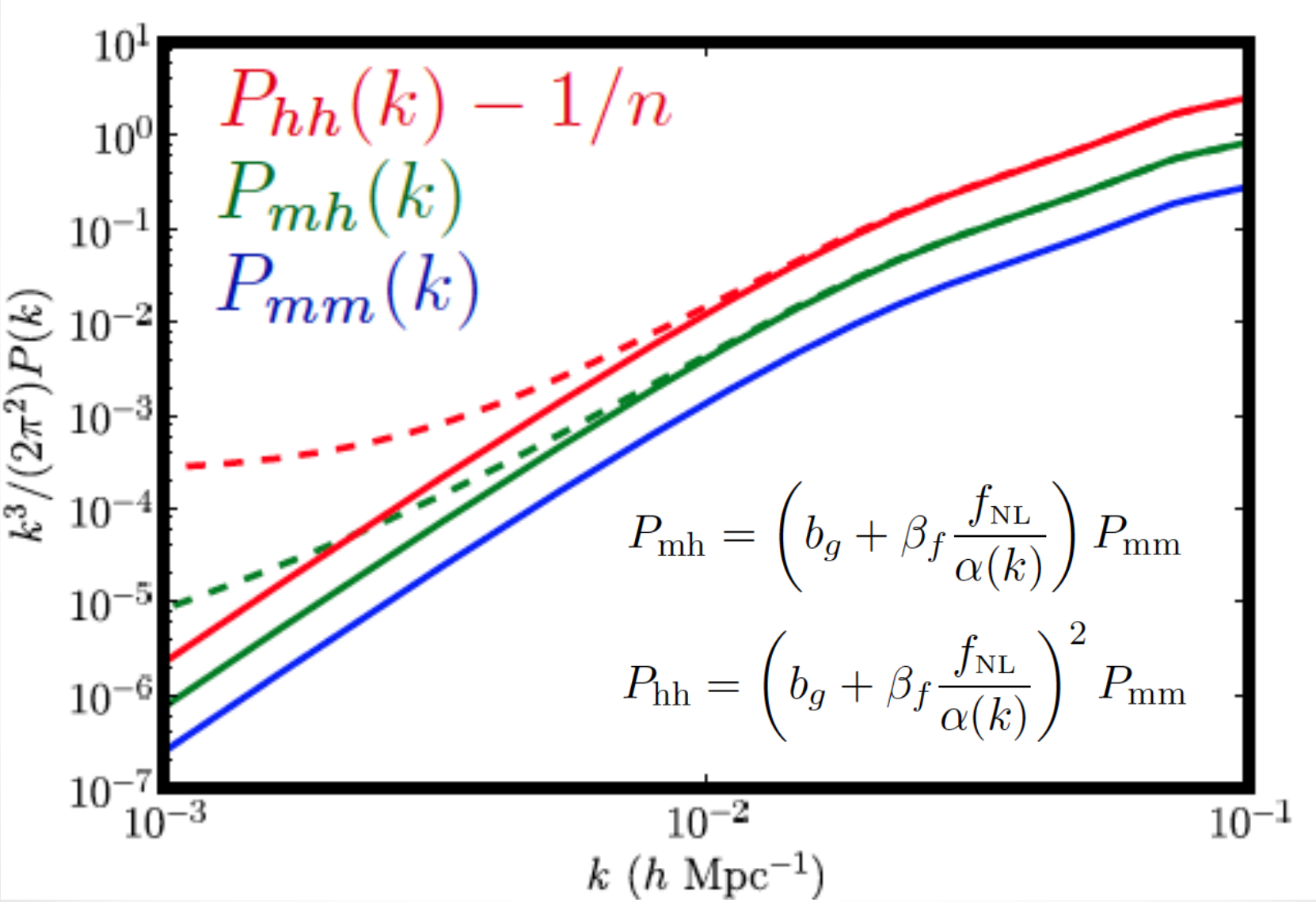
Note that scale dependence is sourced by the *squeezed limit* of the bispectrum:

$$\hat{f}_{\rm NL} \equiv \frac{1}{4} \lim_{k_1 \to 0} \frac{\xi_\Phi^{(3)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)}{P_1 P_2}$$



Dalal et al. 2008
Verde, Matarrese 2009

Clustering in $f_{\rm NL}$ cosmology



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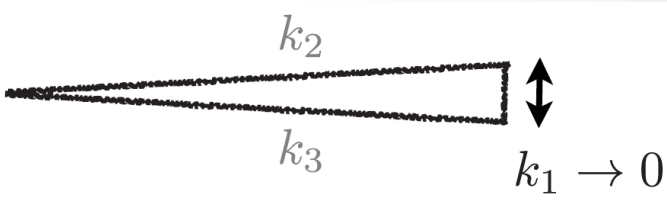

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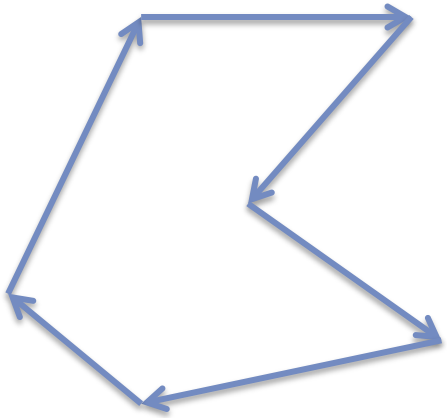
Which shapes contribute to the power spectrum?

$$\langle \Phi_{\boldsymbol{k}_1} \Phi_{\boldsymbol{k}_2} \cdots \Phi_{\boldsymbol{k}_N} \rangle_{\rm c} = (2\pi)^3 \delta_{\rm D}(\boldsymbol{k}_{12\ldots N})\, \xi_{\Phi}^{(N)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \ldots, \boldsymbol{k}_N)$$

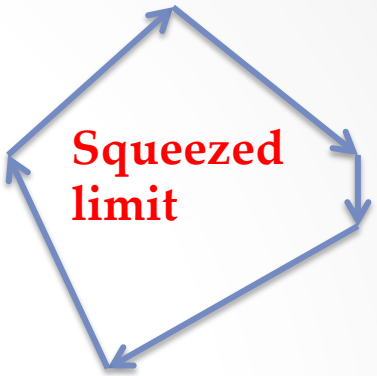


For which configurations will we eventually beat Planck?

General answer

- Matter-Halo correlation:

$$P_{\rm mh}(k,M) = P_{\rm mm}(k)\left(b_g^N(M) + \sum_{n\geq 2}\mathcal{D}_n(M) f_{\hat{1},n}(k,M)\right)$$

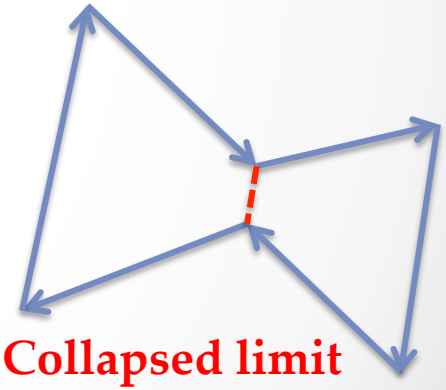


$\vec{k} \to 0$

$$f_{\hat{1},n}(k\to 0) \sim \int_{\boldsymbol{q}_i} \xi_\Phi^{(n+1)}(\boldsymbol{k},\boldsymbol{q}_1,\cdots,\boldsymbol{q}_n)$$

- Halo-Halo correlation:

$$P_{\rm hh}(k,M) = P_{\rm mm}(k)\Bigg(b_g^N(M)^2 + 2b_g^N(M)\sum_{n\geq 2}\mathcal{D}_n(M) f_{1,n}(k,M,\bar{M})\Big|_{M=\bar{M}} + \sum_{m,n\geq 2}\mathcal{D}_m(M)\mathcal{D}_n(\bar{M}) f_{m,n}(k,M,\bar{M})\Big|_{M=\bar{M}}\Bigg)$$



$$f_{n,m}(k\to 0) \sim \int_{\sum \boldsymbol{q}_i = \boldsymbol{k}} \xi_\Phi^{(n+m)}(\boldsymbol{q}_1,\cdots \boldsymbol{q}_n, \boldsymbol{q'}_1,\cdots,\boldsymbol{q'}_n)$$

arXiv:1209.2173
arXiv:1209.2175

Stochastic bias

Note that in general $$\frac{P_{\rm hh}(k)}{P_{\rm mm}(k)} \neq \left(\frac{P_{\rm mh}(k)}{P_{\rm mm}(k)}\right)^2$$

Different bias inferred from $P_{\rm hh}(k)$ and $P_{\rm mh}(k)$

Matter and halo fields are not proportional on large scales

Stochastic bias

Arises when more than one field contribute to curvature and non-Gaussianity:

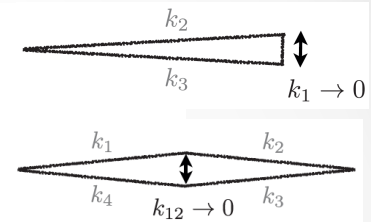
Powerful probe of multi-field inflation

arXiv:1209.2173

τ_{NL} and stochastic bias

$$\xi_\Phi^{(3)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = f_{\rm NL}\left[P_1 P_2 + 5\,\text{perms.}\right] + \mathcal{O}(f_{\rm NL}^3)\ ,$$

$$\xi_\Phi^{(4)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) = 2\left(\tfrac{5}{6}\right)^2 \tau_{\rm NL}\left[P_1 P_2 P_{13} + 23\,\text{perms.}\right]$$



$f_{\rm NL}$ and $\tau_{\rm NL}$ independent parameters in general

$$P_{\rm mh} = \left(b_g + \beta_f \frac{f_{\rm NL}}{\alpha(k)}\right) P_{\rm mm}$$

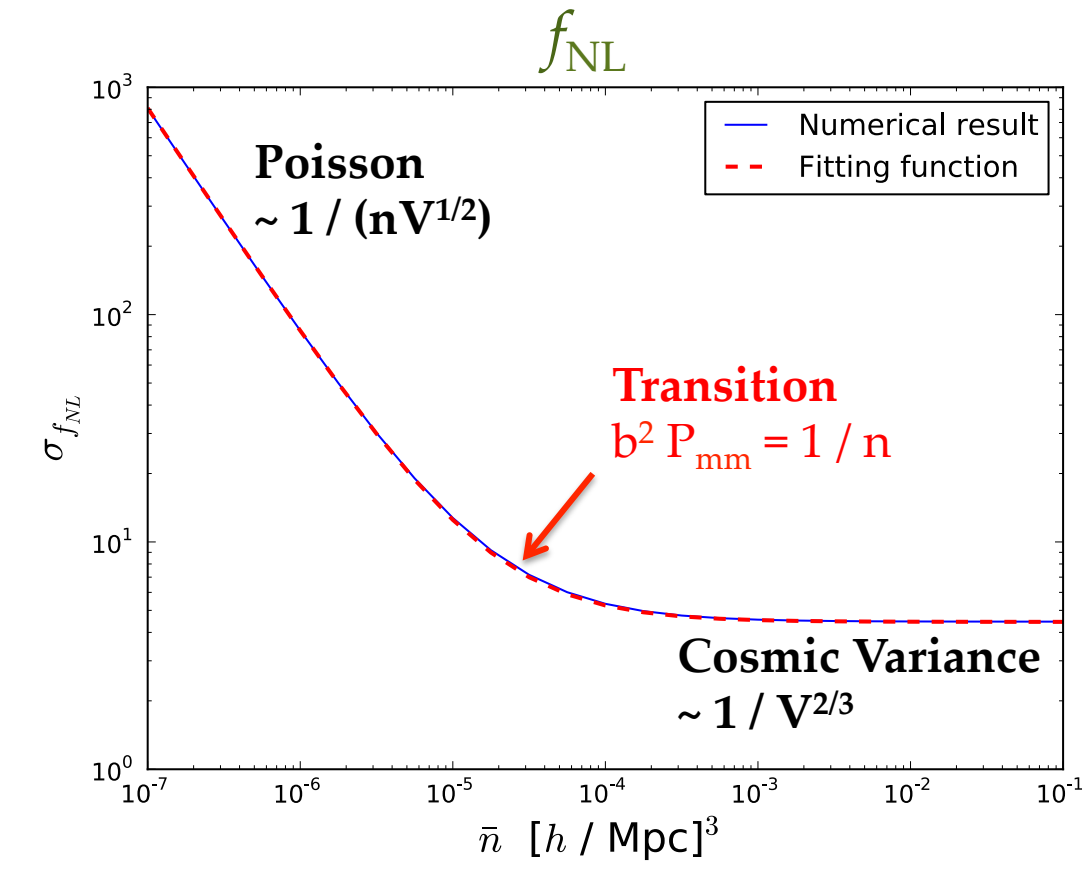
$$P_{\rm hh} = \left(b_g^2 + 2 b_g \beta_f \frac{f_{\rm NL}}{\alpha(k)} + \beta_f^2 \frac{\left(\frac{5}{6}\right)^2 \tau_{\rm NL}}{\alpha^2(k)}\right) P_{\rm mm}$$

Bias is stochastic if

$$\tau_{\rm NL} \neq \left(\frac{6}{5} f_{\rm NL}\right)^2$$

- Detect stochastic bias by comparing $P_{\rm mh}$ and $P_{\rm hh}$
- Note momentum dependence of $P_{\rm hh}$: k^0, k^{-2}, k^{-4}.
- Can separate $f_{\rm NL}$ from $\tau_{\rm NL}$ even with a single tracer.

Forecasts – single tracer



$V = 25\ (\mathrm{Gpc}/h)^3$

$z_m = 0.7, \quad b_g = 2.5$

$M \sim 1.7 \times 10^{13}\ M_o/h$

$k_{max} = 0.03\ \mathrm{h/Mpc}$

Marginalized:

$\sigma(f_{NL}) = 4.5$

$\sigma(\tau_{NL}) = 500$

Planck:

$\tau_{NL} < 2800\ (95\%\ \mathrm{CL})$

$f_{NL}^{local} = 2.7 \pm 5.8$

- LSS is competitive with CMB, even for single bin
- Better using multiple tracers / halo-matter correlation

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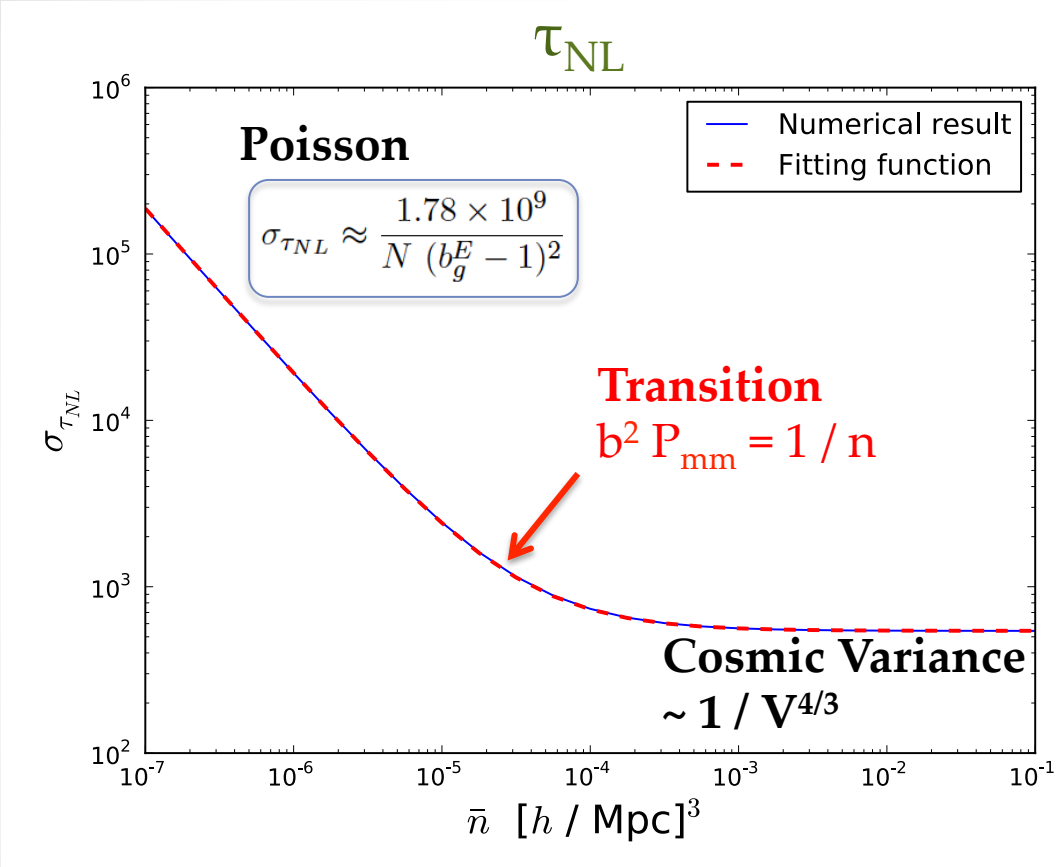

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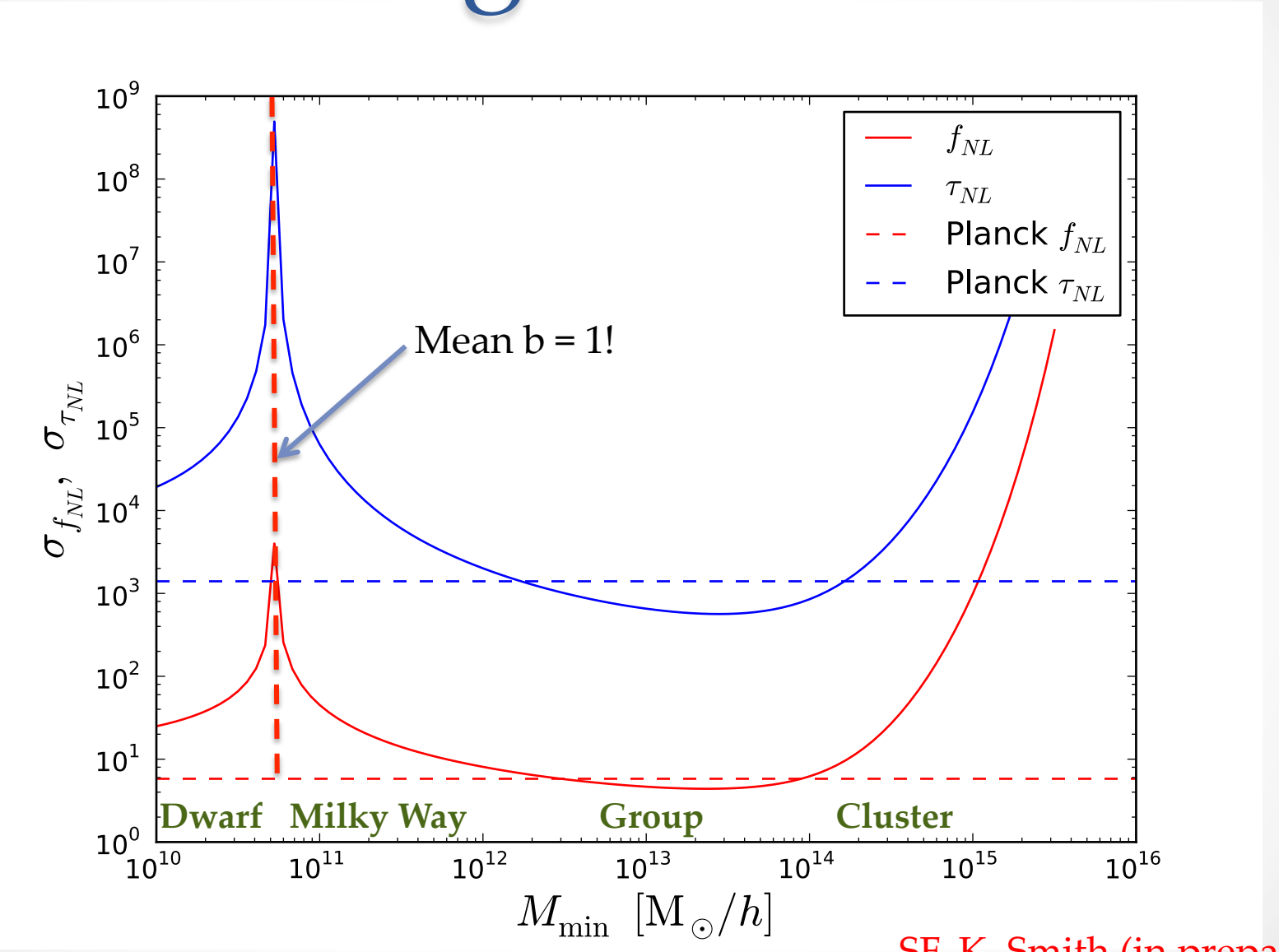
Single tracer



SF, K. Smith (in preparation)

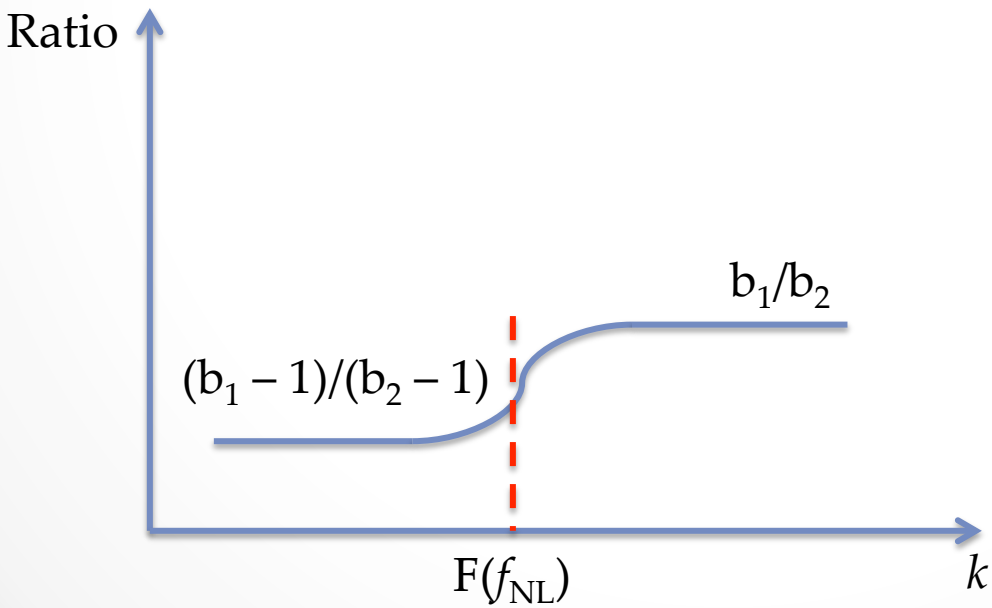
Can we do better?

YES

if we have more than one tracer with different bias

Sample variance cancellation

$$\frac{\delta_{h,1}}{\delta_{h,2}} = \frac{(b_{g,1} + A(b_{g,1}-1)f_{NL}/k^2)\,\cancel{\delta} + \cancel{\epsilon_1}}{(b_{g,2} + A(b_{g,2}-1)f_{NL}/k^2)\,\cancel{\delta} + \cancel{\epsilon_2}}$$



In absence of noise, can measure b_1, b_2 and f_{NL} with no sample variance

Seljak (2008)
Hamaus et al (2010)

Multiple mass bins
Optimal weighting

- Divide sample in N >>1 mass bins $\boldsymbol{\delta}_h = (\delta_{h,1}, \ldots, \delta_{h,N})^T$
- Halo field: local, stochastic tracer $\boldsymbol{\delta}_h = \boldsymbol{b}\,\delta + \boldsymbol{\epsilon}$
- Covariance: $\boldsymbol{C} = \langle \boldsymbol{\delta}_h \boldsymbol{\delta}_h^T \rangle = \boldsymbol{b}\boldsymbol{b}^T P_{mm} + \boldsymbol{E}$
- Error matrix: $\boldsymbol{E} = \langle \boldsymbol{\epsilon}\boldsymbol{\epsilon}^T \rangle$

- In the halo model

$$E_{ij} = \langle \epsilon_i \epsilon_j \rangle = \langle (\delta_i - b_i\delta)(\delta_j - b_j\delta) \rangle = \langle \delta_i \delta_j \rangle - b_i \langle \delta_j \delta \rangle - b_j \langle \delta_i \delta \rangle + b_i b_j \langle \delta^2 \rangle$$
$$= \frac{\delta_{ij}^K}{\bar{n}_i} - \underbrace{b_i \frac{M_j}{\bar{\rho}} - b_j \frac{M_i}{\bar{\rho}} + b_i b_j \frac{\langle n M^2 \rangle}{\bar{\rho}^2}}_{\text{Off diagonal contributions from 1-halo term}}$$

Shot noise

Seljak et al (2009)
Hamaus et al (2010)

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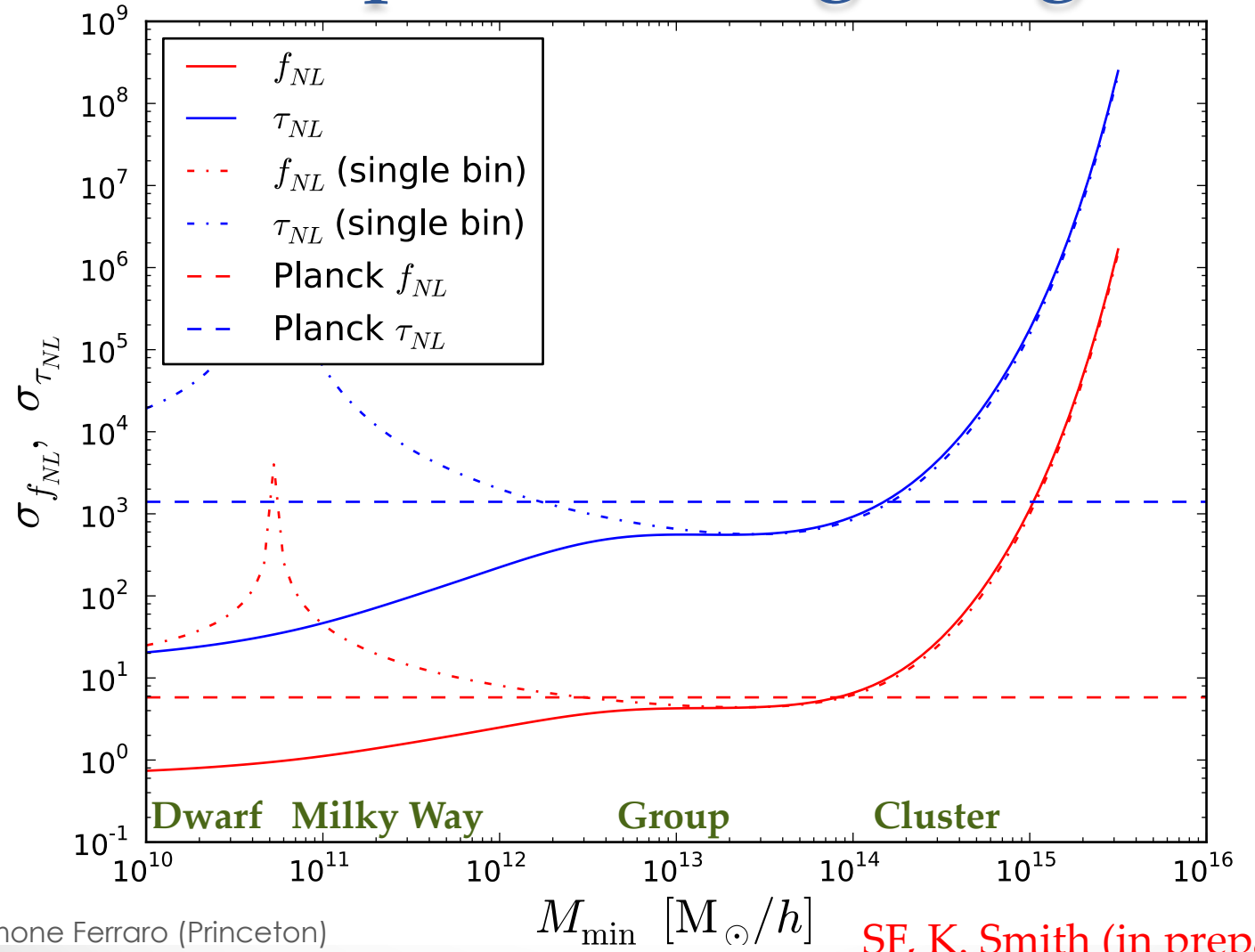

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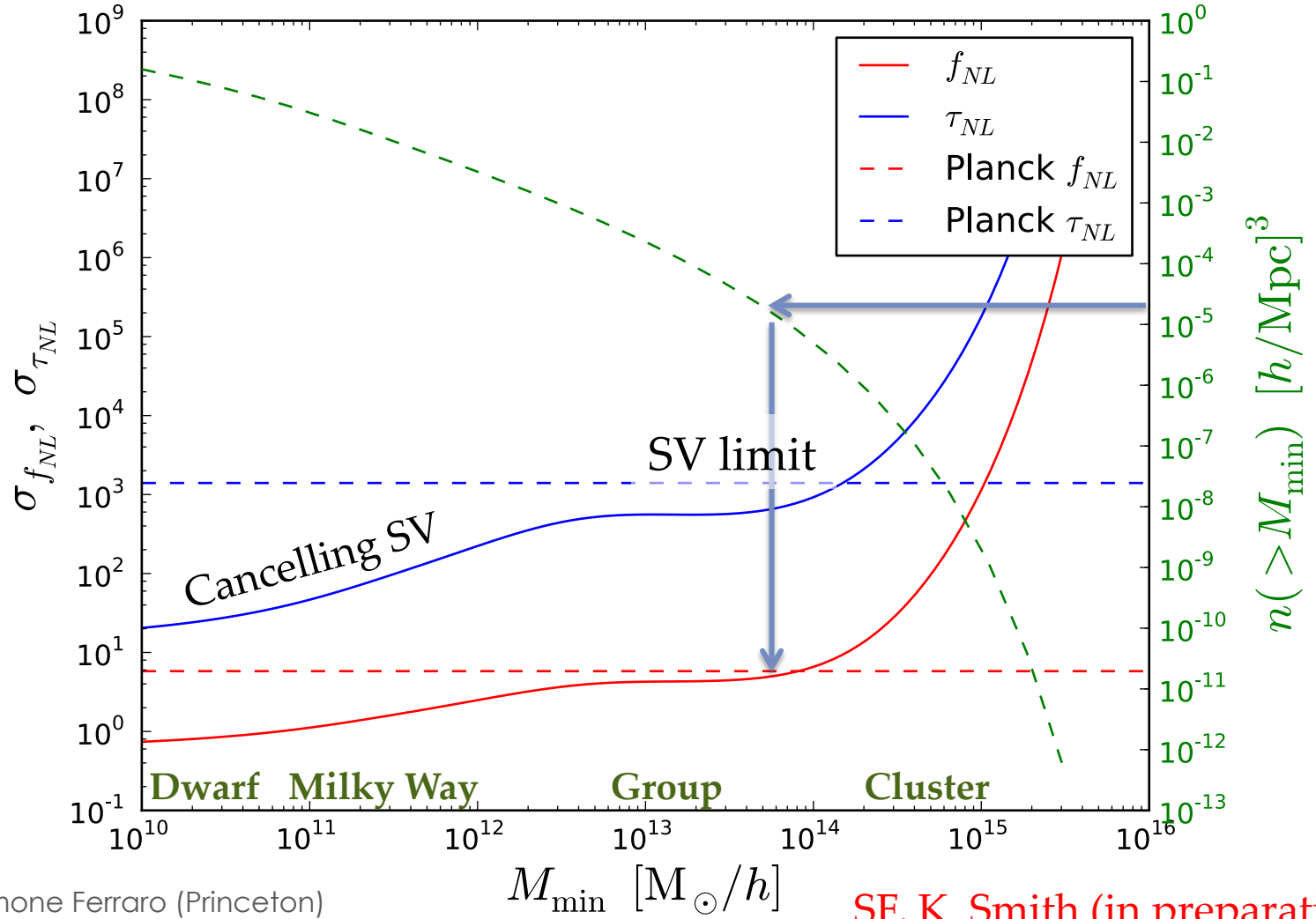
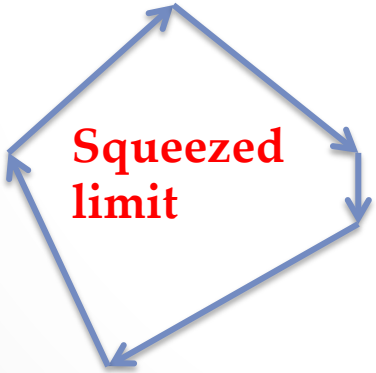


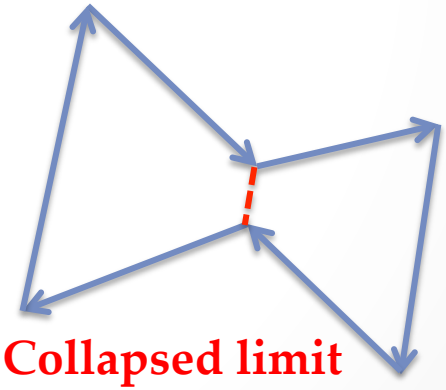
SF, K. Smith (in preparation)

Conclusions

- LSS will eventually do better than CMB, for 'local' or 'collapsed' shapes.
- Next generation of surveys will have a similar improvement to WMAP → Planck
- No strong constrains on other shapes (at least from the power spectrum). Should look at the galaxy bispectrum etc.



$\vec{k} \to 0$



Thank you!

arXiv:1209.2173
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