Forecasting Multi-field Inflation
What halo bias can tell us

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+ work in progress]

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Outline

• Clustering in $f_{NL}$ cosmology
• Clustering with general Non-Gaussian initial conditions
• Stochastic bias
• Examples: Curvaton and $\tau_{NL}$
• Forecasts: single and multiple tracers
• Conclusions
Inflationary perturbations determine late time density field.

Inflationary models can make different predictions for the initial conditions.

Understanding the relationship between dark matter and galaxies is key for LSS to make constrain cosmology.
Constraining inflation

- So far perturbations are... adiabatic, scale invariant, superhorizon.
- How do we distinguish between models?
- Primordial gravitational waves.
- (Non-)Gaussianity: from Planck

\[ f_{\text{local}}^{NL} = 2.7 \pm 5.8 \]
\[ f_{\text{equil}}^{NL} = -42 \pm 75 \]
\[ f_{\text{ortho}}^{NL} = -25 \pm 39 \]
\[ \tau_{NL} < 2800 \text{ (95\% CL)} \]

- Challenge for LSS to improve on this!
Clustering in Gaussian cosmology

- In a Gaussian cosmology, the halo density is proportional to the dark matter density on large scales:

\[ \delta_h = b_g \delta \]

\[ P_{mh}(k) = b_g P_{mm}(k) \]

\[ P_{hh}(k) = b_g^2 P_{mm}(k) \]
Clustering in $f_{NL}$ cosmology

• Local model: $\Phi(x) = \phi(x) + f_{NL}(\phi^2(x) - \langle \phi^2(x) \rangle)$

Generates **scale dependent** halo bias:

\[
P_{mh} = \left( b_g + \beta_f \frac{f_{NL}}{\alpha(k)} \right) P_{mm}
\]

\[
P_{hh} = \left( b_g + \beta_f \frac{f_{NL}}{\alpha(k)} \right)^2 P_{mm}
\]

Note that scale dependence is sourced by the **squeezed limit** of the bispectrum:

\[
\hat{f}_{NL} \equiv \frac{1}{4} \lim_{k_1 \to 0} \frac{\xi_{\Phi}^{(3)}(k_1, k_2, k_3)}{P_1 P_2}
\]

Dalal et al. 2008
Verde, Matarrese 2009

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Clustering in $f_{\text{NL}}$ cosmology

\[ P_{hh}(k) = \frac{1}{n} \]

\[ P_{mh}(k) \]

\[ P_{mm}(k) \]

\[ k^3/(2\pi^2)P(k) \]

\[ k (h \text{ Mpc}^{-1}) \]

\[ P_{mh} = \left( b_g + \beta_f \frac{f_{\text{NL}}}{\alpha(k)} \right) P_{mm} \]

\[ P_{hh} = \left( b_g + \beta_f \frac{f_{\text{NL}}}{\alpha(k)} \right)^2 P_{mm} \]
Clustering in $f_{NL}$ cosmology

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Dalal et al. 2008
Verde, Matarrese 2009
Which shapes contribute to the power spectrum?

\[ \langle \Phi_{k_1} \Phi_{k_2} \cdots \Phi_{k_N} \rangle_c = (2\pi)^3 \delta_D(k_{12\ldots N}) \xi^{(N)}(k_1, k_2, \ldots, k_N) \]

For which configurations will we eventually beat Planck?
General answer

• Matter-Halo correlation:

\[ P_{mh}(k, M) = P_{mm}(k) \left( b_g^N(M) + \sum_{n \geq 2} D_n(M) f_{1,n}(k, M) \right) \]

\[ f_{1,n}(k \to 0) \sim \int_{q_i} \xi^{(n+1)}(k, q_1, \cdots, q_n) \]

• Halo-Halo correlation:

\[ P_{hh}(k, M) = P_{mm}(k) \left( b_g^N(M)^2 + 2 b_g^N(M) \sum_{n \geq 2} D_n(M) f_{1,n}(k, M, \bar{M}) \right|_{M=\bar{M}} \]

\[ + \sum_{m,n \geq 2} D_m(M) D_n(\bar{M}) f_{m,n}(k, M, \bar{M}) \right|_{M=\bar{M}} \]

\[ f_{n,m}(k \to 0) \sim \int_{\sum q_i = k} \xi^{(n+m)}(q_1, \cdots q_n, q'_1, \cdots, q'_n) \]

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arXiv:1209.2173
arXiv:1209.2175
Stochastic bias

Note that in general

\[
\frac{P_{hh}(k)}{P_{mm}(k)} \neq \left( \frac{P_{mh}(k)}{P_{mm}(k)} \right)^2
\]

Different bias inferred from \( P_{hh}(k) \) and \( P_{mh}(k) \)

Matter and halo fields are not proportional on large scales

Stochastic bias

Arises when more than one field contribute to curvature and non-Gaussianity:

Powerful probe of multi-field inflation

arXiv:1209.2173
\( \tau_{\text{NL}} \) and stochastic bias

\[
\begin{align*}
\xi^{(3)}_{\Phi}(k_1, k_2, k_3) &= f_{\text{NL}} [P_1 P_2 + 5 \text{ perms.}] + O(f_{\text{NL}}^3), \\
\xi^{(4)}_{\Phi}(k_1, k_2, k_3, k_4) &= 2 \left( \frac{5}{6} \right)^2 \tau_{\text{NL}} [P_1 P_2 P_13 + 23 \text{ perms.}].
\end{align*}
\]

\( f_{\text{NL}} \) and \( \tau_{\text{NL}} \) independent parameters in general

\[
P_{\text{mh}} = \left( b_g + \beta_f \frac{f_{\text{NL}}}{\alpha(k)} \right) P_{\text{mm}}
\]

\[
P_{\text{hh}} = \left( b_g^2 + 2b_g \beta_f \frac{f_{\text{NL}}}{\alpha(k)} + \beta_f^2 \frac{\left( \frac{5}{6} \right)^2 \tau_{\text{NL}}}{\alpha^2(k)} \right) P_{\text{mm}}
\]

- Detect stochastic bias by comparing \( P_{\text{mh}} \) and \( P_{\text{hh}} \)
- Note momentum dependence of \( P_{\text{hh}} \): \( k^0, k^{-2}, k^{-4} \).
- Can separate \( f_{\text{NL}} \) from \( \tau_{\text{NL}} \) even with a single tracer.

Bias is stochastic if

\[
\tau_{\text{NL}} \neq \left( \frac{6}{5} f_{\text{NL}} \right)^2
\]
Forecasts – single tracer

V = 25 (Gpc/h)^3
z_m = 0.7, b_g = 2.5
M ~ 1.7 x 10^{13} M_o/h
k_{max} = 0.03 h/Mpc

Marginalized:
\sigma(f_{NL}) = 4.5
\sigma(\tau_{NL}) = 500

Planck:
\tau_{NL} < 2800 (95% CL)
f_{NL}^{\text{local}} = 2.7 \pm 5.8

- LSS is competitive with CMB, even for single bin
- Better using multiple tracers / halo-matter correlation

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Forecasts – single tracer

\[ V = 25 \, (\text{Gpc} / h)^3 \]
\[ z_m = 0.7, \quad b_g = 2.5 \]
\[ M \sim 1.7 \times 10^{13} \, \text{M}_\odot / h \]
\[ k_{\text{max}} = 0.03 \, h / \text{Mpc} \]

Marginalized:
\[ \sigma(f_{\text{NL}}) = 4.5 \]
\[ \sigma(\tau_{\text{NL}}) = 500 \]

Planck:
\[ \tau_{\text{NL}} < 2800 \, (95\% \, \text{CL}) \]
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Single tracer

Mean $b = 1!$
Can we do better?

YES

if we have more than one tracer with different bias
Sample variance cancellation

\[
\frac{\delta_{h,1}}{\delta_{h,2}} = \frac{(b_{g,1} + A(b_{g,1} - 1)f_{NL}/k^2) \delta + \epsilon_1}{(b_{g,2} + A(b_{g,2} - 1)f_{NL}/k^2) \delta + \epsilon_2}
\]

In absence of noise, can measure \(b_1\), \(b_2\) and \(f_{NL}\) with no sample variance

Seljak (2008)
Hamaus et al (2010)
Multiple mass bins
Optimal weighting

• Divide sample in $N >> 1$ mass bins

$$\delta_h = (\delta_{h,1}, \ldots, \delta_{h,N})^T$$

• Halo field: local, stochastic tracer

$$\delta_h = b \delta + \epsilon$$

• Covariance:

$$C = \langle \delta_h \delta_h^T \rangle = bb^T P_{mm} + E$$

• Error matrix:

$$E = \langle \epsilon \epsilon^T \rangle$$

• In the halo model

$$E_{ij} = \langle \epsilon_i \epsilon_j \rangle = \langle (\delta_i - b_i \delta)(\delta_j - b_j \delta) \rangle = \langle \delta_i \delta_j \rangle - b_i \langle \delta_j \delta \rangle - b_j \langle \delta_i \delta \rangle + b_i b_j \langle \delta^2 \rangle$$

$$= \frac{\delta_{ij}^K}{n_i} - b_i \frac{M_j}{\bar{\rho}} - b_j \frac{M_i}{\bar{\rho}} + b_i b_j \frac{\langle n M^2 \rangle}{\bar{\rho}^2}$$

Seljak et al (2009)
Hamaus et al (2010)

Off diagonal contributions from 1-halo term

Shot noise
Multiple mass bins
Optimal weighting

\[ f_{NL}, \quad \sigma_{f_{NL}}, \quad \sigma_{\tau_{NL}} \]

\[ \tau_{NL}, \quad \text{Planck } f_{NL}, \quad \text{Planck } \tau_{NL} \]

Dwarf, Milky Way, Group, Cluster

\[ M_{\text{min}} [M_{\odot}/h] \]

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Multiple mass bins
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SV limit
Canceling SV

\[ f_{NL}, \quad \tau_{NL}, \quad \text{Planck } f_{NL}, \quad \text{Planck } \tau_{NL} \]

\[ n(> M_{min}) \left[ h / \text{Mpc}^3 \right] \]

\[ M_{min} \left[ M_\odot / h \right] \]

Dwarf, Milky Way, Group, Cluster

SF, K. Smith (in preparation)
Conclusions

• LSS will eventually do better than CMB, for ‘local’ or ‘collapsed’ shapes.
• Next generation of surveys will have a similar improvement to WMAP → Planck
• No strong constrains on other shapes (at least from the power spectrum). Should look at the galaxy bispectrum etc.

Thank you!

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