No-Scale Supergravity and Inflation after Planck

• Why Supersymmetry and Inflation
• Supergravity and the $\eta$ problem
• No-Scale Supergravity
• Planck and $R+R^2$ inflation
• $R+R^2$ gravity and No-Scale Supergravity
• Stabilization
Old New Inflation

Great idea based on 1-loop corrected SU(5) potential for the adjoint:

\[ V(\sigma) = A\sigma^4 \left( \ln \frac{\sigma^2}{v^2} - \frac{1}{2} \right) \]

\[ A = \frac{5625}{1024\pi^2 g_5^4} \]

Problems:

• Vacuum structure
• Destabilization through quantum fluctuations
• Fine tuning (require curvature to be \( \ll M_X \))
• Density fluctuations - \( \delta \rho/\rho \sim 100 \, g_5^2 \)

Linde; Albrecht, Steinhardt
How SUSY can help

Exact Susy - $V_{1\text{-loop}} = 0$

Broken Susy - 

$$A = \frac{75}{32\pi^2 v^2} g_5^2 m_s^2$$

where

$$m_s^2 = M_B^2 - M_F^2$$

and

$$\frac{m_s^2}{v^2} \sim 2g_5 \frac{m_{3/2}}{v}$$

fixes fine-tuning, $\delta \rho/\rho$, etc. - but isn’t really a model

Ellis, Nanopoulos, Olive, Tamvakis
Hilltop-Inflation and WZ models

\[ V = \lambda (\phi^2 - v^2)^2 \]

require \( \lambda \sim 10^{-12} \) for \( \delta \phi / \phi \)
and \( (v/M_P)^2 > 65/2\pi \) for slow roll

Easily obtained in a WZ model with

\[ W = \frac{\mu}{2} \phi^2 - \frac{\lambda}{3} \phi^3 \]

in a globally supersymmetric model with

\[ V = \left| \frac{dW}{d\Phi} \right|^2 \]

Linde; Albrecht, Brandenberger

Croon, Ellis, Mavromatos
Supergravity

Start with a Kähler Potential

\[ G = K + \ln |W|^2 \]

Minimal N=1 defined by \( K = \phi^i \Phi_i^* \)

and scalar potential

\[ V = e^G \left[ G_i (G^{-1})^i_j G^j - 3 \right] + D - \text{terms} \]

or

\[ V = e^{\phi^i \Phi_i^*} \left[ \left| \frac{\partial W}{\partial \phi^i} + \phi_i^* W \right|^2 - 3|W|^2 \right] + D - \text{terms} \]

for minimal N=1

Typically, \( m^2 \sim H^2 \)

\( \eta \)-problem!
Supergravity

Constructing Models

\[ W = \mu^2 \sum_n \lambda_n \phi^n \]

\( \mu^2 \) fixed by amplitude of density fluctuations, \( \lambda_n \sim O(1) \)

Simplest example,

\[ W = \mu^2 (1 - \phi)^2 \]

\[ V = \mu^4 e^{\phi^2} \left[ 1 + |\phi|^2 - (\phi^2 + \phi^*^2) - 2|\phi|^2 (\phi + \phi^*) \right. \]

\[ + 5|\phi|^2 + |\phi|^2 (\phi^2 + \phi^*^2) - 2|\phi|^2 (\phi + \phi^*) + |\phi^3|^2 \]

\[ \approx \mu^4 (1 - 4\phi^3 + \frac{13}{2} \phi^4 + \cdots) \]
Supergravity

Generic Models

\[ K = SS^* + (\phi - \phi^*)^2 + \ldots \]

with \[ W = Sf(\Phi) \]

resulting in \[ V = |f(\phi)|^2 \]

Easily generates any potential (which is a perfect square)
No-Scale Supergravity

Natural vanishing of cosmological constant (tree level) with the supersymmetry scale not fixed at lowest order. (Also arises in generic 4d reductions of string theory.)

\[ K = -3 \ln(T + T^* - \phi^i \phi_i^*/3) \]

\[ V = e^{\frac{2}{3} K} \left| \frac{\partial W}{\partial \phi^i} \right|^2 \]

Globally supersymmetric potential once \( K \) (canonical) picks up a vev
No-Scale Supergravity

Constructing Models

\[ W = \mu^2 \sum_n \lambda_n \phi^n \]

\( \mu^2 \) fixed by amplitude of density fluctuations, \( \lambda_n \sim O(1) \)

Simplest example,

\[ W = \mu^2 (\phi - \phi^4/4) \]

\[ V = \mu^4 |1 - \phi^3|^2 \]
Planck Results

\[ \epsilon = \frac{1}{2} \left( \frac{V'}{V} \right)^2 \]
\[ \eta = \frac{V''}{V} \]
\[ n_s = 1 - 6\epsilon + 2\eta \]
\[ r = 16\epsilon \]

Trouble for simple supergravity and no-scale models:

\[ \epsilon \approx \frac{1}{72N^4} \ll 1 \quad \text{OK} \]
\[ \eta \approx -\frac{1}{2N} \quad \text{not OK} \]

\[ n_s \sim .933 \]
\[ S = \frac{1}{2} \int d^4 x \sqrt{-g} \left( R + \frac{R^2}{6M^2} \right), \]

where \( M \ll M_P \)

With \( \tilde{g}_{\mu \nu} = \left( 1 + \frac{\varphi}{3M^2} \right) g_{\mu \nu} \)

and \( \varphi' = \sqrt{\frac{3}{2}} \ln \left( 1 + \frac{\varphi}{3M^2} \right) \)

conformally equivalent to:

\[ S = \frac{1}{2} \int d^4 x \sqrt{-g} \left[ \ddot{R} + (\partial_{\mu} \varphi')^2 - \frac{3}{2} M^2 (1 - e^{-\sqrt{2/3} \varphi'})^2 \right] \]

\[ V = \frac{3}{4} M^2 (1 - e^{-\sqrt{2/3} \varphi'})^2 \]
R+R\textsuperscript{2} Inflation

\[ V = \frac{3}{4} M^2 (1 - e^{-\sqrt{2/3}\varphi'})^2 \]

\[ = \mu^2 e^{-\sqrt{2/3}x} \sinh^2 (x/\sqrt{6}) \]

\( x = \varphi/M_P, \quad \mu^2 = 3M^2 \)

Slow Roll parameters:

\[ \epsilon = \frac{1}{3} \operatorname{csch}^2 (x/\sqrt{6}) e^{-\sqrt{2/3}x} , \]

\[ \eta = \frac{1}{3} \operatorname{csch}^2 (x/\sqrt{6}) \left( 2e^{-\sqrt{2/3}x} - 1 \right) \]

\( \mu \) is again set by the normalization of the quadrupole

\[ A_s = \frac{V}{24\pi^2 \epsilon} = \frac{\mu^2}{8\pi^2} \sinh^4 (x/\sqrt{6}) \quad \Rightarrow \mu = 2.2 \times 10^{-5} \text{ for } N = 55 \]

\( x_i = 5.35 \)
For $N=55$, $n_s = 0.965; r = .0035$
No-Scale models revisited

Can we find a model consistent with Planck?  

Start with WZ model again:  

$$W = \frac{\hat{\mu}}{2} \Phi^2 - \frac{\lambda}{3} \Phi^3$$

Assume now that T picks up a vev:  

$$2\langle \text{Re } T \rangle = c$$

$$\mathcal{L}_{\text{eff}} = \frac{c}{(c - |\phi|^2/3)^2} |\partial_\mu \phi|^2 - \frac{\hat{V}}{(c - |\phi|^2/3)^2}$$

Redefine Inflaton to a canonical field $\chi$

$$\phi = \sqrt{3}c \tanh \left( \frac{\chi}{\sqrt{3}} \right)$$
No-Scale models revisited

The potential becomes:

\[
V = \mu^2 \left| \sinh(\chi/\sqrt{3}) \left( \cosh(\chi/\sqrt{3}) - \frac{3\lambda}{\mu} \sinh(\chi/\sqrt{3}) \right) \right|^2 
\]

\[
\hat{\mu} = \mu \sqrt{c/3}
\]

For \(\lambda = \mu / 3\), this is exactly the \(R + R^2\) potential

\[
V = \mu^2 e^{-\sqrt{2/3}x} \sinh^2 (x/\sqrt{6})
\]

\[
\chi = (x + iy)/\sqrt{2}
\]
No-Scale models revisited

How well does this do vis a vis Planck?
Classes of $R+R^2$ in No-Scale Supergravity

Utilizing the no-scale symmetry, we can write

\[ K = -3 \ln \left( 1 - \frac{|y_1|^2 + |y_2|^2}{3} \right) \]

\[ y_1 = \left( \frac{2\phi}{1 + 2T} \right) ; \quad y_2 = \sqrt{3} \left( \frac{1 - 2T}{1 + 2T} \right) \]

or

\[ T = \frac{1}{2} \left( \frac{1 - y_2/\sqrt{3}}{1 + y_2/\sqrt{3}} \right) ; \quad \phi = \left( \frac{y_1}{1 + y_2/\sqrt{3}} \right) \]

with

\[ W(T, \phi) \rightarrow \tilde{W}(y_1, y_2) = \left( 1 + y_2/\sqrt{3} \right)^3 W \]
Classes of $R+R^2$ in No-Scale Supergravity

So is the inflaton $T$ or $\phi$?

1) $T$-fixed ($\phi$-inflaton) or $y_2$-fixed $y_1$-inflaton

Starobinsky potential found when

$$\hat{V} = M^2|\phi|^2|1 - \phi/\sqrt{3}|^2$$

with field redefinition

$$(y_i, \phi) = \sqrt{3} \tanh \left( \frac{\chi}{\sqrt{3}} \right)$$

2) $\phi$-fixed (T-inflaton)

Starobinsky potential found when

$$\hat{V} = 3M^2|T - 1/2|^2$$

or

$$\hat{V} = 12M^2|T|^2|T - 1/2|^2$$

with field redefinition

$$T = \frac{1}{2} e^{2\chi/\sqrt{3}}$$

or $T \rightarrow 1/(4T)$
Classes of $R+R^2$ in No-Scale Supergravity

Example 1:

$$W = M \left[ \frac{y_1^2}{2} \left(1 + \frac{y_2}{\sqrt{3}}\right) - \frac{y_1^3}{3\sqrt{3}} \right]$$

or

$$W = M \left[ \frac{\phi^2}{2} - \frac{\phi^3}{3\sqrt{3}} \right]$$

or (reversing $y_1$ and $y_2$)

$$W = \frac{M}{4} (T - 1/2)^2 (1 + 10T + 2\sqrt{3}\phi)$$

Example 2:

$$W = M y_1 y_2 (1 + y_2/\sqrt{3})$$

or

$$W = \sqrt{3}M \phi (T - 1/2)$$

or (reversing $y_1$ and $y_2$)

$$W = M \left[ \sqrt{3}(T^2 - 1/4)\phi + (T - 1/2)\phi^2 \right]$$

\ldots
Stabilization

Have so far assumed a vev for one of the two fields

\[ K = -3 \ln \left( 1 - \frac{|y_1|^2 + |y_2|^2}{3} + \frac{|y_2|^4}{\Lambda^2} \right), \quad \Lambda < M_P \]

or

\[ K = -3 \ln \left( T + T^* - \frac{|\phi|^2}{3} + \frac{(T + T^* - 1)^4 + d(T - T^*)^4}{\Lambda^2} \right) \]
Summary

• Broad connection between R+R^2 models and no-scale supergravity.

• The Starobinsky model of inflation can be realized with either modulus T or ‘matter’ field φ with a simple WZ superpotential.

• Exact R+R^2 found for specific choices of couplings leading to n_s = 0.965 but r ~ .003 is rather generic.

• To do: supersymmetry breaking; connection to the standard model, reheating....