\[ f(\phi) F^2 \] mechanism and solid inflation: broken spatial invariance during inflation

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- Motivations
- Solid inflation
- The \( \mathcal{L}_\phi - f(\phi) F^2 \) model
- Prolonged anisotropic inflation
- Violation of
\[
\lim_{q \to 0} \langle \zeta_q \zeta_{\vec{k}} \zeta_{-\vec{k}} \rangle = -(n_s - 1) P_\zeta(q) P_\zeta(k)
\]
both in amplitude and shape
Effective field theory ≡ description of a system through the lowest dim. $\hat{O}$ compatible with the underlying symmetry. Very fruitful in many areas of physics, including inflation (Cheung et al '08).

Universe is statistically isotropic and homogeneous (barring anomalies). On the contrary, time translational invariance is broken.

\[ \phi (x) = \phi^{(0)} (t + \pi (x)) \simeq \phi^{(0)} (t) + \partial_t \phi^{(0)} \cdot \pi (x) \]

EFT for $\pi \supset$ operators respecting spatial invariance. Measurements fix their coefficients.

Given improved data, consider broken spatial invariance. New operators and signatures.

In this talk, $2 \neq$ models with common features, not obtained in other models of inflation.
Elastic / Solid inflation

Grizinov '04
Endlich, Nicolis, Wang '12

Medium driving inflation has FT description of a solid. Divide it in cells.

\( \vec{\phi}(t, \vec{x}) \) is the position at time \( t \) of the cell that initially was at \( \vec{x} \).

Solid at rest: \( \phi^i(t, \vec{x}) = x^i \)

Triplet of inflatons; to reconcile with homogeneity & isotropy:

\[
B^{ij} \equiv g^{\mu\nu} \partial_\mu \phi^i \partial_\nu \phi^j , \quad X \equiv \text{tr } B , \quad Y \equiv \frac{\text{tr } B^2}{(\text{tr } B)^2} , \quad Z \equiv \frac{\text{tr } B^3}{(\text{tr } B)^3} , \quad \mathcal{L} = F[X, Y, Z]
\]

(3 independent combs. Normalized so that only \( X \) sensitive to volume)

\[
\Rightarrow \quad \rho = -F , \quad \rho + p = -\frac{2}{3} X F_X , \quad F_X \equiv \frac{\partial F}{\partial X}
\]

To have inflation, \( X F_X \ll F \): solid very weakly affected by expansion
Phonons: $\phi^i = x^i + \pi^i(t, \vec{x})$, $\pi^i = \frac{\partial_i}{\sqrt{-\nabla^2}} \pi_L + \pi_L^i$

Deeply inside the horizon:

$c_s^2 = 1 + \frac{2}{3} \frac{X^2 F_{XX}}{X F_X} + \frac{8}{9} \frac{F_Y + F_Z}{X F_X}$, $c_T^2 = 1 + \frac{2}{3} \frac{F_Y + F_Z}{X F_X}$

Therefore all $X F_X$, $X^2 F_{XX}$, $F_Y + F_Z \ll F$

In spatially flat gauge $\zeta = -H \frac{\delta \rho}{\dot{\rho}} = \frac{1}{3} \partial \cdot \pi$

$$\delta T_{ij, \text{scalar}} = a^2 M_p^2 \dot{H} \zeta \left[ 2 (3 - 2 \epsilon + \eta) \delta_{ij} - \left( 3 + 3 c_s^2 - 2 \epsilon + \eta \right) (3 \delta_{ij} - \delta_{ij}) \right]$$

- $P_\zeta$ slowly growing outside the horizon
- $B_\zeta$ has nontrivial angular dependence in the squeezed limit

Endlich, Nicolis, Wang '12
Vector fields and anisotropic inflation

Initial theoretical interest (Wald ’83; Kaloper ’91; Barrow, Hervic ’05), then WMAP anomaly (now understood to be a systematics).

\[ \mathcal{L} \supset V(A^2) \quad \text{Ford ’89} \]
\[ \mathcal{L} \supset \lambda (A^2 - v^2) \quad \text{Ackerman, Carroll, Wise ’07} \]
\[ \mathcal{L} \supset A^2 R \quad \text{Golovnev, Mukhanov, Vanchurin ’08} \]

In these models, the longitudinal polarization is a ghost.\(^\text{Contaldi, Himmetoglu, MP ’08}\)

Preserve U(1), but \(-\frac{1}{4} F^2 \rightarrow -\frac{f(t)}{4} F^2\) so conformal.

\[ f \propto a^4 \rightarrow \frac{d\rho_B}{d\ln k} \quad \text{scale invariant and frozen, used for magnetogenesis} \quad \text{Ratra ’92} \]

Strong coupling pbm., since \(\alpha \propto f^{-1} \gg 1\) at early times

\[ \text{Duality: } f \propto a^{-4} \rightarrow \text{scale invariant “electric” component (weak coupling)} \quad \text{Demozzi, Mukhanov, Rubinstein ’09} \]

\[ \mathcal{L} = \mathcal{L}_\phi - \frac{f(\phi)}{4} F^2 \quad \text{with } f(\phi(t)) \propto a^{-4} \text{ supports } \vec{A}(t) \neq 0 \quad \text{Watanabe, Kanno, Soda ’09} \]
Expanding $f(\phi) F^2$, $\mathcal{L}_{\text{int}} \supset a^4 \left[ 4\tilde{E}^{(0)} \cdot \delta \tilde{E} \zeta + 2\delta \tilde{E} \cdot \delta \tilde{E} \zeta \right]$

\[ P(\vec{k}) \simeq P(k) \left[ 1 + g_\ast \cos^2 \theta_{\vec{k}, \vec{E}^{(0)}} \right] \]

\[ g_\ast \simeq -\frac{48}{\epsilon} N_{\text{CMB}}^2 \frac{2\rho_{E^{(0)}}}{V(\phi)} \]

\[ Dulaney, Gresham '10; \]
\[ Gumrukcuoglu, Himmetoglu, MP '10 \]
\[ Watanabe, Kanno, Soda '10 \]

$g_\ast = 0.1$ for $\frac{\Delta H}{H} \sim \frac{\rho_{E^{(0)}}}{V(\phi)} \sim 10^{-8}$

Strict relation between $P_\zeta(\vec{k})$ and $B_\zeta$

\[ B_\zeta \rightarrow \mathcal{O}(1) \frac{k_1^3 k_2^3}{k_1^1 k_2^3} , \quad k_1 \ll k_2, k_3 \]

\[ f_{\text{eff. local}}^{\text{NL}} \sim 25 \frac{|g_\ast|}{0.1} \]

Bartolo, Matarrese, MP, Ricciardone '12
From the following new parametrization of the bispectrum of primordial curvature perturbations:

\[ B_\zeta \propto \frac{1 - \cos^2 \theta_{\hat{k}_1, \hat{E}^{(0)}} - \cos^2 \theta_{\hat{k}_2, \hat{E}^{(0)}} + \cos \theta_{\hat{k}_1, \hat{E}^{(0)}} \cos \theta_{\hat{k}_2, \hat{E}^{(0)}} \cos^2 \theta_{\hat{k}_1, \hat{k}_2}}{k_1^3 k_2^3}, \quad k_1 \ll k_2, k_3 \]

Peaked as local in squeezed limit

Nontrivial angular dependence

\[ B_\zeta \Big|_{\text{isotropic measurement}} \propto \frac{1 + \cos^2 \theta_{\hat{k}_1, \hat{k}_2}}{k_1^3 k_2^3} \]

Angular modulation in standard scalar field models from gradient of longest mode, subdominant in \( \frac{k_1}{k_i} \ll 1 \)

Lewis ’11

Motivation for studying

\[ B_\zeta(k_1, k_2, k_3) = \sum_L c_L P_L(\hat{k}_1 \cdot \hat{k}_2) P_\zeta(k_1) P_\zeta(k_2) + (2 \text{ perm}) \]

Shiraishi, Komatsu, MP, Barnaby, ’13

\begin{align*}
\text{\textit{fF}^2 \text{ model}} : & \quad c_0 = \frac{6}{5} f_{NL} \simeq 32 \frac{|g^*|}{0.1}, \quad c_2 = \frac{c_0}{2} \\
\text{solid inflation} : & \quad c_2 \gg c_0
\end{align*}
\[ B_\zeta(k_1, k_2, k_3) = \sum_L c_L P_L(\hat{k}_1 \cdot \hat{k}_2) P_\zeta(k_1) P_\zeta(k_2) + (2 \text{ perm}) \]

\[ c_0 = \frac{6}{5} f_{\text{NL}} \]

Full sky

CV-limited up to \( \ell_{\text{max}} = 2000 \):

\[ \delta c_0 = 4.4 , \; \delta c_1 = 61 \]

\[ \delta c_2 = 13 \; (68\% \text{ CL}) \]

Planck \; (68\% \text{ CL})

\[ c_0 = 3.2 \pm 7 \]

\[ c_1 = 11 \pm 113 \]

\[ c_2 = 3.8 \pm 27.8 \]

\[ \downarrow \]

\[ |g_*| < 0.05 \]
Other analogies besides $B_{\text{squeezed}}$?

Recall $X F_X, X^2 F_{XX} \ll F$ (very weak reaction to volume expansion), and $F_Y + F_Z \ll F$ (very weak reaction to sound deformations).

Prolonged anisotropy

\[
\frac{\Delta H}{H} \propto e^{-\frac{4}{3} c_s^2 T \epsilon H t}
\]

Bartolo, Matarrese, MP, Ricciardone '13

$P_\zeta$ on this background:

\[
g_* = \mathcal{O} \left( \frac{\Delta H}{\epsilon H} \right) \gg \mathcal{O} \left( \frac{\Delta H}{H} \right)
\]

Wald's isotropization theorem ('83): an anisotropic universe with a cosmological constant + a 2nd source with

\[
\mathcal{D} \equiv t^\mu t^\nu T_{\mu\nu}^{2\text{nd}} > 0 \\
S \equiv t^\mu t^\nu \left( T_{\mu\nu}^{2\text{nd}} - \frac{T_{\mu\nu}^{2\text{nd}}}{2} g_{\mu\nu} \right) > 0
\]

isotropizes on a $\sim \sqrt{1/\Lambda}$ timescale

solid: $T_{\mu\nu} = g_{\mu\nu} F(t) - 2 \partial_\mu \phi^i \partial_\nu \phi^j \frac{\partial F}{\partial B_{ij}} \equiv g_{\mu\nu} F(t_{\text{in}}) + T_{\mu\nu}^{2\text{nd}}$

\[
\mathcal{D} = -\left\{ -F(t_{\text{in}}) - [-F(t)] \right\} < 0
\]
Is $\Delta H$ to be expected?

- In $f(\phi)F^2$, frozen and scale invariant $\delta \vec{E}_k$
- Modes produced in first $N_{\text{TOT}} - 60$ e-folds randomly add up to $\vec{E}_{\text{IR}}$
  observed as classical, homogeneous and anisotropic by CMB modes
- Value $\vec{E}_{\text{IR}}$ in our realization drawn by a gaussian of mean 0 and variance
  \[
  \langle \vec{E}_{\text{IR}}^2 \rangle \sim H^4 (N_{\text{tot}} - 60)
  \]
- $|g_*|_{\text{expected}} \gtrsim 0.1 \frac{N_{\text{tot}} - N_{\text{CMB}}}{37}$
  (generically, too anisotropic)

If this model is realized in nature, either $N_{\text{tot}} \sim N_{\text{CMB}}$, or we live
in a patch where $\vec{E}_{\text{IR}} \ll \sqrt{\langle \vec{E}_{\text{IR}}^2 \rangle}$

- Same in solid inflation?
Conclusions

- Two distinct models, characterized by breaking of spatial symmetry

- Common features (anisotropic background, $B_{\text{squeezed}}$, growing $\zeta$), most of which at odds with simplest inflationary expectations

- New class, with perhaps additional new signatures