Cosmic Variance from Mode Coupling

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Nelson, Shandera, 1212.4550 (PRL);
LoVerde, Nelson, Shandera, 1303.3549 (JCAP);
Bramante, Kumar, Nelson, Shandera, 1307.5083;
Work in progress (S. Watson; Sohyun Park)

Linde, Muhkanov; Salopek, Bond; Mollerach et al;
Boubekeur, Lyth; Byrnes et al;
papers related to anomaly...

today: Peloso, Khoury, LoVerde

Shandera, CAP, 23 Sept 2013
The Plan

• Cosmic Variance from a generalized local ansatz

• Big Picture implications for inflation theory
  (important if we want to push observers/data analysis)
Cosmic Variance and the local ansatz:

$$g_{NL} = 10^5$$

LoVerde, Nelson, Shandera, 1303.3549 (JCAP); Nelson, Shandera, 1212.4550 (PRL); Shandera, CAP, 23 Sept 2013
Cosmic Variance and the local ansatz:

\[ g_{NL} = 10^5 \]

\[ f(\psi_G) \rightarrow \phi_G + f_{NL}(B, \tilde{f}_{NL}, \tilde{g}_{NL}, \ldots) \phi_G^2 + g_{NL}(B, \tilde{f}_{NL}, \tilde{g}_{NL}, \ldots) \phi_G^3 + \ldots \]

LoVerde, Nelson, Shandera, 1303.3549 (JCAP);
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Shandera, CAP, 23 Sept 2013
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Planck says NG is small? Or, universe is very big?
Beyond the local ansatz:

Consider a simple scale dependence:

$$\zeta(x) = \phi_G(x) + \sigma_G(x) + \frac{3}{5} f_{NL} \star [\sigma_G(x)^2 - \langle \sigma_G(x)^2 \rangle]$$

$$+ \frac{9}{25} g_{NL} \star \sigma_G(x)^3 + \ldots$$

$$f_{NL}(k) = f_{NL}(k_p) \left( \frac{k}{k_p} \right)^{n_f}$$

Bramante, Kumar, Nelson, Shandera, 1307.5083; Shandera, CAP, 23 Sept 2013
I. Consequences for the power spectrum
Subsampling the generalized local ansatz

\[ P_{\zeta}^{\text{obs}}(k) = P_{\zeta}(k) \left[ 1 + \frac{12}{5} f_{NL}(k) \sigma_{Gl} + \frac{3}{5} f_{NL}^2(k) \left( \sigma_{Gl}^2 - \langle \sigma_{Gl}^2 \rangle \right) \right] \]

Shift to observed spectral index
More simply

Different short wavelength modes couple with different strengths to the (locally constant) background:

Locally observed spectral index is biased in subvolumes.
The spectral index

\( n_{\zeta} = 1, \quad n_f = 0.1 \)

\( n_{\zeta} = 0.96, \quad n_f = 0.1 \)

(solid lines show 0.5 sigma fluctuations)

Bramante, Kumar, Nelson, Shandera, 1307.5083;  
Shandera, CAP, 23 Sept 2013
The spectral index, cont’d

\[ f_{\text{NL}}(k_p) = 5, \langle \zeta_{GL}^2 \rangle = 10^{-4} \]

\[ f_{\text{NL}}(k_p) = 5, \langle \zeta_{GL}^2 \rangle = 10^{-3} \]
Model Building Consequences

$n_s(k_P) \sim n_\zeta = 1$

Strongly NG for $N > 10^8$
for $N > 1000$

Nonpert. for $N > 1000$

Planck 99% inclusion band for $n_s \sim n_\zeta = 1$ and a \(-1\sigma\) background fluct.

$n_s(k_P) \sim n_\zeta = 0.96$

Strongly NG for $N > 1000$
for $N > 100$

Nonpert. for $N > 100$

Planck 99% exclusion region for $n_s \sim n_\zeta = 0.96$ and a \(-1\sigma\) background fluct.

Bramante, Kumar, Nelson, Shandera, 1307.5083;

Shandera, CAP, 23 Sept 2013
How to rule cosmic variance of spectral index out.....
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......of observational relevance:
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Observationally rule out any significant blue tilt in $f_{NL}^{local}$
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......of observational relevance:

Observationally rule out any significant blue tilt in $f_{\text{NL}}^{\text{local}}$

★ Importance of smaller scale probes.
Generalizations

∗ Effect of a general bispectrum on the power spectrum:

\[ \zeta_k = \sigma_{G,k} + \int_{L^{-1}} \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} (2\pi)^3 \delta^3(p_1 + p_2 - k) F(p_1, p_2, k) \sigma_{G,p_1} \sigma_{G,p_2} + \ldots, \]
Generalizations

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- Equilateral bispectrum has no effect
- Bispectrum can weight IR modes differently

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Generalizations

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\]

☑ Equilateral bispectrum has no effect
☑ Bispectrum can weight IR modes differently

★ If tensor modes have an independent clock:

same story \(\text{eg, non Bunch Davies}\)
II. Consequences for the bispectrum and beyond
Scale-dependence in the bispectrum

\[ B_\Phi(k_1, k_2, k_3) = \xi_s(k_3) \xi_m(k_1) \xi_m(k_2) P_\Phi(k_1) P_\Phi(k_2) + 5 \text{ perm}. \]

Self-interactions of one field

Ratio of contributions of each field

\[ \xi_{s,m}(k) = \xi_{s,m}(k_p) \left( \frac{k}{k_p} \right)^{n_f^{(s),(m)}} \]
But...

\[ n_{sq.} \equiv \frac{d \ln B_\zeta(k_L, k_S, k_S)}{d \ln k_L} - (n_s - 1) \]
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★ Just from shift to power spectrum:
\[ n_{sq.} \equiv \frac{d \ln B_{\zeta}(k_L, k_S, k_S)}{d \ln k_L} - (n_s - 1) \]

\[ \Delta n_{sq.}(k) \equiv n_{sq.}^{\text{obs}}(k) - n_{sq.}^{\text{LargeVol.}}(k) \approx -\frac{6}{5} f_{NL}(k_L) \sigma_{Gl} n_f \left( \frac{6}{5} f_{NL}(k_L) \sigma_{Gl} \right) \]
But...

\[ n_{sq.} \equiv \frac{d \ln B_\zeta(k_L, k_S, k_S)}{d \ln k_L} - (n_s - 1) \]

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\* Allowing \( g_{NL}(k) \) adds another shift

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Scaling Patterns
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Self-interactions; eg, local ansatz
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\[ \mathcal{M}_{n,R} = \frac{\langle \delta^n_n \rangle_c}{\langle \delta^2_R \rangle_c^{n/2}} \]

Self-interactions; eg, local ansatz

Hierarchical

\[ \mathcal{M}^{(h)}_n = n! \, 2^{n-3} \left( \frac{\mathcal{M}^{(h)}_3}{6} \right)^{n-2} \]
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Self-interactions; eg, local ansatz

Hierarchical

\[ M_{n}^{(h)} = n! 2^{n-3} \left( \frac{M_3^{(h)}}{6} \right)^{n-2} \]

\[ \propto f_{NL} \mathcal{P}_\zeta^{1/2} \]

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Hierarchical
\[ M_n^{(h)} = n! \, 2^{n-3} \left( \frac{M_3^{(h)}}{6} \right)^{n-2} \]

Extra source, eg, gauge field
\[ \propto f_{NL} P^1_\zeta \]

(Barnaby, Shandera; 1109.2985)
Scaling Patterns

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Extra source, eg, gauge field

Feeder

\[ \mathcal{M}^{(f)}_n = (n - 1)! \, 2^{n-1} \left( \frac{\mathcal{M}^{(f)}_3}{8} \right)^{n/3} \]

(Barnaby, Shandera; 1109.2985)
Scaling Patterns, cont’d

(Quasi-single field)

(Chen, Wang)

Hybrid

\[ M_n^{(h)} \propto (A)^n (B)^{n-2} \]
Cosmic Variance and scaling patterns?

\[ \Phi(\vec{x}) = \phi(\vec{x}) + \psi_G(\vec{x}) + \tilde{f}_{NL} (\psi_G(\vec{x})^2 - \langle \psi_G(\vec{x})^2 \rangle) \]

Gives `feeder' scaling non-Gaussianity
But: in biased subvolumes, back to hierarchical
Cosmic Variance and scaling patterns?

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\Phi(\vec{x}) = \phi(\vec{x}) + \psi_G(\vec{x}) + \tilde{f}_{NL} (\psi_G(\vec{x})^2 - \langle \psi_G(\vec{x})^2 \rangle)
\]

Gives ‘feeder’ scaling non-Gaussianity
But: in biased subvolumes, back to hierarchical

\[
\Phi(\vec{x}) = \phi(\vec{x}) + \tilde{f}_{NL} \psi_G(\vec{x})^p + \ldots
\]

Biased subvolumes: Generate whole local ansatz series, with bias B controlling size of terms

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Implications for Inflation Theory
Power spectrum:

| Inflation | Spectrum of scale-invariant, adiabatic, super horizon modes as “initial conditions” |
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- Inflation

Spectrum of scale-invariant, adiabatic, super horizon modes as “initial conditions”

Bispectrum, trispectrum, etc?
Power spectrum:

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Bispectrum, trispectrum, etc?
Mathematically independent....
Power spectrum:

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Mathematically independent....

....up to relationships of the type \( \tau_{NL} \geq (6/5 f_{NL})^2 \)

(Smith, LoVerde, Zaldarriaga)
Power spectrum:

- Inflation
  - Spectrum of scale-invariant, adiabatic, superhorizon modes as “initial conditions”

Bispectrum, trispectrum, etc?
- Mathematically independent....
  - ....up to relationships of the type $\tau_{NL} \geq \left(\frac{6}{5} f_{NL}\right)^2$

Perturbation theory that obeys usual (effective) field theory notions: patterns
- Non-zero NG, but no patterns in shape or size

(Shandera, CAP, 23 Sept 2013)
Mode Coupling Cosmic Var:
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* Good: can we find a “natural” pattern inflation does not predict? (If not, is that bad for inflation?)
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\* Bad: a random pattern in a big universe may not look random to local observers
Mode Coupling Cosmic Var:

- Derivations didn’t depend on where you got the fluctuations:
  - Good: can we find a “natural” pattern inflation does not predict? (If not, is that bad for inflation?)
  - Bad: a random pattern in a big universe may not look random to local observers

- A familiar aspect of inflation, dressed up in different clothes: IC problem for our observable 60 e-folds region
What about describing only our Hubble patch?

\[
Local: \quad S = \int d^4x \sqrt{-g} \mathcal{L}(X_i, \phi_i, A_{\mu j}, \ldots)
\]

- Apply cosmological principle
- eg, 60 e-folds + non-Bunch Davies is fine tuned.
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\]

• Apply cosmological principle
• eg, 60 e-folds + non-Bunch Davies is fine tuned.

★ With local type mode-coupling, this line is blurred: observer’s version of the landscape needed
More work:
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🌟 Easy to apply this to other models to check what level of confusion can occur (quasi-single field in progress)
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★ Easy to apply this to other models to check what level of confusion can occur (quasi-single field in progress)

★ Is there a useful notion of landscape and measure here? (not quite ‘theory space’? Quantifiable route!)