Chromo-Natural Inflation

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Cosmology After Planck

25 September 2013

PRD: 86, 043530 (2012) with P. Adshead
JHEP: 02, 027 (2013) with E. Martinec & P. Adshead
PRD: 88, 021302 (2013) with E. Martinec & P. Adshead
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We need a flat potential to obtain slow roll.

\[ \eta_V \equiv M_{Pl}^2 \frac{\partial^2 V}{\partial \phi^2} \ll 1 \]

Linde; Steinhardt & Albrecht (1981)
This leads to an *inflationary* hierarchy problem.

The eta problem: \[ \eta_V \equiv M_{Pl}^2 \frac{\partial^2 V}{\partial \phi^2} \ll 1 \]
Magnetic drift may alleviate this hierarchy problem.

\[
V(\phi)
\]

field value, \( \phi \)

potential energy

‘magnetic’ friction via Chern-Simons interaction
The new physics is analogous to the Lorentz force:

\[ \vec{B} = B\hat{z} \]

\[ \ddot{X} + H\dot{X} + \mu^2 X = B\dot{Y} \]
\[ \ddot{Y} + H\dot{Y} + \mu^2 Y = -B\dot{X} \]
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ordinary friction
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potential force
ordinary friction
magnetic force
Large magnetic fields lead to magnetic drift.

Fast mode rapidly damped leaving slow magnetic drift mode.

Long slow spiral down the potential!

\[ \vec{B} = B \hat{z} \]

\[ B = 10\mu \]

\[ H = \frac{\mu^2}{\sqrt{3}} \]
Building a new theory:

\[ \mathcal{L} = \sqrt{-g} \left[ \frac{R}{2} - \frac{1}{2} (\partial \phi)^2 - V(\phi) - \frac{1}{2} \text{Tr} [F^{\mu \nu} F_{\mu \nu}] - \frac{\lambda}{4f} \phi \frac{\epsilon^{\mu \nu \alpha \beta}}{\sqrt{-g}} \text{Tr} [F_{\mu \nu} F_{\alpha \beta}] \right] \]
Start with the basics...

\[ \mathcal{L} = \sqrt{-g} \left[ \frac{R}{2} - \frac{1}{2} (\partial \phi)^2 - V(\phi) - \frac{1}{2} \text{Tr} [F_{\mu\nu} F_{\mu\nu}] - \frac{\lambda}{4f} \phi \frac{\epsilon_{\mu\nu\alpha\beta}}{\sqrt{-g}} \text{Tr} [F_{\mu\nu} F_{\alpha\beta}] \right] \]

Usual inflationary action.

• with something like

\[ V(\phi) = \mu^4 \left( 1 + \cos \left( \frac{\phi}{f} \right) \right) \]

“Natural Inflation” - Freese, Frieman and Olinto ’90
add gauge fields,

\[ \mathcal{L} = \sqrt{-g} \left[ \frac{R}{2} - \frac{1}{2} (\partial \phi)^2 - V(\phi) - \frac{1}{2} \text{Tr} [F_{\mu \nu} F_{\mu \nu}] - \frac{\lambda}{4f} \phi \frac{\epsilon_{\mu \nu \alpha \beta}}{\sqrt{-g}} \text{Tr} [F_{\mu \nu} F_{\alpha \beta}] \right] \]

Action for a vector (gauge) field theory.
and let them interact.

\[ \mathcal{L} = \sqrt{-g} \left[ \frac{R}{2} - \frac{1}{2} (\partial \phi)^2 - V(\phi) - \frac{1}{2} \text{Tr} [F_{\mu\nu} F_{\mu\nu}] - \frac{\lambda}{4f} \phi \frac{\epsilon_{\mu\nu\alpha\beta}}{\sqrt{-g}} \text{Tr} [F_{\mu\nu} F_{\alpha\beta}] \right] \]

Interaction

\[ \phi \epsilon_{\mu\nu\alpha\beta} \text{Tr} [F_{\mu\nu} F_{\alpha\beta}] = \phi \text{Tr} \left[ \vec{E} \cdot \vec{B} \right] \]

- Dimension 5 operator
- Chern-Simons term
Key: a single time derivative.

\[ \varepsilon^{\alpha \beta \gamma \delta} F_{\alpha \beta} F_{\gamma \delta} = d[\text{Something}] \]

integrate action by parts

\[ \mathcal{L} \subset \dot{\phi}[\text{Something}] \]

Like the Lorentz force! \( (\dot{x} \times B) \)
Need a classical, \textit{homogeneous} vector field.

Discovered ca. 1980 for SU(2) fields:

\[ A_0^a = 0 \quad A_i^a = \psi(t)a(t)\delta_i^a \]

solves the non-Abelian gauge field equations of motion on a cosmological background.

\[ E_{\text{chromo}} \propto \dot{\psi} + H\psi \quad B_{\text{chromo}} \propto \tilde{g}\psi^2 \]
Why does this work?

SU(2) ↔ SO(3)

simplest non-Abelian Lie group  spatial rotations
Why does this work?

\[ \text{SU}(2) \leftrightarrow \text{SO}(3) \]

3 gauge fields \leftrightarrow 3 spatial dimensions
We call this Chromo-Natural Inflation.

- Equations of motion:

\[ \ddot{\phi} + 3H \dot{\phi} + V'(\phi) = -3\tilde{g}\frac{\lambda}{f} \psi^2 (\dot{\psi} + H \psi) \]

\[ \ddot{\psi} + 3H \dot{\psi} + (\dot{H} + 2H^2) \psi + 2\tilde{g}^2 \psi^3 = \tilde{g}\frac{\lambda}{f} \psi^2 \dot{\phi} \]

We call this Chromo-Natural Inflation.

\[ \ddot{\phi} + 3H \dot{\phi} + V'(\phi) = -3\tilde{g} \frac{\lambda}{f} \psi^2 (\dot{\psi} + H \psi) \]

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\[
\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = -3\tilde{g}\frac{\lambda}{f}\psi^2(\dot{\psi} + H\psi)
\]

Magnetic drift leads to slow roll.

- In the slow-roll, large $\lambda$ limit, system simplifies.

\[
\begin{align*}
\dot{\phi} &= \frac{f}{\lambda} \left( 2 g \psi + \frac{2 H^2}{g \psi} \right) \\
\dot{\psi} &= -H \psi + \frac{f}{3 \tilde{g} \lambda} \frac{V'(\phi)}{\psi^2}
\end{align*}
\]

- Scalar equation of motion independent of $V'$!

(Martinec, Adshead, MW 2012)
We avoid the eta problem.

Replaced a quantum-unstable tuning:

\[ \eta_V \equiv M_{\text{Pl}}^2 \frac{\partial^2 V}{\partial \phi^2} \ll 1 \]

With (protected) interactions:

\[ M_{\text{Pl}}^2 \frac{\partial^2 V}{\partial \phi^2} \sim 1 \]

and (e.g.) \[ \text{Tr}[F^2] \]

Generic, natural

Protected by gauge-invariance
Gauge fields provide new physical consequences.

New degrees of freedom = new observational handles.
The scalar sector works.

\[ \langle RR \rangle \quad \text{known physics} \]
Even though it looks very complicated...
...we can isolate a “slow” drift mode:

\[ R \{ \]

\[ |\hat{X}|, |\hat{Z}|, |\hat{\delta}\phi| \]

Amplitude

Time

\[-k\tau\]

0.001 0.1 10 1000

|X|  \sqrt{2m_\chi/H\Lambda}  m_\psi  m_\psi\Lambda
Spectral index is a strong function of gauge field effective mass.

\[ m_\psi \equiv \frac{g_\psi}{H} \]
The tensor sector has new features.

\[ ds^2 = -dt^2 + a^2 e^{\gamma_{ij}} dx^i dx^j \]

\[ A_\mu = (0, a(t)\psi(t)\delta^a_i + t^a_i(t, x)) \frac{\sigma_a}{2} \]
Gauge and gravity tensors mix.

\[ \hat{\gamma}^{\pm''} + \left( k^2 - \frac{2}{\tau^2} \right) \hat{\gamma}^{\pm} = C t^{\pm} \]

\[ \hat{t}^{\pm''} + \left( k^2 + \frac{g_\psi \lambda}{f} \dot{\chi} \frac{1}{\tau^2} \right) \hat{t}^{\pm} + k \left( \frac{\lambda}{f} \dot{\chi} + 2 \frac{g_\psi}{H} \right) \frac{1}{\tau} \hat{t}^{\pm} = C \hat{\gamma}^{\pm} \]

P. Adshead, E. Martinec, MW, arxiv: 1301.2598
Gauge and gravity tensors mix.

usual equation

\[
\hat{\gamma}^{\pm''} + \left( k^2 - \frac{2}{\tau^2} \right) \hat{\gamma}^\pm = C t^\pm
\]

\[
\hat{t}^{\pm''} + \left( k^2 + \frac{g\psi}{H} \frac{\lambda}{f} \frac{1}{\tau^2} \right) \hat{t}^\pm + k \left( \frac{\lambda}{f} \dot{\chi} + 2 \frac{g\psi}{H} \right) \frac{1}{\tau} \hat{t}^\pm = C \hat{\gamma}^\pm
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Gauge tensors split; one is \textit{amplified}.

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Gauge tensors split; one is amplified.

• Parity is spontaneously broken by the background.

P. Adshead, E. Martinec, MW, arxiv: 1301.2598
The gauge field feeds the gravitational waves.

“Left-handed” modes

enhancement $\propto \exp(m_\psi - \sqrt{3})$
Chiral GWs give a new observable signature.

Parity-violating Temperature / ‘B’-mode polarization correlation
Gravitational wave production rules out model!

Tensor-to-scalar ratio, $r$, at $k = 0.002 \text{ h/Mpc}$

Gravitational wave spectral tilt
A positive mass for the gauge field could help...
A positive mass for the gauge field could help...

The graph shows the gauge tensor $\omega_+(x, m_\psi)$ as a function of $x = -k\tau$ for different values of $m_\psi$. The Higgs mechanism gives positive gauge mass!

- $m_\psi = \sqrt{2}$
- $m_\psi = 2.5$
- $m_\psi = 4$
- $m_\psi = 6$
Early tests suggest this works, at cost of an extra scalar field.

Tensor-to-scalar ratio

Spectral tilt

(different colors = different “higgs” vev values)
Summary of results.

- Gauge fields can improve inflationary models.
- Original CNI model ruled out by gravitational wave production
- Higgs mechanism allows us to match Planck
- New observables (e.g. chiral gravitational waves) emerge.