Statistical Analysis of Supersymmetry Breaking in Flux Vacua

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Abstract

Based on hep-th/0303194, hep-th/0405279 and

- hep-th/0307049 with Sujay Ashok
- math.CV/0402326 with Bernard Shiffman and Steve Zelditch (Johns Hopkins)
- hep-th/0404116 and to appear with Frederik Denef

1. Predictions from string theory

For almost 20 years we have had good qualitative arguments that compactification of string theory can reproduce the Standard Model and solve its problems, such as the hierarchy problem. But we still seek distinctive predictions which we could regard as evidence for or against the theory.

One early spin-off of string theory, four dimensional supersymmetry, is the foundation of most current thinking in "beyond the Standard Model" physics. Low energy supersymmetry appears to fit well with string compactification. But would not discovering supersymmetry, be evidence against string/M theory?

In recent years, even more dramatic possibilities have been suggested, which would lead to new, distinctive particles or phenomena: large extra dimensions (KK modes), a low fundamental string scale (massive string modes), or rapidly varying warp factors (modes bound to branes, or conformal subsectors).

Any of these could lead to dramatic discoveries. But should we expect string/M theory to lead to any of these possibilities? Would not discovering them be evidence against string theory?

Last year in Durham, I discussed a statistical approach to these and other questions of string phenomenology. Over the last year, our group at Rutgers, and others, have made major progress in developing this approach, with

- Detailed results for distributions of flux vacua (with Denef, Shiffman, and Zelditch).
- Constructions of vacua with all moduli stabilized along lines of KKLT (with Denef and Florea; see Strings 2004).
- Preliminary results on the statistics of
  - supersymmetry breaking scales (MRD, hep-th/0405279 and to appear; see also Susskind, hep-th/0405189; Dine, Gorbatov and Thomas, hep-th/0407043.
  - volume of the extra dimensions (see Strings 2004).

These ideas have already begun to inspire new phenomenological models (*e.g.* Arkani-Hamed and Dimopoulos, hep-th/0405159), and I start to think that fairly convincing predictions could come out of this approach over the next few years.

Much work will be needed to bring this about. But we may be close to making some predictions: those which use just the most generic features of string/M theory compactification, namely the existence of many hidden sectors.

2. Hidden sectors

Before string theory, and during the "first superstring revolution," most thinking on unified theories assumed that internal consistency of the theory would single out the matter content we see in the real world.

In the early 1980's it was thought that $d = 11$ supergravity might do this.

Much of the early excitement about the heterotic string came from the fact that it could easily produce the matter content of E_6, $SO(10)$ or $SU(5)$ grand unified theory.

But this was not looking at the whole theory. The typical compactification of heterotic or type II strings on a Calabi-Yau manifold has hundreds of scalar fields, larger gauge groups and more charged matter. Already in the perturbative heterotic string an extra E_8 appeared. With branes and non-perturbative gauge symmetry, far larger groups are possible.

If we live in a "typical" string compactification, it seems very likely that there are many hidden sectors, not directly visible to observation or experiment.

Should we care? Does this lead to any general predictions?

Hidden sectors may or may not lead to new particles or forces. But what they do generically lead to is a multiplicity of vacua, because of symmetry breaking, choice of vev of additional scalar fields, or other discrete choices.

Let us say a hidden sector allows c distinct vacua or "phases." If there are N hidden sectors, the multiplicity of vacua will go as

$$\mathcal{N}_{vac} \sim c^N .$$

While the many hidden sectors certainly make the detailed study of string compactification more complicated, we should consider the idea that they lead to simplifications as well.

Thus we might ask, what can we say about the case of a large number N of hidden sectors ? Clearly there will be a large multiplicity of vacua.

We only live in one vacuum. However, as pointed out by Brown and Teitelboim; Banks, Dine and Seiberg (and no doubt many others), vacuum multiplicity can help in solving the cosmological constant problem. In an ensemble of $\mathcal{N}_{vac}$ vacua with roughly uniformly distributed c.c. Λ, one expects that vacua will exist with Λ as small as $M_{pl}^4/\mathcal{N}_{vac}$.

To obtain the observed small nonzero c.c. $\Lambda \sim 10^{-122} M_{pl}^4$, one requires $\mathcal{N}_{vac} > 10^{120}$ or so.

Now, assuming different phases have different vacuum energies, adding the energies from different hidden sectors can produce roughly uniform distributions. In fact, the necessary $\mathcal{N}_{vac}$ can easily be fit with $\mathcal{N}_{vac} \sim c^N$ and the parameters $c \sim 10$, $N \sim 100 - 500$ one expects from string theory, as first pointed out by Bousso and Polchinski.

One might regard fitting the observed small nonzero c.c. in **any** otherwise acceptable vacuum as solving the problem, or one might appeal to an anthropic argument such as that of Weinberg to select this vacuum. In the absence of other candidate solutions to the problem, we might even turn this around and call these ideas **evidence** for the hypothesis that we are in a compactification with many hidden sectors.

3. Stringy naturalness

So can we go further with these ideas? Another quantity which can get additive contributions from different sectors is the scale of supersymmetry breaking. Let us call this M^2_{susy} (we will define it more carefully below).

We recall the classic arguments for low energy supersymmetry from naturalness. The electroweak scale m_{EW} is far below the other scales in nature M_{pl} and M_{GUT}. According to one definition of naturalness, this is only to be expected if a symmetry is restored in the limit $m_{EW} \to 0$. This is not true if m_{EW} is controlled by a scalar (Higgs) mass m_H, but can be true if the Higgs has a supersymmetric partner (we then restore a chiral symmetry).

A more general definition of naturalness requires the theory to be stable under radiative corrections, so that the small quantity does not require fine tuning. Again, low energy supersymmetry can accomplish this. Many theories have been constructed in which

$$M_H^2 \sim c M^2_{susy},$$

with $c \sim 1/10$ without fine tuning. Present data typically requires $c < 1/100$, which requires a small fine tuning (the "little hierarchy problem.")

On the other hand, the solution to the cosmological constant problem we accepted above, in terms of a discretuum of vacua, is suspiciously similar to fine tuning the c.c., putting the role of naturalness in doubt.

What should replace it?

The original intuition of string theorists was that string theory would lead to a unique four dimensional vacuum state, or at least very few, such that only one would be a candidate to describe real world physics. In this situation, there is no clear reason the unique theory should be "natural" in the previously understood sense.

With the development of string compactification, it has become increasingly clear that there is a large multiplicity of vacua. The vacua differ not only in the cosmological constant, but in every possible way: gauge group, matter content, couplings, etc. What should we do in this situation?

The "obvious" thing to do at present is to make the following definition:

> An effective field theory (or specific coupling, or observable) T_1 is more natural in string theory than T_2, if the number of phenomenologically acceptable vacua leading to T_1 is larger than the number leading to T_2. (Douglas, 0303194; Susskind, 0406197)

Now there is some ambiguity in defining "phenomenologically" (or even "anthropically") acceptable. One clearly wants $d = 4$, supersymmetry breaking, etc. One may or may not want to put in more detailed information from the start.

In any case, the unambiguously defined information provided by string/M theory is the number of vacua and the distribution of resulting EFT's. For example, we could define

$$\begin{aligned} d\mu[M_H^2, M_{susy}^2, \Lambda] &= \rho(M_H^2, M_{susy}^2, \Lambda) dM_H^2 \; dM_{susy}^2 \; d\Lambda \\ &= \sum_{T_i} \delta(M_{susy}^2 - M_{susy}^2|_{T_i}) \delta(M_H^2 - M_H^2|_{T_i}) \delta(\Lambda - \Lambda|_{T_i}) \end{aligned}$$

a distribution which counts vacua with given c.c., susy breaking scale and Higgs mass, and study the function

$$\rho(10^4 \text{ GeV}^2, M_{susy}^2, \Lambda \sim 0).$$

4. Explicit results for flux vacua

While it may sound overly ambitious to compute a distribution like

$$d\mu[M_H^2, M_{susy}^2, \Lambda],$$

in fact we start to see how to do it for one large class of vacua: compactification of the IIb string on CY with flux, as developed by Giddings, Kachru and Polchinski.

This produces a large class of effective supergravity theories, which can be made completely explicit using techniques developed in the study of mirror symmetry (Candelas et al). For example, the superpotential is the "flux superpotential" of Gukov, Taylor, Vafa and Witten,

$$W = \int \Omega(z) \wedge \left(F^{(3)} + \tau H^{(3)}\right).$$

While this is exact only in the large volume, weak string coupling limit, it is **dual** to a large class of gauge theories which includes many non-perturbative effects, and could be representative of the general situation.

To warm up, let us consider the simplest such problem: the distribution of supersymmetric flux vacua on a a rigid CY, *i.e.* with $b^{2,1} = 0$ (for example, the orbifold $T^6/\mathbb{Z}_3$). We do not discuss stabilizing Kähler moduli here, so the only modulus is the dilaton τ, with Kähler potential $K = -\log \mathrm{Im}\ \tau$.

For the rigid CY, this reduces to

$$W = A\tau + B; \qquad A = a_1 + \Pi a_2; B = b_1 + \Pi b_2$$

with $\Pi = \int_{\Sigma_2} \Omega^{(3)} / \int_{\Sigma_1} \Omega^{(3)}$, a constant determined by CY geometry.

Now it is easy to solve the equation $DW = 0$:

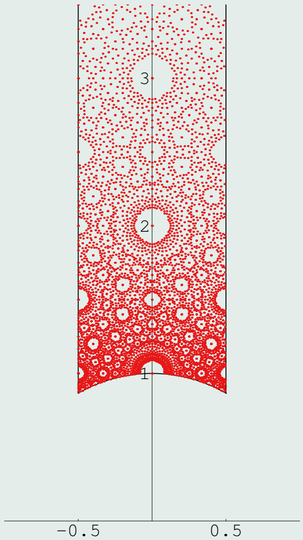
$$\begin{aligned} DW &= \tfrac{\partial W}{\partial \tau} - \tfrac{1}{\tau - \bar\tau} W \\ &= \tfrac{-A\bar\tau - B}{\tau - \bar\tau} \end{aligned}$$

so $DW = 0$ at

$$\bar\tau = -\frac{B}{A}$$

where $\bar\tau$ is the complex conjugate.

Here is the resulting set of flux vacua for $L = 150$ and $\Pi = i$:



This graph was obtained by enumerating one solution of $a_1 b_2 - a_2 b_1 = L$ in each $SL(2, \mathbb{Z})$ orbit, taking the solution $\tau = -(b_1 - ib_2)/(a_1 - ia_2)$ and mapping it back to the fundamental region.

The total number of vacua is $N = 2\sigma(L)$, where $\sigma(L)$ is the sum of the divisors of L. Its large L asymptotics are $N \sim \pi^2 L/6$.

A similar enumeration for a Calabi-Yau with n complex structure moduli, would produce a similar plot in $n + 1$ complex dimensions, the distribution of flux vacua. It could (in principle) be mapped into the distribution of possible values of coupling constants in a physical theory.

The intricate distribution we just described has some simple properties. For example, one can get exact results for the large L asymptotics, by computing a continuous distribution $\rho(z,\tau;L)$, whose integral over a region R in moduli space reproduces the asymptotic number of vacua which stabilize moduli in the region R, for large L,

$$\int_R dz d\tau \ \rho(z,\tau;\ L) \sim_{L\to\infty} N(R).$$

For a region of radius r, the continuous approximation should become good for $L >> K/r^2$. For example, if we consider a circle of radius r around $\tau = 2i$, we match on to the constant density distribution for $r > \sqrt{K/L}$.

Another one complex structure modulus example with $r < \sqrt{K/L}$ was discussed by Girvayets, Kachru, Tripathy 0404243.

Explicit formulas for these densities can be found, in terms of the geometry of the moduli space $\mathcal{C}$. The simplest such result (with Ashok) computes the index density of vacua:

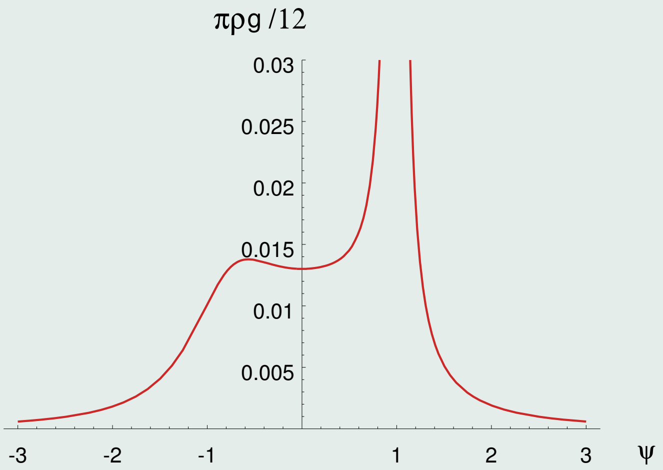
$$\rho_I(z,\tau) = \frac{(2\pi L)^{b_3}}{b_3!\pi^{n+1}} \det(-R - \omega \cdot 1)$$

where ω is the Kähler form and R is the matrix of curvature two-forms. Integrating this over a fundamental region of the moduli space produces an estimate for the total number of flux vacua. For example, for T^6 we found $I \sim 4 \cdot 10^{21}$ for $L = 32$.

This density is "topological" and there are mathematical techniques for integrating it over general CY moduli spaces (Z. Lu and MRD, work in progress). Good estimates for the index should become available for a large class of CY's over the coming years.

5. Distributions of flux vacua

Let us look at the details of the distribution of flux vacua on the mirror quintic ($K = 4$ and $n = 1$), as a function of complex structure modulus:



Note the divergence at $\psi = 1$, the conifold point. It arises because the curvature $R \sim \partial\bar{\partial} \log \log |\psi - 1|^2$ diverges there. The divergence is integrable, but a finite fraction of all the flux vacua sit near it.

The peak near the conifold point can be understood in terms of its **dual gauge theory interpretation** (Maldacena, Klebanov, Strassler, Gopakumar, Vafa, ...), in which the parameter $S = \psi - 1$ above is reinterpreted as the gaugino condensate. Fluxes directly control the gauge coupling g and lead to a distribution $d^2\tau/(\mathrm{Im}\ \tau^2)$ with $\tau = i/g_{YM}^2 + \theta$, *i.e.*

$$d\mu[g] \sim d(g_{YM})^2.$$

The structure of the flux superpotential then reproduces the standard $S = e^{-1/g^2}$, leading to the distribution

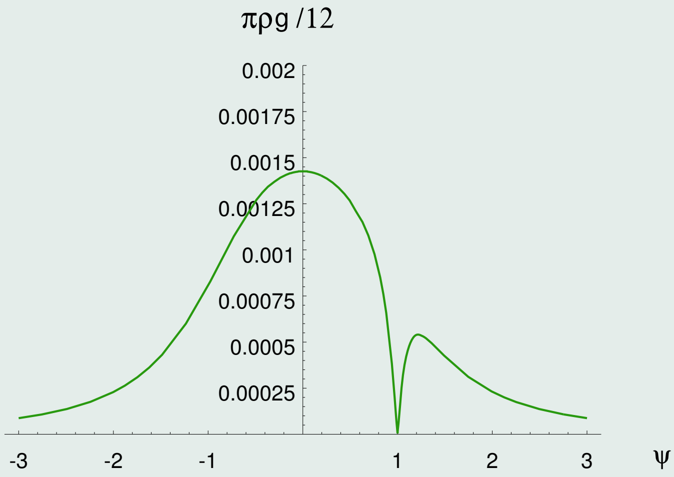
$$d\mu[S] \sim \frac{d^2 S}{|S \log S|^2}.$$

Quantitatively, for the mirror quintic, about 3% of vacua sit near the conifold point, with an induced scale $|\psi - 1| < 10^{-3}$. More generally, the total number of vacua goes as $\mathcal{N}_{vac}|_{S<S_*} \sim \frac{1}{|\log S_*|}$.

Vacua close to conifold degenerations are interesting for model building, as they provide a natural mechanism for generating large scale hierarchies (by dual gauge theory, or in supergravity as in Randall and Sundrum, etc.). We have found that such vacua are common, but are by no means the majority of vacua.

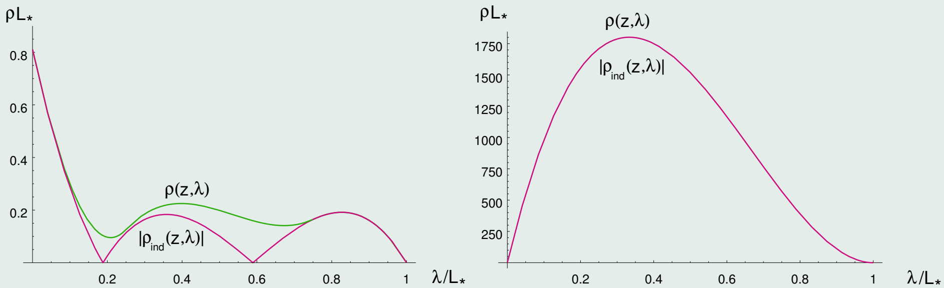
Suppose we go on to break supersymmetry by adding an anti D3-brane, or by other D term effects. The previous analysis applies (since we have not changed the F terms), but now it is necessary that the mass matrix at the critical point is positive.

The distribution of tachyon-free D breaking vacua is



In fact, most D breaking vacua near the conifold point have tachyons (for one modulus CY's), so we get suppression, not enhancement. This is not hard to understand in detail; the mechanism is a sort of "seesaw" mixing between modulus and dilaton, which seems special to one parameter models.

Here is the distribution of (negative) AdS cosmological constants $\hat{\Lambda} = 3e^K|W|^2$, both at generic points (left) and near the conifold point (right). Note that at generic points it is fairly uniform, all the way to the string scale. On the other hand, imposing small c.c. competes with the enhancement of vacua near the conifold point.



The left hand graph compares the total number of vacua (green) with the index (red). The difference measures the number of Kähler stabilized vacua, vacua which exist because of the structure of the Kähler potential, not the superpotential.

6. Susy breaking

D breaking vacua (with $DW = 0$) are described by the earlier results, just we require the vacua to be tachyon free and have near zero c.c. With Denef, we have analyzed F breaking flux vacua in orientifolds in some detail (hep-th/0404116 and to appear). These satisfy

$$0 = \partial_I V = Z_{IJ}\bar{F}^J - 2F_I\bar{W} \qquad \text{and } F \neq 0; V'' > 0 \tag{1}$$

where

$$W = W(z);\ \ F_I = D_I W(z);\ \ Z_{IJ} = D_I D_J W(z).$$

Varying both fluxes and CY moduli scans a certain subspace of these parameters, and the question is what part of this subspace satisifies (1).

In fact (1) can be written as the condition that F_I is an **eigenvector** of a certain matrix constructed from W and Z, and the condition $V'' > 0$ (no tachyons) becomes the condition that F_I has the **lowest positive eigenvalue**.

Since **all** matrices have such eigenvectors, metastable F breaking vacua are **generic**.

This includes the "traditional" vacua in which susy breaking takes place at hierarchically small scales.

But there is nothing in the computations which **requires** the susy breaking to be at small scales. A priori, it might equally well be at high scales. So which is more "natural" in string theory ?

Thus we come back to the problem of computing or estimating the distribution

$$d\mu[M_H^2, M_{susy}^2, \Lambda]$$

at the observed values $M_H \sim 100$ GeV, $\Lambda \sim 0$.

This is a hard problem which cannot (yet) be solved in any detail. To address it, we need to combine existing work and intuitions, **carefully removing the presupposition of all previous work that low scale breaking was required to solve the hierarchy problem**, with the new information from string/M theory.

Of course, the best we could do at present is discuss the distribution of the classes of vacua we know about; there might be others. But consensus has not yet been reached as to what even this predicts.

One idea which most authors agree on (so far) is that, even if a model has large soft mass terms M_0^2, statistical fine tuning will produce the correct Higgs mass. in (roughly) a fraction

$$\frac{M_H^2}{M_0^2} = \int^{M_H^2} \frac{d(M_H^2)}{M_0^2}$$

of the models.

This is plausible if M_H^2 is a sum of independent **positive and negative** contributions. Typically, M_H^2 receives radiative corrections of both signs. Furthermore, unlike the other scalars, the Higgs in the MSSM are non-chiral, leading to the mass terms

$$\mu(|H_1|^2 + |H_2|^2) + m_1^2|H_1|^2 + m_2^2|H_2|^2 + B\mu H_1 H_2,$$

and the $B\mu$ term is a negative contribution to M_H^2.

We also need to **assume** that a finite fraction of models solve the μ problem.

Analogously, one might try to argue that since $M_{susy} \to 0$ leads to $\Lambda \to 0$, the Λ distribution is

$$d\mu[M_{susy}^2, \Lambda] \sim \frac{d\Lambda}{M_{susy}^4} d\mu[M_{susy}^2].$$

However, this is not true: in supergravity,

$$\Lambda = \sum |F|^2 + |D|^2 - 3e^K |W|^2$$

and does **not** go to zero as $F, D \to 0$.

In fact the flux vacua results show that the distribution of the parameter $e^K|W|^2$ (not $|W|$) is fairly uniform, from zero all the way to the string scale. The physics behind this is that the superpotential W is a sum of contributions from the many sectors. This includes supersymmetric hidden sectors, so there is **no reason** W should be correlated to the scale of supersymmetry breaking, and **no reason** the cutoff on the W distribution should be correlated to the scale of supersymmetry breaking.

Such a sum over randomly chosen complex numbers will tend to produce a distribution

$$d^2W = \frac{1}{2} d\theta d(|W|^2) = \frac{1}{2} d\theta d(|W|^2)$$

uniform out to the cutoff scale, which in flux vacua is LM_s^4.

Thus, an arbitrary supersymmetry breaking contribution to the vacuum energy can be compensated by the $-3e^K|W|^2$ term, with no preferred scale. The need to get small c.c. **does not favor a particular scale of susy breaking** in these models.

Thus, the joint distribution goes as

$$d\mu[M_{susy}^2, \Lambda] \sim \frac{d\Lambda}{M_{str}^4} d\mu[M_{susy}^2].$$

Again, all workers agree on this so far.

We now need to estimate $d\mu[M^2_{susy}]$, so we must define M_{susy}. The most universal definition is

$$M^4_{susy} = \sum_i |F_i|^2 + \sum_\alpha D^2_\alpha,$$

the quantity which determines the **gravitino mass** $M^2_{3/2} = M^4_{susy}/M^2_{pl}$.

How is this related to the soft masses M^2_0 which entered the earlier claim that $d\mu[M^2_H] \sim d(M^2_H)/M^2_0$? This relation is model dependent as there are many ways to mediate supersymmetry breaking:

- Generic supergravity contributions (non-renormalizable terms in K; radiative corrections):

$$M^2_0 \sim M^2_{3/2} \sim M^4_{susy}/M^2_{pl}.$$

- Gauge mediation:

$$M^2_0 \sim \lambda^k M^2_{susy},$$

  suppressed by powers of coupling constants.

- others?

A plausible summary of the current understanding (Dine, Gorbatov and Thomas 0407043) is to say that

- If $M_{susy} > (M_H M_{pl})^{1/2} \sim 10^{10}$ GeV, supergravity contributions will dominate, and $M_0^2 \sim M_{3/2}^2 \sim M_{susy}^4/M_{pl}^2$. Then,

$$d\mu[M_H^2, M_{susy}^2] \sim \frac{M_H^2}{M_{susy}^4/M_{pl}^2} \qquad M_{susy}^2 > M_H M_{pl}$$

  in a relatively model independent way.

- If $M_{susy} \leq (M_H M_{pl})^{1/2} \sim 10^{10}$ GeV, we assume that the various mediation mechanisms are generic, allowing models with M_0^2 arbitrarily smaller than M_{susy}^2. Dine *et al* model this with the distribution

$$d\mu[M_H^2, M_{susy}^2] \sim 1 \qquad M_{susy}^2 \leq M_H M_{pl}$$

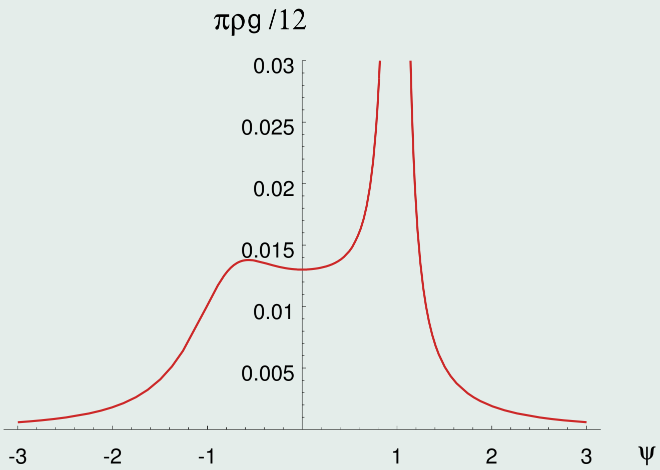
So what is $d\mu[M_{susy}^2]$?

While we have not finished our computations, so far they support the standard intuition that gauge theory non-perturbative effects can drive supersymmetry breaking. Let us grant this:

$$d\mu[F] \sim d\mu[S] \sim d\mu[\psi - 1]$$

in the mirror quintic example we discussed earlier.

Recall that distribution of flux vacua:



The rough description of the resulting distribution is a **large uniform component** added to a **smaller component at hierarchically small scales**,

$$d\mu[F] \sim c\frac{dF}{F} + (1-c)dF.$$

For the mirror quintic, we saw $c \sim .03$.

Another way to say this, is that the $d(g^2_{YM})$ distribution for gauge couplings, allows for a large $g_{YM} \sim 1$ component, in which there is no hierarchically small scale.

The larger point, is that (so far) we see nothing in string/M theory which ties supersymmetry breaking directly with hierarchically small scales. Thus, we must take the possibility of high scale vacua seriously.

Of course, the previous results fit the usual idea that low scale susy breaking could solve the hierarchy problem. In fact we found

$$d\mu[M_H^2, M_{susy}^2, \Lambda] \sim \frac{dM_H^2}{M_{susy}^4/M_{pl}^2}\,\frac{d\Lambda}{M_{str}^4} d\mu[M_{susy}^2] \qquad M_{susy}^2 > M_H M_{pl}$$

which is a **stronger** bias than the naive M_H^2/M_{susy}^2 (but not as strong as including the c.c. would have produced).

The upshot is that high scale breaking will be favored if

$$d[M^2_{susy}] \sim M^{2\alpha}_{susy} dM^2_{susy}$$

with $\alpha > 1$ (since most "phase space" is at large M^2_{susy}).

A uniform distribution will not do this; while a hypothetical d^2F distribution would be on the edge. But there is a very simple effect which in principle could. Namely, since

$$M^4_{susy} = \sum |F|^2 + D^2$$

is a sum of many **positive** terms, if the terms are roughly independent with **any** significant uniform component in their distribution, the overall distribution heavily favors high scale breaking (DD 0404116, MRD 0405279, Susskind, hep-th/0405189).

For example, convolving uniform distributions gives

$$\begin{aligned}\rho(M^2_{susy}) &= \int \prod_{i=1}^{n_F} d^2F \ \prod_{\alpha=1}^{n_D} dD \ dM^4_{susy} \ \delta(M^4_{susy} - \sum |F|^2 - \sum D^2) \\ &\sim (M^2_{susy})^{2n_F+n_D-1} dM^2_{susy}\end{aligned}$$

Combining this with the factor m^2_H/M^2_{susy}, we find that high scale susy breaking is favored if $2n_F + n_D > 2$, a condition surely satisfied by almost all string models.

Now the distribution we suggested earlier was not uniform: there was a second component (with fraction c) counting susy breaking vacua at hierarchically small scales. A rough description of the effects of this is that low scale breaking requires **all** breaking parameters to come from the low scale part of the distribution, a fraction c^n of vacua. If **any** sector sees the uniform component, high scale breaking will result.

This leads to the estimate that high scale breaking would be preferred if the number of moduli satisfies

$$n > \log_{1/c} \frac{M_H^2}{M_{high}^2} \sim 100$$

even taking $c \sim .5$. Since the vast majority of CY's have more than 20 moduli, and we need many moduli to tune the c.c., this argument seems to predict high scale supersymmetry breaking.

There are many potential loopholes in such a claim; for example stringy effects not yet considered might lower the cutoff on the F distributions.

But, having continued our computations, it now appears that it is **not at all true** that the various components in F are independently distributed. The summary I gave of solving the equations for non-supersymmetric vacua,

$$0 = \partial_I V = Z_{IJ}\bar{F}^J - 2F_I\bar{W},$$

was that this amounted to finding eigenvectors of the matrix Z_{IJ}. For a typical matrix without degenerate eigenvalues, the eigenspaces are one dimensional, and this leads to distributions

$$d\mu[F_I] = dF\ \delta(F_I - F\psi_I)$$

where ψ_I is the eigenvector.

This leads to the uniform distribution $d(M^2_{susy})$.

The possibility of power law growth $d\mu[F] \sim F^{2n-2}d^2F$ discussed above corresponds to matrices Z_{IJ} with **degenerate** eigenvalues. While possible, it now appears to us that this is sufficiently non-generic to suppress these components of the distribution.

If this holds up, we would predict

$$d\mu[M_H^2, M_{susy}^2] \sim d(M_H^2) \cdot \frac{dM_{susy}^2}{M_{susy}^4/M_{pl}^2}$$

which, while not as strong as the suppression of high scale models claimed by Dine *et al*, would suffice to favor low scale breaking.

Note that the claim is on the edge – if we get instead a uniform $d^2F \sim M_{susy}^2 d(M_{susy}^2)$, we would obtain $d(M_{susy}^2)/M_{susy^2}$ which is a log uniform distribution, not strongly favoring any scale.

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Many points would have to be nailed down to get a convincing argument one way or the other. Perhaps the two which emerged most clearly from our discussion are to push through the formal computations of distributions of nonsupersymmetric vacua in a variety of constructions, and to understand what cuts off the high end of the distribution of supersymmetry breaking scales.

To name a few more issues, it might be that physics we neglected also puts a lower cutoff on the maximal flux for supersymmetric vacua, it might be that the μ problem is hard to solve, there might be large new classes of nonsupersymmetric vacua, etc. Even if the majority of string/M theory vacua predicted high scale susy, one might try to argue that the initial conditions biased the distribution, etc.

In any case, we explained a simple observation, namely the existence of vacua with high breaking scales in hidden sectors, and the large multiplicity of hidden sectors, which force us to seriously consider the possibility that there are so many high scale models that high scale supersymmetry breaking becomes the natural outcome of string/M theory compactification.

7. Conclusions

We have gone some distance in justifying and developing the statistical approach to string compactification:

We have explicit results for distributions of flux vacua of many types: supersymmetric, non-supersymmetric, tachyon-free. They display a lot of structure, with suggestive phenomenological implications:

- Large uniform components of the vacuum distribution.
- Enhanced numbers of vacua near conifold points.
- Correlations with the cosmological constant.
- Falloff in numbers at large volume and large complex structure.

We have specific IIb orientifold compactifications in which all Kähler moduli are stabilized, and vacuum counting estimates which suggest that all moduli can be stabilized. We are continuing, and are attempting to check **all known consistency conditions** in simple examples, leading to constructions of large numbers of string/M theory vacua.

There are intuitive arguments for some of the most basic properties of the distributions. For example, the $-3|W|^2$ contribution to the supergravity potential is uniformly distributed with a large (at least string scale) cutoff, because of contributions from supersymmetric hidden sectors. Thus, the need to tune the c.c. does not much influence the final numbers.

We start to see the possibility of making real world predictions:

- Large extra dimensions are heavily disfavored with the present stabilization mechanisms.
- Hierarchically small scales (gauge theoretic or warp factor) are relatively common.
- Supersymmetry breaking in hidden sectors may favor high scales of supersymmetry breaking.

At present we think the last of these does not come out, and low scales will be statistically preferred, but the question is still not settled.

Going beyond flux vacua, a next step will be to make arguments for the distribution of gauge groups and representations and other structure in the matter sector, which could give insight into the statistics of models realizing various mediation mechanisms (among other questions).

While many assumptions entered into the arguments we gave, the only essential ones are that

- Our present pictures of string compactification are representative of the real world possibilities.
- The absolute number of relevant string/M theory compactifications is not too high.

With further work, all the other assumptions can be justified and/or corrected, because they were simply shortcuts in the project of characterizing the actual distribution of vacua.

Since interesting results already follow from general properties of the theory, and we now have evidence that the detailed distribution of string/M theory vacua has many simple properties, we are optimistic that a reasonably convincing picture of supersymmetry breaking and other predictions can be developed in time for Strings 2008 at CERN.