Impact of the Top Mass Measurement on Supersymmetry: Present and Future

MCTP Top Quark Symposium

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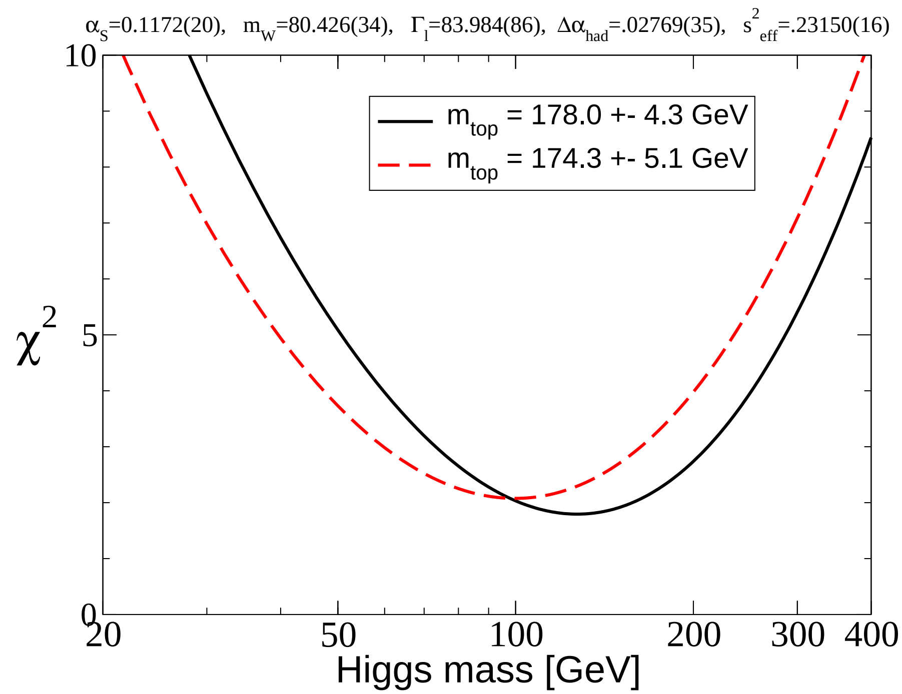
- Why SUSY enthusiasts (should) get nervous if the top mass central values goes lower
- The tension between LEP (no Higgs, no chargino) and the Tevatron ($M_{\rm top} \approx 178$ GeV).
- Impact of the top Yukawa uncertainty:
  - at the electroweak scale
  - connecting to high-scale physics
- A case study: prediction of the sign of the μ parameter in gaugino-mass dominated SUSY breaking

Precision electroweak fits (slightly) favor a heavier top quark and a light Standard Model Higgs boson

(K. Tobe, J. Wells, SPM)

From global fit to LEP1, SLC, Tevatron measurements of $m_W, m_Z, G_F, s^2_{\rm eff}, \Gamma_{\rm leptons}, \Delta\alpha_{\rm hadrons}, \alpha_S$



SUSY effects, and LEP2 measurements above $\sqrt{s} = m_Z$ can affect this weakly, but don't change the trend.

This is a test that SUSY could have failed.

The potential for the Higgs VEVS in Supersymmetry:

$$V = (|\mu|^2 + m_{H_u}^2)v_u^2 + (|\mu|^2 + m_{H_d}^2)v_d^2 - 2bv_u v_d + \frac{g^2 + g'^2}{8}(v_u^2 - v_d^2)^2$$

From the Z boson mass measurement, we already know:

$$v_u^2 + v_d^2 = (174\ \mathrm{GeV})^2$$

Also write: $$v_u/v_d \equiv \tan\beta$$

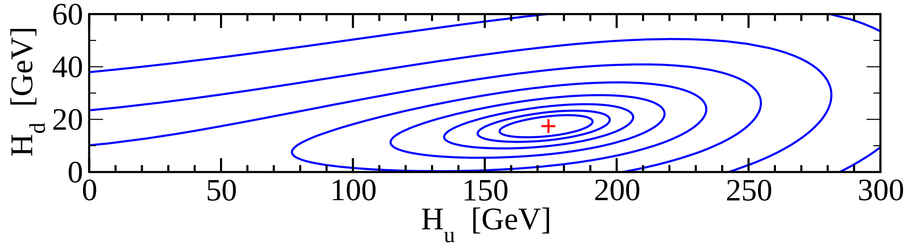
Requires, at tree-level:

$$(|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2) < b^2 < (|\mu|^2 + \tfrac{1}{2}m_{H_u}^2 + \tfrac{1}{2}m_{H_d}^2)^2$$

↑ Non-trivial minimum ↑ Bounded from below

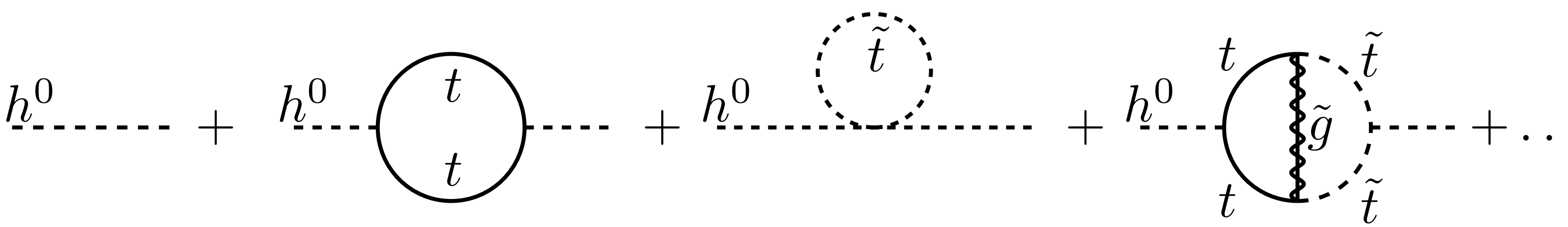
Simplest way to realize this is $m_{H_u}^2 < 0$. Radiative corrections due to a large top Yukawa coupling naturally drives this.

Typical contour map of the Higgs potential in SUSY:



The Standard Model-like Higgs boson corresponds to oscillations along the shallow direction, so very sensitive to radiative corrections.

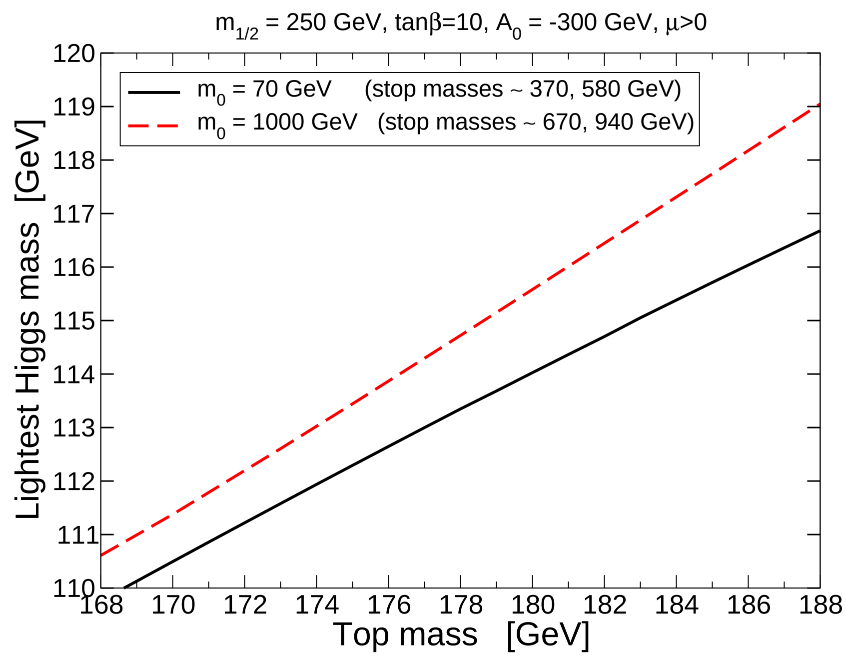
$$m_{h^0}^2 = m_Z^2 \cos^2(2\beta) + \frac{3}{4\pi^2} y_t^2 m_t^2 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \ldots$$



At tree-level:	m_Z^2	pure electroweak
At one-loop:	$y_t^2 m_t^2$	top Yukawa comes in
At two-loop:	$\alpha_S y_t^2 m_t^2$	SUSYQCD comes in
At three-loop:	$\alpha_S^2 y_t^2 m_t^2$	

Direct effect of the measured top mass on the predicted Higgs mass

Consider the Snowmass 2001 Benchmark Model SPS1a$'$. Model parameters consistent with direct limits on superpartners, $B \to s\gamma$, $(g-2)_{\rm muon}$, WMAP dark matter, etc. Vary top mass, all else fixed.



$$\frac{\Delta m_h}{\Delta m_t} \approx 0.35 \qquad (\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} = 460\ {\rm GeV})$$

$$\frac{\Delta m_h}{\Delta m_t} \approx 0.45 \qquad (\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} = 800\ {\rm GeV})$$

An aside: some reasons to be wary of excluding minimal model parameter sets from study just because m_{h^0} comes out "too low":

- Models can be modified in ways that don't necessarily affect superpartner searches, but raise the Higgs mass.

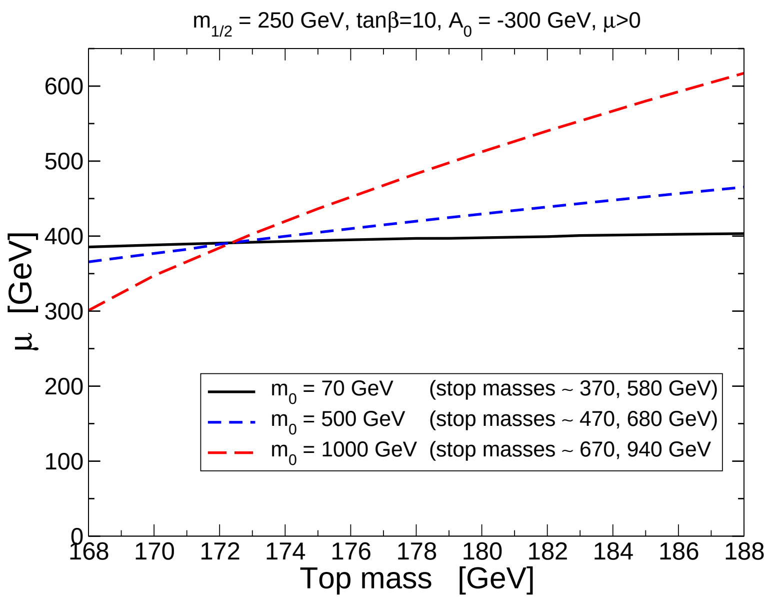
  Example: Add a singlet Higgs S with coupling λ to ordinary Higgs. Then $\Delta m^2_{h^0} = \frac{2\lambda^2 m_W^2}{g^2} \sin^2(2\beta)$

- Lighter pseudoscalars (A^0) are associated with non-Standard-Model-like Higgs couplings; can evade LEP bounds

- Two-loop (mostly known; not all implemented) and three-loop (unknown) effects are important at the 1 GeV level

Reconstruction of unknown parameters related to Electroweak Symmetry Breaking

Suppose we fix all SUSY-breaking parameters (by future measurements of superpartner masses, and a global fit to mSUGRA model parameters). Find dependence of predicted μ as a function of varying m_t:



$$\frac{\Delta\mu}{\Delta m_t} \approx 1 \qquad (\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} = 460\ \mathrm{GeV})$$

$$\frac{\Delta\mu}{\Delta m_t} \approx 5 \qquad (\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} = 560\ \mathrm{GeV})$$

$$\frac{\Delta\mu}{\Delta m_t} \approx 15 \qquad (\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} = 800\ \mathrm{GeV})$$

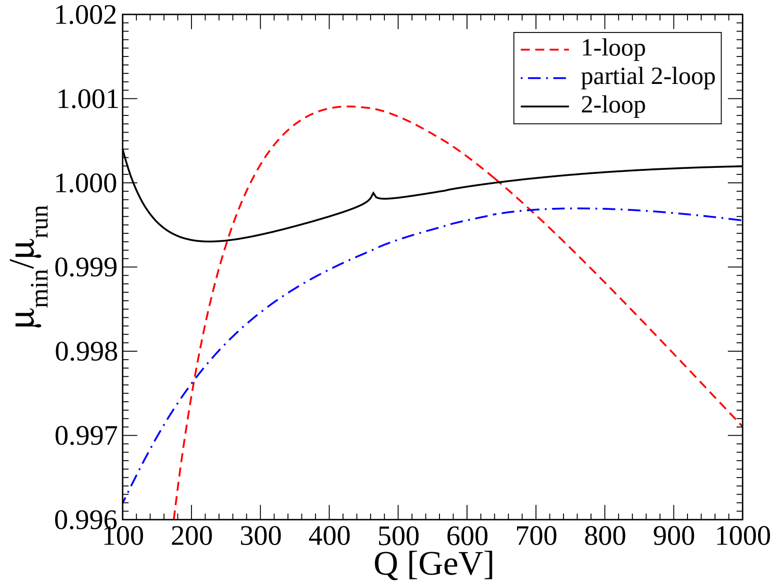
This is important because μ is closely related to the Higgsino masses, $\tilde{C}_2^{\pm}$, $\tilde{N}_3$, $\tilde{N}_4$. These may be hard to get at directly at colliders.

Sensitivity to m_{top} increases dramatically for heavier squarks.

What about pure theory uncertainties?

Complete 2-loop effective potential (SPM, hep-ph/0206136)

Suppose all model parameters except μ and $\tan\beta$ are known with perfect accuracy; then use electroweak symmetry breaking to predict them:



"run" means μ obtained by minimizing effective potential at benchmark renormalization scale $Q_0 = 640$ GeV, then run to scale Q.

"min" means μ obtained by minimizing using parameters directly at Q.

In the ideal case of an exact calculation, $\mu_{\rm min}/\mu_{\rm run} = 1$ exactly.

Fine tuning?

Consider the equations that relate the Lagrangian parameters $m^2_{H_u}$, $m^2_{H_d}$, μ, b to $m_Z, \tan\beta$:

$$
\begin{aligned}
m_Z^2 &= \frac{1}{\cos(2\beta)}\left[m^2_{H_u} - m^2_{H_d} + b(\tan\beta + \frac{1}{\tan\beta})\right] \\
\sin(2\beta) &= \frac{2b}{m^2_{H_u} - m^2_{H_d} + 2|\mu|^2}
\end{aligned}
$$

In order to make $m^2_{h^0}$ large enough to evade LEP2, must make top squark masses large. This tends to imply:

$$
|m^2_{H_u}|, \;\; |m^2_{H_d}| \gg m_Z^2
$$

Significant cancellation is needed!

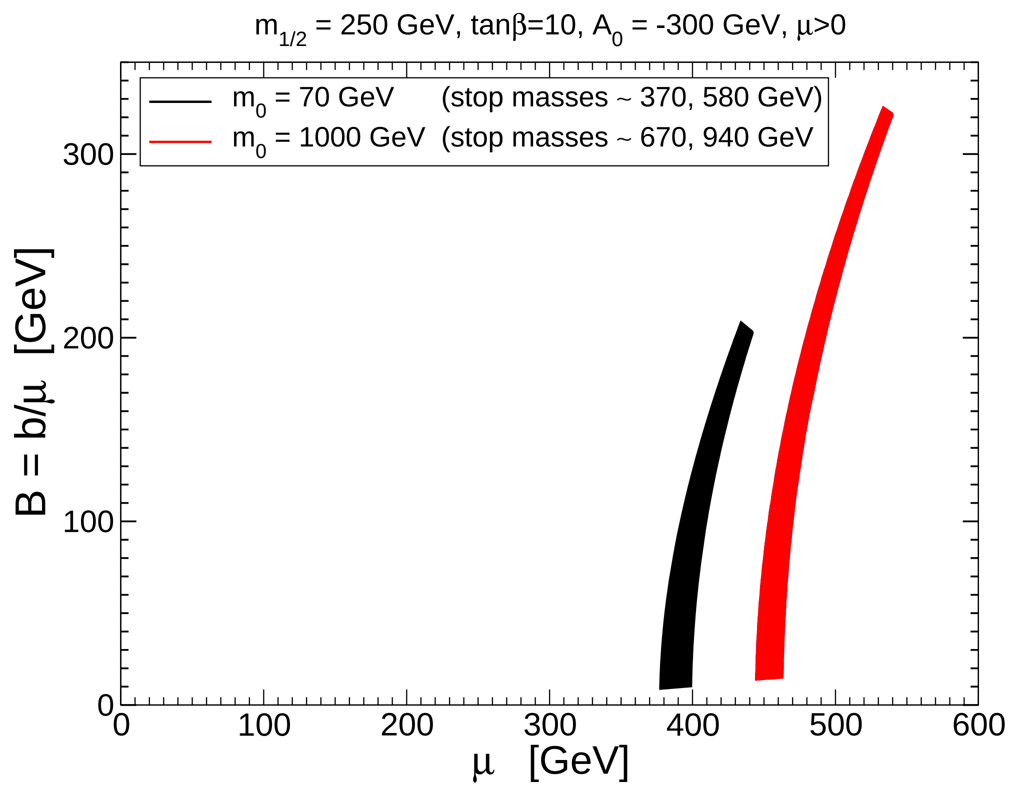
In order to study this, go back to the SPS1a$'$ Benchmark Model, since it barely escapes LEP2 bounds for the charginos, and is borderline for the Higgs.

An exercise: consider "m_Z", determined by the effective potential, as a function of the dimensionful parameters appearing in the Lagrangian. Let the input parameters μ, $B = b/\mu$ float from their benchmark values.

For what ranges of values of the inputs do we get a stable vacuum with:

$$0 < \text{“}m_Z\text{”} < 2m_Z \ ?$$

Answer:

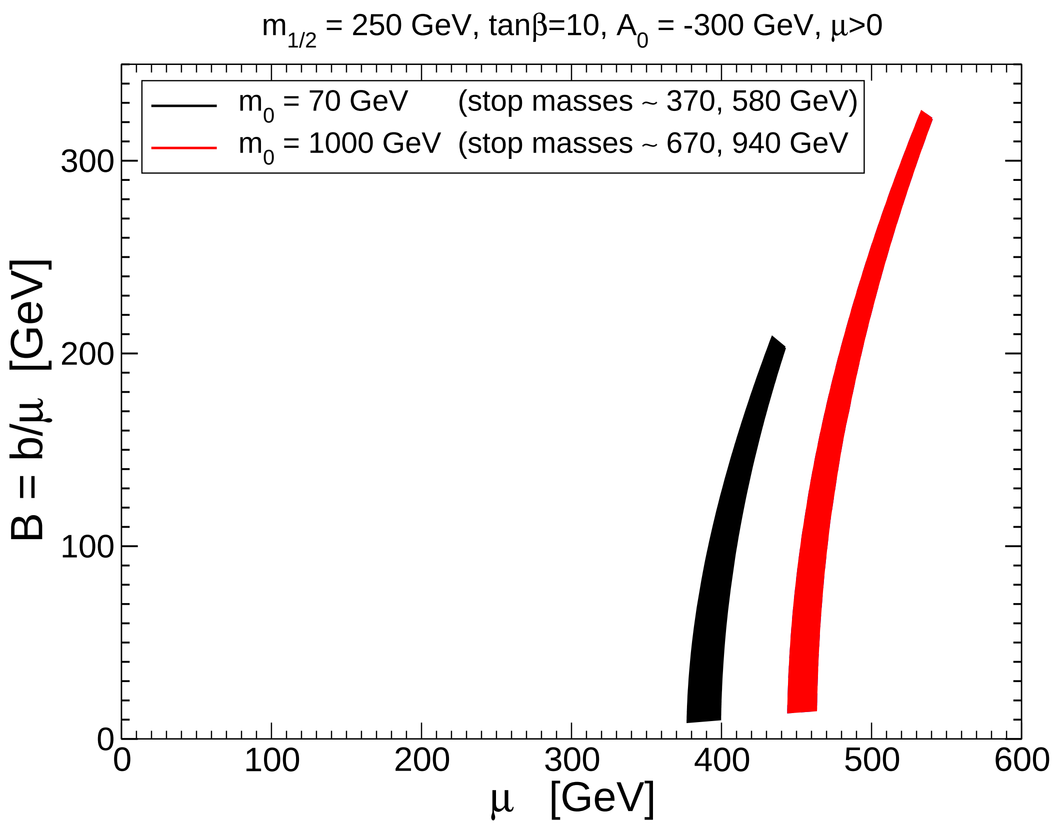


(This turns out not to depend very strongly on m_{top}. Results just shift slightly.)

My Fine Tuning Criterion:

Would I wager 1 year's salary, against $500, that a theory graduate student couldn't hit it with a dart from the front row of the lecture room, when projected on the screen?

Answer in this case: No, I would not. Therefore, not fine-tuned.



In common practice, the phase of μ is considered to be a free input parameter to models. However, in a complete model of SUSY breaking, this is not true; the phase or sign of μ is properly to be determined as an output, and one sign may be disallowed on purely theoretical gounds. **The most important uncertainty to be reduced is the top Yukawa coupling.**

The phase of μ appears non-trivially in the neutralino, chargino, sfermion sectors:

$$\text{Neutralino mass matrix} = \begin{pmatrix} M_1 & 0 & -* & * \\ 0 & M_2 & * & -* \\ -* & * & 0 & -\mu \\ * & -* & -\mu & 0 \end{pmatrix}$$

$$\text{Chargino mass matrix} = \begin{pmatrix} M_2 & * \\ * & \mu \end{pmatrix}$$

Each $*$ is a real, positive quantity $< m_Z$, and M_1 and M_2 can be taken real, positive as a prediction of the model. This fixes the μ sign convention (same as Haber+Kane).

There are hints from present data that μ might be **positive**:

- $(g-2)_{\rm muon}$ experiment result favors $\mu > 0$ in simplest models.
- $B \rightarrow S\gamma$ easiest to accomodate if $\mu > 0$.

However, the following arguments do not use these hints.

When we enforce electroweak symmetry breaking, by convention, b is taken real, positive. We can also always choose the gluino mass M_3 to be real and positive, by convention. With these conventions:

$$\text{Sign}(\mu) = \text{Sign}(b/\mu M_3).$$

Models that predict b therefore, in principle, predict the sign of μ, which is then not a free parameter.

Most programs just ignore b, and pick it at the electroweak scale to get the right VEV. Instead, let's pick boundary conditions for it to predict $\text{Sign}(\mu)$.

A large class of models, motivated by the SUSY flavor and CP problems:

Gaugino Mass Dominance

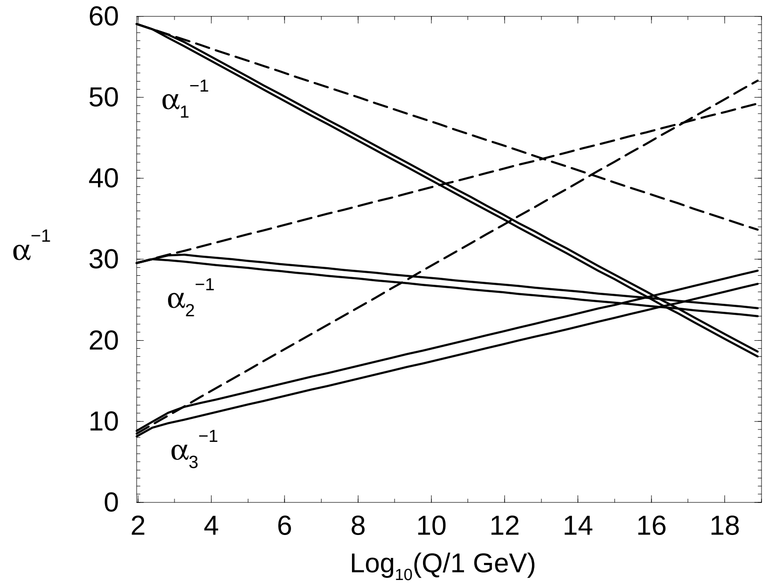
At some very high scale Q_0 (= $M_{\rm GUT}$? = $M_{\rm Planck}$?), all SUSY breaking is dominated by a common gaugino mass.

$m_{1/2} \neq 0$	Common gaugino mass
$a_t = a_b = a_\tau \approx 0$	(scalar)^3 couplings
$m^2_\phi \approx 0$	scalar masses
$b \approx 0$	Higgs mass^2

Unknown physics at scale above $M_{\rm GUT}$ will change these boundary conditions. For simplicity, ignore that for now, put in later.

To predict $\mathrm{Sign}(\mu)$, we run $\mathrm{Sign}(b/\mu M_3)$ from high scales down to the electroweak scale...

A reason to be optimistic that this is meaningful is the apparent unification of gauge couplings at $M_{\rm GUT} = 2 \times 10^{16}$ GeV with SUSY.



In my opinion, this is not to be construed as compelling evidence for SUSY; accidents happen.

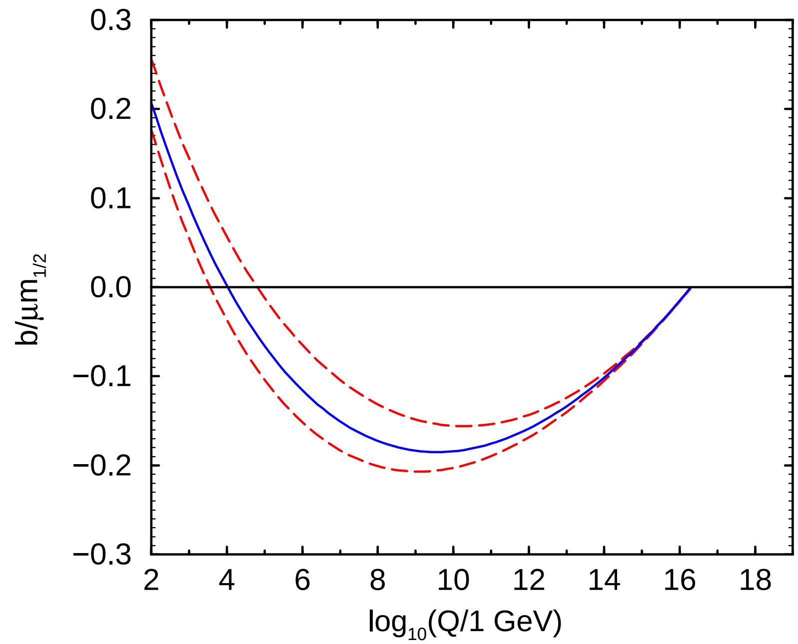
However, if we accept SUSY as a given, then it suggests that we can extrapolate model parameters from M_W to $M_{\rm GUT}$.

What happens above $M_{\rm GUT}$ (trans-GUT physics) is more difficult to guess at.

Renormalization group running

Central values and ranges for important quantities:

$$
\begin{aligned}
m_{\text{top}} &= 175 \pm 8 \text{ GeV} \\
\alpha_S^{\overline{\text{MS}}}(m_Z) &= 0.118 \pm 0.003 \\
m_b^{\overline{\text{MS}}}(m_Z) &= 2.88 + 16(0.118 - \alpha_S) \\
&\quad \pm 0.10 \text{ GeV}
\end{aligned}
$$



In this simple version, the sign of μ is **always** positive.

Why did this happen?

$$
\begin{aligned}
Q\frac{d}{dQ}(a_t/y_t) &= \frac{1}{16\pi^2}\left[\frac{32}{3}g_3^2 M_3 + 12 a_t y_t + \ldots\right] \\
Q\frac{d}{dQ}(b/\mu) &= \frac{1}{16\pi^2}\left[6 g_2^2 M_2 + 6 a_t y_t + \ldots\right]
\end{aligned}
$$

At large renormalization scales Q, gaugino masses are dominant, force a_t/y_t and b/μ **negative**. But, continuing to smaller scales, $a_t y_t$ dominates, forces b/μ **positive** again.

In the Real World, things won't be so simple. We started with exact gaugino mass dominance at the GUT scale, but unknown physics between M_{GUT} and M_{Planck} will modify this. Assume this new physics is flavor-blind (otherwise we have a SUSY flavor problem).

To take the trans-GUT-scale physics into account, allow for modified boundary conditions at M_{GUT}. The running of b/μ depends most strongly on:

$$
\begin{aligned}
B_0/m_{1/2} &\equiv b/\mu m_{1/2} \\
A_0/m_{1/2} &\equiv a_t/y_t m_{1/2} \; (\approx a_b/y_b m_{1/2} \approx a_\tau/y_\tau m_{1/2})
\end{aligned}
$$

Allow:

- All gaugino masses $m_{1/2} < 400$ GeV
- All scalar masses $< m_{1/2}$ (very weak dependence on this)
- Solve for $\tan\beta$; allow if couplings perturbative
- Require superpartner masses > 100 GeV
- $m_{\mathrm{top}}, m_{\mathrm{bottom}}, \alpha_S$ scanned over

Parameterization of ignorance of trans-GUT physics

Running of a_f/y_f and b/mu depends on how MSSM is embedded into the gauge group above the GUT scale. Also depends on how strong the gauge couplings are above that scale.

Fortunately, at least when perturbation theory is valid, uncertainties nearly cancel out of the most important ratios for determining Sign(μ):

$$(A_{0t}/B_0, A_{0b}/B_0) = \begin{cases} (1.5, 1.5) & E_6\text{-like} \\ (1.75, 1.75) & SO(10)\text{-like} \\ (2.0, 1.75) & SU(5)\text{-like} \\ (2.56, 2.44) & \text{MSSM-like} \\ (2.88, 2.88) & SU(4)_{\rm PS}\text{-like} \end{cases}$$

The ratios $A_0/m_{1/2}$ and $B_0/m_{1/2}$ are harder to predict with confidence, and so are kept as free parameters.

Scan over A_0, B_0, $m_{1/2}$, m_0, at $M_{\rm GUT}$, and $m_{\rm top}$, $m_{\rm bottom}$, α_S from present data with uncertainties.

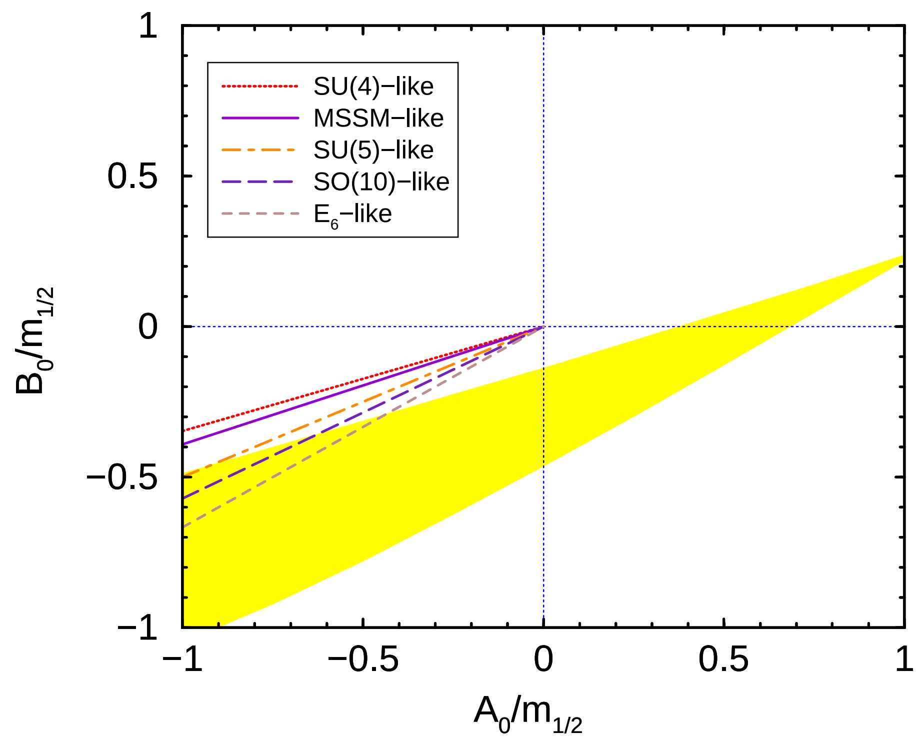
Upper unshaded region requires $\mu > 0$,

Lower unshaded region requires $\mu < 0$,

Yellow shaded region allows both signs for μ:

The origin $(A_0, B_0) = (0, 0)$ is the case of strict gaugino mass dominance at $M_{\rm GUT}$.

The reason for the existence of the yellow "overlap" region is mostly due to the present uncertainty in the top mass.



What if we knew the top Yukawa coupling with perfect accuracy?

Set $m_{\rm top}$ equal to its central value, repeat the exercise:

Upper unshaded region requires $\mu > 0$,

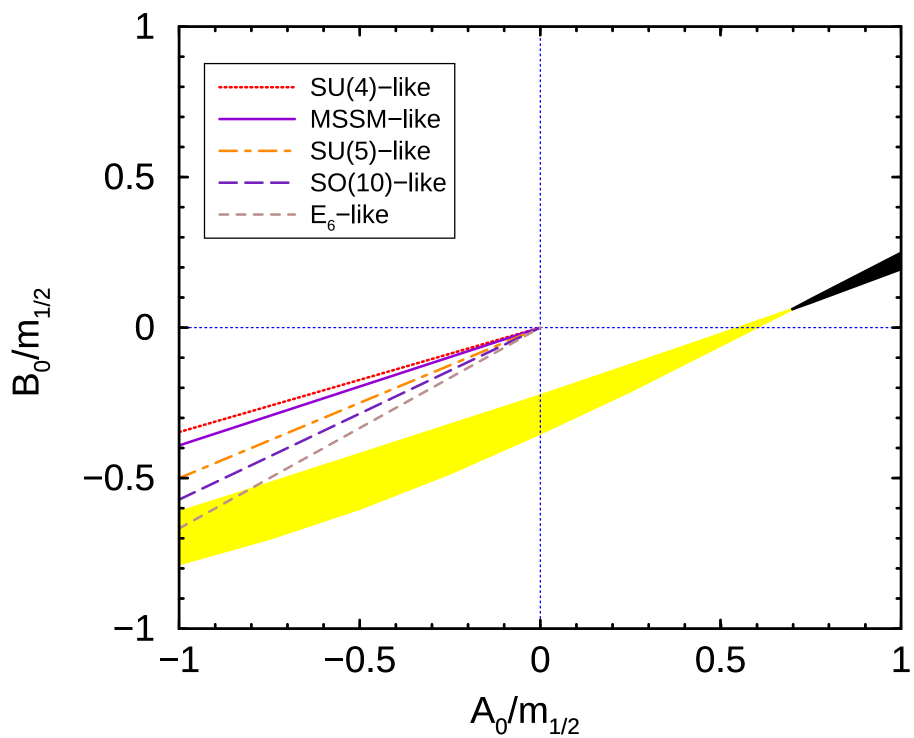
Lower unshaded region requires $\mu < 0$,

Yellow shaded region allows both signs for μ,

Black region has no solution:

The region in which we cannot predict the sign of μ in this class of models shrinks substantially.

Unfortunately, the shaded region is not reducible further...



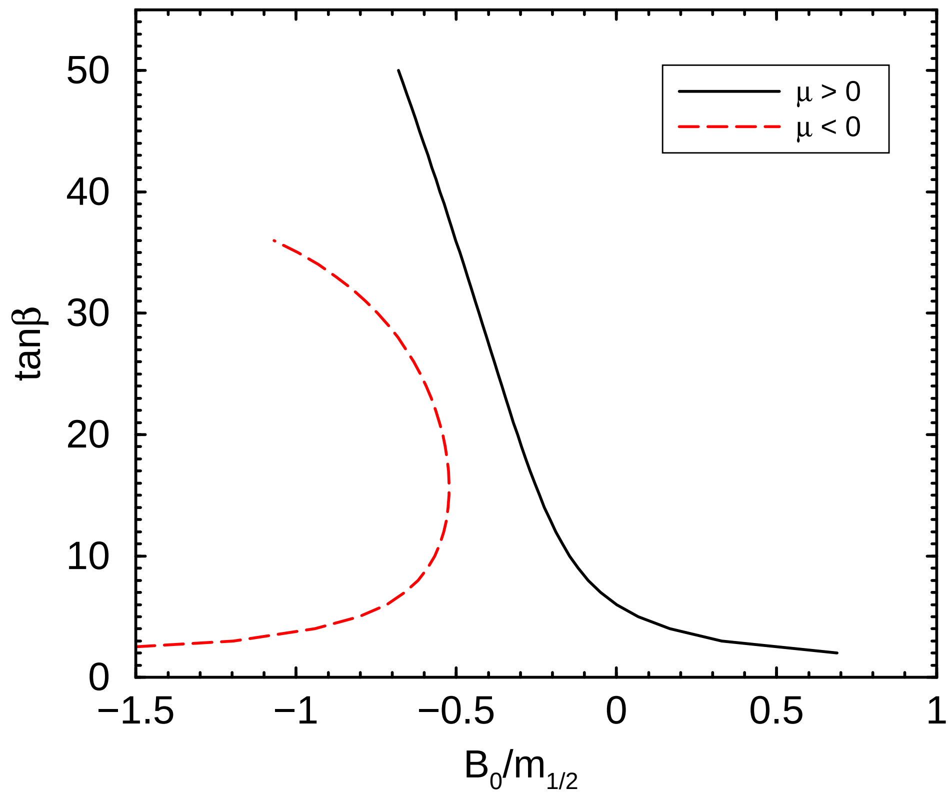
The overlap in the regions allowing both $\mu > 0$ and $\mu < 0$ is sometimes an irreducible ambiguity, even with perfect knowledge of the top Yukawa coupling.

Example model:

$m_{1/2} = 400$ GeV,

$A_0/m_{1/2} = -0.75$,

$m_0^2/m_{1/2}^2 = 0.5$



For $-0.7 \lesssim B_0/m_{1/2} \lesssim -0.5$, have three solutions for $\tan\beta$. One has positive μ, two have negative μ.

But this is the exception rather than the rule.

Outlook

- The top quark mass is the key to quantitative understanding of electroweak symmetry breaking, and its connection to SUSY breaking, in supersymmetry
- SUSY is more comfortable if the experimental top quark mass moves up, rather than down!
- Every bit of accuracy on the top mass, width, and couplings will be useful
- Theoretical calculations will be required to two and three loops