Obstacles to Running Up

(in the MSSM and Beyond)

and Some Solutions

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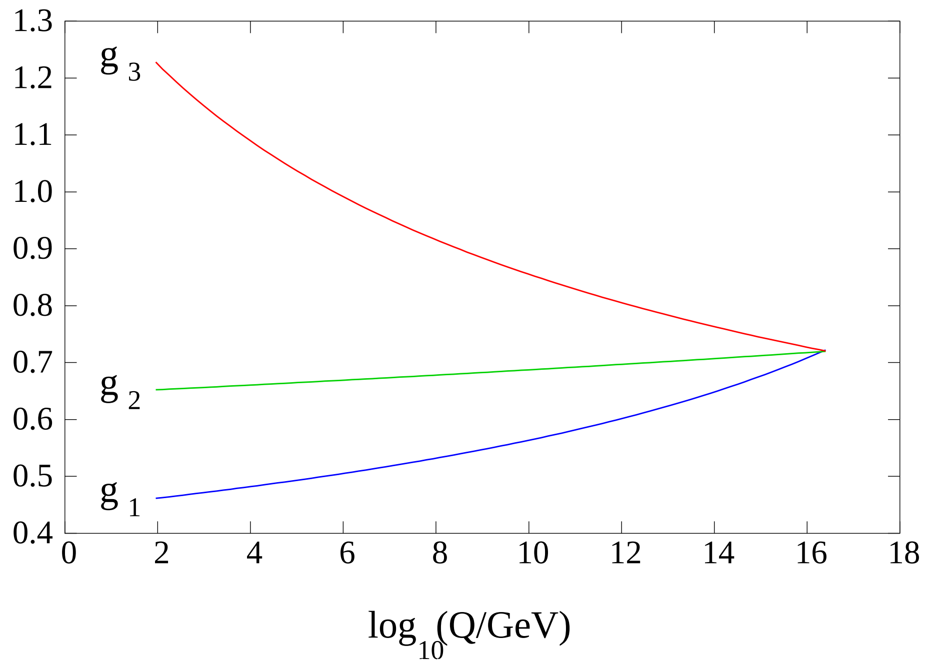
Motivation = Unification

Unification!

- The gauge couplings of the MSSM unify at high energy:

$$M_{GUT} \simeq 2 \times 10^{16} \text{ GeV}, \qquad g_{GUT} \simeq 0.7.$$



- M_{GUT} is suggestively close to M_{Pl}.

Unification!!

- Each matter generation of the MSSM has the quantum numbers of a complete $SU(5)$ multiplet.

$$\begin{aligned}\bar{\mathbf{5}} &= (D^c, L) = \left(\bar{\mathbf{3}}, \mathbf{1}, \frac{\mathbf{1}}{\mathbf{3}}\right) \oplus \left(\mathbf{1}, \mathbf{2}, -\frac{\mathbf{1}}{\mathbf{2}}\right) \\ \mathbf{10} &= (Q, U^c, E^c) = \left(\mathbf{3}, \mathbf{2}, \frac{\mathbf{1}}{\mathbf{6}}\right) \oplus \left(\bar{\mathbf{3}}, \mathbf{1}, -\frac{\mathbf{2}}{\mathbf{3}}\right) \oplus \left(\mathbf{1}, \mathbf{1}, \mathbf{1}\right)\end{aligned}$$

- Adding a heavy singlet neutrino field to each generation, these combine into a single $\mathbf{16}$ of $SO(10)$.

$$\mathbf{16} = \bar{\mathbf{5}} \oplus \mathbf{10} \oplus \mathbf{1}.$$

- Supersymmetry also provides a natural explanation for the large disparity between M_{GUT} and M_{ew}.

A Grand Desert?

- Adding new physics to the MSSM below M_{GUT} typically destroys the successful unification relations.

- The hints of unification suggest a grand desert, with no new physics between M_{ew} and M_{GUT}.

$\Rightarrow$ It may be possible to extract information about physics near M_{Pl} by making measurements near M_{ew}.

Obstacles to Running Up

- Renormalization group (RG) running is needed to extrapolate the parameter values at M_{ew} to their values at M_{GUT}.

- Obstacles:

  1. Extracting SUSY parameters from data is challenging.

     events $\rightarrow$ masses/couplings $\rightarrow$ Lagrangian parameters

  2. Experimental and theoretical uncertainties in parameter values at M_{ew} can become magnified by the RG flow.

     Some combinations of parameters are better than others.

  3. New intermediate scale physics can modify the predictions one would get assuming a grand desert.

     Unification strongly constrains the possibilities.

Outline

- We consider two particular obstacles to RG running.

  1. Sensitivity to input uncertainties within the MSSM.

     → MSSM running and the S term.

  2. New intermediate scale physics.

     → complete GUT multiplets at an intermediate scale.

- For both cases, we examine the effects on the low-to-high RG running of various Snowmass (SPS) mSUGRA points.

Assumptions

1. We work to one loop in the RG equations (for now).
2. Only third generation Yukawas are taken into account.
3. Flavour universality:

$$m_{ij}^2 = \begin{pmatrix} m_{12}^2 & 0 & 0 \\ 0 & m_{12}^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}, \quad etc. \ \ldots$$

Strategy

1. Specify g_i, y_i, $\tan\beta$ at M_{ew} and run up to M_{GUT} within the MSSM.
2. Input (mSUGRA) values of soft parameters at M_{GUT}.
3. Run parameters down to M_{ew} in the MSSM.
4. Impose electroweak relations to fix $|\mu|$ and $|B\mu|$.
5. Run all parameters back to M_{GUT}, adding input errors or new physics.

MSSM Running and the S Term

The S Term

- The one loop MSSM RG evolution of the soft masses is given by [*e.g.* Martin+Vaughn '94]

$$(16\pi^2)\frac{dm_i^2}{dt} = \tilde{X}_i - \sum_{a=1}^{3} 8\, g_a^2\, C_i^a\, |M_a|^2 + \frac{6}{5}\, g_1^2\, Y_i\, S,$$

where

$$S = Tr(Ym^2) = m_{H_u}^2 - m_{H_d}^2 + tr(m_Q^2 - 2m_U^2 + m_E^2 + m_D^2 - m_L^2).$$

- $S = 0$ in mSUGRA and many gauge-mediated models.

- S evolves homogeneously. At one loop,

$$(16\pi^2)\,\frac{dS}{dt} = \frac{66}{5}\, g_1^2\, S.$$

- If S vanishes at one scale, it vanishes at all scales.
  If S is non-zero at M_{ew}, it runs large in the UV.

- If S grows very large, it can dominate the running.

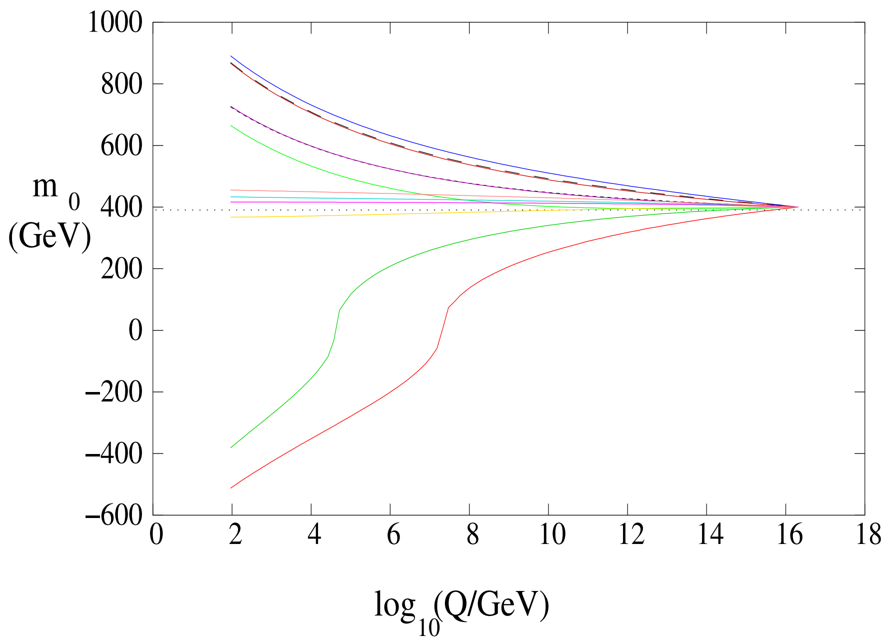
  e.g. SPS-4

  $m_0 = 400$ GeV, $m_{1/2} = 300$ GeV, $A_0 = 0$, $\tan\beta = 50$.

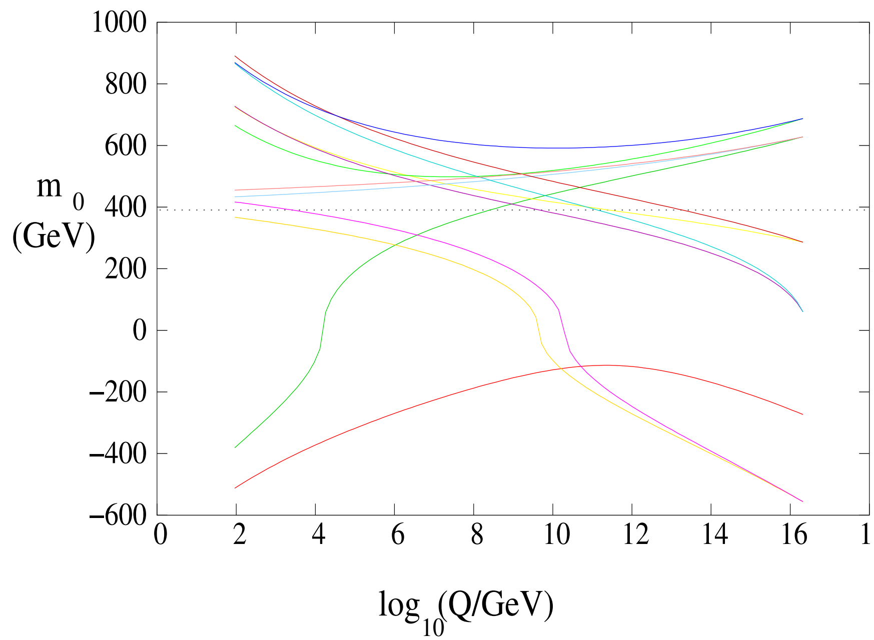
  $\Rightarrow m^2_{U_2} \simeq (800 \text{ GeV})^2$ at M_{ew}.

  Suppose $m^2_{U_2}(M_{ew})$ is not determined.

RG with $m^2_{U_2}(M_{ew}) = (800 \text{ GeV})^2$



RG with $m^2_{U_2}(M_{ew}) = (1600 \text{ GeV})^2$



S and a Hypercharge FI Term

- The S term is related to a hypercharge Fayet Iliopoulos (FI) term.
- Consider the MSSM augmented by such a FI term, ξ:

$$\begin{aligned} \mathscr{L} &= \frac{1}{2}D_1^2 + \xi\, D_1 + D_1\sqrt{\frac{3}{5}}g_1 \sum_i \bar{\phi}_i Y_i \phi^i - \sum_i \tilde{m}_i^2 |\phi^i|^2 + \ldots \\ &\longrightarrow -\frac{1}{2}\xi^2 - \frac{3}{5}\frac{g_1^2}{2}\left(\sum_i Y_i |\phi^i|^2\right)^2 - \sum_i \left(\tilde{m}_i^2 + \sqrt{\frac{3}{5}}g_1 Y_i\, \xi\right)|\phi^i|^2 + \ldots \end{aligned}$$

- The net effect is to shift the soft masses:

$$m_i^2 = \tilde{m}_i^2 + g_Y Y_i\, \xi.$$

- Only the shifted masses m_i^2 are observable, not $\tilde{m_i}^2$ or ξ individually.

Running ξ

- There are two convenient ways to do the RG running:

  1. Run the shifted masses m_i^2 alone. (D_1 eliminated.)
     - The running of m_i^2 is the same as before.
  2. Run $\tilde{m}_i^2$ and ξ separately. (D_1 uneliminated.)
     - The RG running of $\tilde{m}_i^2$ is the same as m_i^2, but without the S term.
     - ξ evolves at one-loop according to [Jack,Jones, Parsons '00]

$$\frac{d\xi}{dt} = \frac{\xi}{g_1}\frac{dg_1}{dt} + \frac{2\,g_1}{16\pi^2}\sqrt{\frac{3}{5}}\,Tr(Y\tilde{m}^2).$$

     - This is inhomogeneous - $Tr(Y\tilde{m}^2) \neq 0$ generates a ξ.

- The S term in the running of m_i^2 corresponds to the running of the FI term in the uneliminated formalism.

- ξ doesn't affect the other soft parameters until three loop order.

Uncertainties due to S

- The running of the soft masses can be very sensitive to the value of S.
- Since S depends on all the soft masses, there can be a large uncertainty in its value.

  e.g. 1. One of the soft masses is undetermined.

  e.g. 2. Some of the soft masses have large uncertainties.
- In terms of $\tilde{m}_i^2$ and ξ, there is a theory ambiguity.
  For each set $\{m_i^2\}$, there is an equivalence class of possible $\{\tilde{m}_i^2, \xi\}$.
- An invariant combination in both cases is

$$Y_i m_j^2 - Y_j m_i^2.$$

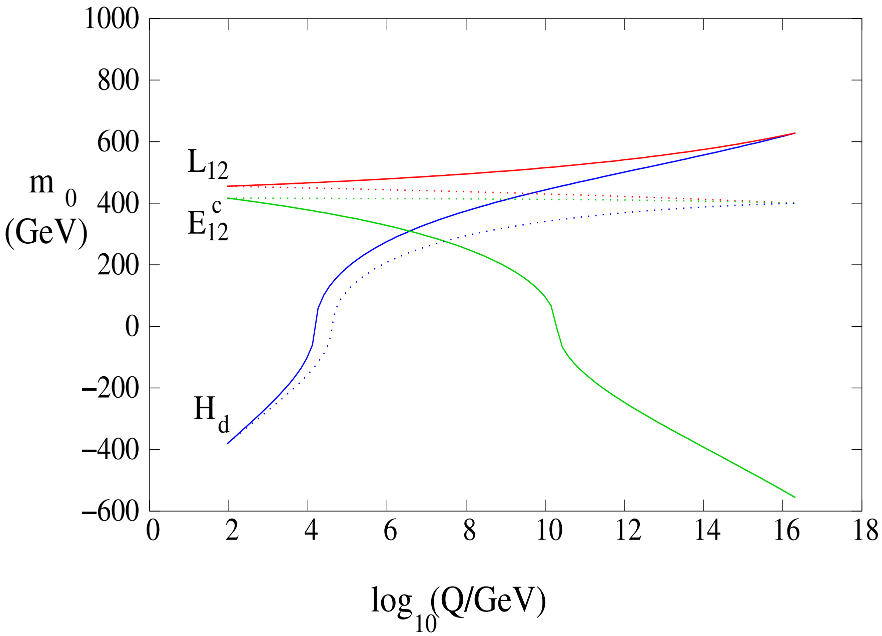
  for any $i \neq j$.
- Even without running, $S \neq 0$ provides interesting information.
  (mSUGRA+ξ? [de Gouvêa, Murayama, Friedland '98])

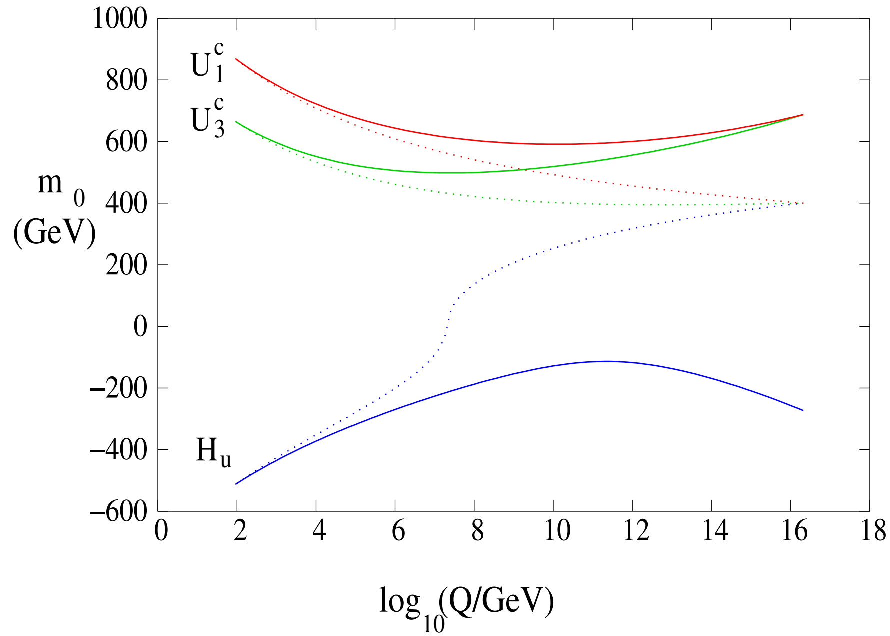
e.g. 1. SPS-4 with an undetermined soft mass.

Suppose $m^2_{U_2}$ is very poorly determined.

$m^2_{U_2} \simeq$ (800 GeV)^2 in mSUGRA at M_{ew}.

Running with $m^2_{U_2} =$ (1600 GeV)^2 instead,





The combination $Y_i\, m^2_j - Y_j\, m^2_i$ is unchanged.

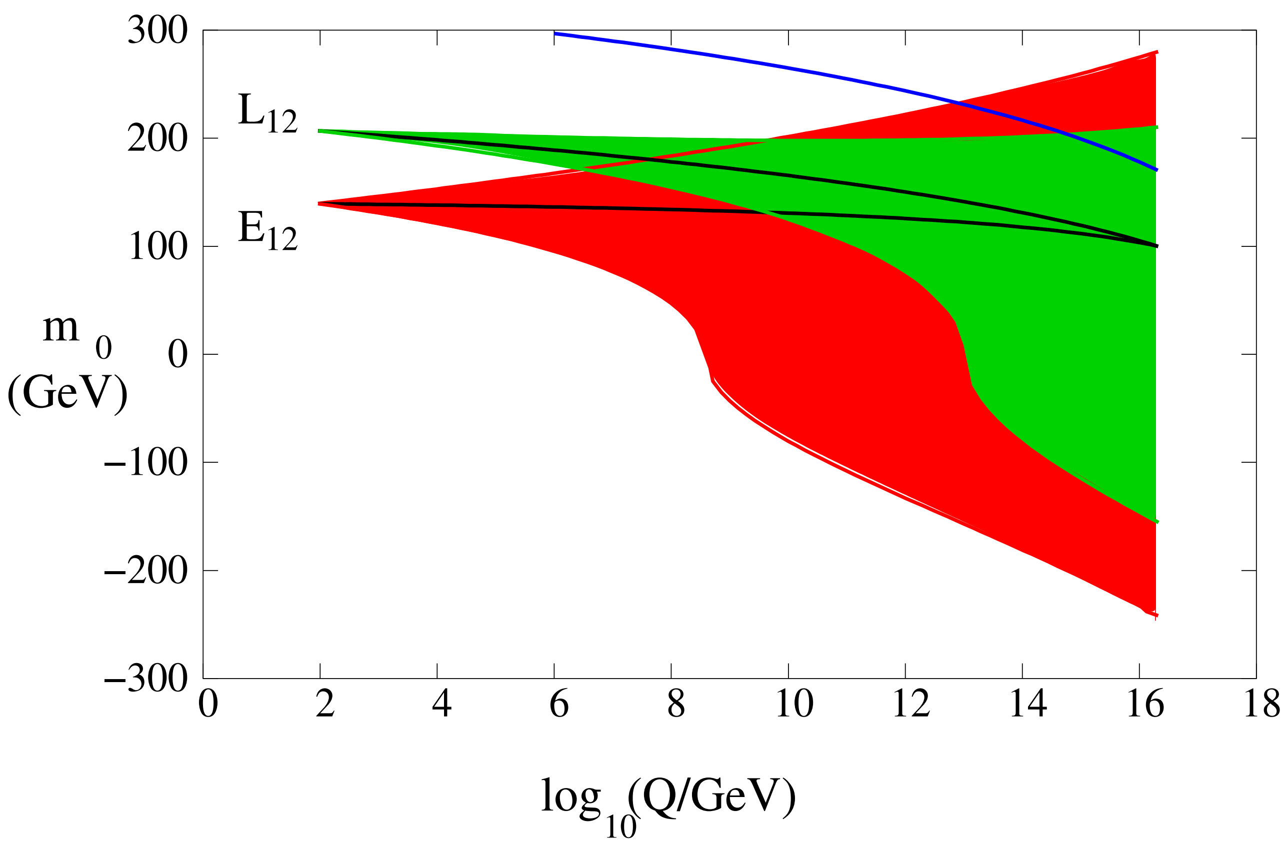
e.g. 2. SPS-1a with parameter uncertainties.

$m_0 = 100$ GeV, $m_{1/2} = 250$ GeV, $A_0 = -100$ GeV, $\tan\beta = 10$.

Assume 20% error in $\sqrt{m^2_{Q_3}}$, $\sqrt{m^2_{U_3}}$, $\sqrt{m^2_{D_3}}$, at M_{ew}.

The S term induces a large uncertainty in the running of the slepton masses.

This does not affect $m^2_{L_{12}} + \frac{1}{2} m^2_{E_{12}}$.



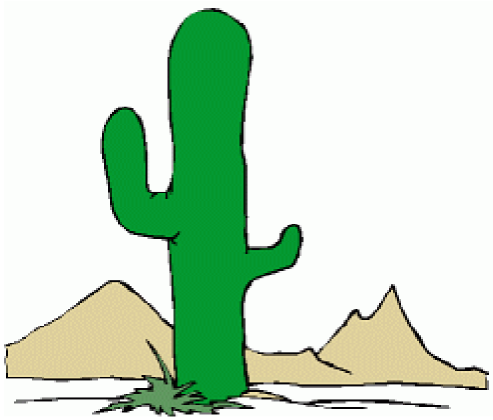
The Upshot

- Scalar soft masses can be sensitive to the value of $S(M_{ew})$.
- Since S depends on all MSSM soft masses, it is hard to pin down.
- This is particularly relevant to the slepton soft masses:
  - They can perhaps be deduced from LHC data.
  - Their running is not sensitive to uncertainties in m_t, α_s.
  - They are sensitive to $S \neq 0$ since $|Y| = 1/2,\ 1$.
- Uncertainties due to S cancel out in $Y_i m_j^2 - Y_j m_i^2$.

New Intermediate Scale Physics

Life in the Desert

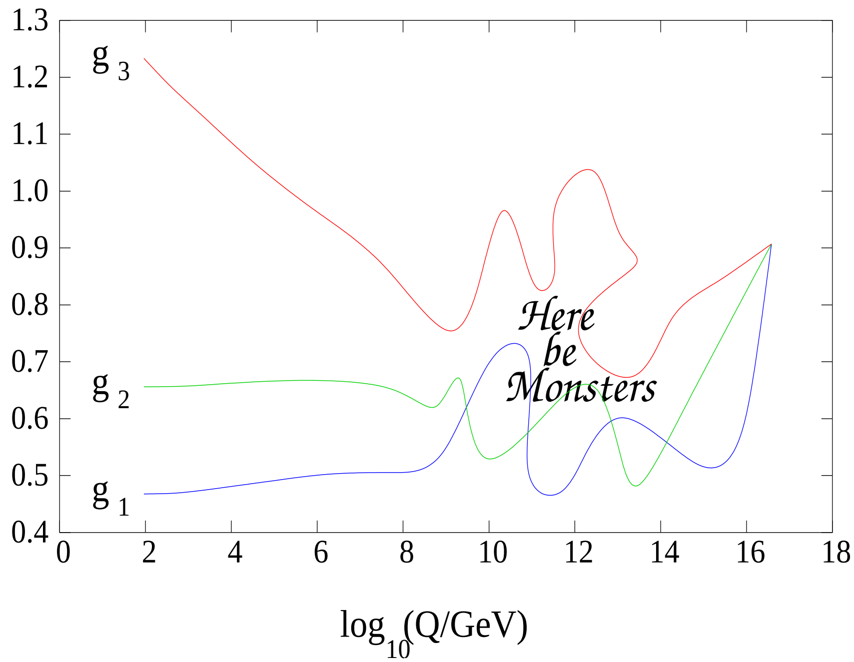
- A grand desert is not the only possibility consistent with unification.



- If the new physics consists of gauge singlets or complete GUT multiplets, unification will be about as good as in the MSSM

- Examples:
  - Gauge singlets for a μ term, or to induce small neutrino masses.
  - Gauge-mediated models often contain several GUT multiplets.
  - Extended gauge structures associated with the GUT group.

- New intermediate scale physics can modify the high scale predictions one would get assuming a grand desert.
- Arbitrarily complicated new physics can ruin the naive predictions arbitrarily badly.



- In many cases, certain combinations of parameters are not affected by the new physics.
- In other cases, the new physics can be inferred from low-scale measurements.

  (*i.e.* heavy singlet neutrinos and lepton flavour violation.)

Complete GUT Multiplets

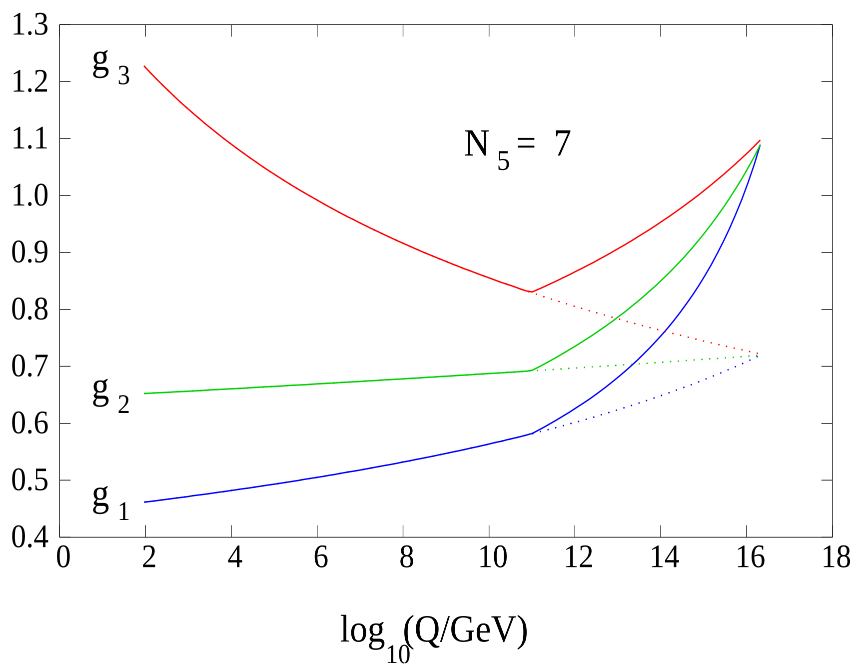
- Consider the MSSM augmented by N_5 sets of $\mathbf{5} \oplus \bar{\mathbf{5}}$ multiplets,

$$W \supset \tilde{\mu}\, 5 \cdot \bar{5},$$

  with $\tilde{\mu} \simeq 10^{11}$ GeV.

- We assume that all other superpotential couplings are small.

- At one loop, this preserves unification and its scale M_{GUT}, but increases the value of $g(M_{GUT})$,



e.g. 1. SPS-5 with $N_5 = 7$ extra $\mathbf{5} \oplus \bar{\mathbf{5}}$'s, $\tilde{\mu} = 10^{11}$ GeV.

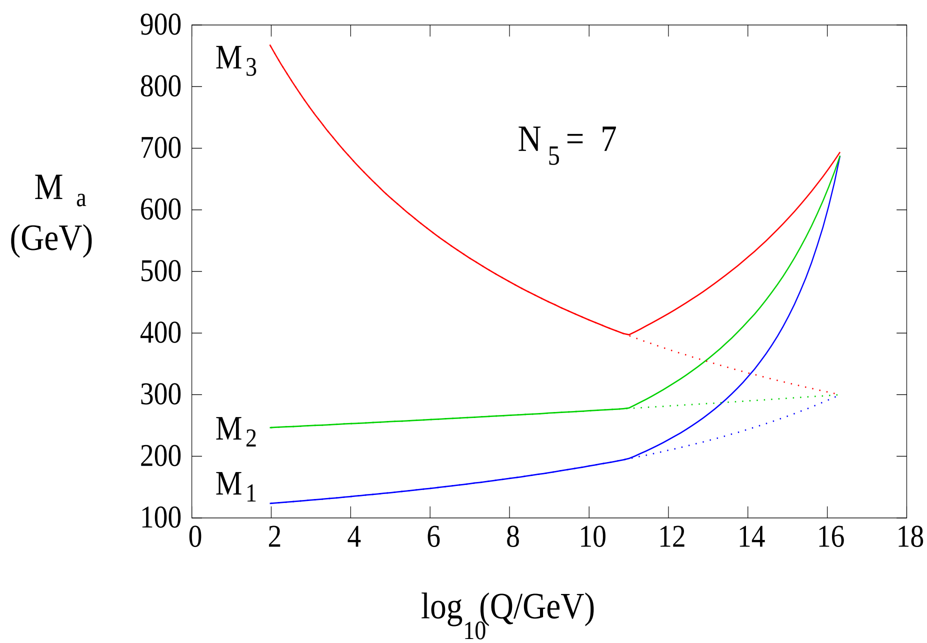
$m_0 = 150$ GeV, $m_{1/2} = 300$ GeV, $A_0 = -1000$ GeV, $\tan\beta = 5$.

Gaugino Masses

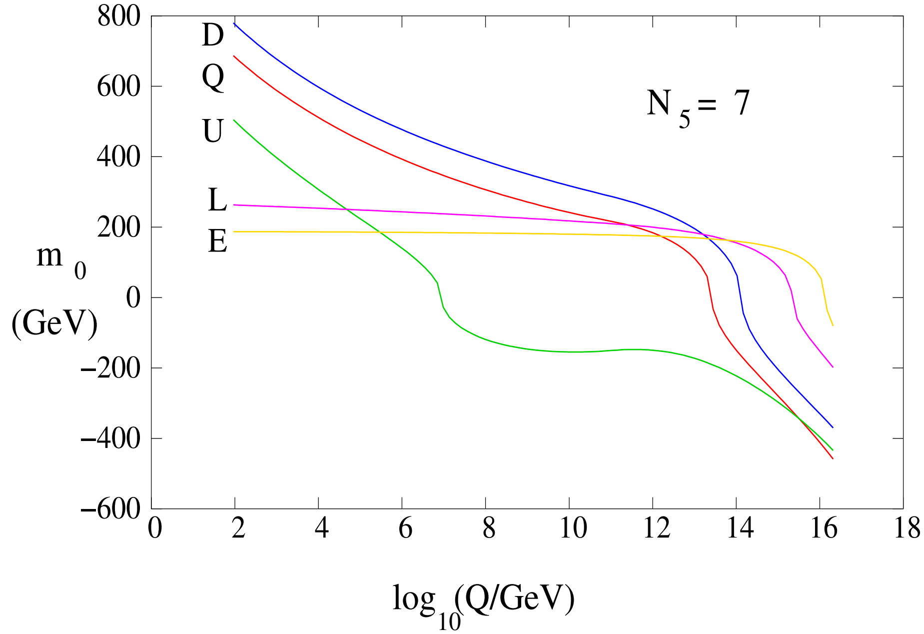
M_a/g_a^2 is still scale-independent at one loop.

The shift in the GUT scale value is

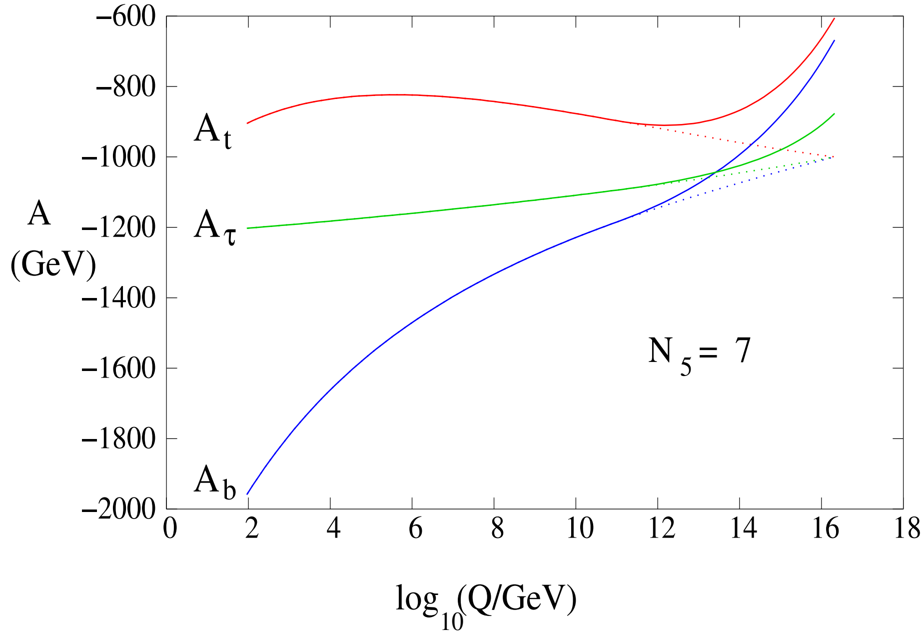
$$M_a(M_{GUT}) = M_a^0(M_{GUT}) \cdot \left[1 - \frac{N_5\, \alpha_G^0}{2\pi} \ln\left(\frac{M_{GUT}}{\tilde{\mu}}\right) \right]^{-1}$$



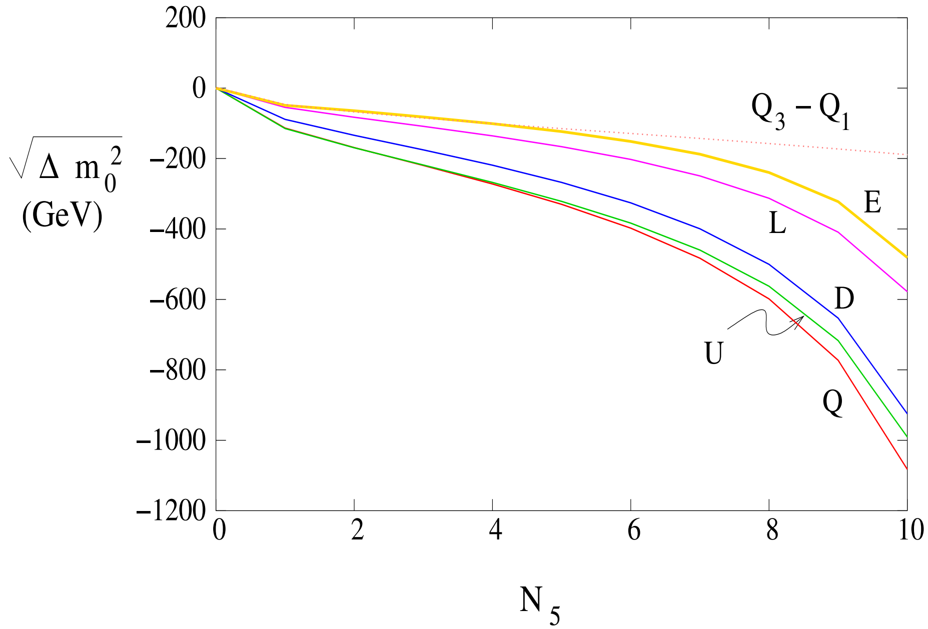
Soft Scalar Masses



Trilinear A Terms



- The naive extrapolation breaks down because of the new multiplets.
- The net shift, $\Delta m_0^2 = m_i^2(M_{GUT}) - m_0^2$, for SPS-5 is



- Useful invariants are more difficult to come by.
  $m^2_{\tilde{f}_3} - m^2_{\tilde{f}_1}$ is less sensitive to N_5, but not perfect.
- The variations could be even larger if there are sizeable Yukawa couplings between MSSM fields and the exotics.
- Even with the constraint of unification, the new physics can have a sizeable effect on the RG evolution.

Summary

- Extracting Lagrangian parameters from LHC data will be challenging. Running these parameters up will also require some thinking.

- Low-scale parameter uncertainties can get magnified by the RG running.
  $\longrightarrow$ slepton masses and the S term.

- New intermediate scale physics can change the predictions one would obtain assuming a grand desert.
  $\longrightarrow$ new physics that preserves unification can have a large effect.

- Even with these challenges, it will still be possible to test specific models against LHC data. (*e.g.* P.Kumar's talk)

- In this regard, it is important to look for combinations of parameters that are insensitive to uncertainties in the low scale values, and that won't destabilize the RG running.

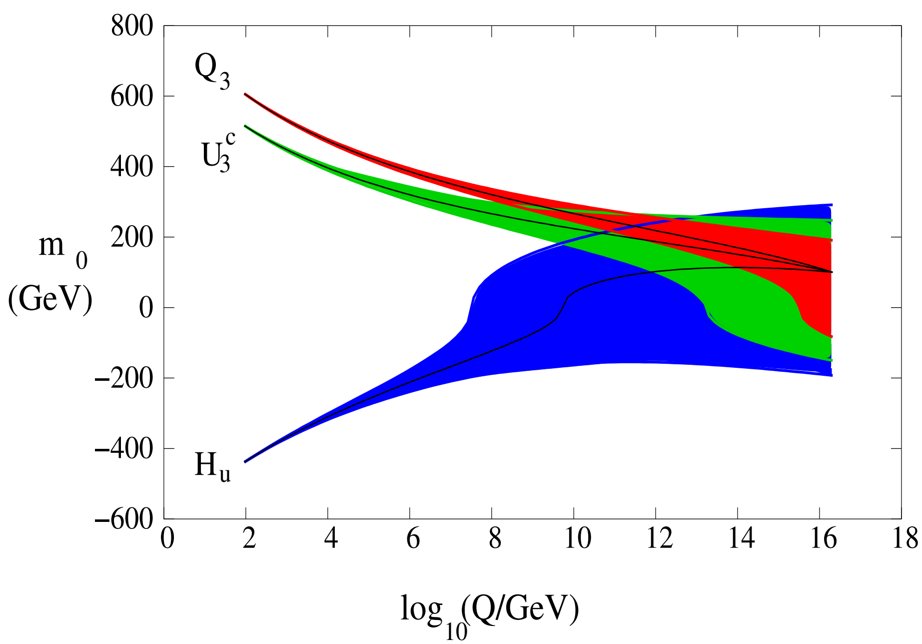
Extra Slides

SM Input Uncertainties

- In addition to SUSY parameter uncertainties, SM parameter uncertainties can be significant.
- $\alpha_s(M_Z)$ and $m_t(m_t)$ are particularly important, and they will still have substantial errors after the LHC.

e.g. For $m_t(m_t) = (175 \pm 1)$ GeV, $\alpha_s(M_Z) = (0.120 \pm 0.002)$,

we find for SPS-1a



- The largest effect comes from changes in the running of y_t.
- Fortunately, these errors are correlated.

  $\rightarrow m_{Q_3}^2$, $m_{U_3}^2$ shift together.
- The other soft terms are much less sensitive to α_s and y_t.
- Better determinations of α_s and m_t will still help a lot.

SPS Points

Point	m_0	$m_{1/2}$	A_0	$\tan\beta$	$sgn(\mu)$
1a	100	250	-100	10	$+$
1b	200	400	0	30	$+$
2	1450	300	0	10	$+$
3	90	400	0	10	$+$
4	400	300	0	50	$+$
5	150	300	-1000	5	$+$

B.C. Allanach *et.al.* [hep-ph/0202233]