Cosmological Singularities from Matrices

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Space-time from Matrices

- In string theory space and time are not fundamental, but derived concepts which emerge out of more fundamental structures.
- In a few cases we have some hint of what this structure could be – these are situations where the space-time physics has a holographic description – usually in terms of a field theory of matrices.
- These are in fact descriptions of closed string dynamics in terms of open strings

Examples

Closed String Theory	Open String Theory
2 dimensional strings	Matrix Quantum Mechanics
M theory/ critical string in light cone gauge	SUSY Matrix Quantum Mechanics/ 1+1 YM
Strings in $AdS_5 \times S^5$	3+1 dimensional N=4 Yang-Mills

We will describe some recent attempts to construct toy models of cosmological singularities in each of these

2d Closed String from Double scaled Matrix Quantum Mechanics

- $M_{ij}(t)$ - $N \times N$ hermitian matrix. This is the degree of freedom of open strings joining D0 branes

$$S = \int dt \frac{1}{2} Tr[(D_t M)^2 + M^2]$$

- Gauging – states are singlet under SU(N)
- Eigenvalues are fermions. Single particle hamiltonian $$H = \frac{1}{2}(p^2 - x^2)$$
- Density of fermions

$$\partial_x \phi(x,t) = \frac{1}{N} \mathrm{Tr} \delta(M(t) - x \cdot I)$$

To leading order in 1/N, the dynamics of the scalar field is given by the action

$$S = N^2 \int dx dt \, [\frac{1}{2} \frac{(\partial_t \phi)^2}{(\partial_x \phi)} - \frac{\pi^2}{6} (\partial_x \phi)^3 - (\mu - \frac{1}{2} x^2) \partial_x \phi]$$

- This collective field theory would be in fact the field theory of closed strings in two dimensions – the space dimension has emerged out of the matrix
- The fundamental quantum description is in terms of fermions
- Collective field theory used to find the emergent space-time as seen by closed strings – at the semiclassical level

Physics of the ground state

- The ground state is a filled fermi sea – for which the collective field is static

$$\partial_x \phi_0 = \frac{1}{\pi}\sqrt{x^2 - 2\mu}$$

- Fluctuations $\phi(x,t) = \phi_0(x) + \eta(x,t)$ are described at the semiclassical level by two scalar fields living in the two regions $|x| \geq \sqrt{2\mu}$ with $\eta(\pm\sqrt{2\mu}, t) = 0$

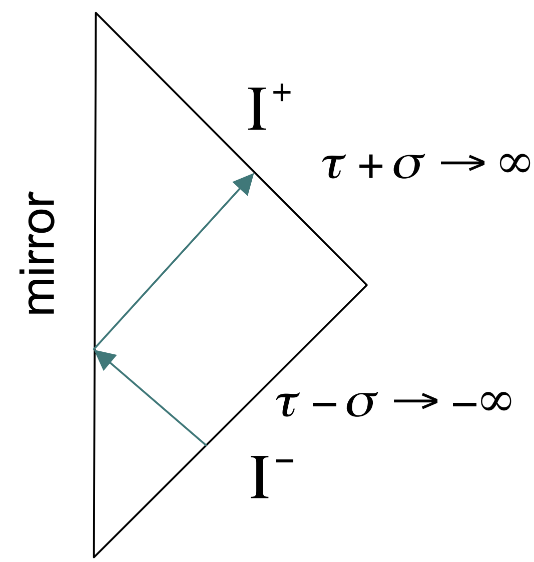
- In fact, at the semiclassical level these fluctuations may be thought to live in a relativistic space-time.
- In terms of coordinates

$$t = \tau \qquad x = \sqrt{2\mu}\cosh\sigma$$

$$H = \frac{1}{2}\int d\sigma\{[\Pi^2 + (\partial_\sigma\eta)^2] + \frac{1}{\mu\sinh^2\sigma}[\Pi^2\partial_\sigma\eta + \frac{\pi^2}{3}(\partial_\sigma\eta)^3]\}$$

These two massless scalars are related to the only two dynamical fields of 2d string theory by a transform which is non-local at the string scale.

Both these scalars live in the same space-time.



$I^\pm$ Weakly coupled

Space-like boundaries

S.R.D. and J. Karczmarek, PRD D71 (2005) 086006

- The infinite W_∞ symmetry of the theory may be used to find time-dependent classical solutions –

  (Karczmarek and Strominger; S.R.D., J. Davis, F. Larsen and P. Mukhopadhyay)
- Fluctuations around such solutions are once again massless scalars, but the global nature of the space-time can be rather non-trivial.
- One of these examples

$$\partial_x \phi_0 = \frac{1}{\pi(1+e^{2t})}\sqrt{x^2 - (1+e^{2t})}$$

$$\partial_t \phi_0 = -\frac{xe^{2t}}{1+e^{2t}}\,\partial_x \phi_0$$

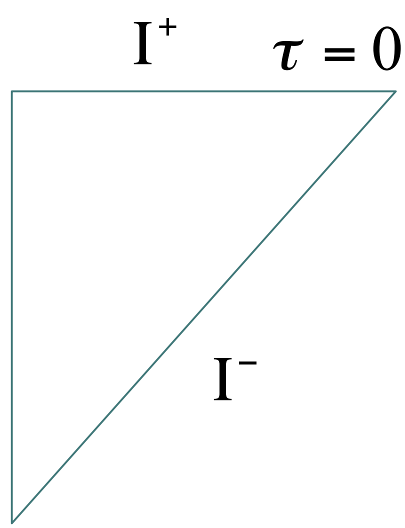
- The semiclassical space-time perceived by these fluctuations are again best described in terms of Minkowskian coordinates τ, σ

$$x = \cosh\sigma\sqrt{1+e^{2t}}$$ $$e^{\tau} = e^{t}/\sqrt{1+e^{2t}}$$

- As the fundamental time of the problem t runs over its full range, the time τ stops

$$-\infty \le t \le \infty \quad \Leftrightarrow \quad -\infty \le \tau \le 0$$

It appears that there is a space-like boundary



Note : $x \to \infty$ over the entire I^+

- This is geodesically incomplete.
- Normally one would simply extend the space-time to complete it.
- However in this case there is a fundamental definition of time provided by the matrix model – the time t - It does not make sense to extend the space-time beyond this boundary

- Several other examples of this type
- World-sheet formulation not settled. We have a proposal – space-like tachyon condensation

Details of tachyon condensation

- Use Macroscopic loops to guess the perturbation to the world-sheet action which represent such classical solutions

$$T(\phi,t) = 1 - \sqrt{2\mu\left(1+e^{2t}\right)^{-1}}e^{-\phi}K_1\left(\sqrt{2\mu\left(1+e^{2t}\right)}e^{-\phi}\right)$$

- At early times this is the usual Liouville wall

$$T(\phi \to +\infty, t \to -\infty) = \mu e^{-2\phi}\left(\phi + const\right)$$

- Generally this represents a space-like tachyon condensation

$$T(\phi >> 0, t < \phi) = \frac{e^{2t}}{1+e^{2t}} + \mu e^{-2\phi}\left(\phi + const - \ln\sqrt{1+e^{2t}}\right)$$

Beyond semiclassical approximation

(S.R.D. and Luiz dos Santos)

- What is really happening is that unlike the ground state, the future boundary is not a weakly coupled region. In fact the hamiltonian ∂_τ is again

$$H = \frac{1}{2}\int d\sigma\{[\Pi^2 + (\partial_\sigma\eta)^2] + \frac{1}{\sinh^2\sigma}[\Pi^2\partial_\sigma\eta + \frac{\pi^2}{3}(\partial_\sigma\eta)^3]\}$$

- Except at the very edge there is no true space-time interpretation in this region
- However the fermion theory is perfectly well defined

- The time-dependent background is in fact a non-normalizable state of the fermion theory

$$|\alpha> = e^{i\alpha W_{02}} |\mu>$$

- Various expectation values in this state may be calculated in terms of corresponding quantities in the ground state. For example the fermion density

$$<\rho>_\alpha = \frac{\mathrm{Re}}{\sqrt{1+\alpha e^{2t}}} \int_0^\infty \frac{ds\, e^{i\mu s}}{(-4\pi i \sinh s)^{1/2}} \exp[i \frac{x^2}{2(1+\alpha e^{2t})} \tanh^2 \frac{s}{2}]$$

- Expressions like this show that the exact answer differs significantly from the semiclassical expression over almost the entire I^+
- There is no S-Matrix. However the time evolution of the wave function seems to make sense

Lesson

- The open string time – in this case the time of the matrix model - can go over the full range
- The closed string time – the time which is perceived by fluctuations in a semiclassical interpretation - can be terminated
- At the end of this semiclassical time, there is no valid relativistic interpretation of the model – though the model itself seems to make sense

IIB Matrix Big Bangs

S.R.D., J. Michelson *Phys.Rev.D72(2005)086005,*
S.R.D, J. Michelson, *hep-th/0602099*

- Something similar happens in Matrix Big Bangs of *Craps, Sethi and Verlinde*
- We will discuss this for IIB pp-wave backgrounds with two compact directions - x^- and a space x^8

$$
\begin{aligned}
ds^2 &= 2dx^+dx^- - 4\mu^2[(x^1)^2 + \cdots (x^6)^2](dx^+)^2 \\
&\quad -8\mu x^7 dx^8 dx^+ + [(dx^1)^2 + \cdots (dx^8)^2] \\
F_{+1234} &= F_{+5678} = \mu\ e^{Qx^+} \\
\Phi &= -Qx^+
\end{aligned}
$$

$$
IIB : g_s, l_s
$$

$$
x^- \approx x^- + 2\pi\ R \qquad x^8 \approx x^8 + 2\pi\ R_B
$$

- The holographic theory is a 2+1 dimensional SU(J) Yang Mills theory on a torus (ρ,σ)

$$\begin{aligned}\mathcal{L} = \mathrm{Tr}\ \{\ & \frac{1}{2}[(D_\tau X^a)^2 - (D_\sigma X^a)^2 - e^{2Q\tau}(D_\rho X^a)^2] \\ & + \frac{1}{2(G_{YM}e^{Q\tau})^2}[F_{\sigma\tau}^2 + e^{2Q\tau}(F_{\rho\tau}^2 - F_{\rho\sigma}^2)] \\ & - \frac{\mu^2}{2}[(X^1)^2 + \cdots (X^6)^2 + 4(X^7)^2] + \frac{(G_{YM}e^{Q\tau})^2}{4}[X^a, X^b]^2 \\ & - \frac{4\mu}{(G_{YM}e^{Q\tau})} e^{Q\tau} X^7\ F_{\rho\sigma} - 4\mu i (G_{YM}e^{Q\tau}) X^7\ [X^5, X^6]\end{aligned}$$

$$\sigma \approx \sigma + 2\pi \frac{l_s^{\ 2}}{R} \qquad \rho \approx \rho + 2\pi \frac{l_s^{\ 2}}{R} g_s \qquad G_{YM}^{\ \ 2} = \frac{RR_B^{\ 2}}{g_s l_s^{\ 4}}$$

$$G_{YM} \to G_{YM} e^{Q\tau}$$

$$\partial_\rho \to \partial_\rho e^{Q\tau}$$ ← New feature In IIB

- At sufficiently early times

(i) only diagonal X's survive

(ii) The gauge field $F_{\mu\nu}$ gets dualized into a scalar field – so we have 8 scalars now

(iii) The effective size of the ρ direction is small – becomes a 1+1 dimensional theory

(iv) This 1+1 dimensional theory becomes the world-sheet theory of the original IIB string moving in this background in the $x^+ = \tau$ gauge

(v) The rank of the gauge group J becomes identified with the momentum in x^- direction

$$p_- = J / R$$

Details of Dualization

J. Michelson, (unpublished)

- In the regime where the fields become abelian, introduce an auxilliary field – add $\frac{1}{2}\epsilon^{\mu\nu\lambda}\partial_\mu\phi F_{\nu\lambda}$
- Integrate out the gauge field

$$\mathcal{L}' = -\frac{1}{2}[\sum_{a=1}^{7}(\partial_\mu X^a)^2 + G_{\rm YM}^2(\partial_\mu\phi)^2] - 2\mu^2[\sum^{6}(X^i)^2 + 4(X^7)^2] + 4G_{\rm YM}\mu\ X^7\partial_\tau\phi$$

- Perform a field redefinition

$$X^i = Y^i, \qquad i = 1,\cdots,6,$$

$$X^7 = Y^7\ \cos(2\mu\tau) + Y^8\ \sin(2\mu\tau),$$

$$G_{\rm YM}\phi = -Y^7\ \sin(2\mu\tau) + Y^8\ \cos(2\mu\tau)$$

- Final form

$$\mathcal{L}_{diag} = -\frac{1}{2}\sum_{I=1}^{8}(\partial_\mu Y^I)^2 - 2\mu^2\sum_{I=1}^{8}(Y^I)^2$$

- Generically such a space-time interpretation is not valid. This is specifically true near $\tau \to -\infty$ - here the coupling of the YM theory is weak and nonabelian configurations are important.
- From the point of view of the YM theory this is the far past –
- From the point of view of the space-time string theory *forcibly extrapolated* to $x^+ \to -\infty$ this appears as a "beginning of time"
- Once again the open string (YM) time runs over the full range – while the closed string time appears to begin.

However at this beginning the space-time interpretation is itself breaking down.

Matrix Membranes

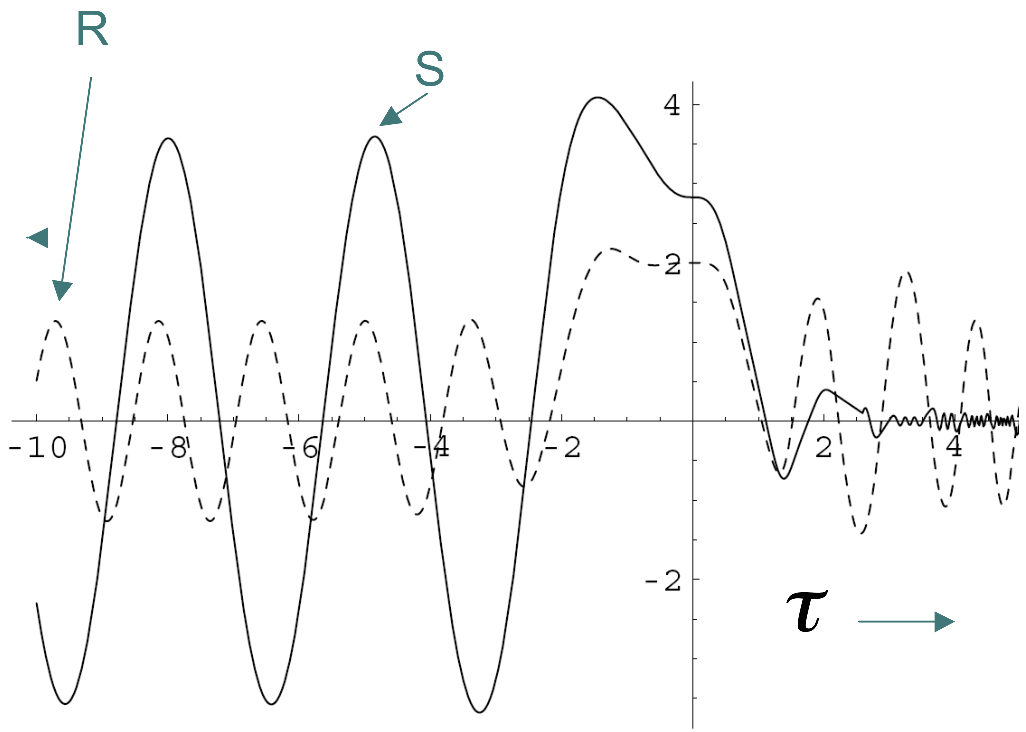
- The quantity G_{YM}^2/μ acts as a semiclassical parameter in the theory
  $\mu/G_{YM}^2 >> 1$: classical solutions representing **fuzzy ellipsoids**

$$X^5(\tau,\sigma) = S(\tau)J^1$$

$$X^6(\tau,\sigma) = S(\tau)J^2$$

$$X^7(\tau,\sigma) = R(\tau)J^3$$

$$[J^a, J^b] = i\varepsilon_c{}^{ab} J^c$$



Even though R oscillates, the size of the fuzzy ellipsoid always goes to zero at late times

Brane Production

- The effect of factors of $e^{Q\tau}$ in front of ∂_ρ may be thought of as a time-dependent size of the circle.
- States of the YM theory are labelled by (m,n)

  m = momentum along σ

  n = momentum along ρ

(m,0) : states of F-strings

(0,n) : states of D- strings

(m,n) = (p,q) strings

In the 1+1 theory in (τ,σ) ,states with $n \neq 0$

are KK modes with a time dependent mass

$$m_n{}^2 = 4\mu^2 + \left(\frac{nR}{g_s l_s{}^2}\right)^2 e^{2Q\tau}$$

- This implies particle [(p,q) string] production.
- The "out" vacuum at late times is a squeezed state of "in" particles

$$|0\rangle_{\rm out} = \prod_{n,m} \{(1-|\gamma_m|^2)^{1/4} \ \exp[\frac{1}{2}\gamma_m^\star a^{\dagger I,({\rm in})}_{m,n} \ a^{\dagger I,({\rm in})}_{-m,-n}]\} \ |0\rangle_{\rm in}$$

$$_{\rm out}\langle 0|a^{\dagger I,({\rm in})}_{m,n} a^{I,({\rm in})}_{m,n}|0\rangle_{\rm out} = \frac{1}{e^{\frac{2\pi\omega_m}{Q}} - 1}$$

$$\gamma_m = \frac{\beta_m^\star}{\alpha_m} = -ie^{-\frac{\pi\omega_m}{Q}} \qquad \omega_m^2 = 4\mu^2 + \frac{m^2R^2}{l_B^4}$$

- In other words, if we require the state at late times to contain only **fundamental** strings, the state near the big bang must be a squeezed state of (p,q) strings

- **Does this say anything about the issue of initial conditions ?**

Big Bangs and AdS/CFT

S.R.D, K. Narayan, J. Michelson and S. Trivedi, hep-th/0602107

- The IIB pp-wave has another dual – a large R-charge sector of a 3+1 dim YM theory – or rather some quiver version of the theory.
- Can we address the issue of singularities in this AdS/CFT language ?
- This seems to require construction of the supergravity background *before performing a Penrose limit* – we have not yet succeeded in doing that
- But this led us to find an infinite class of time dependent backgrounds which have natural CFT duals

- The supergravity solutions are

$$ds^2 = (\frac{r^2}{R^2})\tilde{g}_{\mu\nu}dx^\mu dx^\nu + (\frac{R^2}{r^2})dr^2 + R^2 d\Omega_5^2$$

$$F_{(5)} = R^4(\omega_5 + *_{10}\omega_5) \qquad \phi(x^\mu)$$

- This is a solution if $g_{\mu\nu}(x^\mu)$ and $\phi(x^\mu)$ obey

$$\tilde{R}_{\mu\nu} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi \qquad \partial_\mu(\sqrt{-\det(\tilde{g})}\ \tilde{g}^{\mu\nu}\partial_\nu\phi) = 0$$

- These are deformations of $AdS_5 \times S^5$ .In fact they are near-horizon limits of deformations of the full 3-brane geometry
- There are similar geometries which are deformations of $AdS_m \times S^n$

Examples

- It is easy to find lots of solutions of this form – e.g. Kasner-like geometries with space-like singularities.

- A particularly interesting set of solutions are those with potential null singularities

$$d\tilde{s}^2 = e^{f(X^+)}(-2dX^+dX^- + dx_2^2 + dx_3^2)$$

$$\phi = \phi(X^+)$$

In this case we must have

$$\frac{1}{2}(f')^2 - f'' = \frac{1}{2}(\partial_+\phi)^2$$

Solutions retain half of the supersymmetries

Pick a $f(X^+)$, find $\phi(X^+)$

Looks like Liouville + c=1 matter

This class of solutions also discussed in

Chu and Ho, hep-th/0602054

Details of supersymmetry

- The null solutions retain the following susy's

$$\Gamma^4 \epsilon = \epsilon, \qquad \gamma^+ \epsilon = 0, \qquad \epsilon = Z^{-1/8} e^{f/4} \eta$$

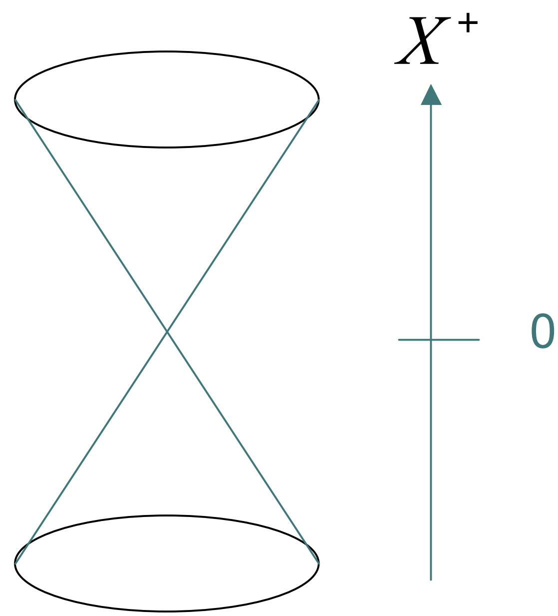
- where

$$\Gamma^4 = i\Gamma^{0123} \quad Z = Z(x^m) = \frac{R^4}{r^4}$$

- There are examples where the string coupling is always bounded

$$e^{f(X^+)} = \tanh^2 X^+$$

$$e^{\phi} = g_s \left(\tanh \frac{X^+}{2} \right)^{\sqrt{8}}$$



Even though curvature invariants vanish, there is a singularity at $X^+ = 0$. This is reached by Geodesics at finite proper time.

Here the string coupling vanishes

At $X^+ = \pm\infty$ the space-time is pure $AdS_5 \times S^5$

In such backgrounds there is a natural CFT dual

- Note that we have turned on a non-normalizable mode. This means we have sources in the gauge theory
- The natural dual is in fact the gauge theory which lives in the metric $\tilde{g}_{\mu\nu}$ and has a coupling $e^{\phi/2}$ - *may be seen e.g. from DBI action of a 3-brane in this background*
- The supersymmetries of the bulk translate to those in this candidate CFT.
- Correlation functions of suitably dressed operators are non-singular at $X^{+} = 0$.
- Interesting question : **How does the gauge theory encode the space-time singularity ?**