Generalized $N = 1$ compactifications and Mirror symmetry

Thomas W. Grimm

University of Wisconsin, Madison

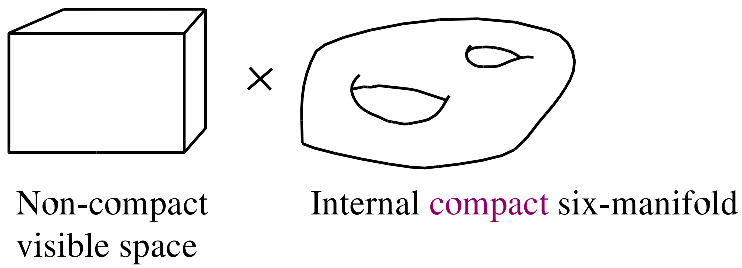
based on: [hep-th/0602241] Iman Benmachiche, TWG
[hep-th/0507153] TWG

Ann Arbor, Michigan, April 2006

Introduction and Motivation

➪ Phenomenology

String Theory $\longrightarrow$ Four-dimensional $N=1$ Supergravity



- Study compactifictions on generalized geometries recently introduced by Hitchin. The four-dimensional $N=1$ theories take a particularly elegant form and arise in very general reductions!
  $\Rightarrow$ explore generic features of $N=1$ compactifications – Landscape of String vacua?

- Address moduli problem: generate potential for massless scalar fields due to background fluxes and non-Calabi-Yau geometries

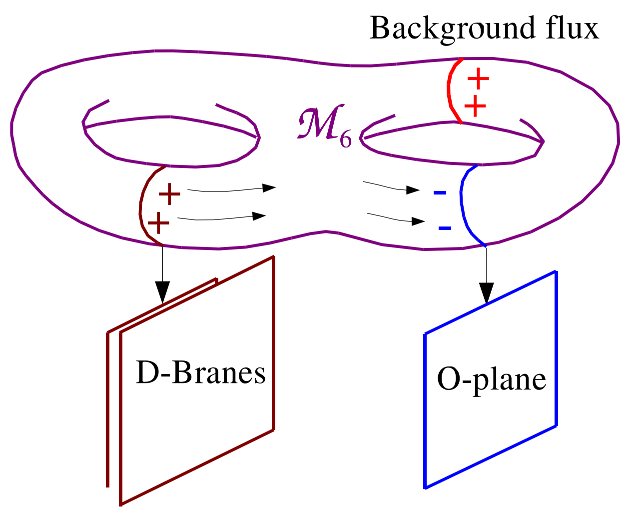
- Discuss dualities in these generalized geometries in the presence of background flux

- Four-dimensional Gauge theory and specific models

➪ A realization in Type II String theory: necessity of orientifolds

- minimal supersymmetry: background $M_{1,3} \times \mathcal{M}_6$
  $\mathcal{M}_6$ – special manifold

- moduli stabilization:
  background fluxes
  non-Calabi-Yau geometry



- non-Abelian gauge groups:
  space-time filling D-branes

$\Rightarrow$ consistency: orientifold planes

Generalized geometries and the orientifold projection

➪ The background manifold $\mathcal{M}_6$:

$\mathcal{M}_6$ chosen such that the four-dim theory possesses $N = 2$ susy

Number of (independent) globally defined spinors $\eta^1, \eta^2 \dots$ on $\mathcal{M}_6$ determines number of four-dimensional supersymmetries.

- In absence of flux $\delta\Psi_M = 0$, $\delta\lambda = 0$:

  Calabi-Yau manifold: $$\boxed{\nabla_m \eta = 0 \qquad \Leftrightarrow \qquad dJ = 0 \quad d\Omega = 0}$$

  J - real Kähler two-form, Ω - holomorphic three-form

- In the presents of flux, one globally defined spinor η: Strominger, Hull, ...

  $SU(3)$ structure manifold: $$\boxed{\nabla_m \eta = \mathrm{Flux} \qquad \Leftrightarrow \qquad dJ \neq 0 \quad d\Omega \neq 0}$$

  J - globally def. (1,1)-form, Ω - globally def. (3,0)-form

  Relation of η to J, Ω : $$J_{mn} = \eta_+^\dagger \gamma_{mn} \eta_+ \qquad \Omega_{mnp} = \eta_+^\dagger \gamma_{mnp} \eta_-$$

- More general ansatz allows for two globally defined spinors η^1, η^2 which can locally coincide.

Gates,Hull,Rocek; Graña,Minasian,Petrini,Tomasiello

Jeschek,Witt; Graña,Louis,Waldram

Generalized manifolds with $SU(3) \times SU(3)$ structure: Hitchin, Gualtieri, Witt

$$TM_6 \quad \rightarrow \quad \boxed{E \;\cong\; TM_6 \oplus T^*\mathcal{M}_6 \qquad \text{generalized tangent bundle}}$$

– existence of η^1, η^2 reduce structure group of E: O(6,6) is T-dualy group

$$SO(6,6) \quad \rightarrow \quad SU(3) \times SU(3)$$

– define complex even and odd forms which encode the geometry of $\mathcal{M}_6$:

$$\Phi^{\mathrm{ev}/\mathrm{odd}} = \sum_{i=1}^{6} \eta^{\dagger\,2}_{+-} \gamma_{m_1 \ldots m_i} \eta^1_+ \; dx^{m_1} \wedge \ldots \wedge dx^{m_i}$$

These forms are 'pure' and satisfy certain $SU(3) \times SU(3)$ conditions.

– special cases for $SU(3)$ structure manifold:

$$\Phi^{\mathrm{ev}} = e^{iJ} \qquad\qquad \Phi^{\mathrm{odd}} = \Omega$$

➪ The orientifold projection (Type IIA example): Acharya,Aganagic,Brunner,Hori,Vafa,...

Bosonic Type IIA spectrum

NS-NS: $\hat{\phi},\ \hat{G}_{MN},\ \hat{B}_2$	R-R: $\hat{C}^{\rm odd} = \hat{C}_1 + \hat{C}_3 + \hat{C}_5 + \hat{C}_7 + \hat{C}_9$

- mod out (gauge-fix) discrete symmetries of the string theory:

$\mathcal{O} = (-)^{F_L}\,\Omega_p\,\sigma^*$

1) world sheet parity Ω_p

2) geometric symmetry σ of $\mathcal{M}_6$: $\sigma^2 = 1$ (identity on $M_{3,1}$)

- demand $N = 1$ supersymmetry $\quad \lambda(\omega_{2n}) = (-1)^n \omega_{2n} \qquad \lambda(\omega_{2n-1}) = (-1)^n \omega_{2n-1}$

$$\sigma^* \Phi^{\rm odd} = \lambda(\bar{\Phi}^{\rm odd}) \qquad \sigma^* \Phi^{\rm ev} = \lambda(\Phi^{\rm ev})$$

Calabi-Yau case: σ is anti-holomorphic and isometric involution – $O6$ planes.

- truncate spectrum such that: $\mathcal{O}(\text{Field}) = \text{Field}$

$$\sigma^* \hat{\phi} = \hat{\phi} \qquad \sigma^* \hat{B}_2 = -\hat{B}_2 \qquad \sigma^* \hat{C}^{\rm odd} = \lambda(\hat{C}^{\rm odd})$$

The four-dimensional theory of the $N = 1$ chiral multiplets

➪ Four-dimensional spectrum: expand ten-dimensional forms in appropriate eigenspaces of σ^*

$$\text{forms on } \mathcal{M}_6 : \qquad \Lambda^{\text{ev}} = \Lambda^{\text{ev}}_+ \oplus \Lambda^{\text{ev}}_- \qquad \Lambda^{\text{odd}} = \Lambda^{\text{odd}}_+ \oplus \Lambda^{\text{odd}}_-$$

- R-R sector

$$\text{scalars} \quad C^{\text{odd}}_{(0)} = \hat{C}^{\text{odd}}|_{\Lambda^{\text{odd}}_+} \qquad \text{two-forms} \quad C^{\text{odd}}_{(2)} = \hat{C}^{\text{odd}}|_{\Lambda^{\text{odd}}_-}$$

A similar expansion is performed on even forms Λ^{ev} yielding vectors and three-forms. Infinite set of scalars, two-form as well as vectors, three-form related by duality condition on the ten-dimensional field strengths.

- NS-NS sector

$$\varphi^{\text{odd}} = e^{-\hat{\phi}} e^{-\hat{B}_2} \wedge \Phi^{\text{odd}} \qquad \varphi^{\text{ev}} = e^{-\hat{B}_2} \wedge \Phi^{\text{ev}}$$

Together with the four-dimensional graviton the forms $\varphi^{\text{ev/odd}}$ encode all degrees of freedom in the NS-NS sector. The B-field appears through the natural action of $SO(6,6)$ on forms ($so(6,6) \cong \Lambda^2 T^* \oplus \Lambda^2 T \oplus \text{End}T$).
Not all degrees of freedom are independent, e.g.

$\rightarrow$ $\text{Im}(\varphi^{\text{odd}}) = *_6 \, \text{Re}(\varphi^{\text{odd}})$ is function of $\text{Re}(\varphi^{\text{odd}})$ only — Hitchin

➪ The scalar field space:

1. How do they combine into $N=1$ chiral multiplets?

- correct D-brane couplings: combine

$$\varphi_c^{\rm odd} \equiv e^{-\hat{B}_2} \wedge \hat{C}^{\rm odd} + i\, \mathrm{Re}(\varphi^{\rm odd})\, |_{\Lambda_+^{\rm odd}}$$

linear in the $N=1$ complex scaler fields

- $\varphi^{\rm ev}(t)$ is holomorphic function of complex scalars t^a (Calabi-Yau example $e^{\hat{B}_2+iJ}=e^t$)

Hitchin; Graña, Louis, Waldram

2. What is the metric on the scalar field space?

- $N=1$ susy $\rightarrow$ Kähler metric, i.e. $G_{AB}=\partial_A \bar{\partial}_B K$
- Kähler potential:

$$K(t,\varphi_c^{\rm odd}) = -\ln\Big(\int_{\mathcal{M}_6} \varphi^{\rm ev}\wedge\bar{\varphi}^{\rm ev}\Big) - 2\ln\Big(\int_{\mathcal{M}_6} \varphi^{\rm odd}\wedge\bar{\varphi}^{\rm odd}\Big)$$

- first term: as in $N=2$ scale invariant functional on even forms   Hitchin; Graña, Louis, Waldram
- second term: Kähler space inside the $N=2$ quaternionic manifold,
  metric encoded by functional of $\mathrm{Re}(\varphi^{\rm ev})$

➪ The $N=1$ superpotential:

The background fluxes and non-Calabi-Yau geometry induce a potential for the scalar fields.

- We allow for non-trivial NS-NS background flux H_3 and R-R fluxes $F^{\rm ev}$ on $\mathcal{M}_6$

$$H_3 = \langle d\hat{B}_2 \rangle_{\mathcal{M}_6} \ , \qquad\qquad F^{\rm ev} = \langle d\hat{C}^{\rm odd} \rangle_{\mathcal{M}_6}$$

- The manifold with $SU(3)\times SU(3)$ structure generically has $d\varphi_c^{\rm odd} \neq 0$ and $d\varphi^{\rm ev} \neq 0$. This deviation from a Calabi-Yau manifold contributes to the potential.

- The induced superpotential can be derived by a fermionic reduction.

$$W(t,\varphi_c^{\rm odd}) = \int_{\mathcal{M}_6} (F^{\rm ev} + d_H \varphi_c^{\rm odd}) \wedge \varphi^{\rm ev}$$

  Here $d_H = d + H_3\wedge$ denotes the H-twisted differential. This superpotential reduces to the known cases on general Calabi-Yau and $SU(3)$ structure orientifolds.

  TWG,Louis; Villadoro,Zwirner; Graña,Louis,Waldram

- A similar analysis can be performed for type IIB set-ups: essentially exchanges the role of even and odd forms

Mirror symmetry / T-duality with fluxes: A conjecture

➪ The Question:
What is the mirror dual of type IIB Calabi-Yau $O3/O7$ orientifolds with background fluxes?

- Mirror symmetry/T-duality is believed to map H-flux to a non-trivial mirror geometry.
- In type IIB Calabi-Yau orientifolds fluxes induce the Gukov-Vafa-Witten superpotential

$$W_{GVW} = \int_Y (F_3 - \tau H_3) \wedge \Omega(z)$$

- Let us restrict to a simple case: one complex structure modulus z
  $F_3 = 0$ and $H_3 = m\alpha_1 + e\beta^1$
  In large complex structure limit:

$$W_{GVW} = -\tau(e\, z + m\, z^2)$$

Has the H-flux e and m an $SU(3) \times SU(3)$ mirror geometry?

➪ Perform mirror symmetry (T-dulality in three directions SYZ):
H_3 has maximally two legs into the T-dualized directions (the 'Q-space' Shelton, Taylor, Wecht)

- Mirror deformation due to electric flux e: The complex three-form Ω_3 is not anymore closed.
  $SU(3)$ structure mirror ('half-flat') Gurrieri, Louis, Micu, Waldram; Fidanza, Minasian, Tomasiello

$$d\text{Re}(\Omega_3) \propto e$$

- Mirror deformation due to magnetic flux m: A non-trivial one-form Ω_1 on the mirror space is needed:

$$d\text{Re}(\Omega_1) \propto m$$

  Such a one-form is present on an appropriate $SU(3) \times SU(3)$ manifold!
  Can use the superpotential calculated above

$$W_{SU(3)\times SU(3)} = \int d\varphi_c^{\text{odd}} \wedge \varphi^{\text{ev}}(t) = -N^0(e\,t + m\,t^2)$$

- Origin of Ω_1? Recall $so(6,6) \cong \Lambda^2 T^* \oplus \Lambda^2 T \oplus \text{End}T$ $\Rightarrow$ $\beta \in \Lambda^2 T$ two-vector

  $\Omega_1 + \Omega_3 = e^{-\beta}\Omega_3$: Can β correspond to non-commutativity of $\mathcal{M}_6$?
  Kapustin, Mathai, Rosenberg

Conclusions

- discussed compactification of type II supergravity on orientifolds of $SU(3) \times SU(3)$ manifolds
  - determined four-dimensional spectrum without finite truncation, NS-NS and R-R sector is encoded by specific odd and even forms on $\mathcal{M}_6$
  - Kähler potential consists of the two Hitchin functionals on $\mathcal{M}_6$
  - holomorphic superpotentials for the geometry due to fluxes and non-Calabi-Yau geometry
- commented on mirror symmetry/T-duality of flux compactifications
  - mirror spaces of Calabi-Yau compactifications with H-flux are generically generalized $SU(3) \times SU(3)$ manifolds