The Decay of Nearly Flat Space

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hep-th/0603105

Motivation

Landscape

Many Vacua

Probability of each vacuum $\left\{ \begin{array}{l} \text{Enumerate vacua} \\ \text{Understand dynamics} \end{array} \right.$

Ergodic Evolution (Banks & Johnson hep-th/0512141)

Causal patch description of eternal inflation

$\Lambda_{min} > 0$ "ground" state, all others are fluctuations

Probability ~ Lifetime ~ Entropy

$\Gamma \rightarrow 0$ for $\Lambda \rightarrow 0$ to stabilize Λ_{min} dS

but, $\Gamma \neq 0$ (discontinuous) at $\Lambda = 0$

What we did

Investigate CdL equations

- Consider singular “solutions”
- General properties
- Map “solution” space

Γ continuous as $\Lambda \rightarrow 0$

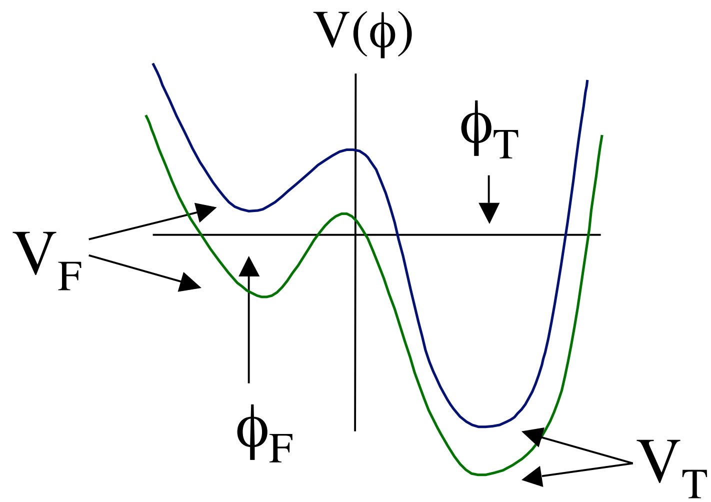
⇨ If $\Gamma \rightarrow 0$, $\Lambda = 0$ limit is stable

(See also Banks, Johnson, & Aguirre hep-th/0603107)

CdL Tunneling Review

Scalar coupled to gravity

$$S = \int d^4x \sqrt{g} \left(\frac{-M_p^2}{16\pi} R + \frac{1}{2}(\nabla\phi)^2 + V(\phi) \right)$$



Euclidean instanton

$\Gamma \sim \exp(-S_I + S_{BG})$

SO(4) symmetry

metric: $ds^2 = dt^2 + \rho^2(t) d\Omega_3^2$

S^3

Lorentzian dynamics

expanding bubble of true vacuum

- $V_T > 0$ ➔ dS
- $V_T = 0$ ➔ open FRW
- $V_T < 0$ ➔ big crunch

Equations of Motion

$$\ddot{\phi} + 3\frac{\dot{\rho}}{\rho}\dot{\phi} = V'(\phi)$$

→ Particle in potential -V(ϕ) with friction ~ $H = \frac{\dot{\rho}}{\rho}$

$$\ddot{\rho} = -\frac{4\pi}{3M_p^2}\rho\left(\dot{\phi}^2 + V(\phi)\right)$$

$$\dot{\rho}^2 - 1 = \frac{8\pi}{3M_p^2}\rho^2\left(\frac{\dot{\phi}^2}{2} - V(\phi)\right)$$

} Coupled to FRW

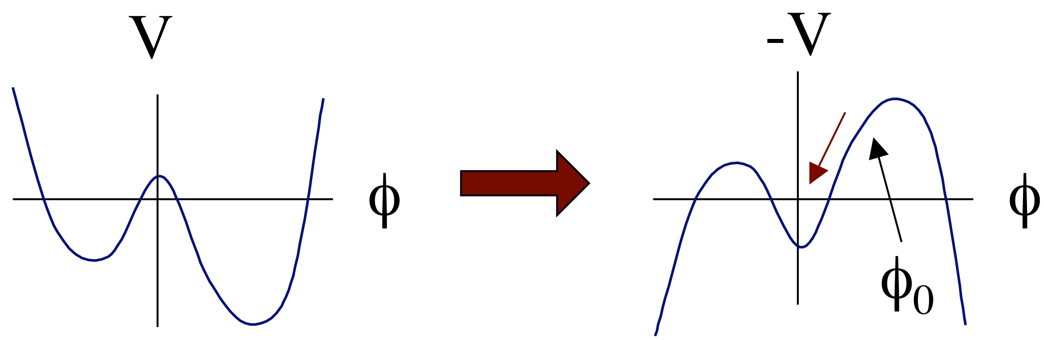
Boundary Conditions at ρ = 0 poles

$\dot{\phi} = 0$ → Continuous

$\dot{\rho}^2 = 1$ → Smooth

Solutions - Noncompact

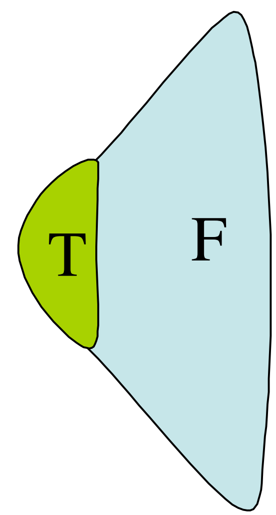
$V_f \leq 0$:



R^4 topology, one pole at $t = 0$

$\phi \to \phi_f$ as $t \to \infty$ ⇒ EAdS ($V_F < 0$) or Flat ($V_F = 0$)

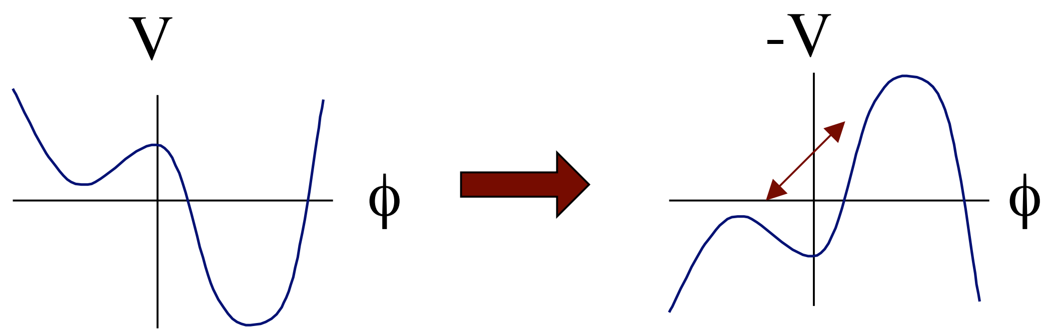
$\dot{\rho} > 0$ ⇒ $\dot{E} < 0$

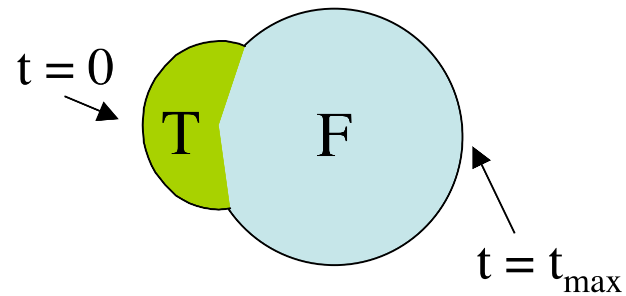


May not reach ϕ_f ⇒ False vac. stable

Solutions - Compact

$V_f > 0$:



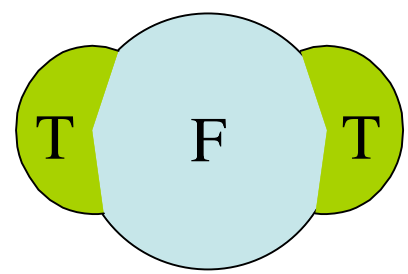


$S^4 \sim$ EdS, two poles at $t = 0$, t_{max}

$\dot{\rho}$ crosses 0 at equator $\Rightarrow$ E$\uparrow$ (anti-friction)

Tunneling solution always exists

Multiple passes - $P \geq 0$



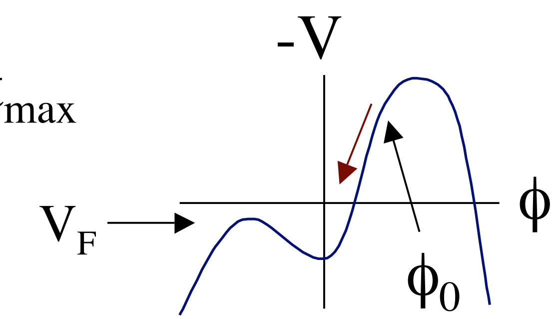
P=2

Properties of "Solutions"

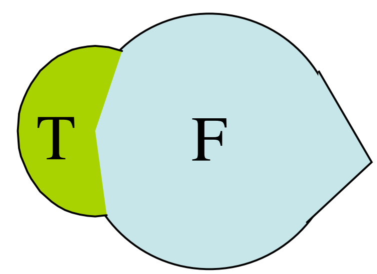
"Solution" - solve with (V_F, ϕ_0) ➡ singular **or** regular

- Generically compact with singularity at t_{max}
- $\phi \rightarrow \pm\infty$ for singular "solutions"
- Across reg. compact "sol'n" $\Delta P = 1$

$$\Delta E \begin{cases} > 0 \Rightarrow \phi \rightarrow -\infty \\ < 0 \Rightarrow \phi \rightarrow \infty \text{, extra pass} \end{cases}$$



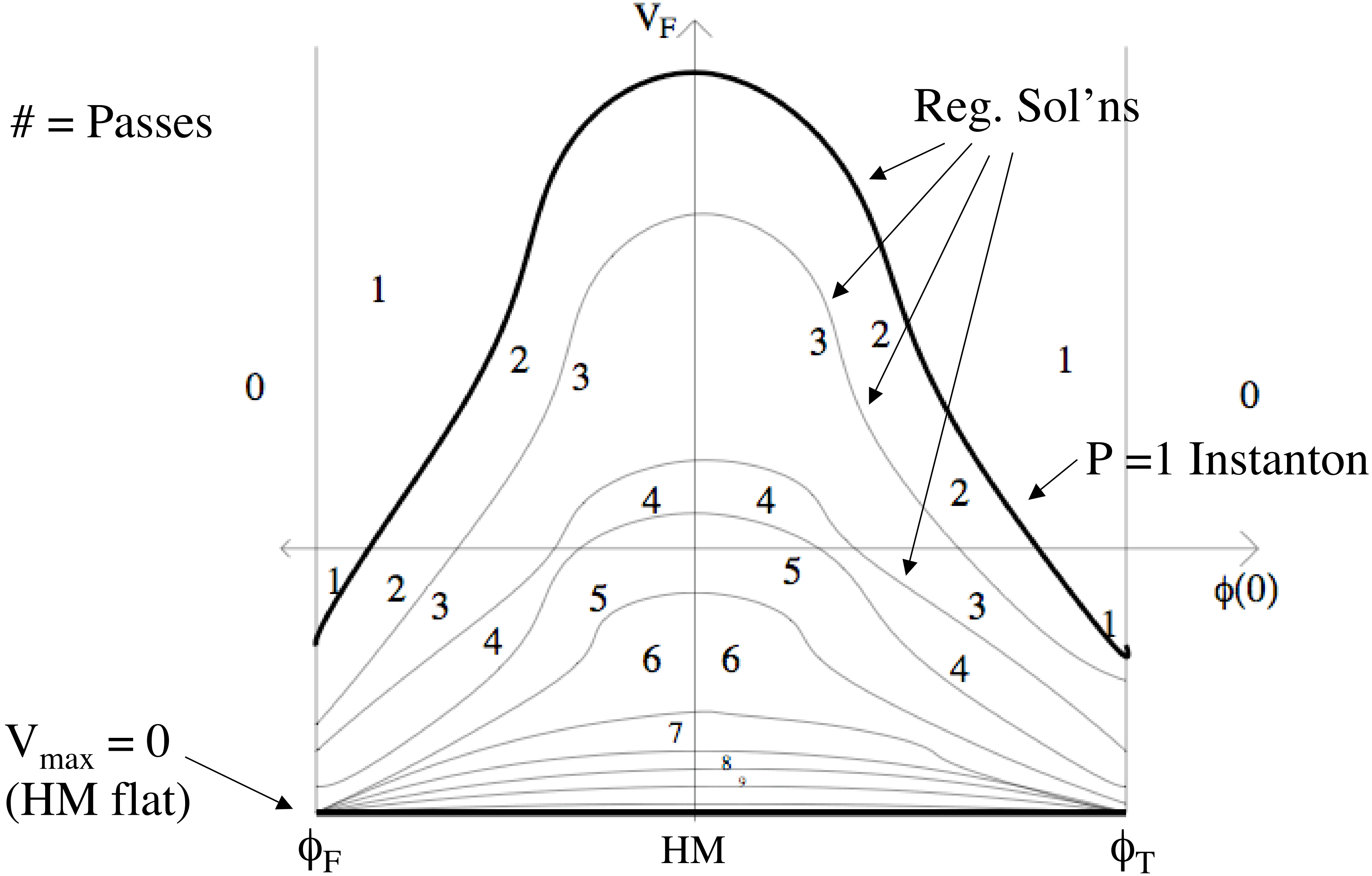
- Across non-compact soln $\Delta P = ?$
- Between ϕ_0^1 and ϕ_0^2 with $\Delta P \neq 0$, reg. sol'n



Singular "solution"

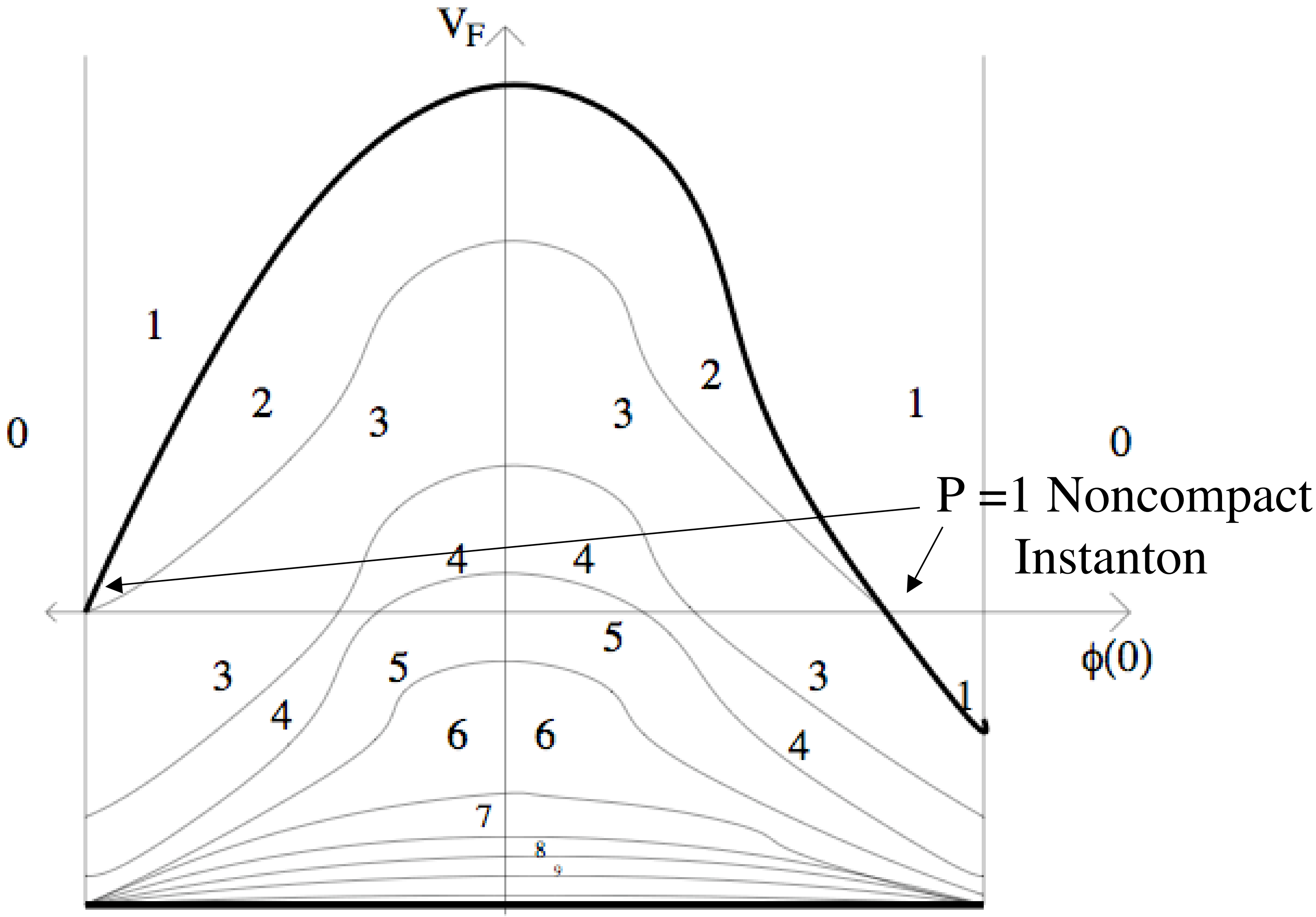
Solution space

No Λ=0 tunneling



Solution space

Λ=0 tunneling

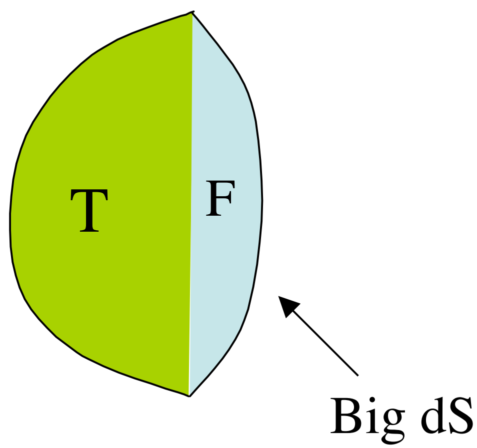


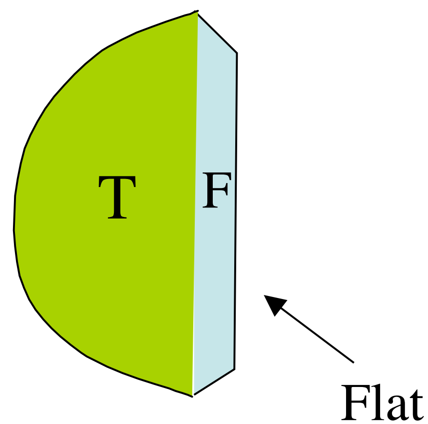
$V_F \rightarrow 0$ Limit

Stable $V_F = 0$ False Vacuum

No noncompact solution (by assumption)

Reg. Compact $V_F > 0$ → Reg. Compact $V_F = 0$





S_I finite → S_I finite

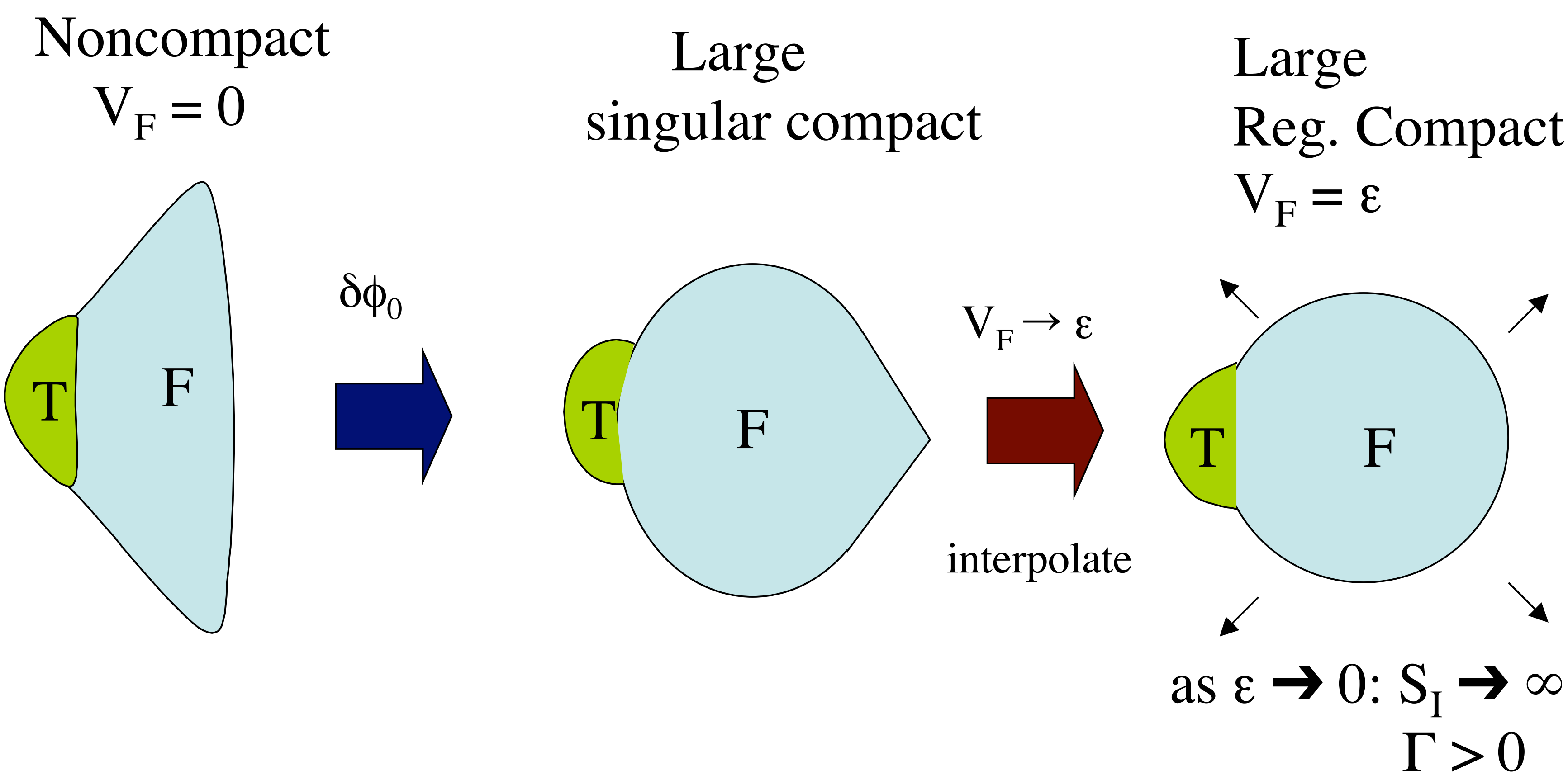
$(S_{BG} = S_F = \infty) \Rightarrow \Gamma = 0$

$V_F \rightarrow 0$ Limit

Unstable $V_F = 0$ False Vacuum

Noncompact solution exists (by assumption)

Limit discontinuous - hard to perturb



Summary

- Smooth $V_F \rightarrow 0$ limit
  if $\Gamma \rightarrow 0 \Rightarrow$ stable flat space

- Ergotic landscape doubtful

- "Solution" space - rich structure