A-Model Correlators from the Coulomb Branch

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Summary

- A new look at an old topological field theory: A-twist of $\mathcal{N} = (2,2)$ SUSY NLSM of maps $\Phi : \mathbb{P}^1 \to X$; X a smooth, compact toric variety.

World-sheet Instantons $\leftrightarrow$ (genus 0) Gromov-Witten Invariants
Correlators $\leftrightarrow$ Generating functions

- Witten : X a *phase* in the Kähler moduli space of a GLSM

- Topological correlators computed *exactly* by semiclassical expansion in a particular phase;
  - Geometric Phase: instanton sums of Morrison and Plesser (sums and combinatorics)
  - *Non-Geometric Phase*: *massive Coulomb vacua* (algebraic expressions)

Précis of Gauged Linear Sigma Models

- A GLSM is an $\mathcal{N} = (2,2)$, $d = 2$, $G \simeq [\mathrm{U}(1)]^r$ gauge theory;
- Parameters: F-I terms r^a and θ-angles θ^a; $\quad q_a := exp(-2\pi r^a + i\theta^a)$, $a = 1, \ldots, r$.

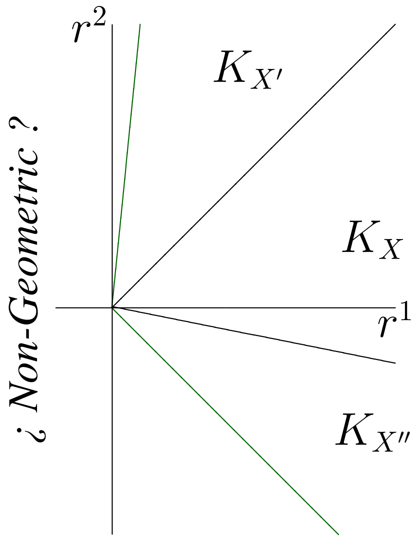
(Bosonic) Field Content:

- n Matter fields ϕ^i, charges Q^i_a;
- Gauge: $v_{a;\mu}$, σ_a.

(Classical) Potential:

$$V(\phi,\sigma) = Q^a_i Q^b_i \sigma_a \bar{\sigma}_b |\phi^i|^2 + \tfrac{e_0}{2} D^a D^a,$$
$$D^a = Q^a_i |\phi^i|^2 - r^a.$$

$X \simeq \{D^a = 0\}/G$, $r^a \in K_X$ Kähler class on X



Twisting the GLSM and Summing the Instantons

- GLSM has $\mathrm{U}(1)_+ \times \mathrm{U}(1)_-$ R-symmetry; $\mathrm{U}(1)_V$ non-anomalous;
- Modify energy-momentum tensor by $\mathrm{U}(1)_V$ current; shifts spins of fermions

Consequences:

- Twisted theory has BRST charge $\mathcal{Q}$, and $\mathcal{Q}$-cohomology spanned by $\sigma_a(z)$.
- Topological Correlators: $\langle \sigma_{a_1}(z_1) \cdots \sigma_{a_k}(z_k) \rangle$
  - independent of world-sheet metric (RG-invariant!);
  - depend on metric of X only through q_a.
  - Path integral localizes onto gauge instantons:

    $$\langle \sigma_{a_1} \cdots \sigma_{a_k} \rangle = \sum_{\mathbf{n}} \#(\mathbf{n}; a_1, \ldots, a_k) \prod_a q_a^{n_a}, \quad n_a = -\frac{1}{2\pi} \int f_a,$$

    $\#(\mathbf{n}; a_1, \ldots, a_k)$: a Gromov-Witten invariant of X.
  - Morrison and Plesser gave a toric prescription to compute $\#(\mathbf{n}; a_1, \cdots, a_k)$ in any geometric phase.

Some Remarks on the Correlators

- Evaluation of $\#(\mathbf{n}; a_1, \cdots, a_k)$ involves non-trivial combinatorics;
- Computing the sums can be cumbersome.

Can we circumvent these problems?

- $\langle F(\sigma)\rangle$ are rational functions of the q_a.
- Quantum cohomology relations:

$$\langle \prod_{i|Q_i^a>0} (\xi_i)^{Q_a^i} F(\sigma)\rangle = q_a \langle \prod_{i|Q_i^a<0} (\xi_i)^{-Q_a^i} F(\sigma)\rangle, \quad \xi_i = \sum_a Q_i^a \sigma_a.$$

Is there a simple way to understand these properties?

There *is* a sophisticated way: Batyrev , Szenes and Vergne have shown these properties to hold by explicitly analyzing the combinatorics.

We provide a simple derivation.

Massive Coulomb Vacua

Is SUSY broken in the Non-Geometric "phase" ? No. Quantum vacua emerge.

Integrate out matter fields at one loop: this leads to an effective twisted superpotential for the gauge multiplets, $\widetilde{W}(\Sigma)$. Equations of motion for σ :

$$\frac{\partial \widetilde{W}}{\partial \sigma_a} = \prod_{i|Q_i^a>0} (\xi_i)^{Q_a^i} - q_a \prod_{i|Q_i^a<0} (\xi_i)^{-Q_a^i}.$$

- These vacua are *reliable* in the Non-Geometric region.
- Idea: in Non-Geometric region, path integral localizes onto these σ-vacua.
- We performed the localization computation in spirit of Vafa's *Topological Landau-Ginzburg Models*. A σ-vacuum $\sigma_a = \hat{\sigma}_a$ contributes

$$H(\hat{\sigma}) = \left(\mathrm{Hess}\, \widetilde{W}(\hat{\sigma}) \prod_i \xi_i(\hat{\sigma}) \right)^{-1} \quad \text{to the path integral.}$$

Result and Some Applications

$$\langle F(\sigma) \rangle = \sum_{\hat{\sigma}} F(\hat{\sigma}) H(\hat{\sigma})$$

- Rationality of correlators and quantum cohomology follow trivially,
- Often a more efficient computational procedure,
- Can use to compute A model correlators for C-Y hypersurfaces in X
- For non-compact X most phases have both gauge instantons and massive Coulomb vacua. This combination can lead to a failure of the quantum cohomology relations.
- Unlike gauge instantons, trivial to extend to genus g Riemann surface:

$$\langle F(\sigma) \rangle_g = \sum_{\hat{\sigma}} F(\hat{\sigma}) H(\hat{\sigma})^{1-g}.$$

  Unfortunately, no new Gromov-Witten invariants, but a packaging for combinatorics.

Further Directions

Is there a pure Landau-Ginzburg description? Is it given by the Hori-Vafa Abelian duality?

Can we couple this story to topological world-sheet gravity and generate new Gromov-Witten invariants?