Black Hole Vacua

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based on work with Andrew Chamblin
Motivation

- String Theory is supposed to solve quantum gravity problems
  - e.g. Black hole singularity?
• Fidkowski, Hubeny, Kleban and Shenker suggested using AdS/CFT

- Consider geodesics that bounce off singularity
- AdS/CFT relates this to correlators
Generalities

Consider

\[ ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-1}^2 \]

Where \( f(r) \)

- monotonically increasing function
- singular at \( r = 0 \)
- zero at \( r = r_+ > 0 \) [horizon]

Examples:

- Schwarzchild: \( f(r) = 1 - \frac{\omega_d M}{r^{d-2}} \)
- AdS-Schwarzchild: \( f(r) = \frac{r^2}{\ell^2} + 1 - \frac{\omega_d M}{r^{d-2}} \)
Technicalities:
Define tortoise coordinate: 
\[ r^* = \int_0^r \frac{dr'}{f(r')} + \frac{\pi i}{f'(r_+)} \]
Double null coordinates:
\[ u = t - r^* \quad \quad v = t + r^* \]
Kruskal coordinates:
\[ U = e^{-\frac{f'(r_+)}{2} u} \quad \quad V = e^{\frac{f'(r_+)}{2} v} \]
Kruskal coordinates ⇔ Penrose diagram:

- Schwarzshchild \((r^*(r = \infty) = \infty)\)
- BTZ black hole \((r^*(r = \infty) = 0)\)
- AdS\(_5\)-Schwarzshchild \((0 < r^*(\infty) < \infty)\)
  
* Which Penrose ⇔ asymptotics of \(r^*\)
Symmetries:

- $\frac{\partial}{\partial t}$ is timelike (in asymptotic regions) Killing vector

- $S^{d-1}$ symmetries

  \[ \frac{1}{2}(U + V) \]

  - $T \rightarrow -T$ (vertical reflection of Penrose)
    - inverts time ($t$), preserves $S^{d-1}$
    - fixed points at $T = 0$

  \[ \frac{1}{2}(-U + V) \]

  - $Z \rightarrow -Z$ (horizontal reflection of Penrose)
    - inverts time, preserves $S^{d-1}$
    - fixed points at $Z = 0$

- antipodal map of $S^{d-1}$ (not on the Penrose diagram)
  - does not affect time; symmetry of $S^{d-1}$
  - no fixed points
Antipodal Map

Consider combination

\[ T \to -T, \ Z \to -Z, \ \text{antipodal map on} \ S^{d-1} \]

- acts freely
- preserves direction of time
- preserves (obvious) symmetries of the space-time
  - commutes with \( \frac{\partial}{\partial t} \)
  - commutes with \( S^{d-1} \) symmetries

Call this the **antipodal** map
Black Hole Vacua

- $J = R_T R_Z P \Rightarrow \mathbb{Z}_2$ “antipodal” map
  - Commutes with all symmetries

- Cf. de Sitter space:

- $\exists \mathbb{Z}_2$ antipodal map $X \to -X$ on covering space
  - Commutes with all symmetries
  - Can partially correlate point and antipodal point

  * $\Rightarrow$ Mottola-Allen transformation
  * Defines one complex-parameter family of vacua:
    - $\alpha$-Vacua
Consider a scalar field on this spacetime.

Can choose modes $\phi_n$ so that

$$\phi_n(x_A) \rightarrow \phi_n(x)^*$$

antipodal map:
- positive frequencies $\leftrightarrow$ negative frequencies
- (Antipodal map includes time reversal)

Can Bogoliubov standard vacuum:

$$b_n = \cosh \alpha a_n - e^{-i\gamma} \sinh \alpha a_n^\dagger$$
$$b_n^\dagger = \cosh \alpha a_n^\dagger - e^{i\gamma} \sinh \alpha a_n$$

- One complex parameter family of vacua
- Preserve all (obvious) symmetries

No reason to choose one over another

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α Vacua

These are exactly like dS α-vacua
  ● including construction

But dS α-vacua are frowned upon:
  ● Causality problems
  ● Unphysical Poles
  ● Pinch Singularities
  ● Not Thermal

We need not have these problems!
Causality

$\alpha$-vacua

- send in signal from south pole
- correlation propagates from north pole
  $\Rightarrow$ Causality problems?

- Yes! Naïvely, lightcones only intersect on horizon, \textbf{but}

- dS gets taller (gravitational backreaction)

- $\Rightarrow$ lightcones intersect!

But for \textbf{black holes},

- gravitational backreaction increases horizon size
- lightcones only intersect inside horizon

Leblond, Marolf, Myers
Unphysical Poles

Consider
\[ \langle \phi(x_1)\phi(x_2)\phi(x_3) \rangle \sim \int dy G^F(x_1,y)G^F(x_2,y)G^F(x_3,y) \]

For α vacua, poles when \( y \) is on lightcone of point or antipodal point

- Divergences if, say, also \( x_2 \) on lightcone of \( x_3A \)
  - coincident poles
- Surprising if \( x_1, x_2, x_3 \) causally connected

But for black holes this requires one of \( x_1, x_2, x_3 \) to be inside horizon.
Pinch Singularities

\[ \sim \int dx \int dy \, G_{\alpha \gamma}^F(x, y) G_{\alpha \gamma}^F(y, x), \]
\[ \sim \ldots + \int dx \int dy \, \sinh^2 2\alpha \, G_0^F(x, y) G_0^F(y, x)^* + \ldots \]

Has both \( i\epsilon \) prescriptions

\[ \Rightarrow \] can't evade singularity!

Change time-ordering prescription?

No! (In principle) calculate string loops (\textit{unlike} dS!)

\[ \Rightarrow \] No pinch singularities
Thermality

In an $\alpha$-vacuum

\[
\frac{P_{\alpha\gamma}(E_i \rightarrow E_j)}{P_{\alpha\gamma}(E_j \rightarrow E_i)} = \left| \frac{\cosh \alpha + \sinh \alpha e^{i\gamma e^{\beta \Delta E} \frac{\beta}{2}}}{\cosh \alpha + \sinh \alpha e^{i\gamma e^{-\beta \Delta E} \frac{\beta}{2}}} \right|^2 e^{-\beta \Delta E}.
\]

- Only thermal (temperature $\beta$) if $\alpha = 0$
- Contradicts detailed balance?
  
  \[- \text{i.e. } \rho(E_i)P(E_i \rightarrow E_j) \neq \rho(E_j)P(E_j \rightarrow E_i)\]

NO! Just means *nonequilibrium, steady state.*
Holography?

- For AdS-Schwarzschild, have AdS/CFT.
- Two boundaries $\Rightarrow$ two CFTs
- Ordinary vacuum $\Leftrightarrow$ Pure state of (doubled) CFT
  - Trace over CFT$_1$ $\Rightarrow$ thermal state of CFT

For $\alpha$-vacuum, Bogoliubov CFT:

\[
\begin{align*}
   b^\dagger_1 &= \cosh \alpha a^\dagger_1 - e^{i\gamma} \sinh \alpha a_2,
   \\
   b^\dagger_2 &= \cosh \alpha a^\dagger_2 - e^{i\gamma} \sinh \alpha a_1
\end{align*}
\]

Note "1" and "2" no longer bdy$_1$ and bdy$_2$.

$\text{Tr}_2 \Rightarrow$ nonthermal density matrix
Agrees with nonthermal formula on AdS side!

Sim. propagators $\leftrightarrow$ correlation functions
Using AdS/CFT, compute $S = -\text{Tr} \rho \ln \rho$:

- **Entropy at low temperature:**

- **Entropy at high temperature:**
Conclusions

- Black holes have $\alpha$-Vacua
  - Very general
  - For AdS-Schwarzschild, can use CFT as well
- Can avoid problems of dS $\alpha$-Vacua
- Compute $\alpha$-dependent entropy from CFT
  - What does it mean???