PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

with David Mateos, Rowan Thomson
& Andrei Starinets

Cool D3/D7 Thermodynamics

Outline:

1. AdS/CFT with D7-branes
   → fundamental matter

2. AdS/CFT thermodynamics

3. Probe brane thermodynamics
   → self-similar embeddings
   → first-order phase transition

4. Conclusions/Future Directions

Maldacena; Witten; Gubser, Klebanov & Polyakov; …..

AdS/CFT from D3-branes:

d=10 Type IIb superstrings in $AdS_5 \times S^5$
with N units of RR flux

equivalent to

d=4 ***N*** =4 U(N) super-Yang-Mills

$$\frac{1}{2}\left(R^2/\alpha'\right)^2 = g_{YM}^2 N \equiv \lambda \gg 1$$
$$2\pi\, g_s = g_{YM}^2 \ll 1$$

supergravity limit ~ large-N with strong `t Hooft coupling

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all adjoint fields!

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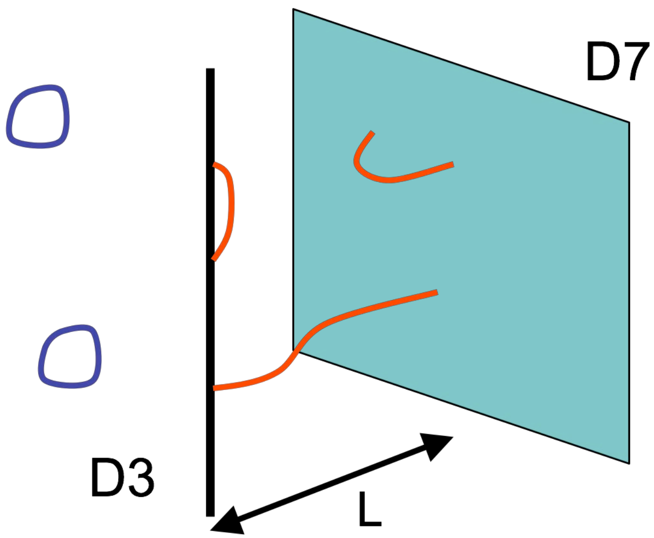
Decoupling limit of N D3-branes with k D7-branes

(Karch and Katz)

N coincident D3-branes:



d=4 U(N) o.s., d=8 U(k) o.s.
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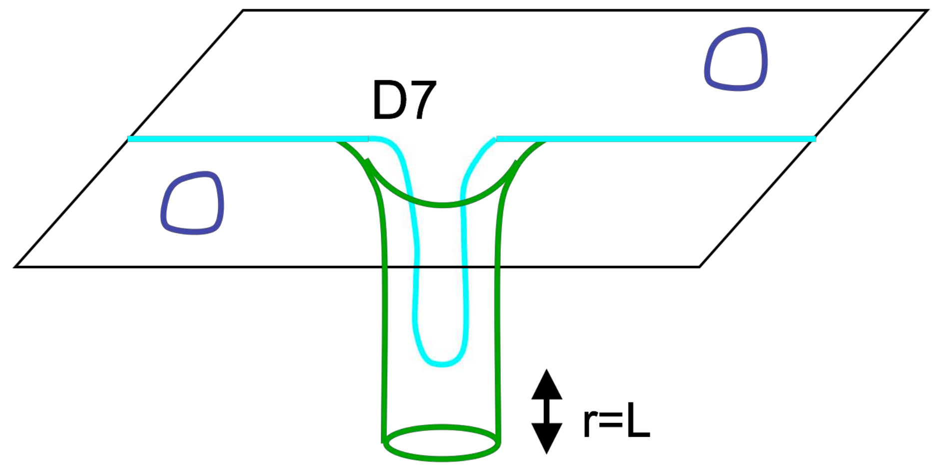

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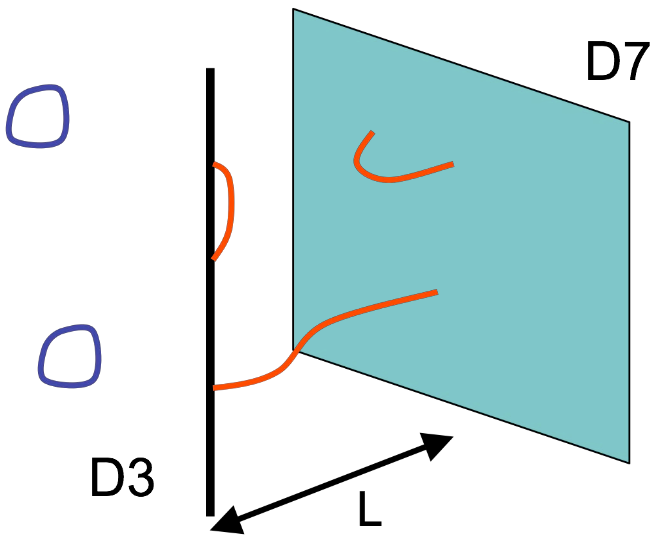

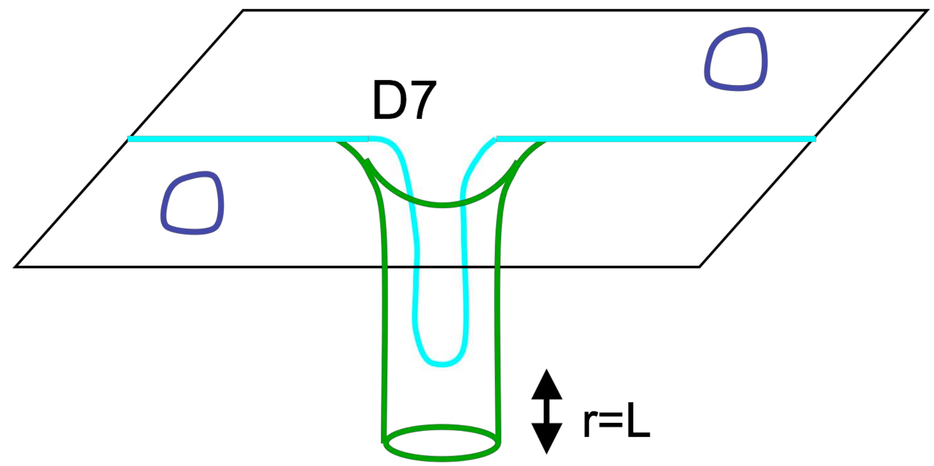

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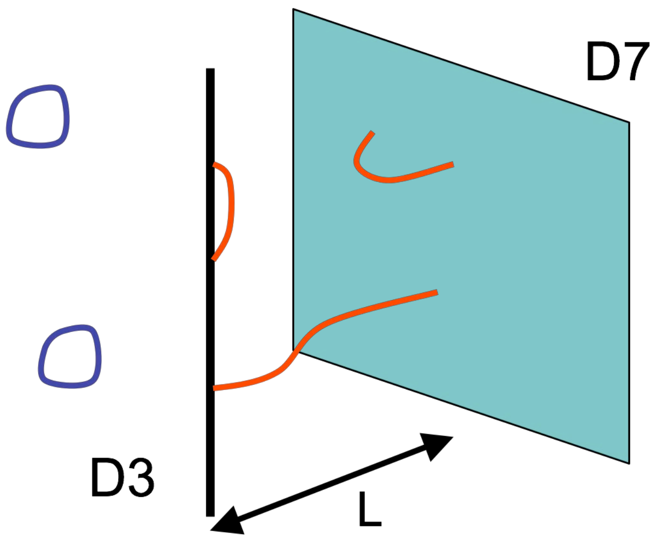

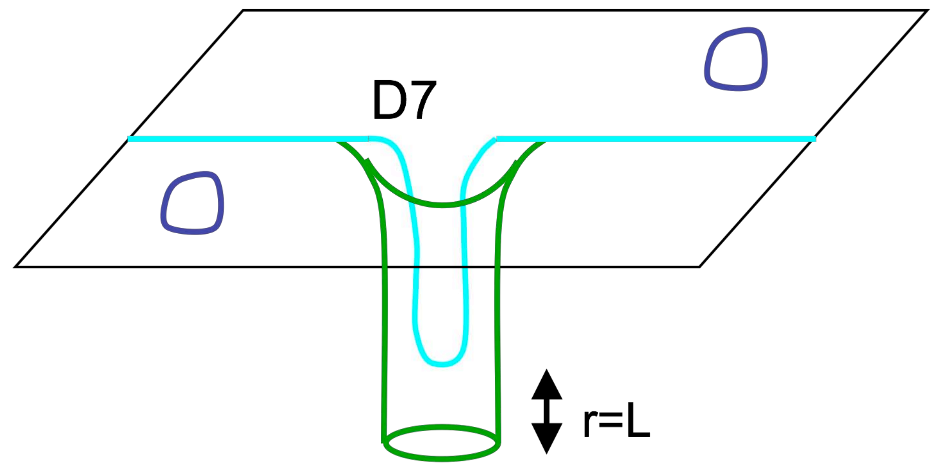

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Decouples asymptotic (closed) strings

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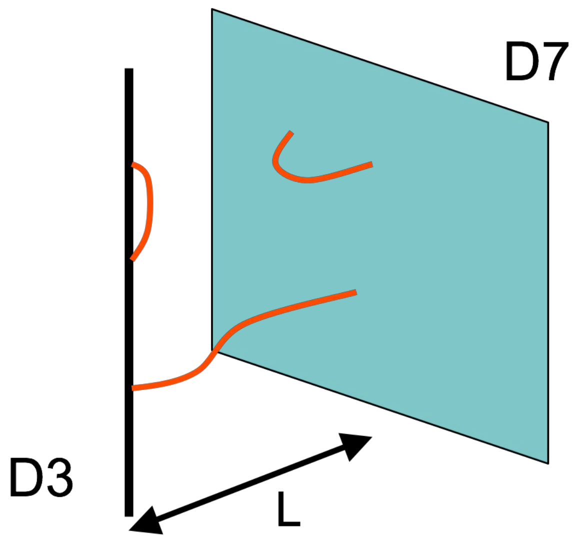

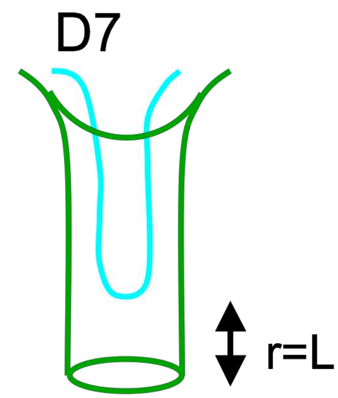

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Reduces brane theory to field theory

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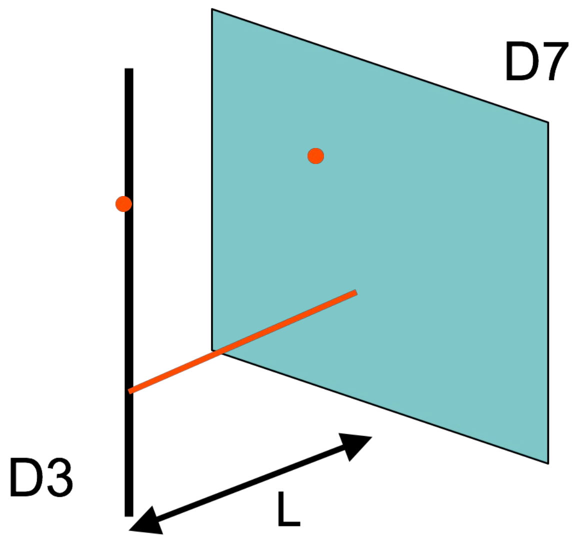

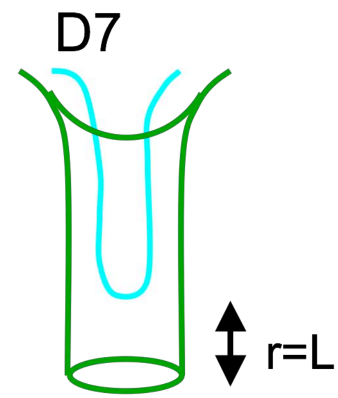

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Minimum mass for 3-7 strings: $m_q = \frac{L}{2\pi\alpha'}$

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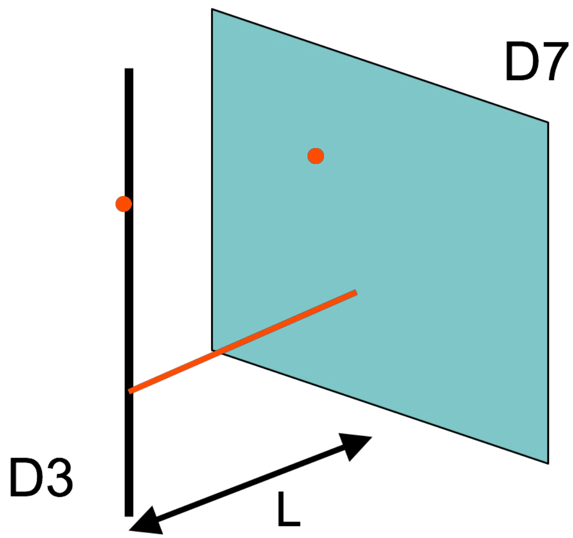

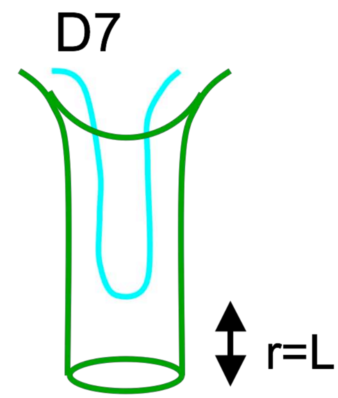

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Gauge coupling on D7's: $g_{YM}^2 \sim g_s\, \alpha'^2 \to 0$

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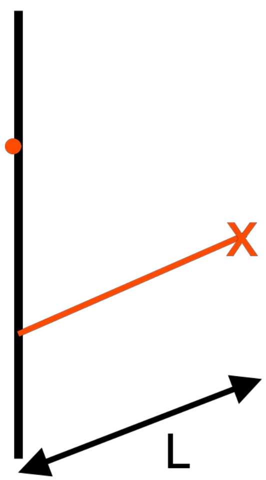

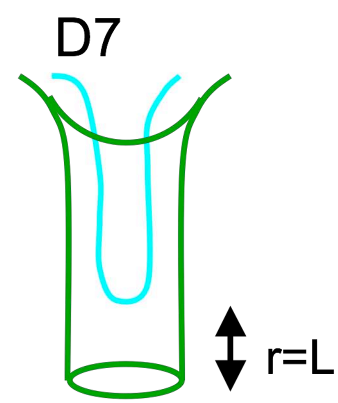

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- throat geometry: AdS_5 X S^5 with k D7's
- brane theory → $\mathcal{N}$=4 U(N) SYM with fundamental matter

Field theory:

$\mathcal{N}$=4 U(N) super-Yang-Mills

vector:	A_μ	adjoint in U(N)
4 Weyl fermions:	$\lambda, \psi_{1,2,3}$	
3 complex scalars:	$\phi_{1,2,3}$	

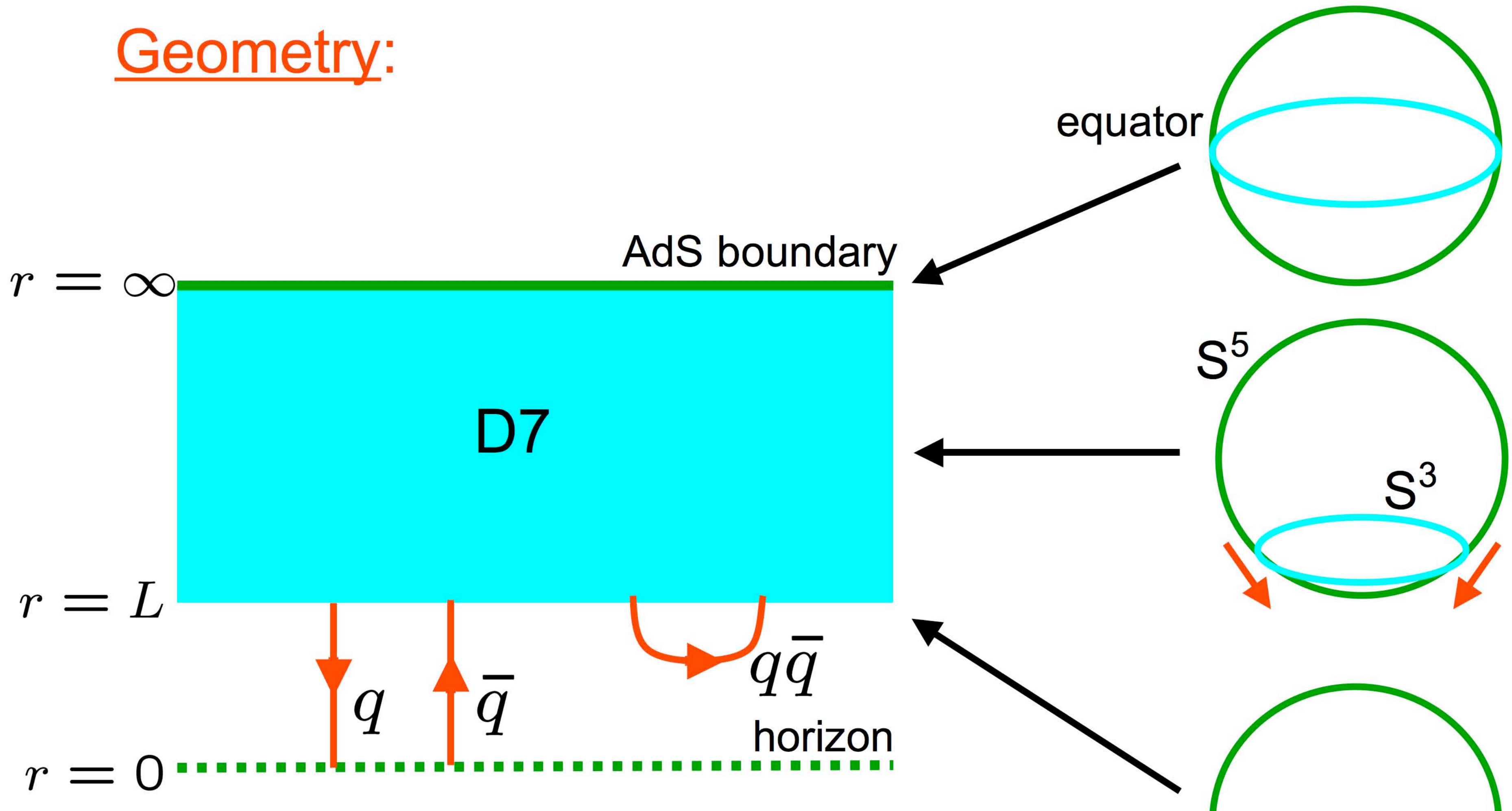
coupled to: k massive $\mathcal{N}$=2 hypermultiplets

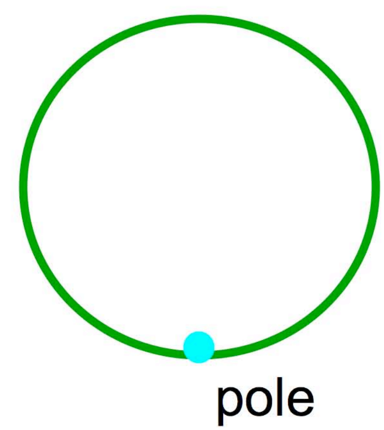
2 complex scalars :	$q_\pm$	fund. in U(N) & global U(k)
2 Weyl fermions:	$\chi_\pm$	

- SUSY: $\mathcal{N}$=4 $\rightarrow$ $\mathcal{N}$=2
- SO(6) $\rightarrow$ SO(4) = $SU(2)_L$ X $SU(2)_R$ $\{ \text{X } U(1)_R \}_{m=0}$

$$Q_i \quad (0, \frac{1}{2}, 1); \quad q_\pm \quad (0, \frac{1}{2}, 0); \quad \chi_\pm \quad (0, 0, \mp 1)$$

Geometry:





SO(4) = rotational symmetry of S^3

{ $U(1)_R$ = additional rot. sym. for L=0 }

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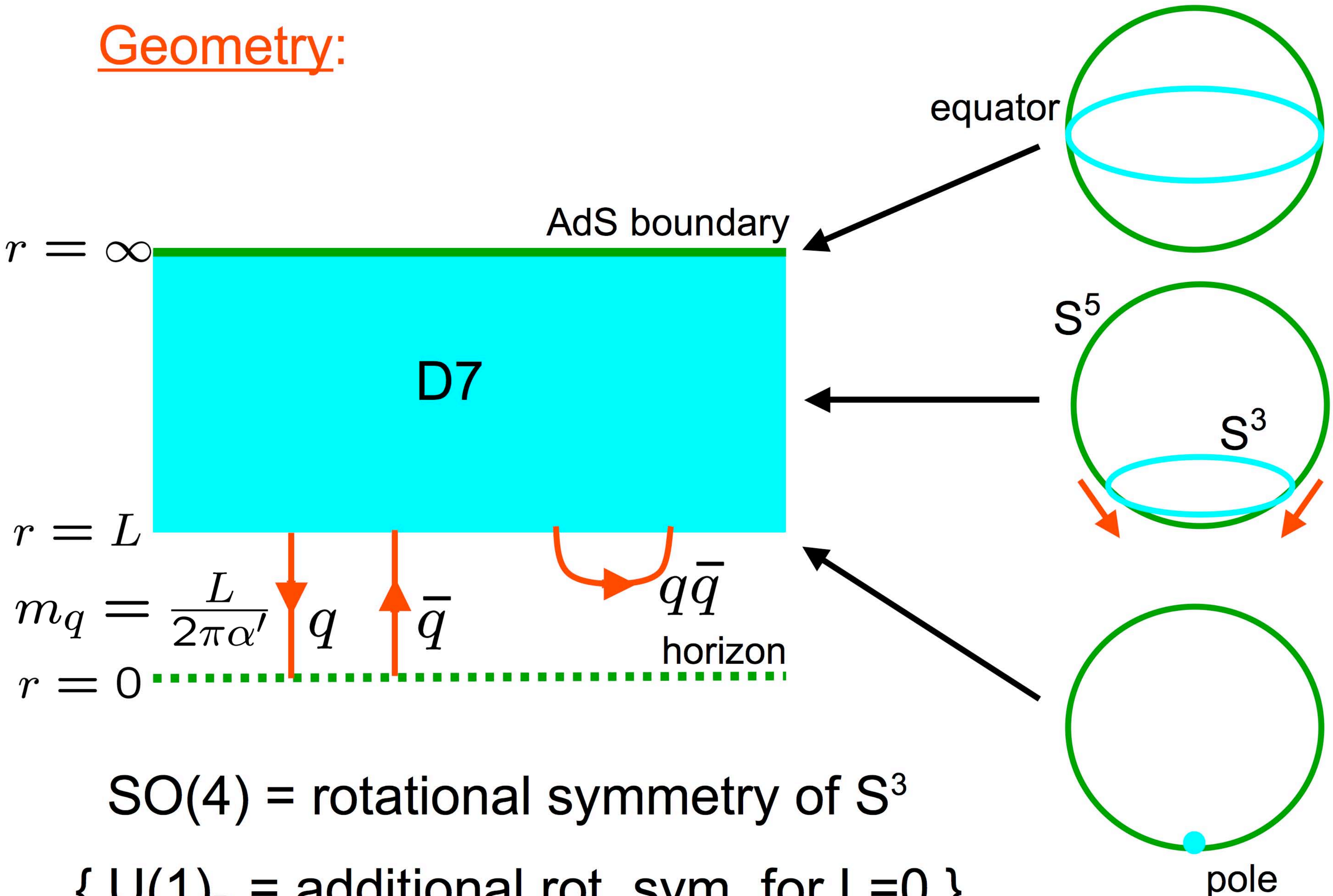

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Witten; …..

AdS/CFT thermodynamics:

Gauge theory thermodynamics = Black hole thermodynamics

Replace AdS_5 with (planar) AdS black hole:

$$ds^2 = \frac{r^2}{R^2}\left(- \quad dt^2 + d\vec{x}^{\,2}\right) + \frac{R^2}{r^2} dr^2$$

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Wick rotate and apply techniques of euclidean path integral:

smoothness at $r=r_o$: $\Delta\tau = \beta = \frac{\pi R^2}{r_o} = \frac{1}{T}$

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$$I_E = -\frac{1}{16\pi G_5}\int d^5x\sqrt{g}\left(R(g) + \frac{12}{R^2}\right)$$ *+ G-H term + counter-terms*

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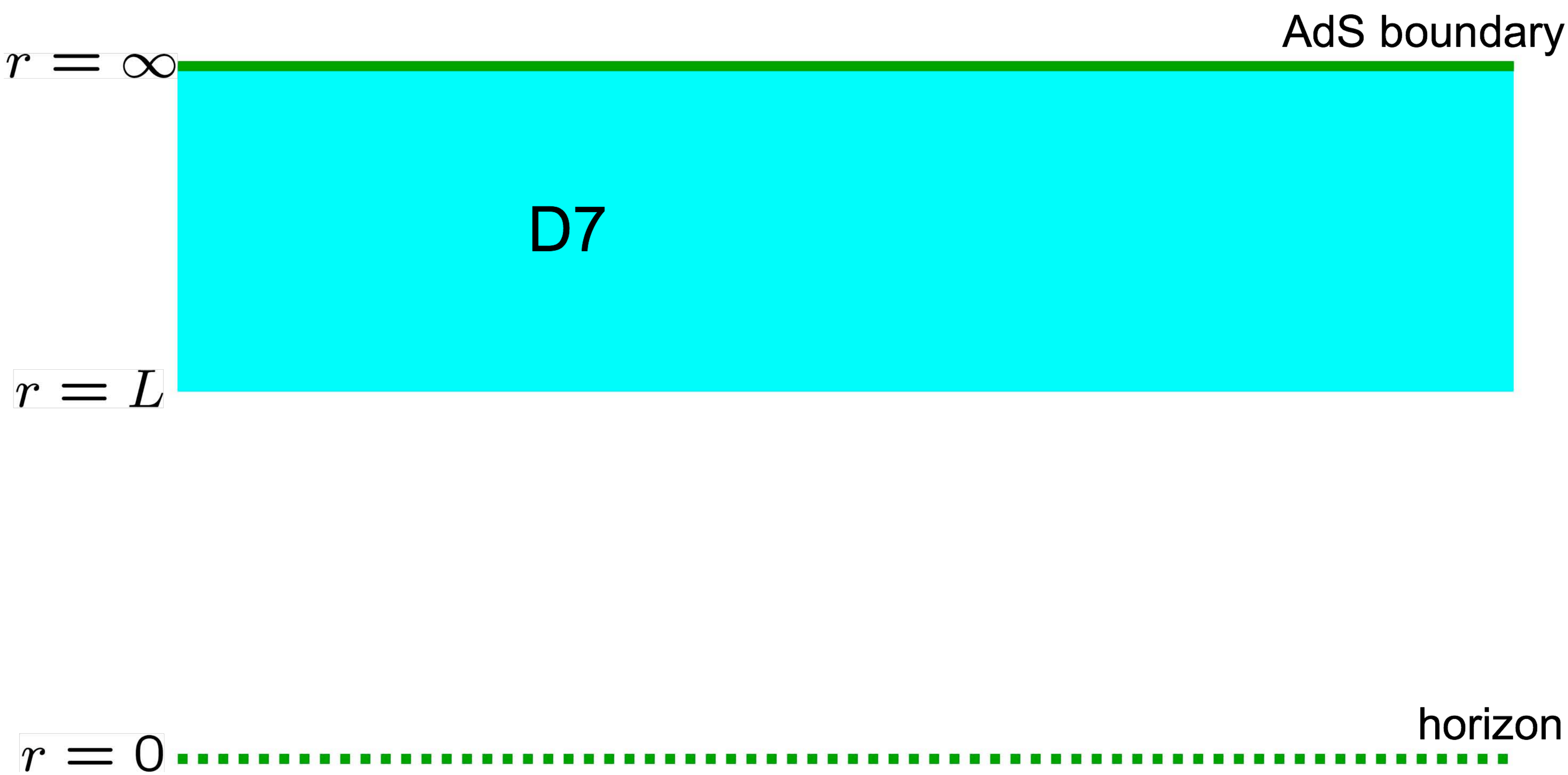
free-energy density: $f = I_E/\beta V_x = -\frac{\pi^3 R^3}{16 G_5}\frac{1}{\beta^4}$

entropy density: $s = \beta^2\partial_\beta f = \frac{\pi^3 L^3}{4G_5}\frac{1}{\beta^3} = \frac{\pi^2}{4}N^2 T^3$

Babington, Erdmenger, Evans, Guralnik and Kirsch

AdS/CFT thermodynamics with D7-branes:

put D7-branes in AdS black hole background



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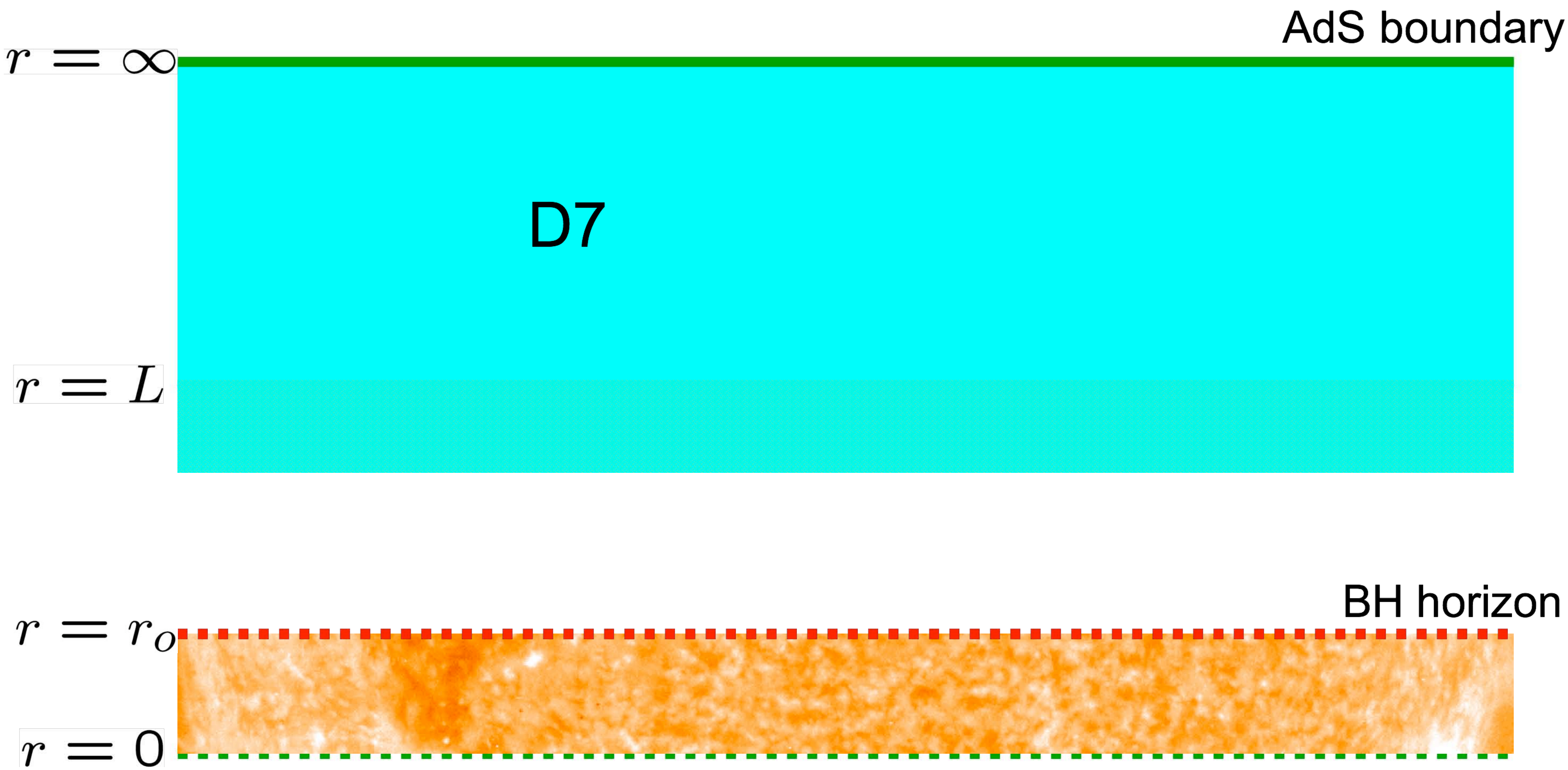

for L sufficiently larger than r_o, D7 smoothly closes at finite r

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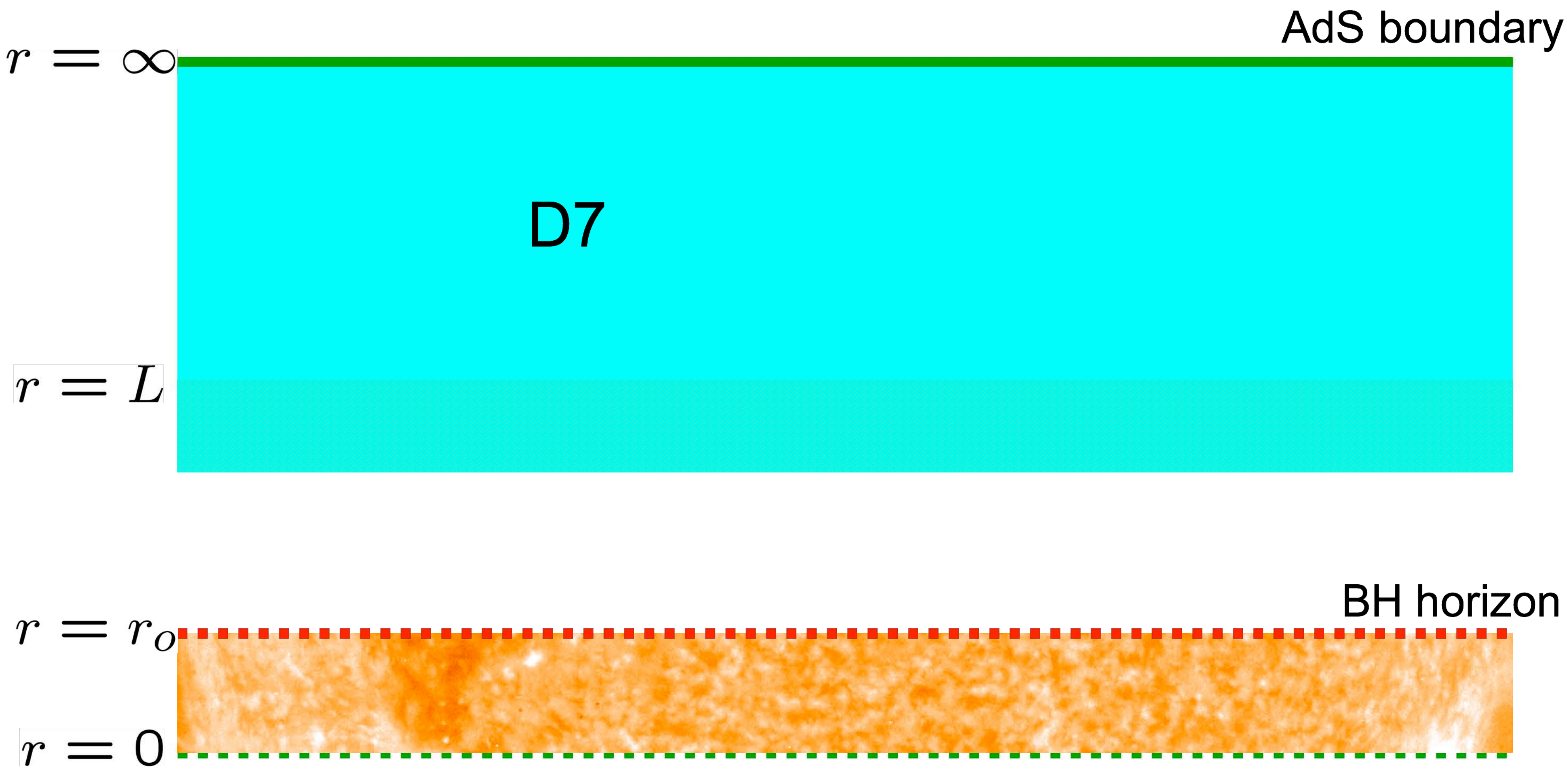

for m_q sufficiently larger than T , D7 smoothly closes at finite r

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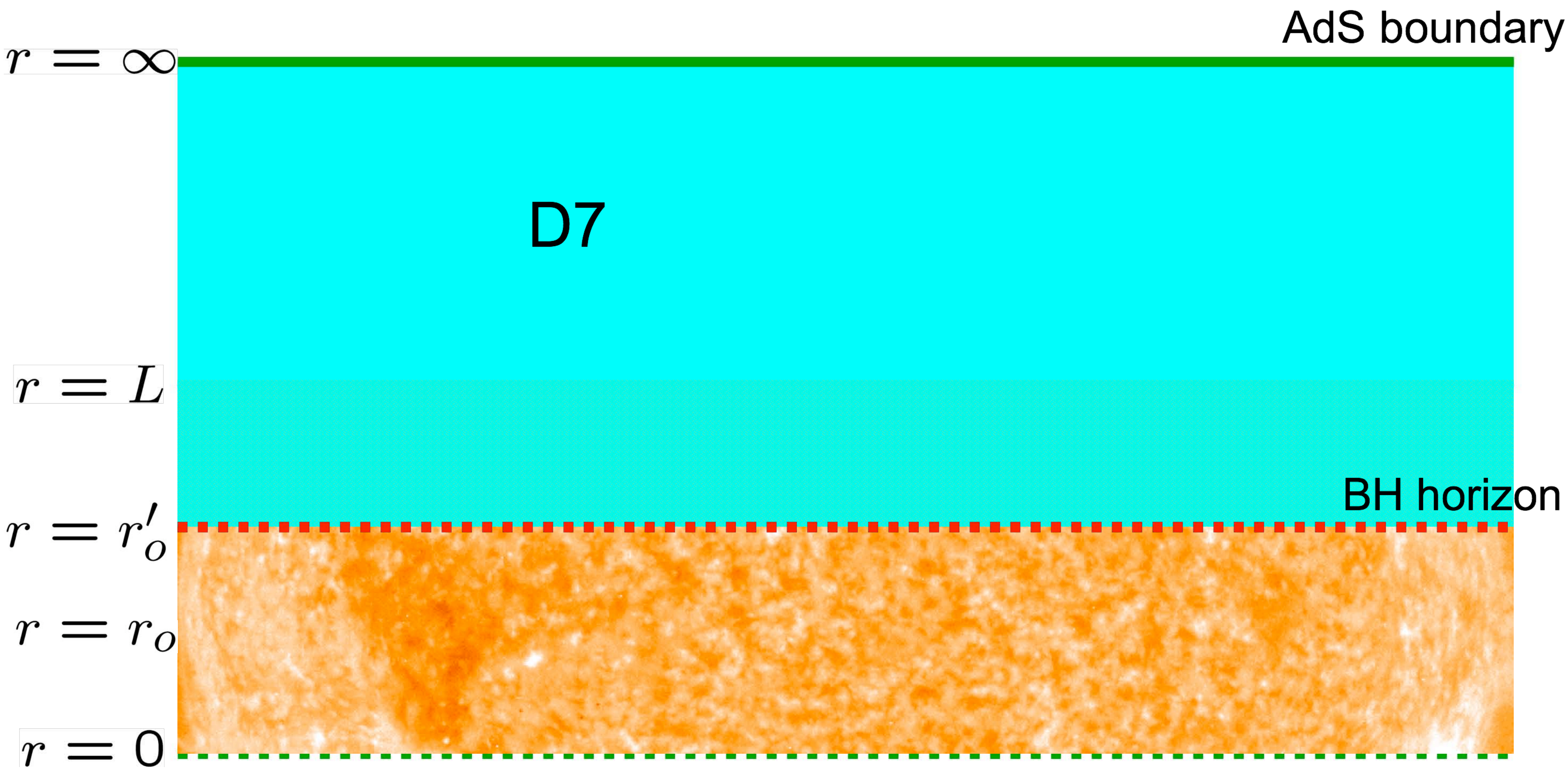

for L ≤ r_o , D7 extends into the black hole without closing off

Babington, Erdmenger, Evans, Guralnik and Kirsch

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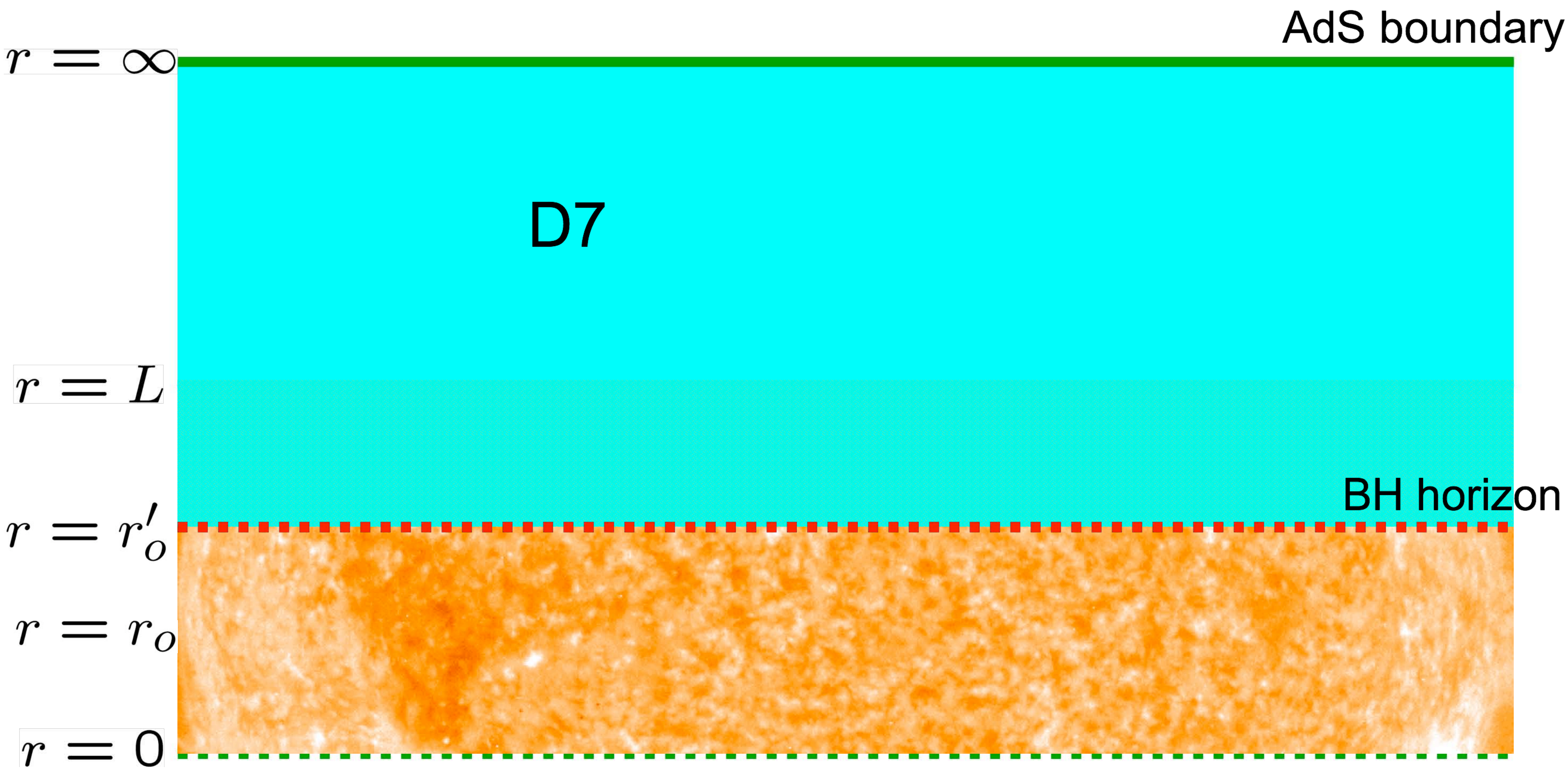

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AdS/CFT thermodynamics with D7-branes:

Thermal background:

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Thermal background:

D7 embedding: θ(r)

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AdS/CFT thermodynamics with D7-branes:

Thermal background: improved radius $\rho^2 = r^2 + \sqrt{r^2 - r_o^2}$

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For SUSY background (r_o=0), $\psi(\rho) = \frac{R_\infty}{\rho}$

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$$m_q = \frac{R_\infty}{2\pi\ell_s^2}$$

$$\langle\bar{\psi}\,\psi\rangle = -3\pi^4\ell_s^2 T_7 r_o^3\,\tilde{c} = -\frac{3\pi}{32}\sqrt{\lambda}NT^3\,\tilde{c}$$

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D7 embedding: $\psi(\rho) = \sin\theta(\rho)$

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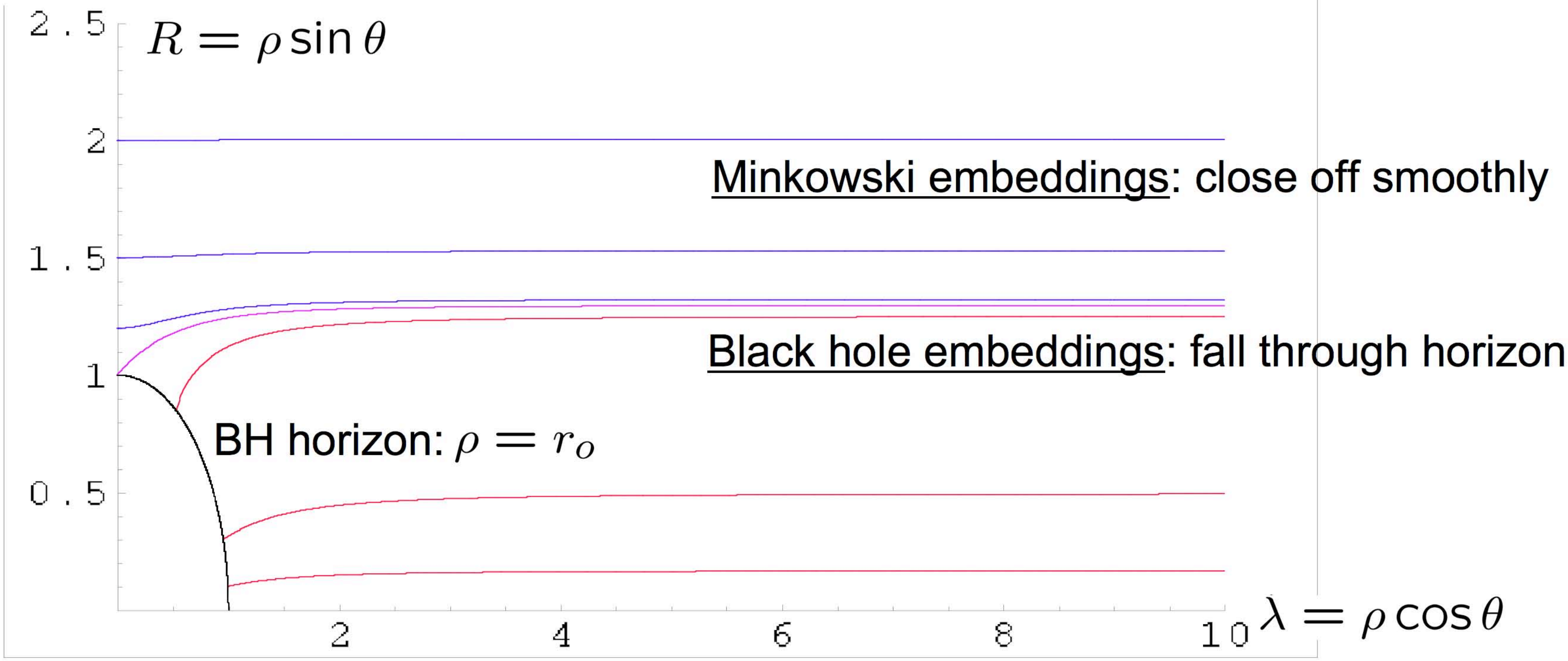
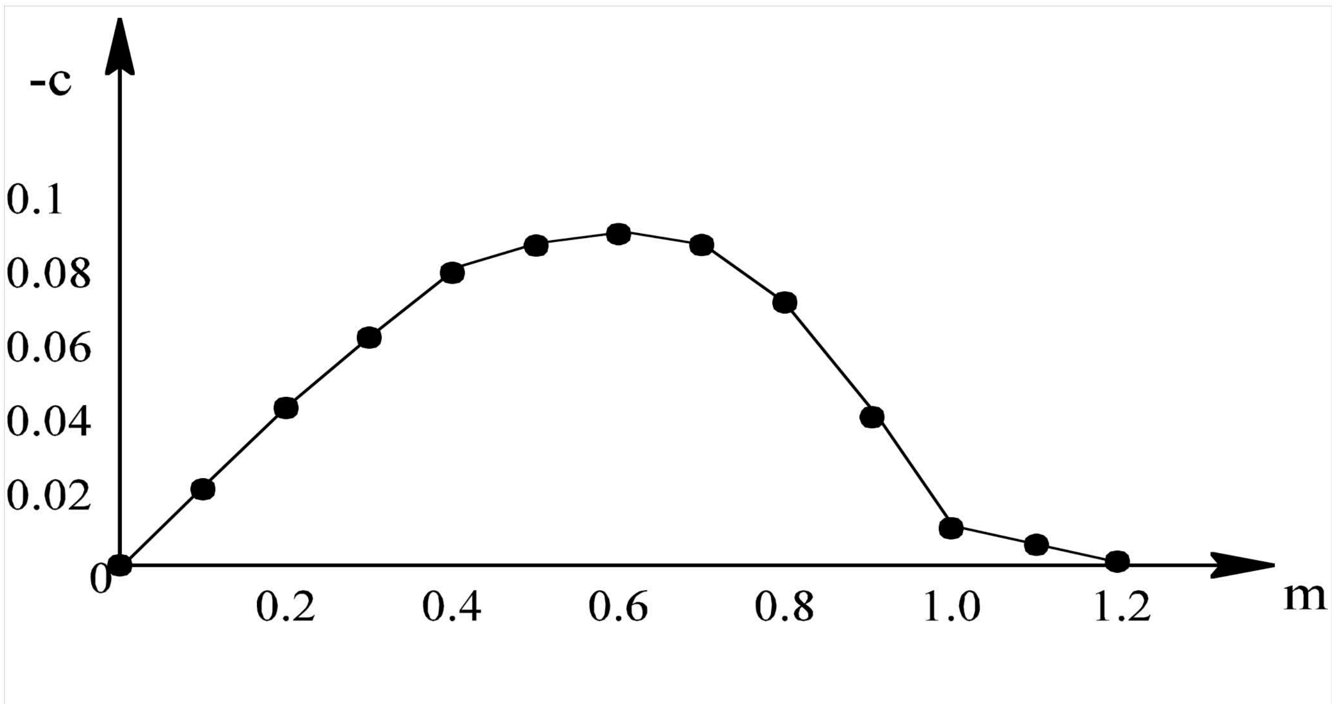
Numerical results:



Babington, Erdmenger, Evans, Guralnik and Kirsch
[arXiv:hep-th/0306018]

Thermal D3/D7 System:

Quark condensate $\langle\bar{\psi}\psi\rangle$ versus quark mass M_q/T



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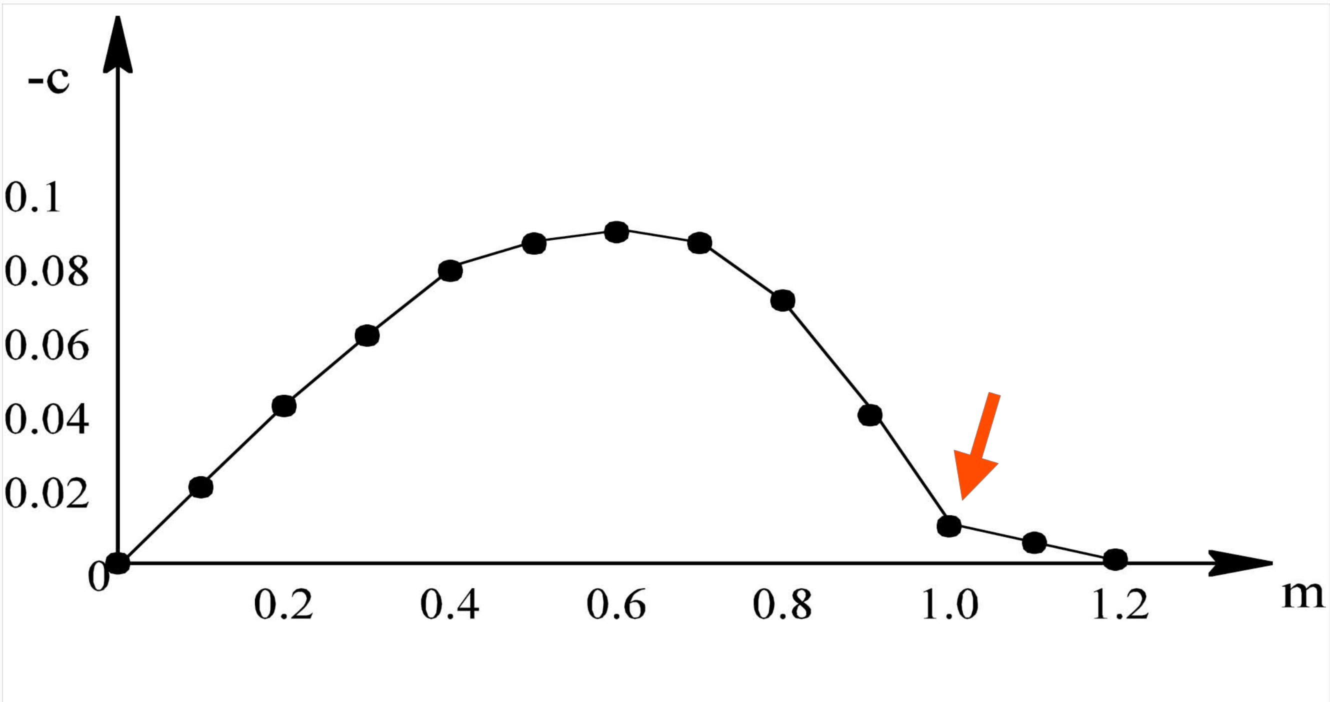

third-order transition? – needs refinement

Babington, Erdmenger, Evans, Guralnik and Kirsch
[arXiv:hep-th/0306018]

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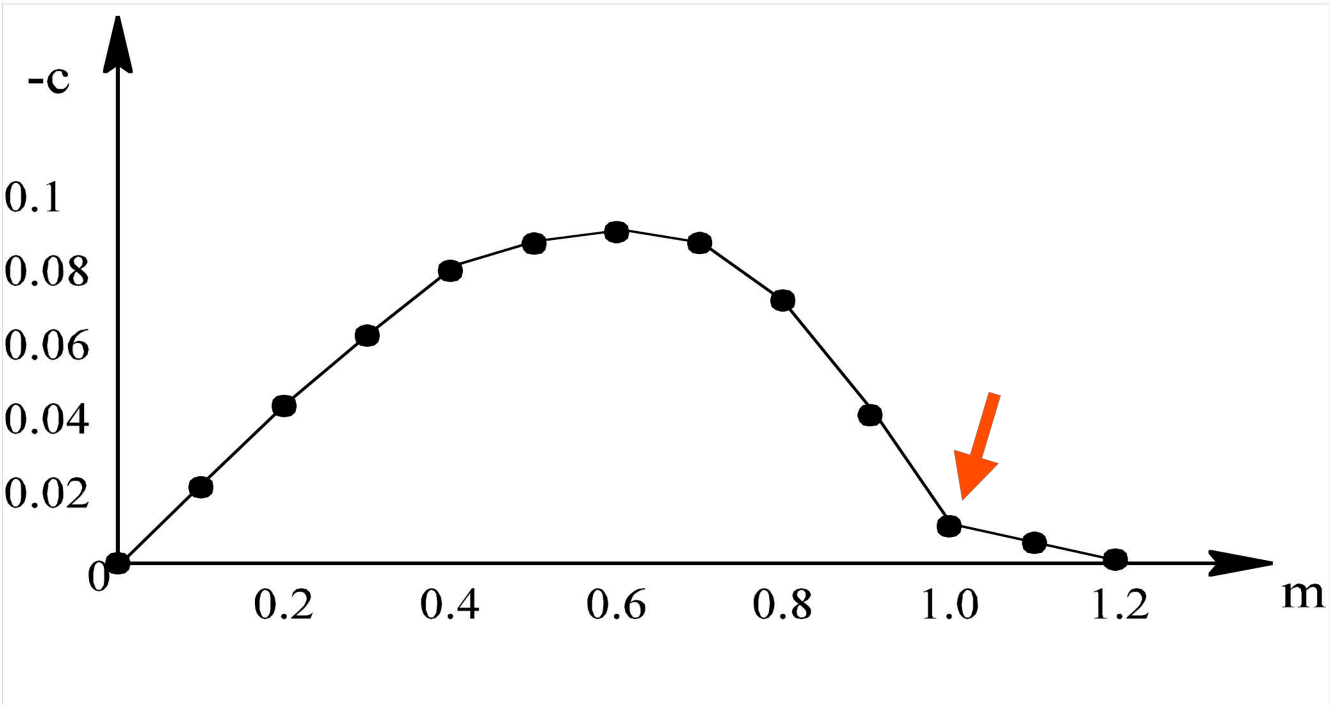


critical behaviour and first-order phase transition

Frolov, Larsen and Christensen
[arXiv:hep-th/9811148]

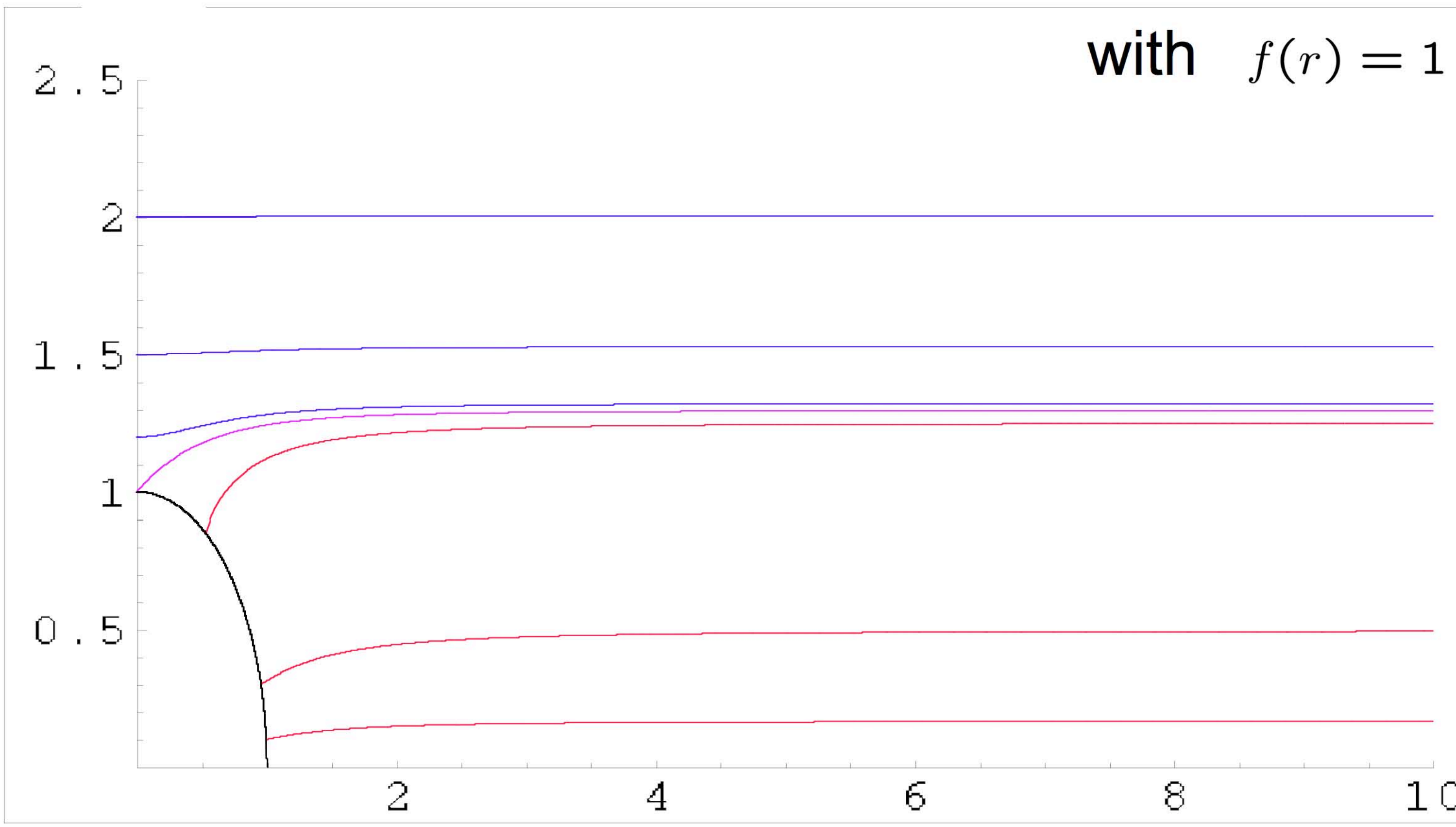
Critical Self-Similar Embeddings:

Thermal background:

D7 embedding: θ(r)

$$ds^2 = \frac{r^2}{R^2}\left(-f(r)\,dt^2 + d\vec{x}^{\,2}\right) + \frac{R^2}{r^2}\frac{dr^2}{f(r)} + R^2\left[d\theta^2 + \sin^2\theta\,d\phi^2 + \cos^2\theta\,d\Omega_3^2\right]$$

with $f(r) = 1 - \frac{r_0^4}{r^4}$



Critical Self-Similar Embeddings:

Frolov, Larsen and Christensen
[arXiv:hep-th/9811148]

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Critical Self-Similar Embeddings:

$$r = r_0 + \frac{1}{2}\kappa Z^2 \ , \quad \theta = \frac{\pi}{2} - \frac{R}{L} \ , \quad \vec{x} = \frac{L}{r_0}\vec{y}$$

with $\kappa = 2\frac{{r_0}^2}{R^4}$

$$ds^2 = -\kappa^2 Z^2\, dt^2 + dZ^2 + dR^2 + R^2\, d\Omega_3^2 + d\vec{y}^2 + L^2\, d\phi^2$$

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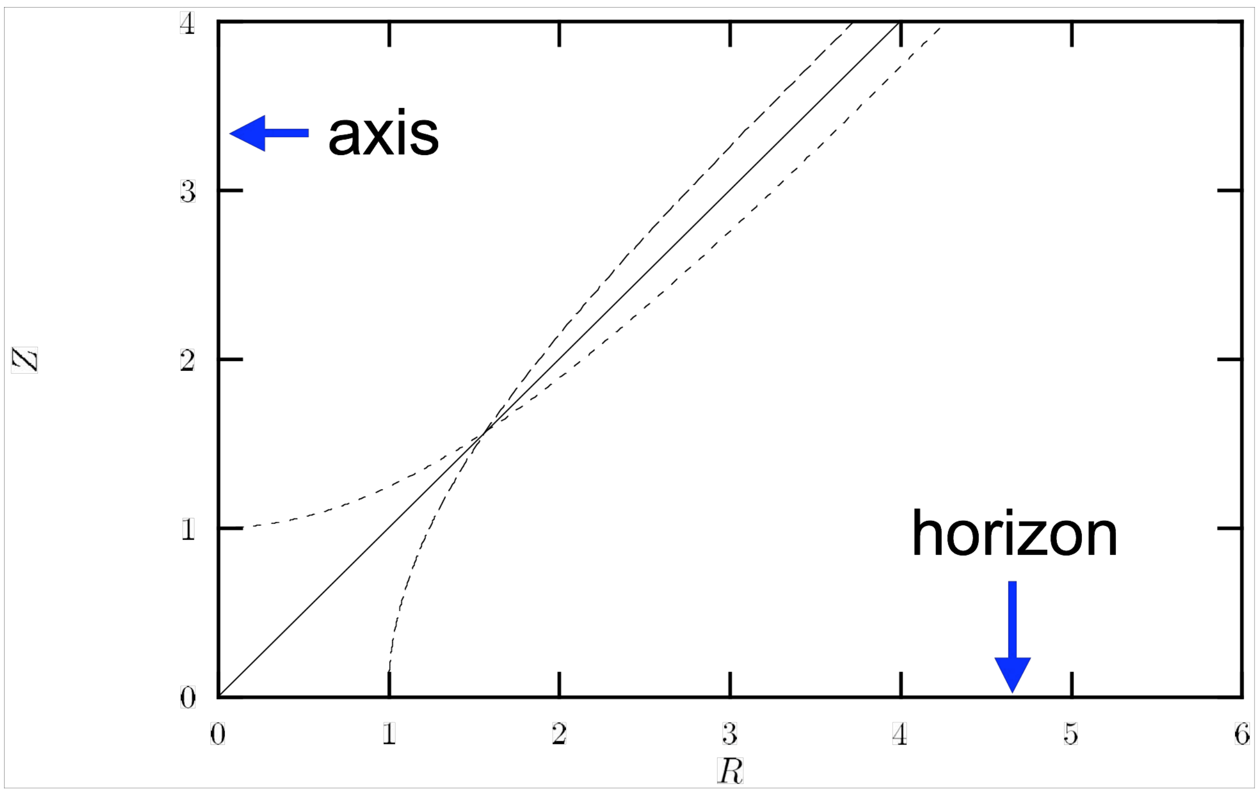

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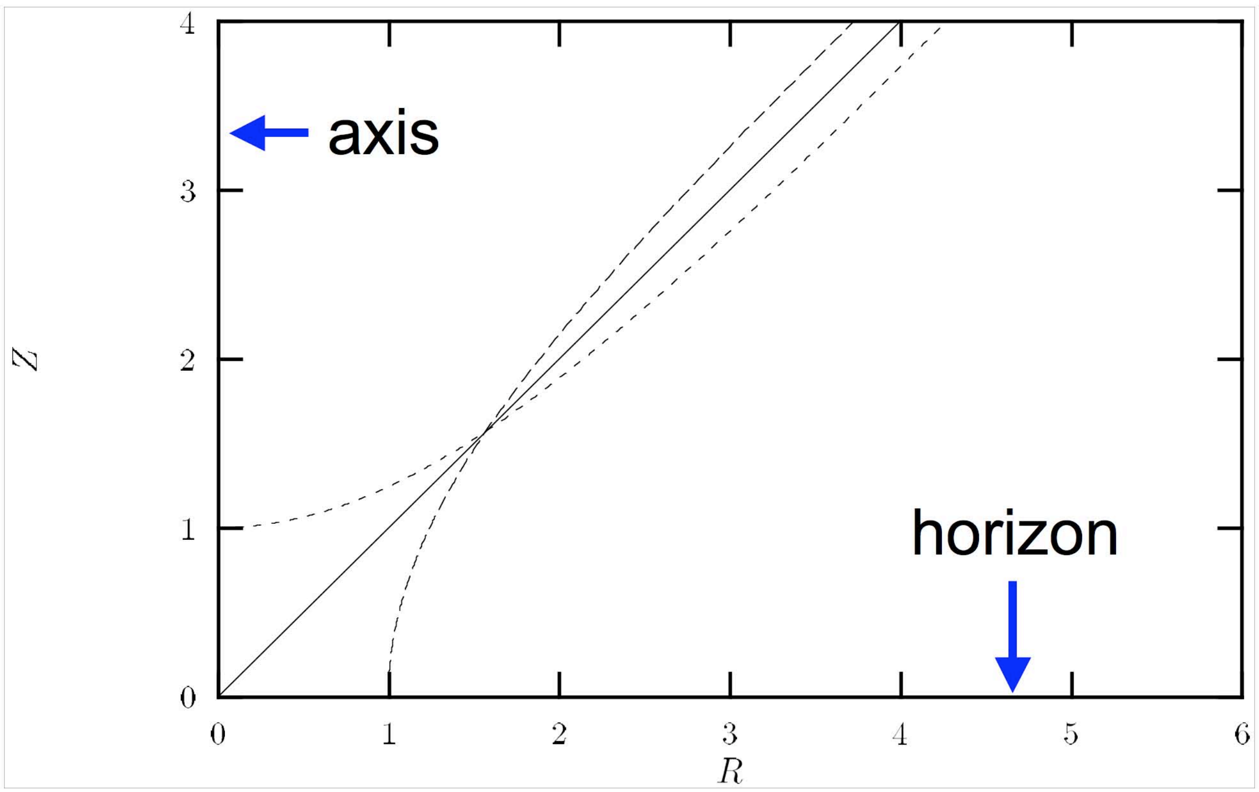
D7 embedding: R(Z)



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embedding eqn: $$Z\,R\,R'' + (R\,R' - 3\,Z)\left(1 + R'^2\right) = 0$$

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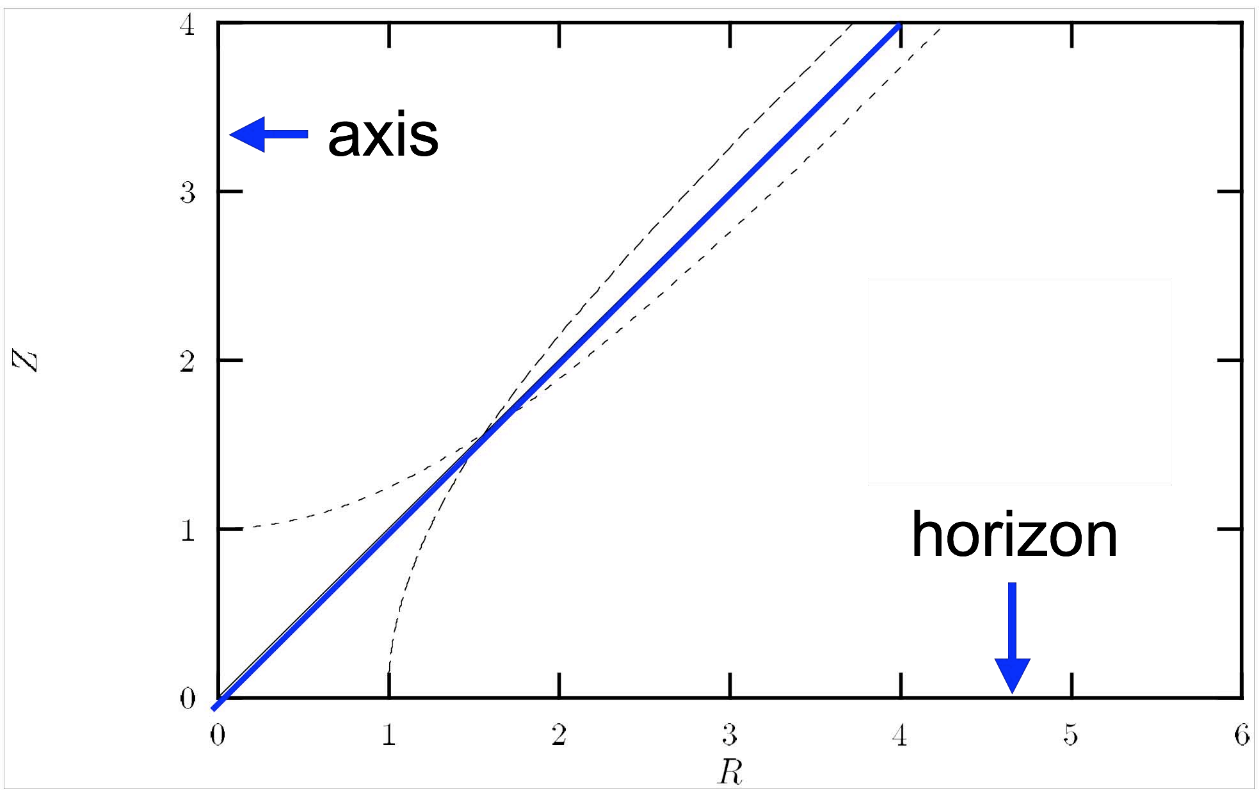

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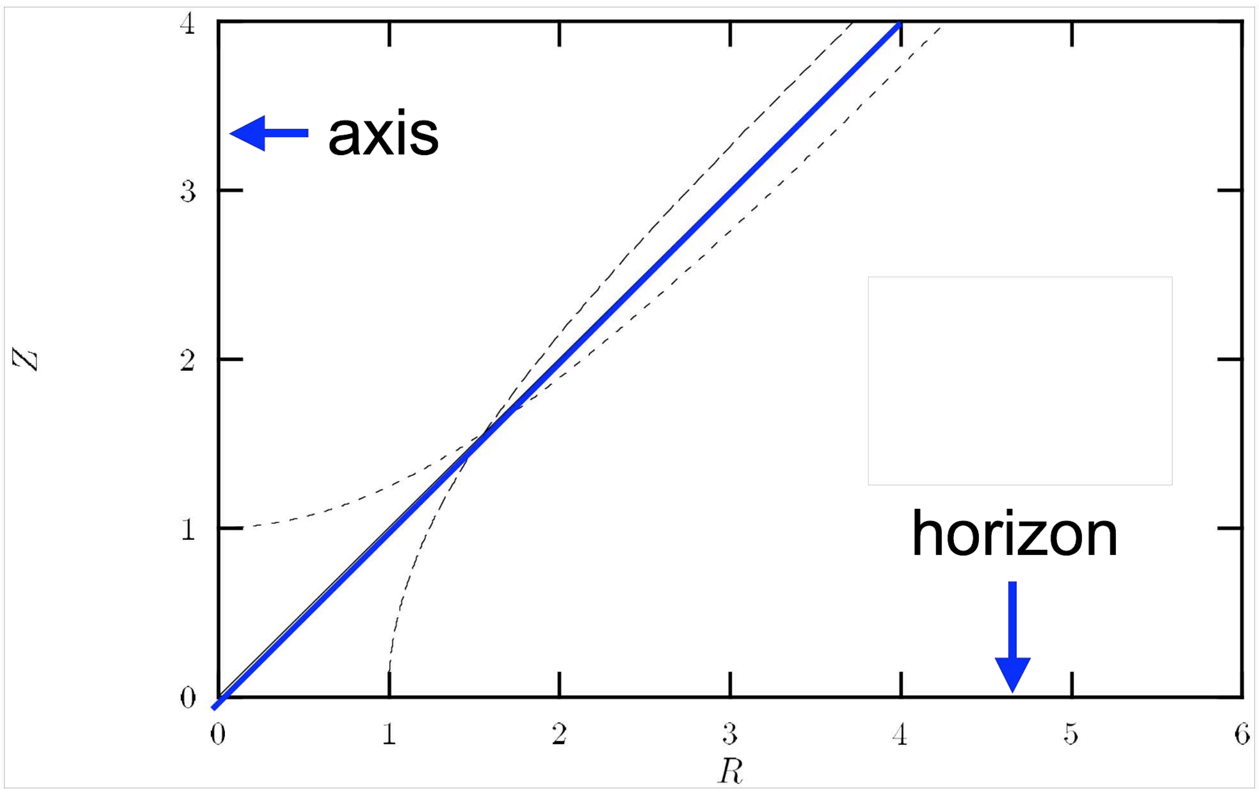

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attractor: linearize $R = \sqrt{3}Z + p(Z)$ (far from *Z=0*)

$$Z^2\,p'' + 4Z\,p' + 4\,p = 0$$

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fixing fiducial $\widetilde{R}_0 = 1$, then $\lambda = 1/R_0$

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asymptotic parameters: $\begin{pmatrix} \delta m_q \\ \delta c \end{pmatrix} = \mathcal{A}\begin{pmatrix} a \\ b \end{pmatrix}$

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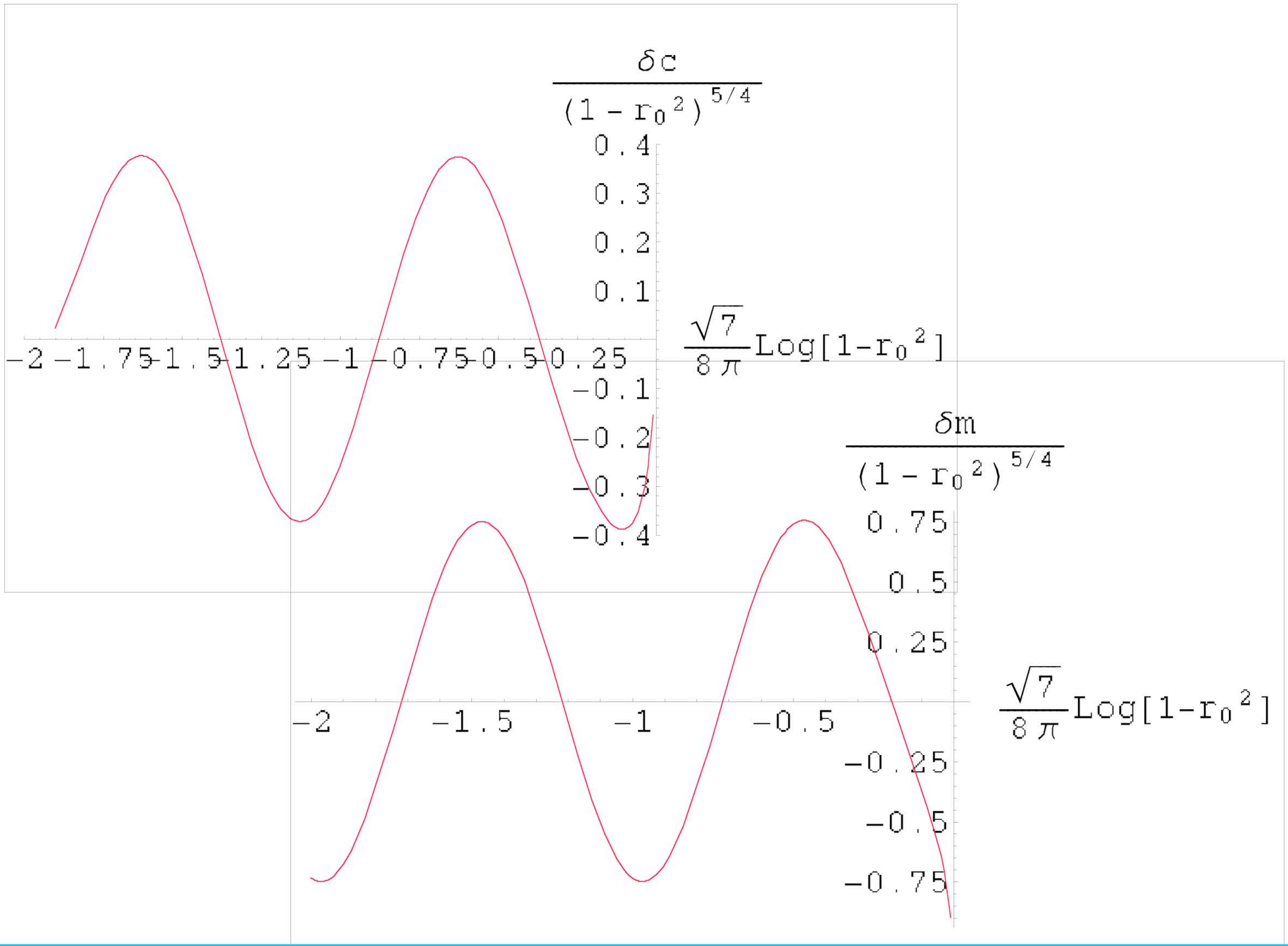
$R_0^{-5/2}\,\delta m_q$, $R_0^{-5/2}\,\delta c$ are periodic functions of $\sqrt{7}/4\pi\log R_0$



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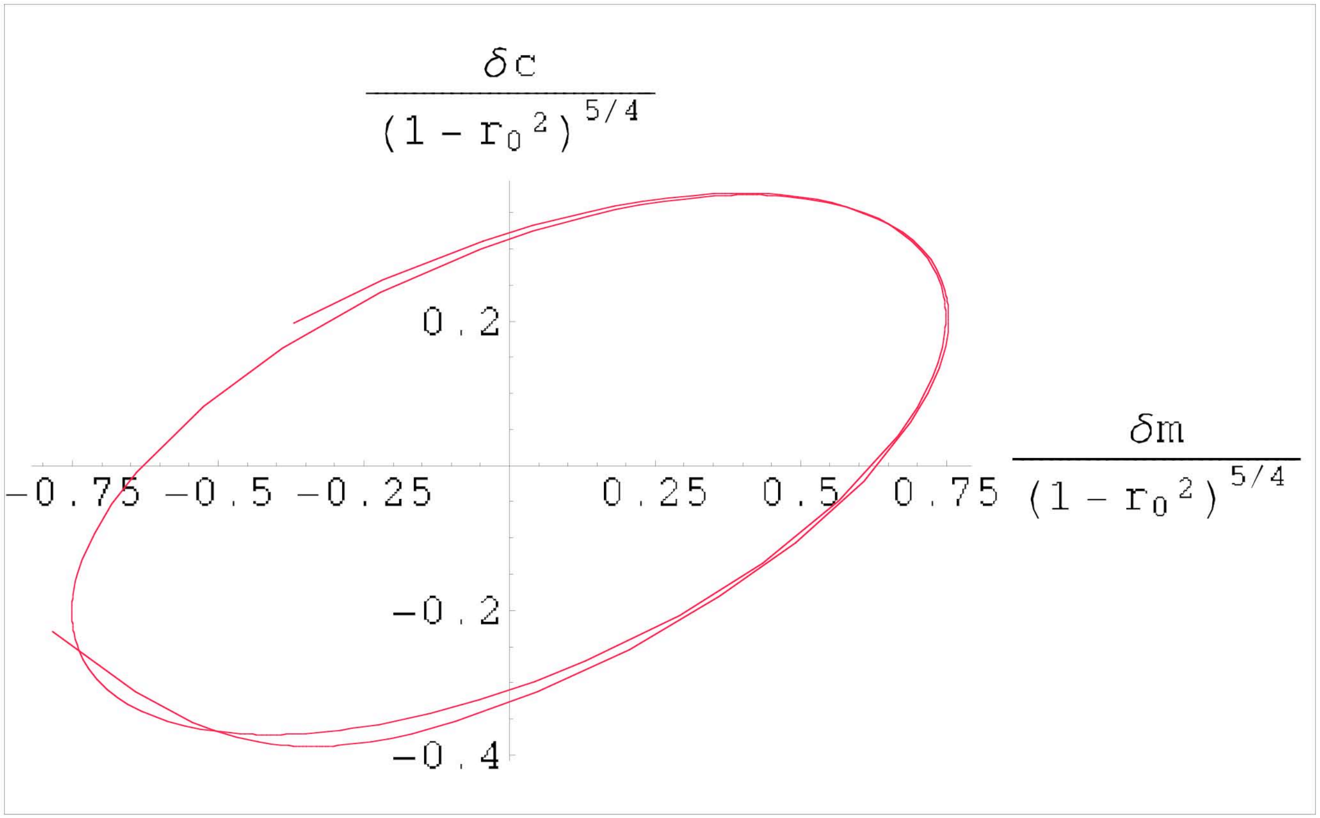

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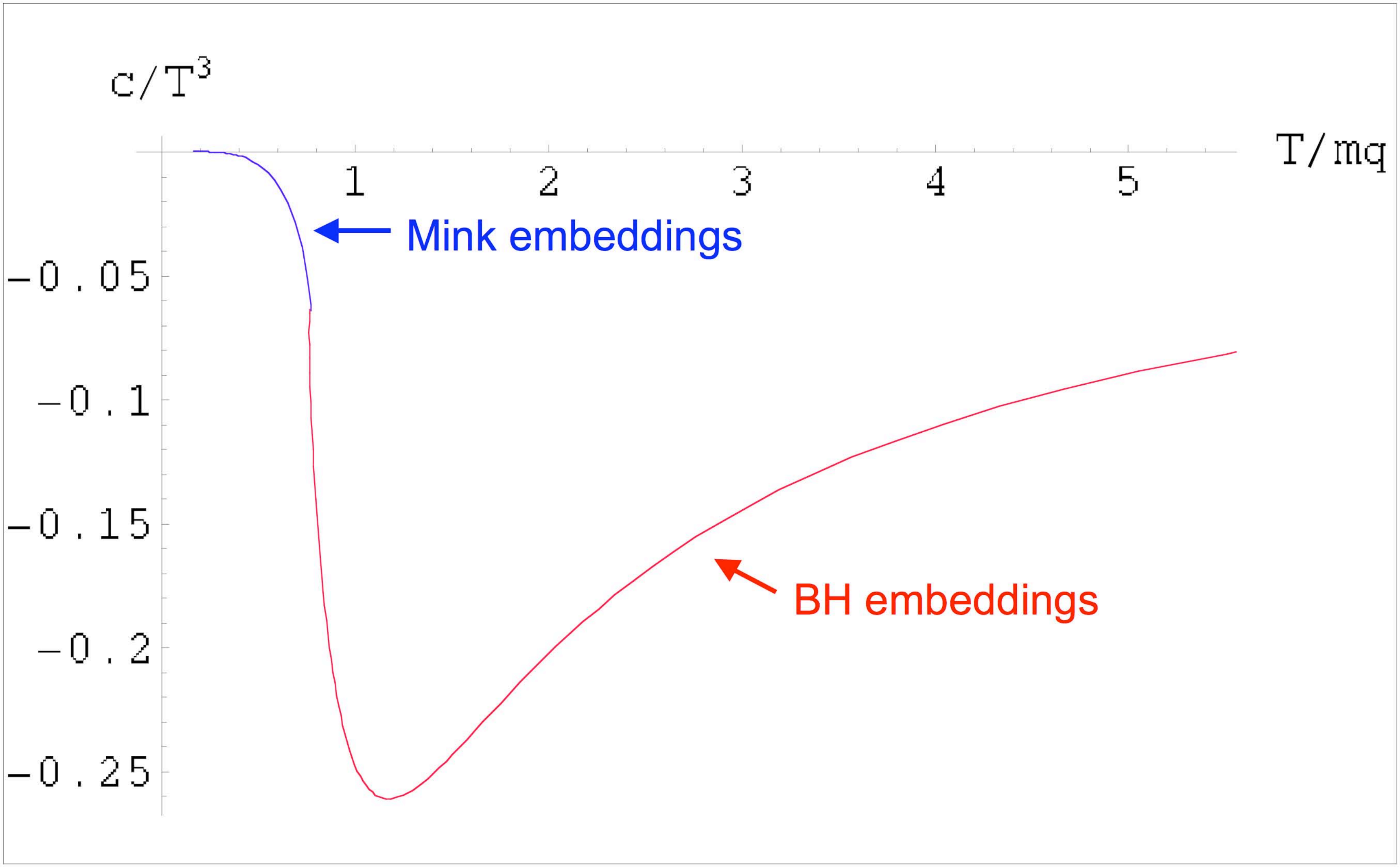
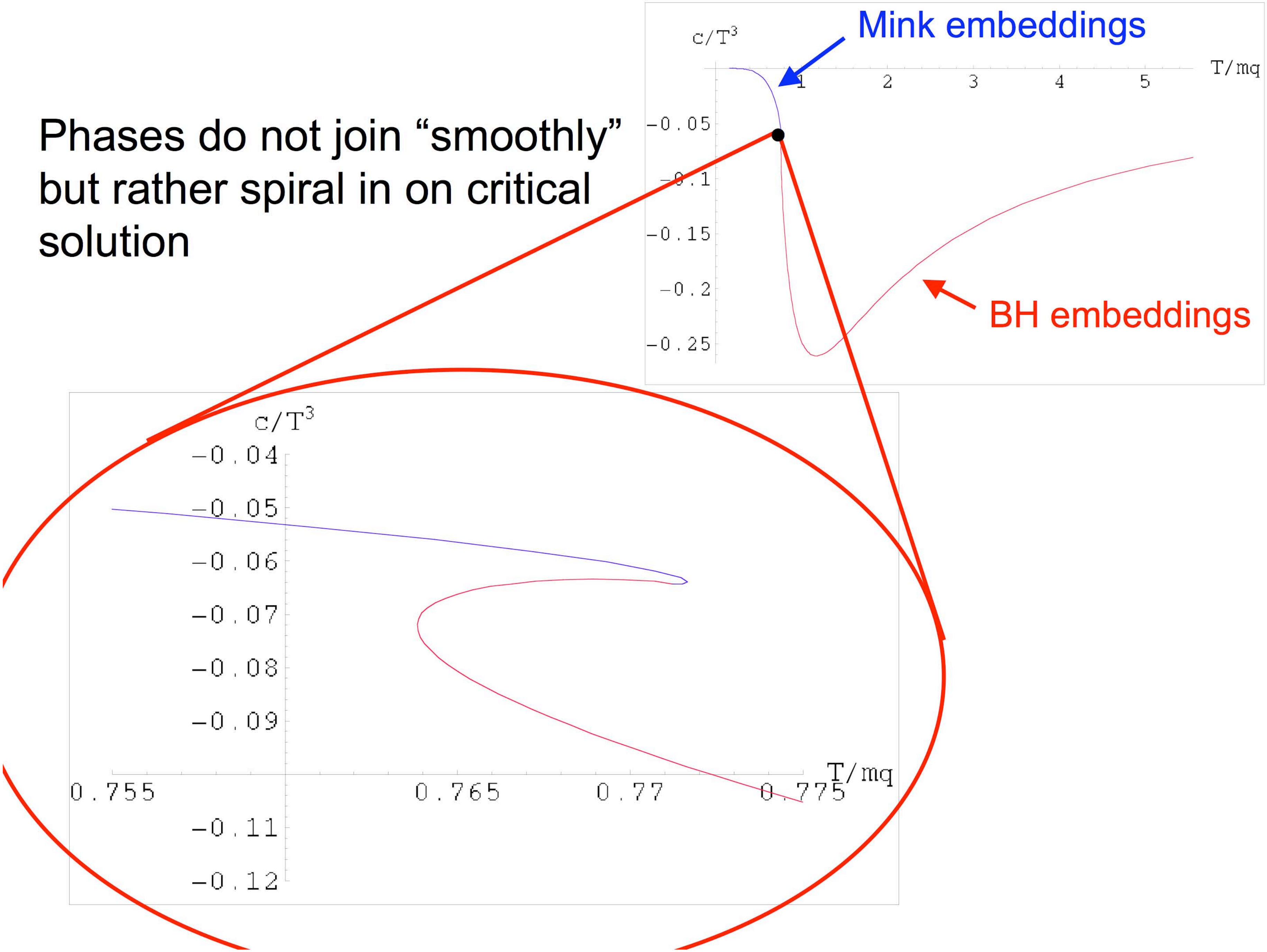


c/T^3
T/mq
1
2
3
4
5
-0.05
-0.1
-0.15
-0.2
-0.25
Mink embeddings
BH embeddings

Phases do not join "smoothly" but rather spiral in on critical solution



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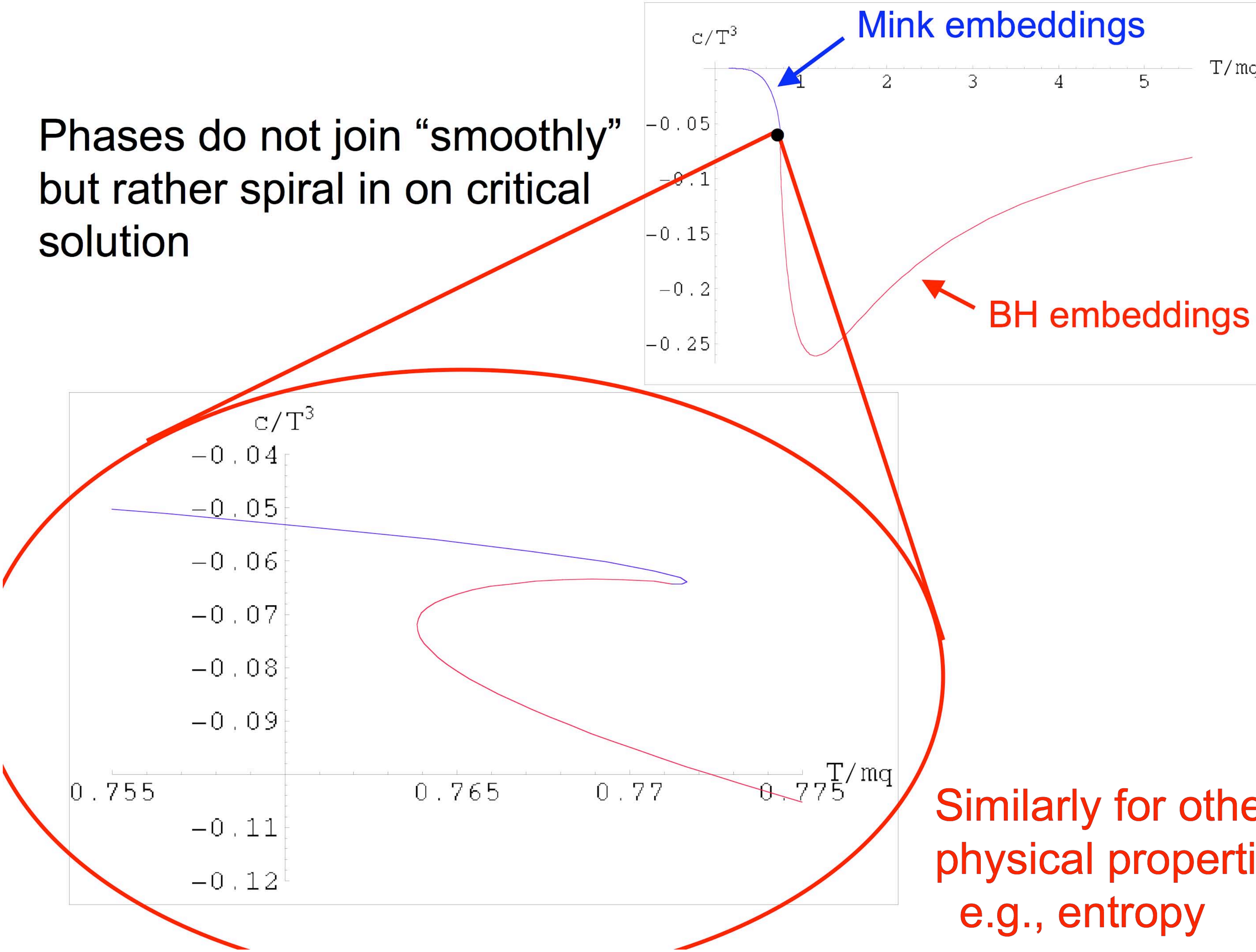

Similarly for other physical properties!
e.g., entropy

Brane Thermodynamics:

- calculate D7-brane contributions to thermal quantities using same Euclidean action techniques

$$I_{\mathrm{D7}} = T_7 \int d^8\sigma \sqrt{\det P[G]_{ab}}$$

$$-\frac{\pi^2}{2} R^4 T_7 \int d^4\sigma \sqrt{\gamma} \left(1 - 2\psi^2 + \psi^4\right)$$

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need boundary counter-terms!

(Karch, O'Bannon and Skenderis)

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$$\frac{I_{\rm D7}}{V_x} = A\, G\left(\frac{R_\infty}{r_o}\right) - \frac{A}{4}\left[\left(\frac{\rho_{min}^2}{r_o^2} - \frac{R_\infty^2}{r_o^2}\right)^2 - 4\frac{R_\infty}{r_o}\tilde{c}\right]$$

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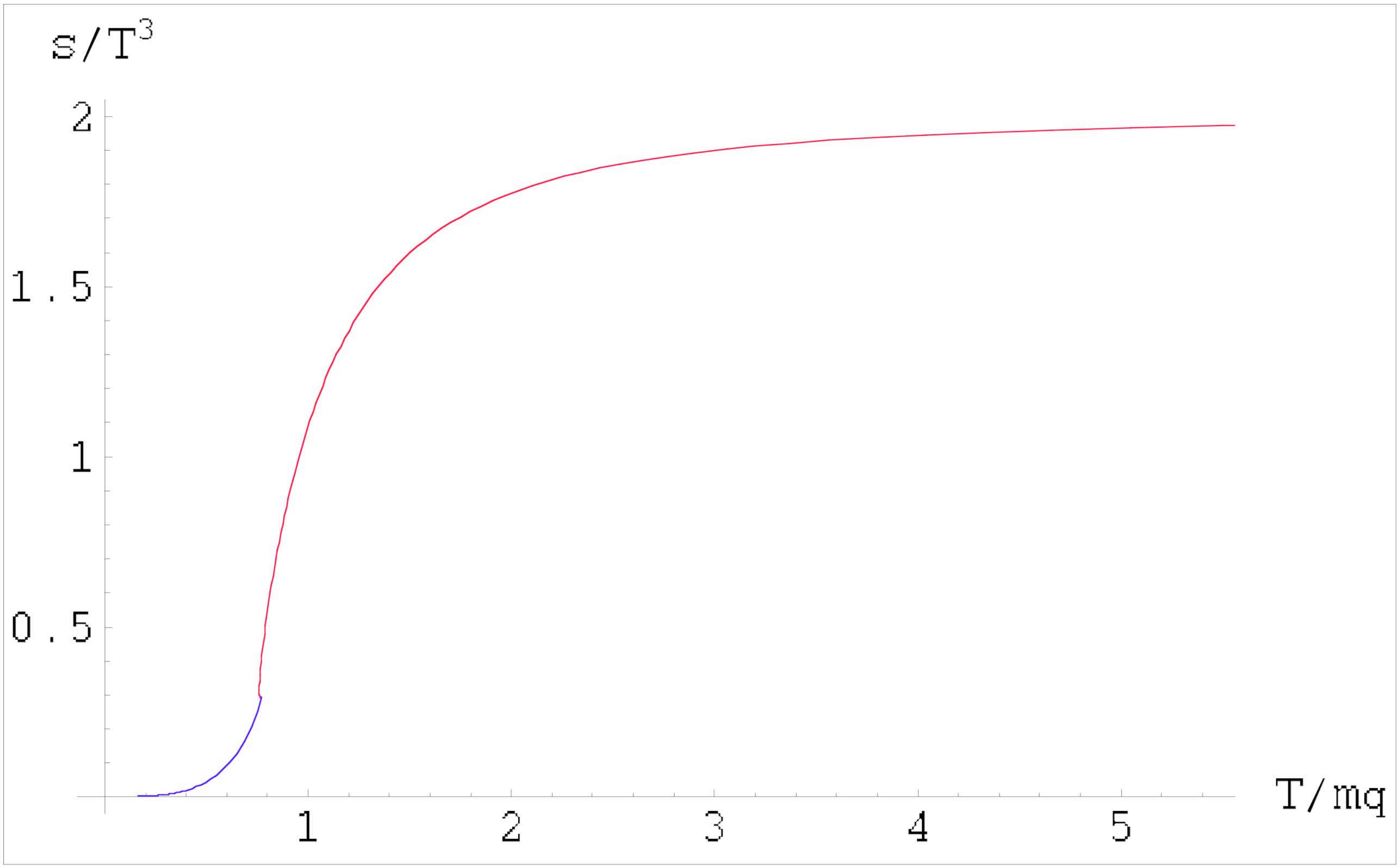
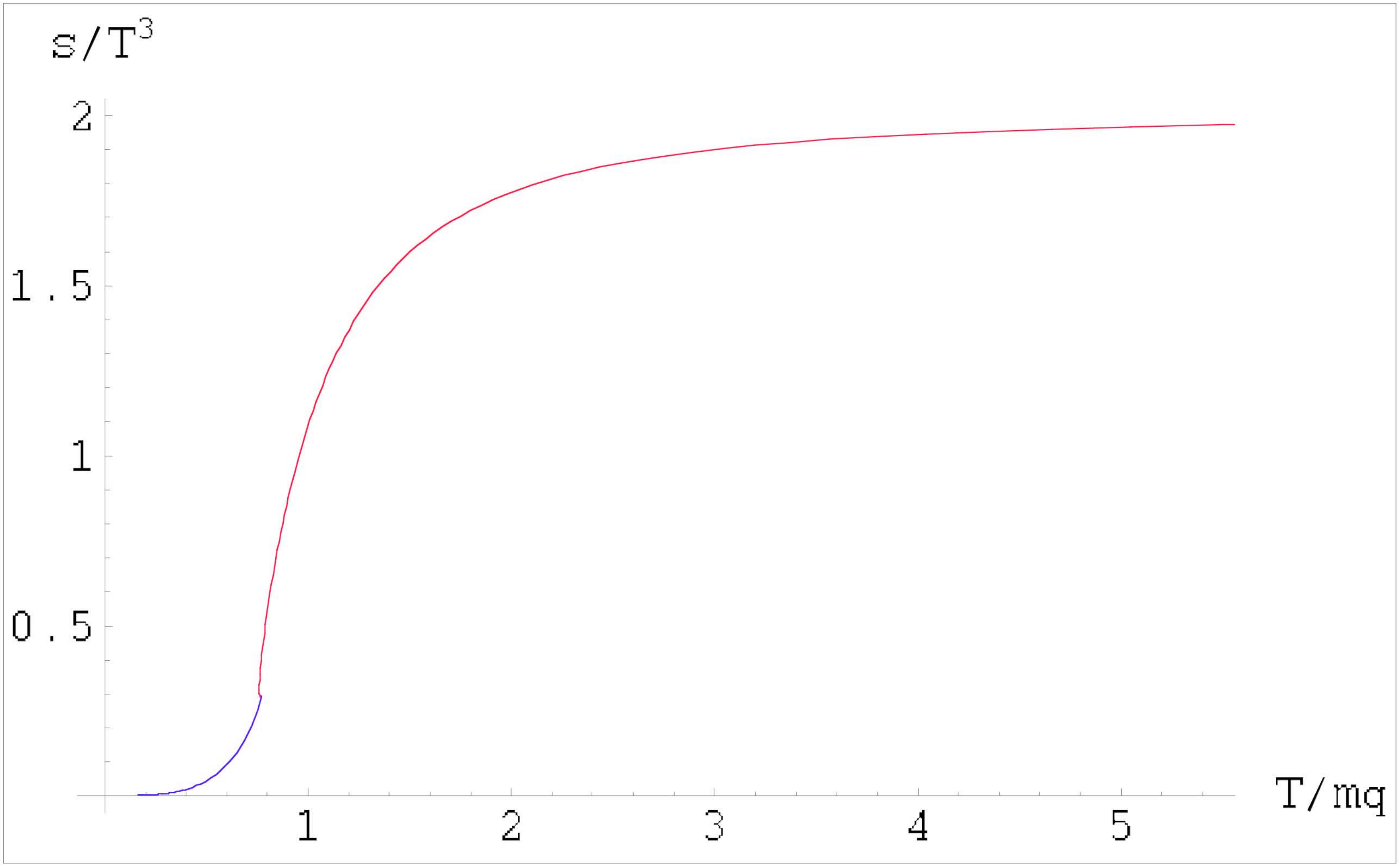
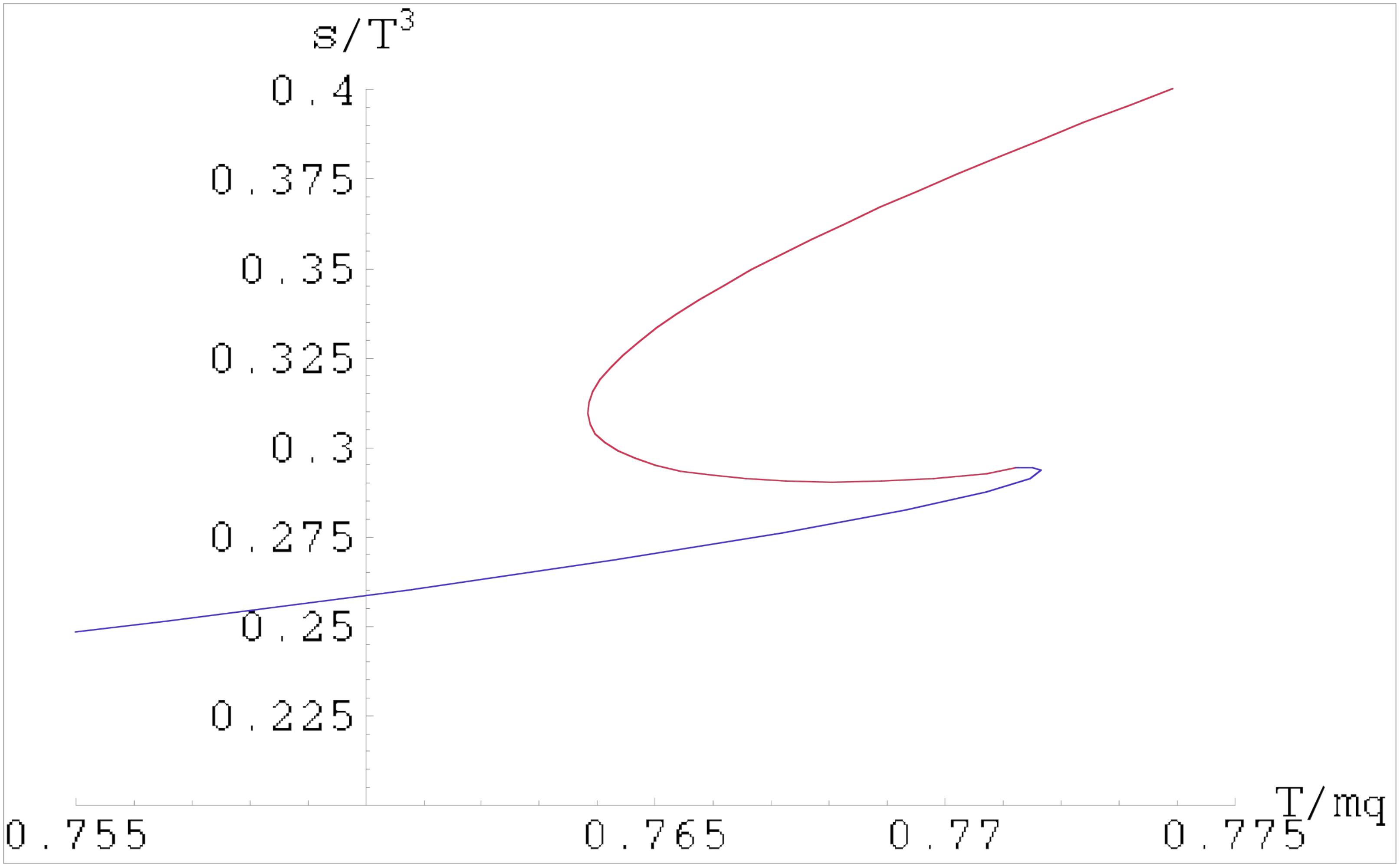


s/T^3
2
1.5
1
0.5
1
2
3
4
5
T/mq

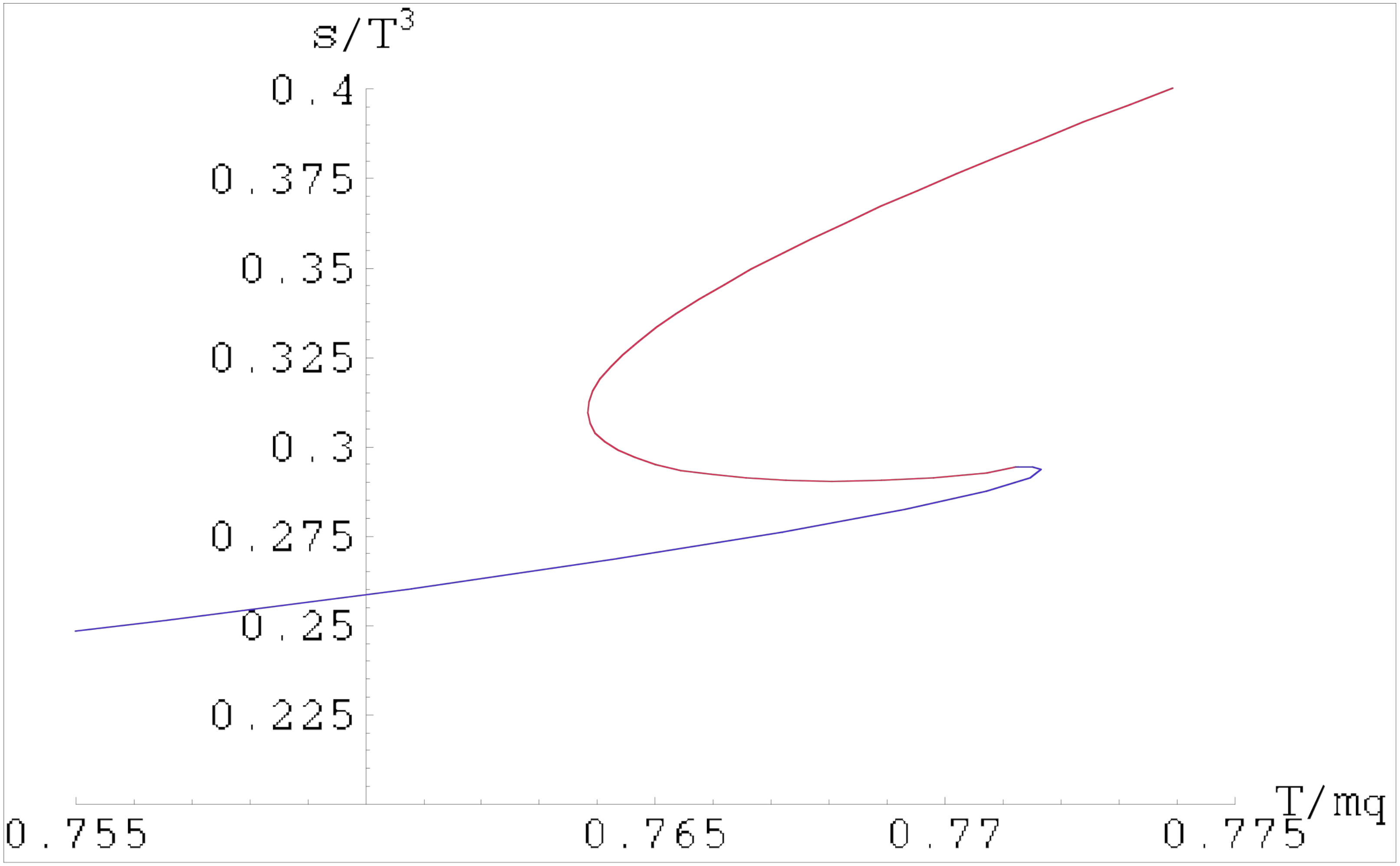
Brane entropy:



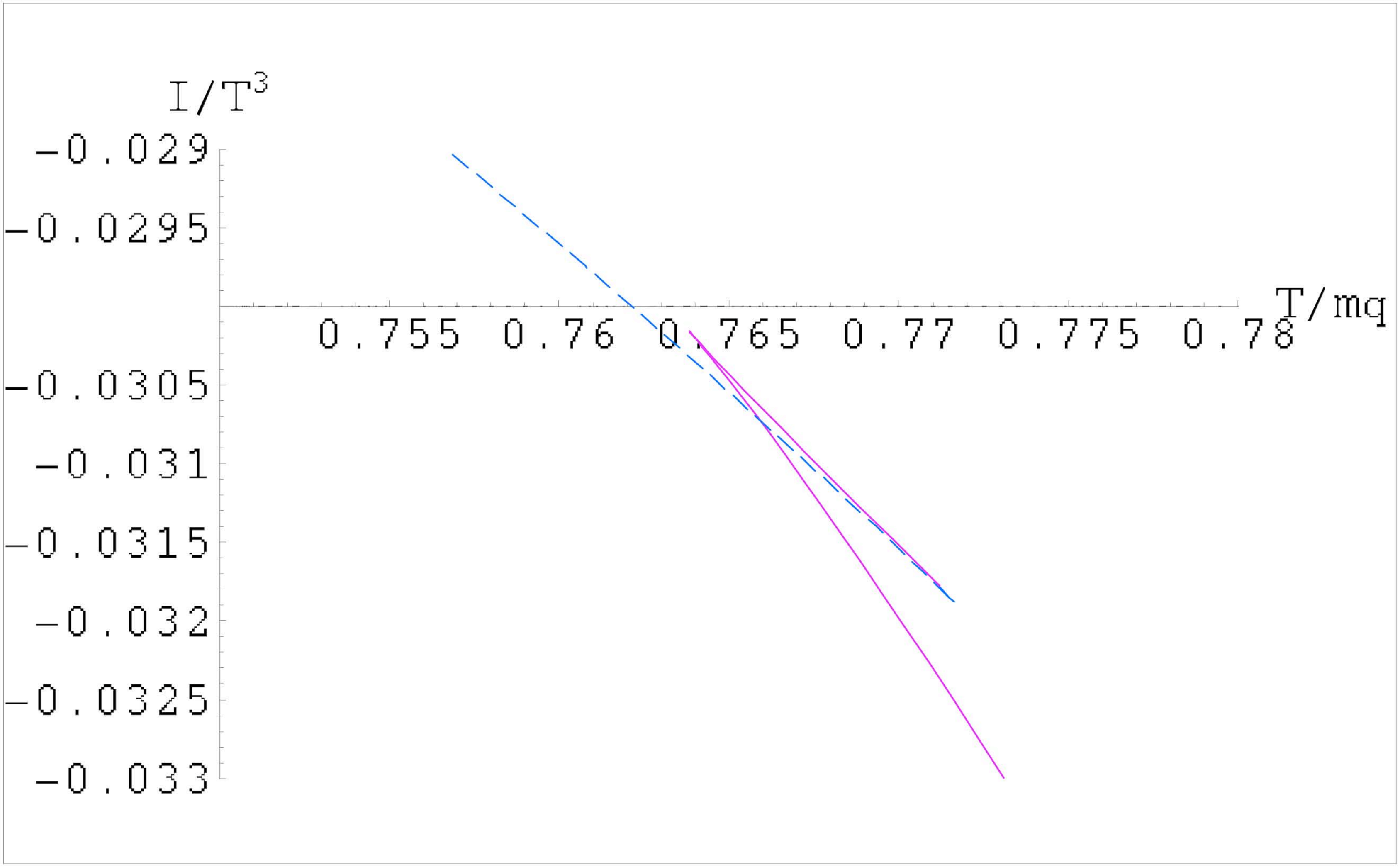


s/T^3
0.4
0.375
0.35
0.325
0.3
0.275
0.25
0.225
T/mq
0.755
0.765
0.77
0.775

Brane entropy:



Action density:



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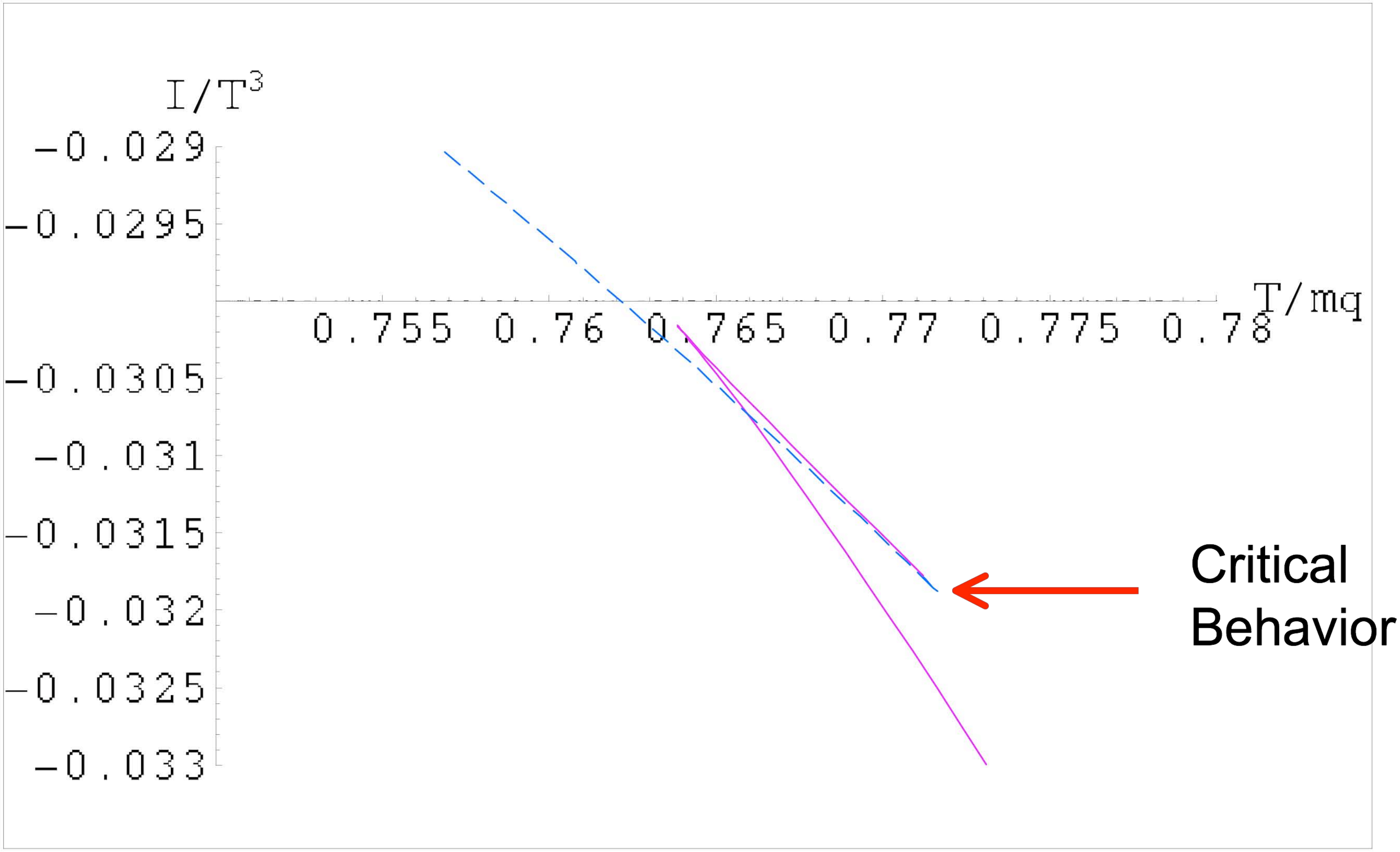

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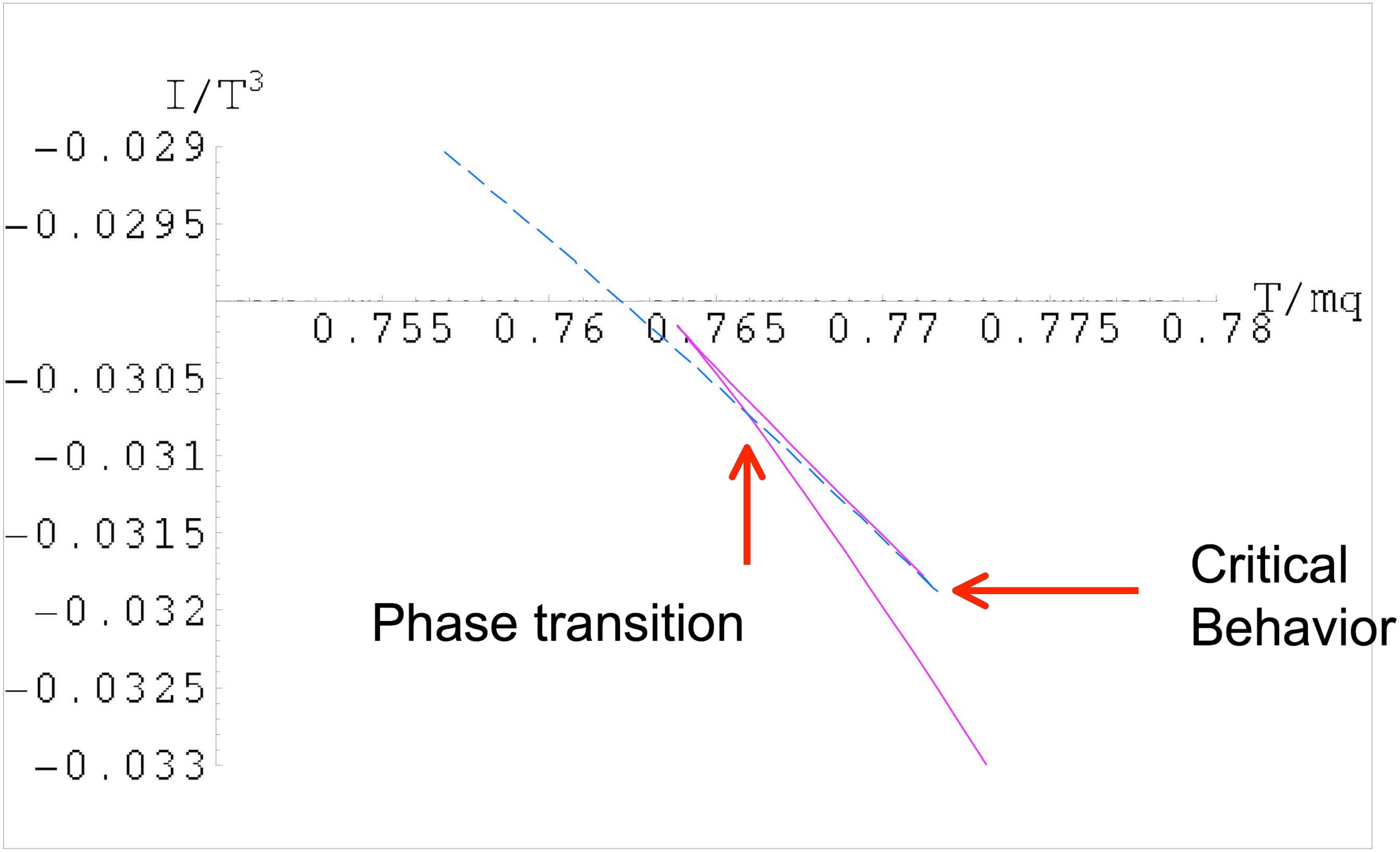
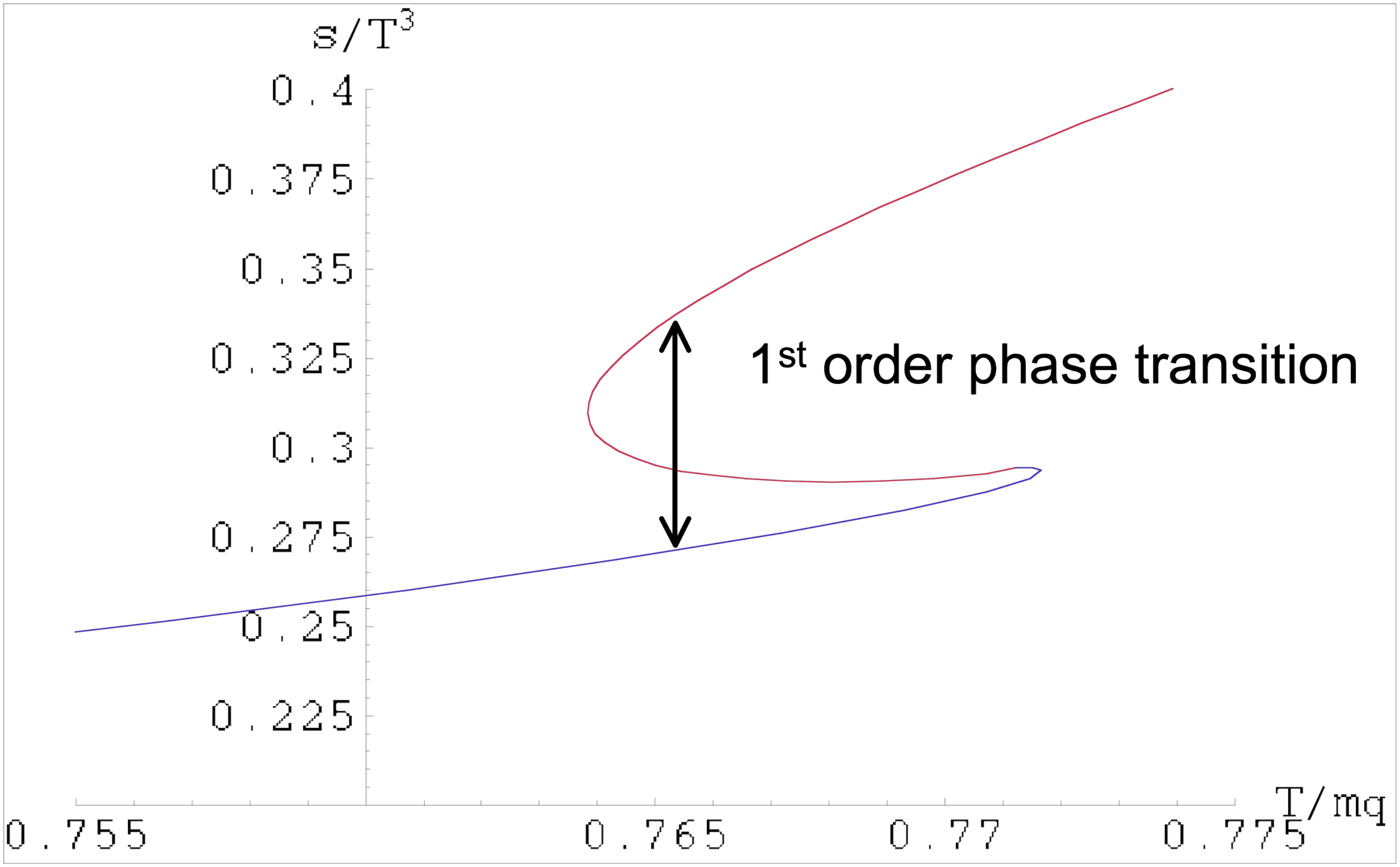
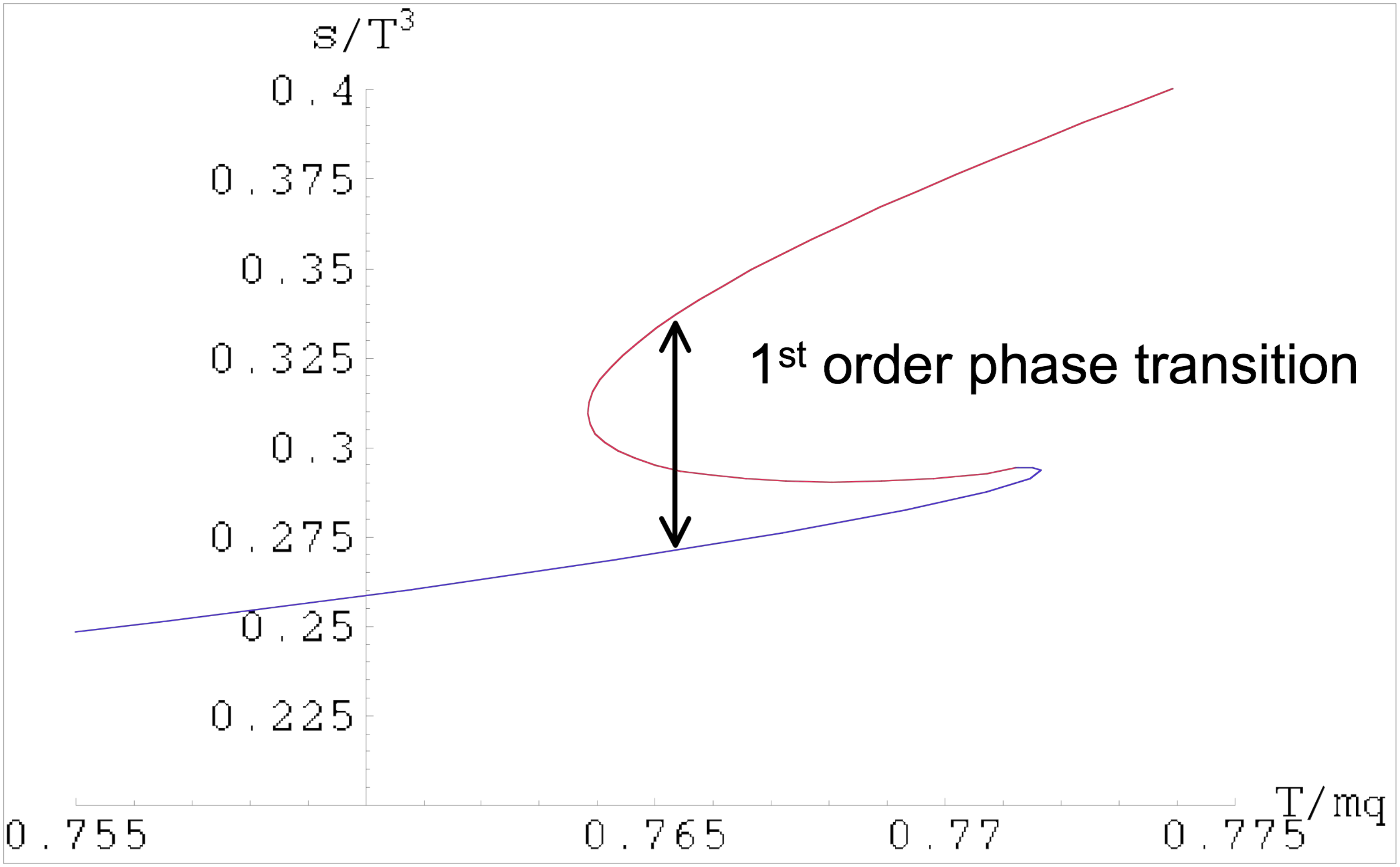



s/T^3
0.4
0.375
0.35
0.325
0.3
0.275
0.25
0.225
0.755
0.765
0.77
0.775
T/mq
1st order phase transition

Brane entropy:



Conclusions/future directions:

- Self-similar embeddings are universal for probe branes in black hole bkgd's (eg, D4/D6 "same")

  → universal feature for many gauge/gravity dualities with fundamental matter

- first order phase transition also seems universal
- probing self-similar region??
- how robust are these features?

  → finite N_f/N_c? Hawking radiation?

- (Brane) Entropy = (Horizon) Area ?? (okay)
- viscosity: $\eta/s = 1/4\pi$?? (work in progress)