M-THEORY AND THE STRING GENUS EXPANSION

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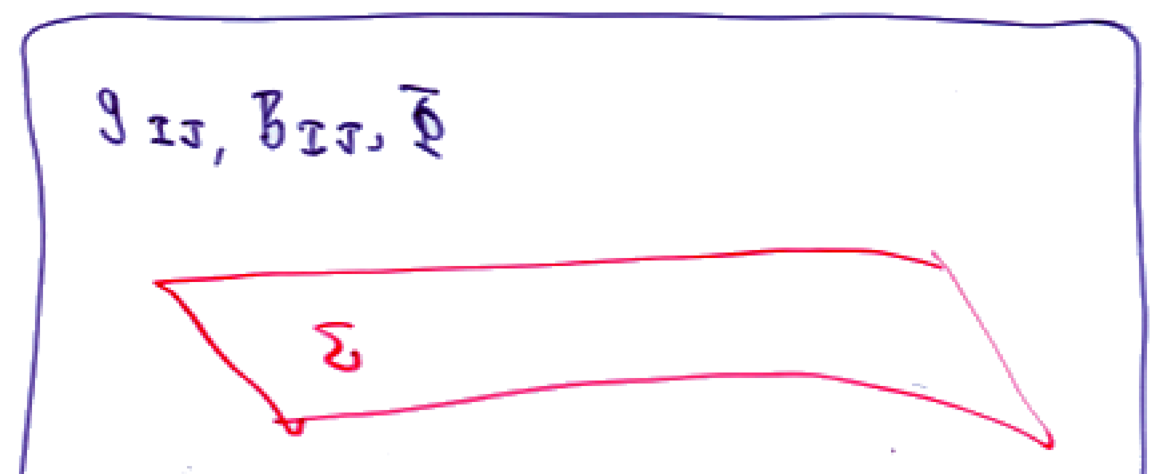
IIA String

$$I = \frac{1}{4\pi\alpha'} \int_\Sigma d^2\sigma \, \tilde{\gamma}^{1/2} \Big[\tilde{\gamma}^{ij} \partial_i X^I \partial_j X^J g_{IJ} + \tilde{\epsilon}^{ij} \partial_i X^I \partial_j X^J B_{IJ} + \cdots + \alpha' \, {}^{(2)}R(\tilde{\gamma}) \, \Phi \Big]$$

Fradkin-Tseytlin term



$\Phi = \text{const}$

Then Σ is weighted by

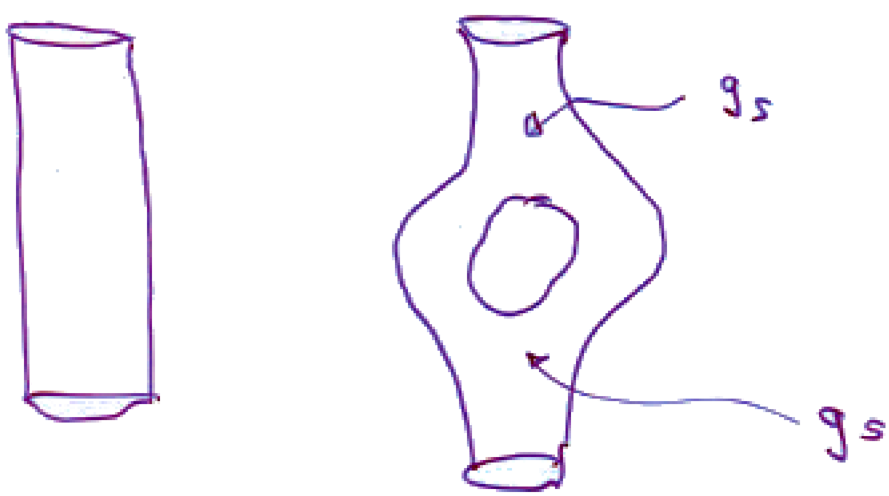
$$e^{-\frac{1}{4\pi}\int {}^{(2)}R \,.\, \Phi}$$

By Gauss-Bonnet theorm

$$\int {}^{(2)}R = 4\pi\chi = 8\pi(1-g)$$

A surface of genus g picks up a factor of

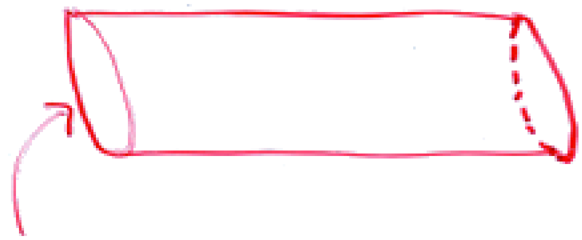
$$e^{+2g\Phi}$$



$$g_s = e^{\Phi}$$

M-theory :

Spacetime



KK circle — radius R_{11}

$d=11$ SG

$$\int -R + \frac{1}{48}F^2 + \text{fermions} + CS$$

$$\downarrow$$

$$\int e^{-2\phi}\left(-R + \frac{1}{12}H^2 + \frac{1}{2}(\nabla\phi)^2\right) + \left(\frac{1}{4}F_{(2)}^2 + \frac{1}{48}F_{(4)}^2\right.$$

$$+ \ldots$$

$$\Rightarrow \quad e^{\Phi} = (R_{11})^{3/2} = g_s$$

Thus dilaton dependent piece of IIA string action $\sim \frac{3}{2}\chi \ln R_{11}$

M2 - brane

$$\frac{T_{M2}}{2} \int d^3\sigma \; \gamma^{1/2} \Big(\gamma^{\mu\nu} \partial_\mu X^I \partial_\nu X^J g_{IJ} - 1$$
$$+ \epsilon^{\mu\nu\rho} \partial_\mu X^I \partial_\nu X^J \partial_\rho X^K C_{IJK} \Big)$$
$$+ \cdots$$

- κ - symmetry $\Rightarrow$ Einstein equations
- Reduction to $d = 10$ + dualisation gives D2 action
- Reduction when wrapped on KK circle gives IIA action although no obvious sign of the Fradkin - Tseytlin term.

Path Integral (Polyakov)

$$Z \sim \sum_{\text{topologies}} \int \frac{\mathcal{D}X\,\mathcal{D}\gamma}{\mathrm{Vol}(\mathrm{Diff}_0)}\, e^{-S_{M2}(X,\gamma)}$$

$$\Big\downarrow \text{ Gauge fix. Integrate over pure diffeos}$$

$$\sim \sum_{\text{topologies}} \int J\, \mathcal{D}(\text{moduli})\,\mathcal{D}X\, e^{-S_{M2}}$$

Jacobian from changing variables from $\gamma_{\mu\nu}$ to Diffeos + moduli.

Topology — $S^1 \times \Sigma$

genus g Riemann surface

wrapped on the Kaluza-Klein circle

Gauge choice : $\sigma^3 = X^{11} + f(\sigma^1, \sigma^2)$

morphs into the string conformal factor and decouples (Achucarro, Kapusta + Stelle)

$\gamma_{i3} = 0$

$$S = \frac{1}{4\pi\alpha'} \int \tilde{\gamma}^{1/2} \Big(\tilde{\gamma}^{ij} \partial_i X^I \partial_j X^J \, g_{IJ} + \epsilon^{ij} \partial_i X^I \partial_j X^J \, B_{IJ} \Big)$$

with $\displaystyle \alpha' = \frac{1}{2\pi T_{M2} R_{11}}$

independently of f.

$$\tilde{\gamma} = e^{\delta v} \, e^{2\sigma} \, \hat{\gamma}$$

$\perp$ orbits of $\mathrm{Weyl}(\Sigma) \times \mathrm{Diff}_0(\Sigma)$

Weyl transformation

δv generates a diffeomorphism

Explicitly –

Action of Weyl + Diffeos on γ is, infinitesimally

$$\delta\gamma_{ij} = \left(2\delta\sigma + \nabla^k \delta v_k\right)\gamma_{ij} + \underbrace{\left(\nabla_i \delta v_j + \nabla_j \delta v_i - \gamma_{ij}\nabla_k \delta v^k\right)}_{\equiv\, 2P_1(\delta v)_{ij}}$$

P_1 maps vectors to tracefree symmetric tensor

P_1^+ maps tracefree symmetric tensors to vectors

$$(P_1^+ h_{ij}) = -2\nabla^i h_{ij}$$

$\ker P_1 \equiv$ Conformal Killing vectors.

$\ker P_1^+ \equiv$ Moduli

	$\dim \operatorname{Ker} P_1$	$\dim \operatorname{Ker} P_1^\dagger$
$g = 0$	6	0
$g = 1$	2	2
$g \geq 2$	0	$6g - 6$

Riemann – Roch theorem

$$\dim \ker P_1^\dagger - \dim \ker P_1 = 6g - 6$$

$g \geq 2$

$$Z = \int DX \, D[\text{moduli}] \, J \, \frac{d\nu \, d\sigma}{\mathrm{Vol}(\mathrm{Diff} \times \mathrm{Weyl})} \, e^{-S}$$

$$J = \left(\det P_1^{+} P_1 \right)^{1/2}$$

Measure on moduli space:

$$\| \delta h_{\mu\nu} \|^2 = \int d^3\sigma \, \gamma^{1/2} \, \gamma^{\mu\rho} \gamma^{\nu\sigma} \, \delta h_{\mu\nu} \delta h_{\rho\sigma}$$

for M2

$$\Downarrow$$

Normalised moduli for M2

$$\| \delta h_{\mu\nu} \| = (R_{11})^{1/2} \, \| \delta h_{ij} \|$$

$$\Uparrow$$

Normalised moduli for string.

$$\mathcal{D}[\text{moduli}]_{M2} = R_{11}^{\frac{1}{2}\dim\ker P_1^\dagger} \mathcal{D}[\text{moduli}]_{IIA}$$

Now if $\dim\ker P_1 \neq 0$ $(g = 0, 1)$

$$(\det P_1^\dagger P_1)^{1/2} \longrightarrow (\det{}' P_1^\dagger P_1)^{1/2}$$

where $'$ means integrate only over the space orthogonal $\ker P_1$.

Norm of conformal Killing vectors scales in the same way as the moduli.

$$\mathcal{D}[\gamma_{\mu\nu}] \to \frac{1}{\mathrm{Vol}(\mathrm{Ker}\,P_1)} \left(\det{}' P_1^{\dagger} P_1\right)^{1/2} dv\, d\sigma\, d[\text{moduli}].$$

So, universally

$$\int \frac{\mathcal{D}\gamma}{\mathrm{Vol}(\mathrm{Diff}_0)} = \int \frac{\mathcal{D}\tilde{\gamma}}{\mathrm{Vol}(\mathrm{Diff} \times \mathrm{Weyl})} R_{11}^{\frac{1}{2}(\dim\ker P_1^{\dagger} - \dim\ker P_1)}$$

so by Riemann-Roch

this factor is

$$(R_{11})^{-\frac{3}{2}\chi}$$

$$= e^{-\Phi\chi}$$

Result :

- Careful consideration of M2 path integral reproduces the Fradkin – Tseytlin term.