Discrete Symmetries of Quiver Theories and Wrapped Branes
hep-th/0602094

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Great Lakes Strings
April Fool’s Day
Pattern first recognized in hep-th/9811048: (Gukov, Rangamani, Witten)

D3 on Orbifold 6D Backgrounds $\rightarrow$ Quiver Gauge Theories
Orbifold $\mathbb{Z}_n$ Backgrounds $\rightarrow$ Cycles Valued in $\mathbb{Z}_n$
Branes may wrap these cycles.
Number Operators of Wrapped Branes have AdS/CFT Dual
Quiver Gauge Theories Have $\mathbb{Z}_n$ symmetries
Discrete Symmetries $\rightarrow$ NONCOMMUTATIVE
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Overview

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$Y^{p,q}$ Geometries/CFT Duals

New infinite class of theories $Y^{p,q}$ geometries

(Gauntlett, Martelli, Sparks, Waldram (0403002))

\[
ds_1^2 = H^{-\frac{1}{2}} dx^\mu dx_\mu + H^{\frac{1}{2}} \left( dr^2 + r^2 \left( ds_{Y^{p,q}}^2 \right) \right)
\]  \(1\)

When $\text{GCD}(p, q) = a \neq 1$ these are orbifold geometries.

Quiver diagram given by (Martelli, Sparks (0411238))

\[
\left( \sigma \tilde{\sigma} \tau \ldots \ldots \ldots \right) (\ldots) (\ldots) \ldots \ 
\]

(\((p-q)/a) \tau-\text{type, } (q/a) \sigma-\text{type})

\text{a-times}
\(Y^{p,q}\) Geometries/CFT Duals

New infinite class of theories \(Y^{p,q}\) geometries (Gauntlett, Martelli, Sparks, Waldram (0403002))

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Quiver diagram given by (Martelli, Sparks (0411238))

\[
\underbrace{(\sigma\tilde{\sigma} \tau \cdots \cdots \cdots \cdots \cdots)}(\cdots)(\cdots)\cdots
\]

(2)

\[(p-q)/a \quad \tau-\text{type}, \quad (q/a) \quad \sigma-\text{type}\]

\[a-\text{times}\]
\( Y^{p,q} \) Geometries/CFT Duals

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\[ds_1^2 = H^{-\frac{1}{2}} dx^\mu dx_\mu + H^{\frac{1}{2}} \left( dr^2 + r^2 \left( ds_{Y^{p,q}}^2 \right) \right) \]  \hspace{1cm} (1)

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\[
\begin{align*}
\left( \sigma \tilde{\sigma} \tau \cdots \cdots \cdots \cdots \cdots \right) & (\cdots) (\cdots) \cdots \\
\underbrace{((p-q)/a) \tau - \text{type}} \quad \underbrace{(q/a) \sigma - \text{type}} \quad \text{a-times}
\end{align*}
\]  \hspace{1cm} (2)
\( Y^{p,q} \) Geometries/CFT Duals

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ds_10^2 = H^{-\frac{1}{2}}dx^\mu dx_\mu + H^{\frac{1}{2}} \left( dr^2 + r^2 \left( ds_{Y^{p,q}}^2 \right) \right) \tag{1}
\]

When \( \text{GCD}(p, q) = a \neq 1 \) these are orbifold geometries. Quiver diagram given by (Martelli, Sparks (0411238))

\[
\left( \sigma \tilde{\sigma} \tau \ldots \ldots \ldots \ldots \right) (\ldots) (\ldots) \ldots \tag{2}
\]

\( \left( (p-q)/a \right) \tau \)-type, \( (q/a) \sigma \)-type

\( a \)-times

Unit cells
Our Work

Example: $Y^{2,0}$: Diagram

Symmetries $A : (1, 2, 3, 4) \rightarrow (3, 4, 1, 2)$
$B : (1, 1, \omega, \omega^{-1})$ and $C : (\omega, \omega^{-1}, \omega^{-1}, \omega)$ with $\omega^{2N} = 1$

These satisfy (up to the COGG)

$$A^2 = B^2 = C^2 = 1, \quad AB = BAC, \quad C \text{ commutes} \quad (3)$$

and is a finite Heisenberg Group
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hep-th/0602094, hep-th/0603114
Our Work

Example: $Y^{2,0}$: Diagram

![Diagram](image)

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General $Y^{p,q}$ ($p$, $q$ not coprime)

We find this to be a general pattern, even for complicated $Y^{p,q}$!

We work out explicitly:
Conclusions

For a large class of theories, we find that \textit{Wrapped Brane Number Operators DO NOT COMMUTE!}
(Worked on by D. Belov and G. Moore)
We later generalize this to even the non-conformal case!
(hep-th/0603114)
(also, see hep-th/0412193 Herzog, Ejaz, Klebenov for non-conformal generalizations)