Supersymmetric Flavour Universality in String Theory

Joseph P. Conlon (Cavendish Laboratory & DAMTP, Cambridge)

LHC New Signatures Workshop
University of Michigan, January 2008

arXiv:0710.0873 (JC)
The MSSM Flavour Problem

The LHC experiments hope to discover supersymmetry. But why should any new susy signatures occur at all?

One major problem:
The MSSM Flavour Problem

The LHC experiments hope to discover supersymmetry.
But why should any new susy signatures occur at all?

One major problem:

Why hasn’t supersymmetry been discovered already?

Compare with

- The $c$ quark - predicted before discovery through its contribution to FCNCs (the GIM mechanism)
- The $t$ quark - mass predicted accurately through loop contributions at LEP I.
The MSSM gives new contributions to $K_0 - \bar{K}_0$ mixing.
The MSSM generates new contributions to $BR(\mu \rightarrow e\gamma)$. 
Universality in Effective Field Theory

SUSY is a happy bunny if soft terms are flavour universal.

\[ m_{Q,\alpha\bar{\beta}}^2 = m_Q^2, \]
\[ A_{\alpha\beta\gamma} = AY_{\alpha\beta\gamma}, \]
\[ \phi_{M_1} = \phi_{M_2} = \phi_{M_3} = \phi_A. \]

Why should this be?

Answer to flavour problem significantly affects all of susy phenomenology.
Soft Terms in Supergravity

- There exists lore that gravity (=string)-mediated susy breaking always suffers from large FCNCs.

- Soft terms come from expanding $K$ and $W$ in powers of matter fields $C^\alpha$ and moduli fields $\Phi$,

\[
W = \hat{W}(\Phi) + \mu(\Phi)H_1H_2 + \frac{1}{6}Y_{\alpha\beta\gamma}(\Phi)C^\alpha C^\beta C^\gamma + \ldots, \\
K = \hat{K}(\Phi, \bar{\Phi}) + \tilde{K}_{\alpha\bar{\beta}}(\Phi, \bar{\Phi})C^\alpha C^{\bar{\beta}} + [ZH_1H_2 + h.c.] + \ldots, \\
f_a = f_a(\Phi).
\]

- To compute soft terms, need to know $\tilde{K}_{\alpha\bar{\beta}}(\Phi, \bar{\Phi})$, $Y_{\alpha\beta\gamma}(\Phi)$ and $f_a(\Phi)$.
Soft Terms in Supergravity

Soft scalar masses $m^2_{i\bar{j}}$ and trilinears $A_{\alpha\beta\gamma}$ are given by

$$\tilde{m}^2_{\alpha\bar{\beta}} = (m^2_{3/2} + V_0) \tilde{K}_{\alpha\bar{\beta}}$$

$$-\tilde{F}^m \tilde{F}^n \left( \partial_m \partial_n \tilde{K}_{\alpha\bar{\beta}} - (\partial_m \tilde{K}_{\alpha\bar{\gamma}}) \tilde{K}^{\bar{\gamma}\delta} (\partial_n \tilde{K}_{\delta\bar{\beta}}) \right)$$

$$A'_{\alpha\beta\gamma} = e^{\tilde{K}/2} F^m \left[ \tilde{K}_m Y_{\alpha\beta\gamma} + \partial_m Y_{\alpha\beta\gamma} ight.$$  
$$- \left( (\partial_m \tilde{K}_{\alpha\bar{\rho}}) \tilde{K}^{\bar{\rho}\delta} Y_{\delta\beta\gamma} + (\alpha \leftrightarrow \beta) + (\alpha \leftrightarrow \gamma) \right) \left. \right] .$$

$$M_a = \frac{F^m \partial_m f_a}{\text{Re}(f_a)}$$
Universality in Effective Field Theory

Sufficient conditions for flavour universality:

1. Hidden sector factorises

\[ \Phi_{\text{hidden}} = \Psi_{\text{susy}} \oplus \chi_{\text{flavour}}. \]

2. The kinetic terms are decoupled

\[ \mathcal{K}(\Phi, \bar{\Phi}) = \mathcal{K}_1(\Psi + \bar{\Psi}) + \mathcal{K}_2(\chi, \bar{\chi}), \]

3. The superpotential Yukawas depend only on \( \chi_{\text{flavour}} \):

\[ Y_{\alpha\beta\gamma}(\Psi, \chi) = Y_{\alpha\beta\gamma}(\chi). \]

4. The gauge couplings depend only on \( \Psi \) fields:

\[ f_a(\Psi, \chi) = \sum_i \lambda_i \Psi_i. \]
5. The visible metric factorises.

\[ \mathcal{K}_{\alpha\beta}(\Psi, \bar{\Psi}, \chi, \bar{\chi}) = h(\Psi + \bar{\Psi})k_{\alpha\beta}(\chi, \bar{\chi}) \]

6. \( \Psi \) breaks susy, \( \chi \) does not:

\[ D_{\Psi_i} W \neq 0, D_{\chi_j} W = 0. \]

If all these assumptions hold, susy breaking generates flavour universal soft terms.
Universality in Effective Field Theory

5. The visible metric factorises.

\[ \mathcal{K}_{\alpha \bar{\beta}}(\Psi, \bar{\Psi}, \chi, \bar{\chi}) = h(\Psi + \bar{\Psi})k_{\alpha \bar{\beta}}(\chi, \bar{\chi}) \]

6. \( \Psi \) breaks susy, \( \chi \) does not:

\[ D_{\Psi_i} W \neq 0, \quad D_{\chi_j} W = 0. \]

If all these assumptions hold, susy breaking generates flavour universal soft terms.

Totally *ad hoc* in effective field theory!
Universality in String Theory

String theory knows this structure!

- Calabi-Yau moduli space factorises in this fashion.
- Kähler $(T)$ and complex structure $(U)$ moduli have factorised moduli spaces

\[ IIB : \Psi_{susy} \rightarrow T, \chi_{flavour} \rightarrow U, \]
\[ IIA : \Psi_{susy} \rightarrow U, \chi_{flavour} \rightarrow T. \]

- In flux compactifications susy breaking factorises:
  \[ F^T \neq 0, \quad F^U = 0. \]
1. Hidden sector factorises

\[ \Phi_{\text{hidden}} = \Psi_{\text{susy}} \oplus \chi_{\text{flavour}}. \]
1. Hidden sector factorises

\[ \Phi_{\text{hidden}} = \Psi_{\text{susy}} \oplus \chi_{\text{flavour}}. \]

\[ \Phi_{\text{moduli}} = \Psi_{\text{Kähler(T)}} \oplus \chi_{\text{complex structure(U)}}. \]

The moduli space of Calabi-Yau manifolds has two distinct, factorised parts: Kähler and complex structure moduli.
2. The kinetic terms are decoupled

\[ \mathcal{K}(\Phi, \bar{\Phi}) = \mathcal{K}_1(\Psi + \bar{\Psi}) + \mathcal{K}_2(\chi, \bar{\chi}), \]
2. The kinetic terms are decoupled

\[ \mathcal{K}(\Phi, \bar{\Phi}) = \mathcal{K}_1(\Psi + \bar{\Psi}) + \mathcal{K}_2(\chi, \bar{\chi}), \]

The IIB moduli space Kähler potential is

\[ K = -2 \ln(\mathcal{V}(T + \bar{T})) - \ln \left( \int \Omega \wedge \bar{\Omega}(U, \bar{U}) \right) - \ln(S + \bar{S}) \]

The kinetic terms factorise into \( T \) and \( U \) parts.
The imaginary part of $T$ is axionic.

$T$ has a perturbative shift symmetry,

$$ T \rightarrow T + i\epsilon. $$

This shift symmetry is unbroken in both world-sheet ($\alpha'$) and space-time ($g_s$) perturbation theory.

Perturbative quantities depend only on $(T + \bar{T})$. 
3. The superpotential Yukawas depend only on $\chi_{flavour}$:

$$Y_{\alpha\beta\gamma}(\Psi, \chi) = Y_{\alpha\beta\gamma}(\chi).$$
3. The superpotential Yukawas depend only on $\chi_{\text{flavour}}$:

$$Y_{\alpha\beta\gamma}(\Psi, \chi) = Y_{\alpha\beta\gamma}(\chi).$$

Perturbativity would require $Y_{\alpha\beta\gamma}(T) \sim T^{\lambda}$.

The shift symmetry $T \rightarrow T + i\epsilon$ forbids this.

In perturbation theory, $Y_{\alpha\beta\gamma}(T, U) = Y_{\alpha\beta\gamma}(U)$. 

Supersymmetric Flavour Universality in String Theory – p. 14/2
Example:

For toroidal compactifications, the superpotential Yukawas take the following form,

\[ Y_{ijk}(U) = \vartheta \left[ \begin{array}{c} \delta_{ijk}^r \\ 0 \end{array} \right] (0; U^r I_{ab}^r I_{bc}^r I_{ca}^r) \]

- No dependence on \( T \)
- A very complicated (exponential) dependence on \( U \).
4. The gauge couplings depend only on $\Psi$ fields:

$$f_a(\Psi, \chi) = \sum_i \lambda_i \Psi_i.$$
4. The gauge couplings depend only on $\Psi$ fields:

$$f_a(\Psi, \chi) = \sum_i \lambda_i \Psi_i.$$ 

D7 brane gauge coupling:

$$f_a = \frac{T}{4\pi}.$$ 

- No U-dependence
- Linear dependence on $T$. 
5. The visible metric factorises.

\[ \mathcal{K}_{\alpha\bar{\beta}}(\Psi, \bar{\Psi}, \chi, \bar{\chi}) = h(\Psi + \bar{\Psi})k_{\alpha\bar{\beta}}(\chi, \bar{\chi}) \]
5. The visible metric factorises.

\[ \mathcal{K}_{\alpha\bar{\beta}}(\Psi, \bar{\Psi}, \chi, \bar{\chi}) = h(\Psi + \bar{\Psi})k_{\alpha\bar{\beta}}(\chi, \bar{\chi}) \]

This comes from the universal scaling property of physical Yukawa couplings

\[ \hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha'\beta'\gamma'}}{(\tilde{K}_{\alpha\alpha'}\tilde{K}_{\beta\beta'}\tilde{K}_{\gamma\gamma'})^{\frac{1}{2}}} \]

This arises from the origin of physical Yukawa couplings as due to wavefunction overlap.
Example: For toroidal compactifications, the Kähler metric is

\[ K_{ij}^{ab} = \delta_{ij} S^{-1/4} ( (T^1 + \bar{T}^1)(T^2 + \bar{T}^2)(T^3 + \bar{T}^3) )^{-1/4} \times \prod_{I=1}^{3} U_I^{-1/2} \left( \frac{\Gamma(\theta_{1ab}^1)\Gamma(\theta_{2ab}^2)\Gamma(1 - \theta_{1ab}^1 - \theta_{2ab}^2)}{\Gamma(1 - \theta_{1ab}^1)\Gamma(1 - \theta_{2ab}^2)\Gamma(\theta_{1ab}^1 + \theta_{2ab}^2)} \right)^{1/2} \]

This has

- At leading order a universal scaling dependence on \((T + \bar{T})\)
- Subleading (universal) corrections at \(\mathcal{O}(\alpha'^2)\)
6. $\Psi$ breaks susy, $\chi$ does not:

$$D_{\Psi_i} W \neq 0,\ D_{\chi_j} W = 0.$$
Universality in String Theory

6. $\Psi$ breaks susy, $\chi$ does not:

$$D_{\Psi_i} W \neq 0, D_{\chi_j} W = 0.$$ 

- This is a statement about the vacuum structure and is equivalent to $F^T \neq 0, F^U = 0$.
- The $T$ fields break supersymmetry and the $U$ fields do not.
- In IIB flux compactifications these conditions are satisfied.
Moduli Stabilisation: Fluxes

\begin{align*}
\hat{K} &= -2 \ln \left( \mathcal{V}(T + \bar{T}) \right) - \ln \left( i \int \Omega \wedge \bar{\Omega}(U) \right) - \ln \left( S + \bar{S} \right), \\
W &= \int G_3 \wedge \Omega(U), \\
V &= e^{\hat{K}} \left( \sum_{U,S} \hat{K}^{\alpha\bar{\beta}} D_{\alpha} W D_{\bar{\beta}} \bar{W} + \sum_{T} \hat{K}^{i\bar{j}} D_{i} W D_{\bar{j}} \bar{W} - 3|W|^2 \right) \\
&= e^{\hat{K}} \left( \sum_{U,S} \hat{K}^{\alpha\bar{\beta}} D_{\alpha} W D_{\bar{\beta}} \bar{W} \right)
\end{align*}

Stabilise \( S \) and \( U \) by solving \( D_S W = D_U W = 0 \).
Moduli Stabilisation: Fluxes

\[
\hat{K} = -2 \ln \left( \mathcal{V}(T_i + \bar{T}_i) \right), \\
W = W_0. \\
V = e^{\hat{K}} \left( \sum_T \hat{K}^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right) \\
= 0
\]

No-scale model:
- vanishing vacuum energy
- broken susy \((F^T \neq 0, F^U = 0)\)
- \(T\) unstabilised
- Goldstino is breathing mode \(g_{\mu\nu} \rightarrow \chi^2 g_{\mu\nu}\)
Moduli Stabilisation: Fluxes

- Susy breaking factorises: $F^T \neq 0, F^U = 0$.

- Goldstino is breathing mode and is manifestly flavour universal.
  The breathing mode modifies the normalisation but not the structure of Yukawa couplings.

- The factorisation of supersymmetry breaking persists when all moduli are stabilised (e.g. large volume models)
Corrections

- Factorisation holds at leading order.
- Factorisation is inherited from the underlying $\mathcal{N} = 2$ structure and holds in the large-radius limit.
- It is broken by e.g. loop corrections that are present in $\mathcal{N} = 1$ compactifications.
- Corrections are loop-suppressed, but very hard to compute explicitly.
Conclusions

- String flux compactifications give a natural solution to the MSSM flavour problems.

- Calabi-Yau moduli space factorises into Kähler and complex structure moduli.

- One sector generates flavour, the other generates susy breaking.

- IIB flux compactifications explicitly realise this factorisation.