# Photons and Quantum Information Processing

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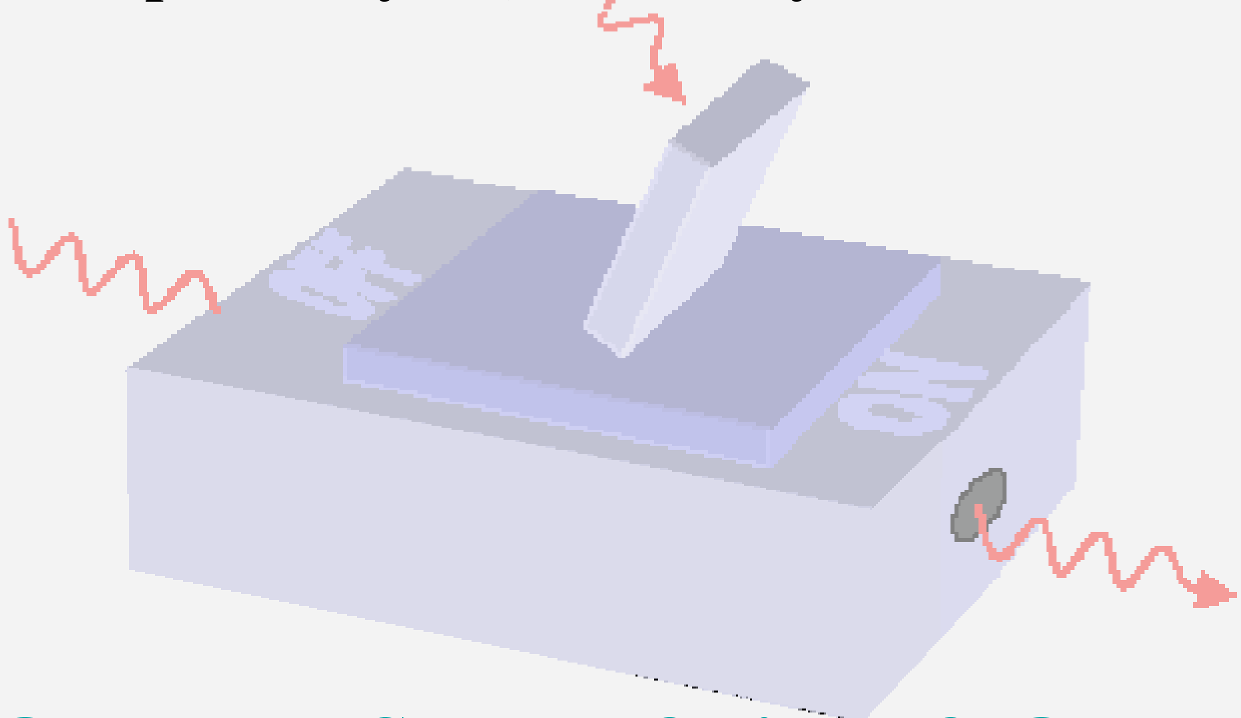


## Talk 1: Quantum States of Light & Q. Interference
## Talk 2: Quantum Measurement & Info. w/ Photons

Michigan Quantum Summer School 2008

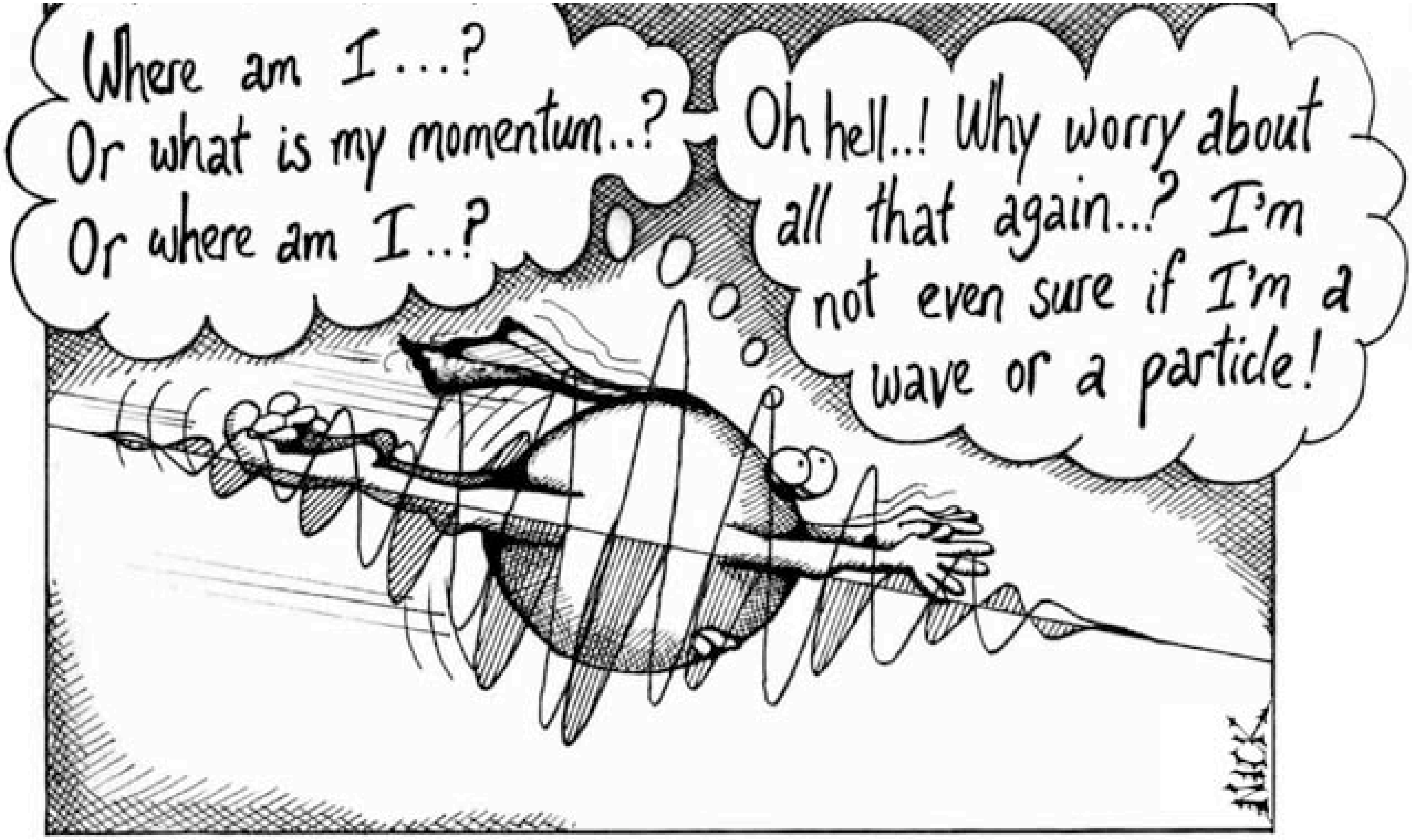
# FIRST TOPIC: *Interruptions*

## MAKE THEM!

VALID REASONS TO INTERRUPT ME:

I’m going too fast.
I’m going too slow.
You want to correct my grammar.
You disagree with something I said.
I seem to disagree with something I’ve said.
You have a question about something I’ve said.
You have a question about something completely unrelated.

# What is a photon anyway?




Photon self-identity problems.

# What did all those famous guys mean?

**"Two photons never interfere with each other – a single photon only ever interferes with itself" (P.A.M. Dirac)**

**"A single photon has no phase"**

Remember: the energy of a photon (of given colour) is completely certain.

$$\bar{\mathcal{H}}|1\rangle = \frac{3\hbar\omega}{2}|1\rangle$$

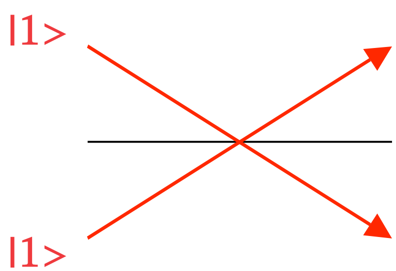
Energy eigenstate = stationary state: $\langle A(t)\rangle = \langle 1|e^{iEt/\hbar}Ae^{-iEt/\hbar}|1\rangle = \langle A(0)\rangle.$

In a single-photon state, there "are" oscillations with a known amplitude... but since their time origin is completely uncertain, so is E(t).

Compare an eigenstate of the harmonic oscillator- the maximum X is related to the energy, but the particle's equally likely to be at +X or -X.

# What did Dirac mean?

Two photons meeting at a beam-splitter don't interfere (in the usual sense).




$$a_o = \frac{a_1 + a_2}{\sqrt{2}}$$

$$\langle n_o \rangle = \langle a_o^\dagger a_o \rangle = \frac{1}{2} \langle \underbrace{a_1^\dagger a_1}_{n1} + \underbrace{a_2^\dagger a_2}_{n2} + \underline{\underline{a_1^\dagger a_2 + a_2^\dagger a_1}} \rangle$$

The cross terms change $n_1$ and $n_2$; unless these numbers are uncertain, these terms vanish.

Optical phase actually refers (roughly) to the quantum phase *difference* between the amplitude to have *n* photons and the amplitude to have *n+1*.

# Uncertainty relations?

What is phase, except time?
What is photon-number, except energy?

$$\phi = \omega t$$

$$E \approx n\hbar\omega$$

$$\Delta E \Delta t = (\Delta n \hbar\omega)(\Delta\phi/\omega) = \hbar \Delta n \Delta\phi$$

$$\Delta n \Delta\phi \geq \frac{1}{2}$$

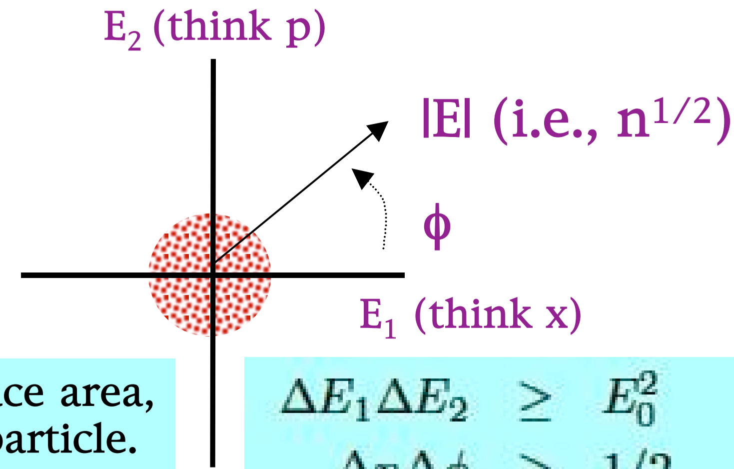
# Phase-space picture

A classical phasor...

complex $E = |E|\, e^{i\phi}\, e^{-i\omega t}$

$= E_1 \cos \omega t + E_2 \sin \omega t$

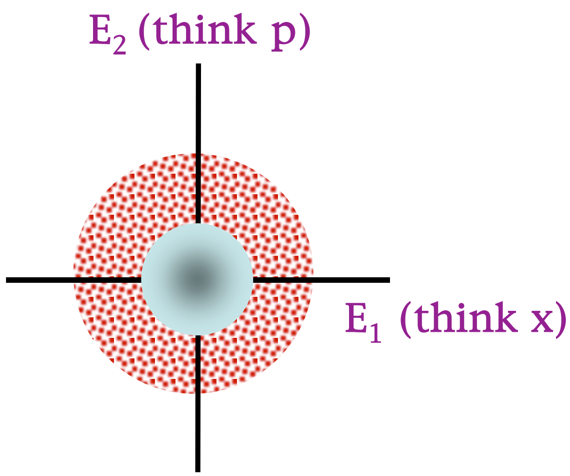
Cf. SHO: $x(t) = x_0 \cos \omega t + p_0 \sin \omega t$



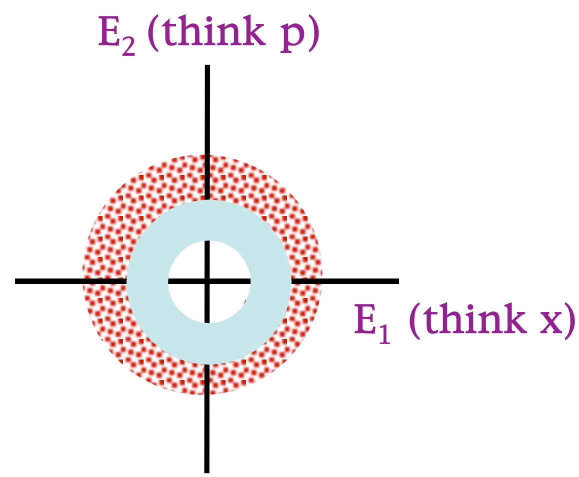

minimum phase-space area,
as for X and P of a particle.

$$\Delta E_1 \Delta E_2 \geq E_0^2$$
$$\Delta n \Delta \phi \geq 1/2$$

The vacuum isn't empty:

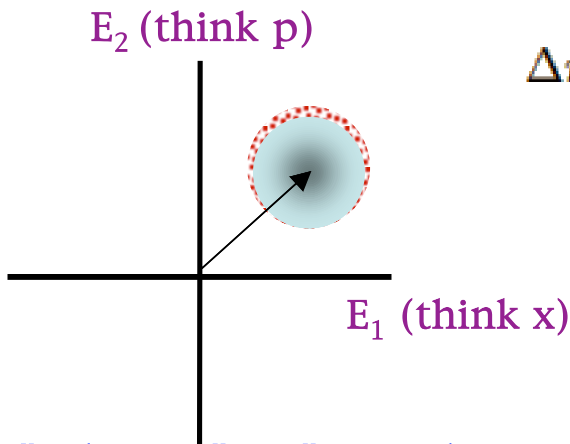



A phase-space distribution
for a 0-photon state




A phase-space distribution
for a 1-photon state

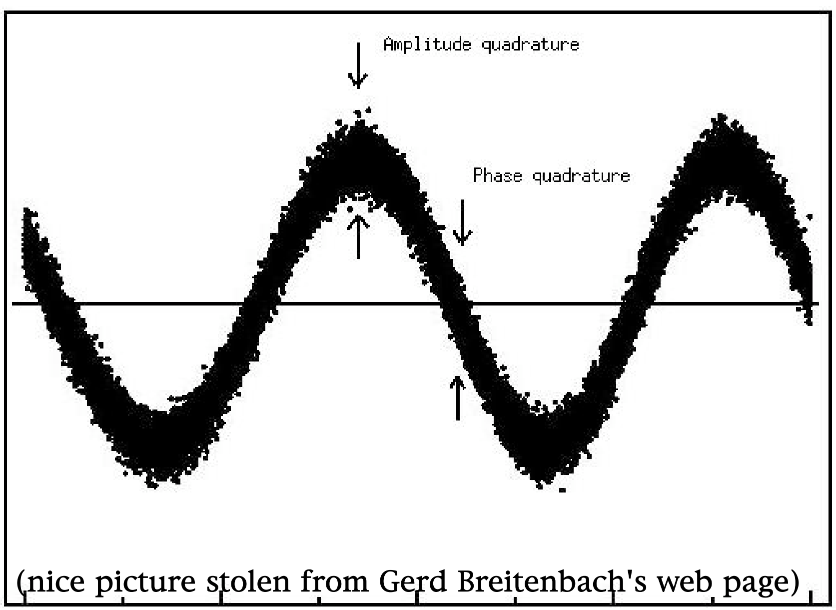
# The coherent state




$\Delta n \sim \sqrt{n}$ (uncorrelated particles $\rightarrow$ Poisson statistics)

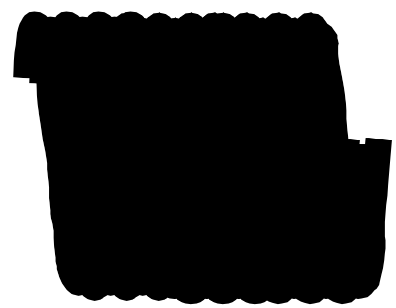
Minimum uncertainty product: $\Delta\phi \sim 1/2\sqrt{n}$.

A "coherent" or "quasiclassical" state.




(nice picture stolen from Gerd Breitenbach's web page)

The vacuum state
(also α=0 coherent state
or n=0 number state)



# How does one measure these things?

With a radio-frequency field, of course, one measures E(t) with an antenna.

Not possible at optical frequencies - we can only detect power/energy/photons.

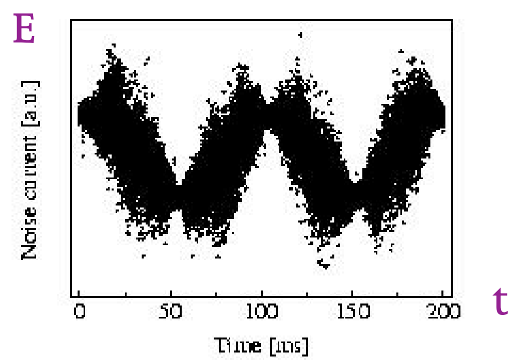
Homodyning/heterodyning: interfere a signal with a strong oscillator.

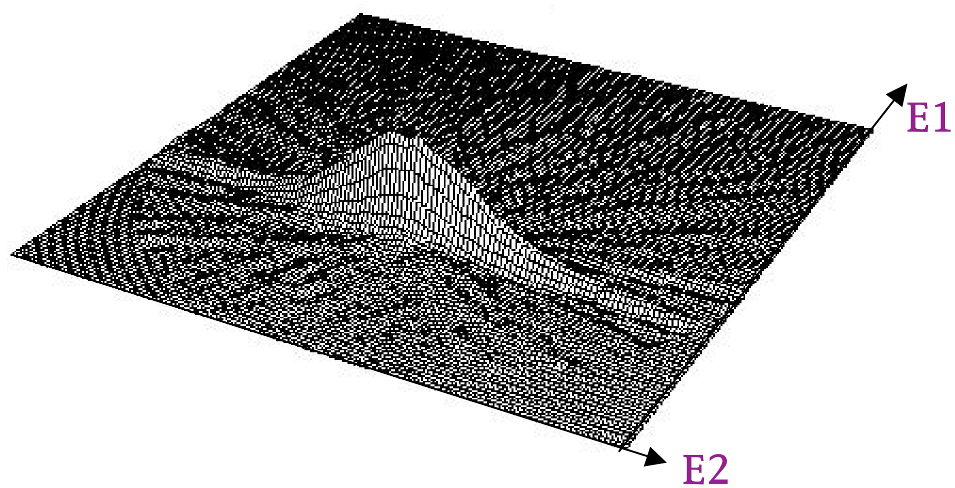
$$\left| E_s e^{i\phi} + |E_{LO}| \right|^2 \longrightarrow |E_{LO}|^2 + 2\,|E_{LO}| E_s \cos\phi + \ldots$$

By varying phase of local oscillator, can measure different quadratures just by measuring intensity.

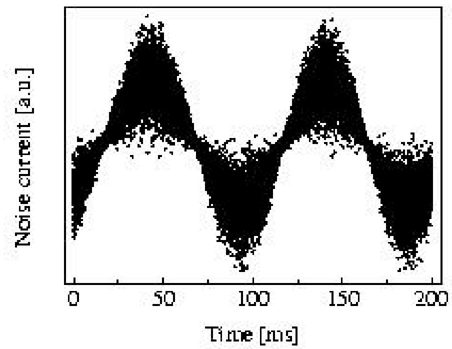
Note: not so different from RF; your oscilloscope has a local oscillator too.

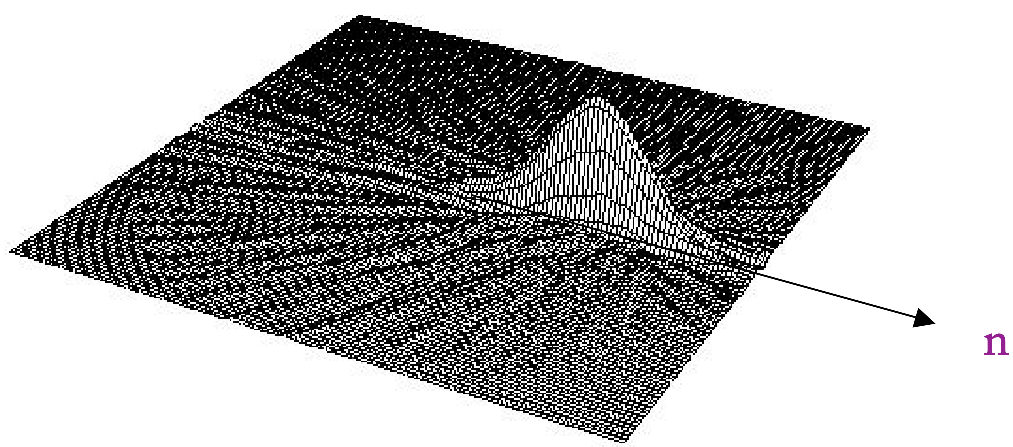
# Amplitude-squeezed state



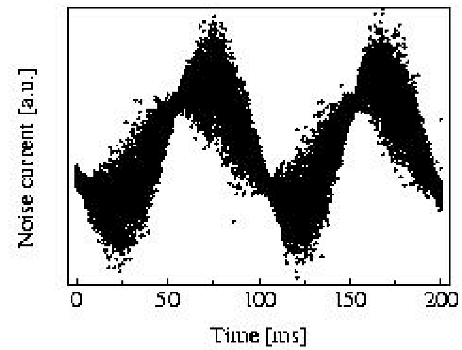



# Phase-squeezed state

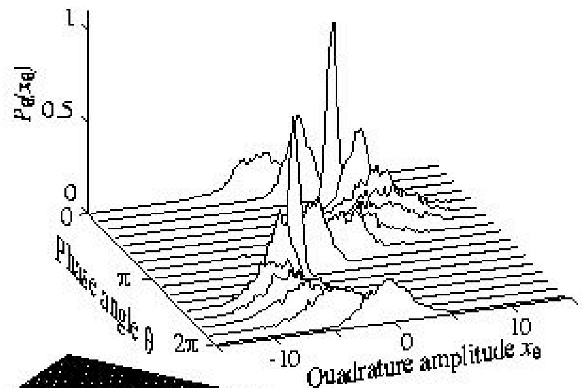


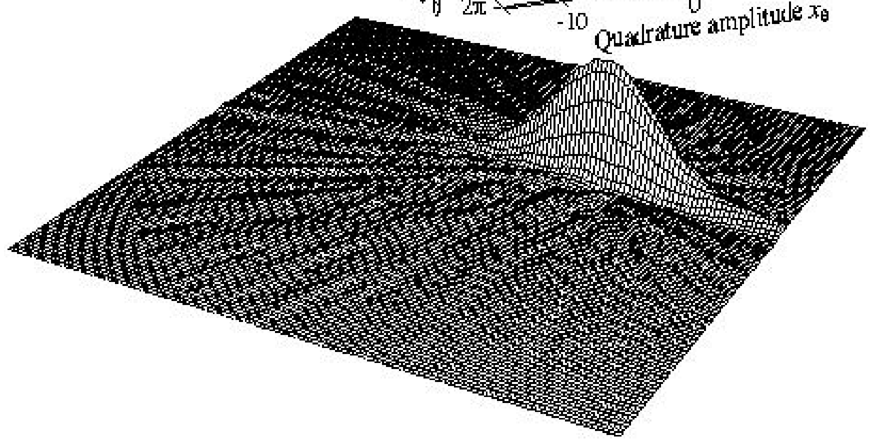



# Quadrature-squeezed state



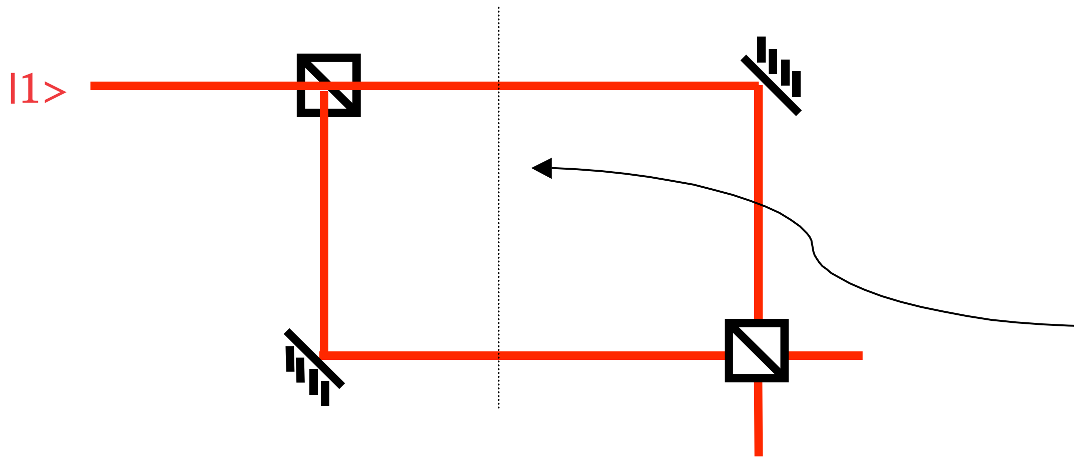

Noise trace
Marginals
Wigner function



# Back to Bohr & Einstein (and Young and Taylor, and Clauser & Grangier & Aspect...)

So, does a single photon exhibit interference, or not?!




Entangled state |01> + |10>; no definite number in either arm.

$$\Delta n_1 = \Delta n_2 = \frac{1}{2}$$

$$\Delta \phi_{1,2} \geq 1$$

$$\Delta(n_1 + n_2) = 0$$

$$\Delta(\phi_1 + \phi_2) \to \infty$$

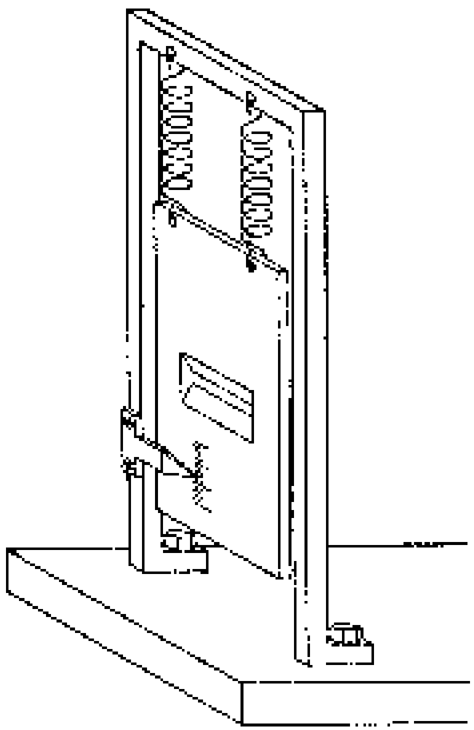
$$\Delta(n_1 - n_2) = 1$$

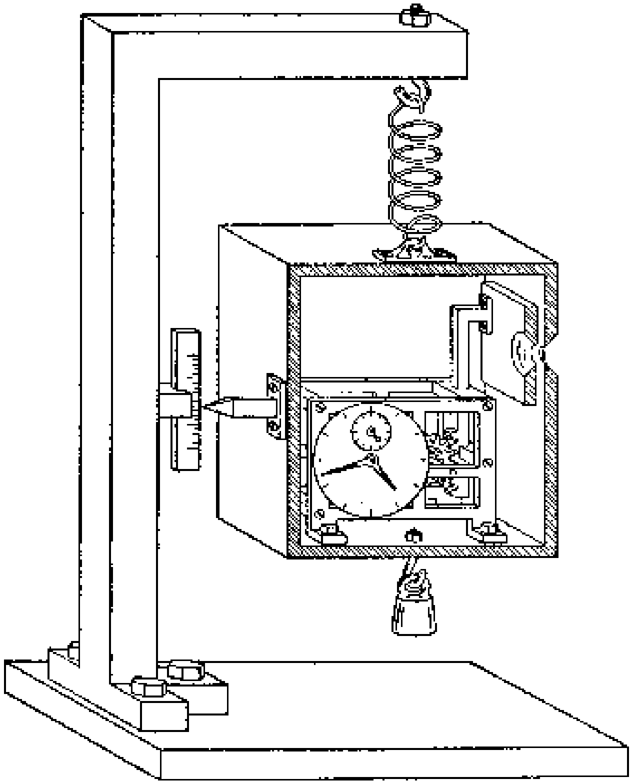
$$\Delta(\phi_1 - \phi_2)$$

# The Bohr-Einstein debates

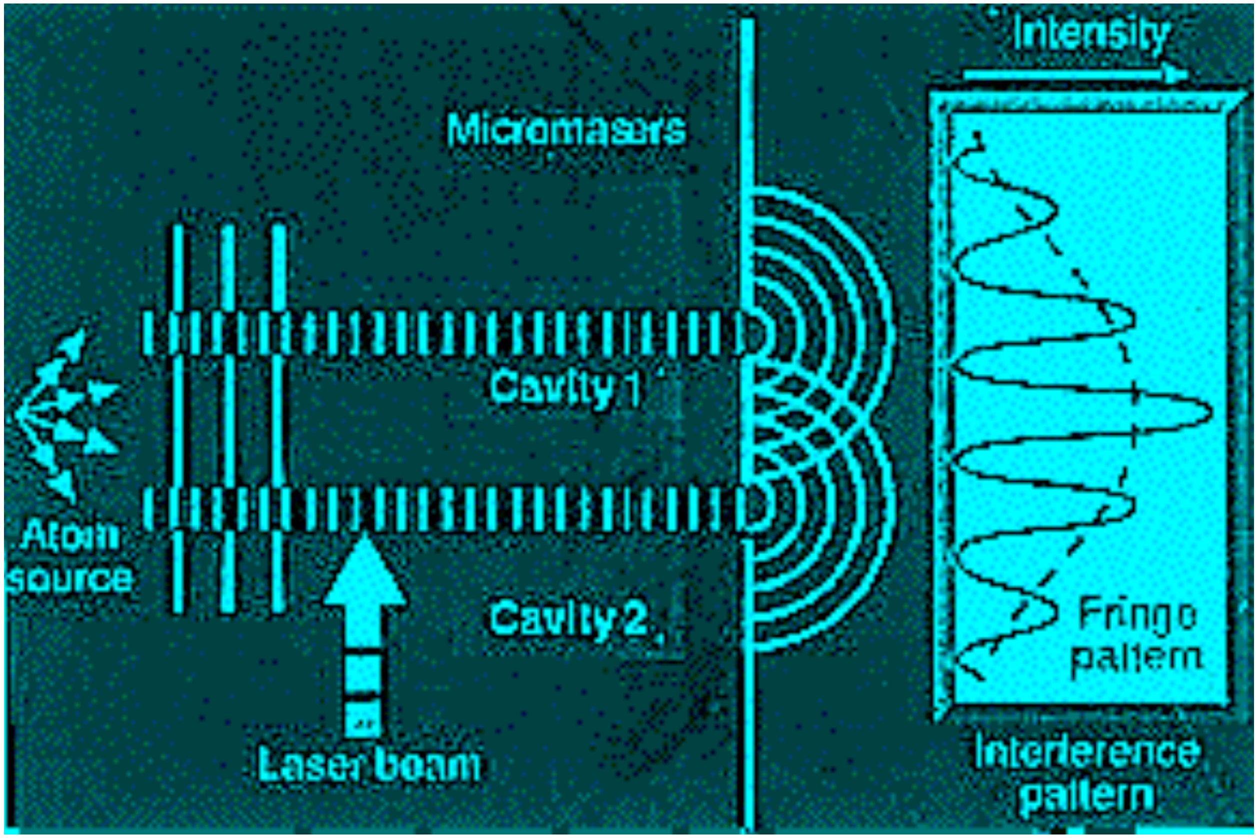
**How can a particle go through both slits at once?**
**If I measured which one it went through, how could interference occur between the two of them?**

# More and more schemes to measure Welcher Weg (which way) the particle goes...





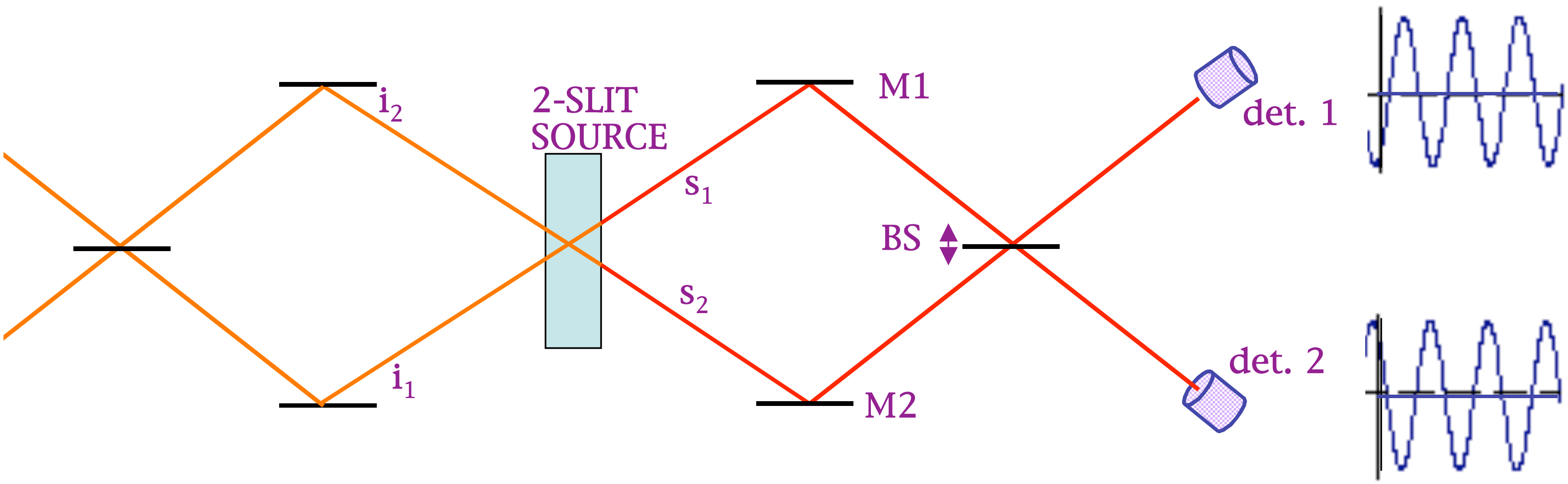
# Quantum Eraser
# (Scully, Englert, Walther)




**Suppose we perform a which-path measurement using a *microscopic* pointer, z.B., a single photon deposited into a cavity. Is this really irreversible, as Bohr would have all measurements? Is it sufficient to destroy interference? Can the information be "erased," restoring interference?**

# A microscopic measurement

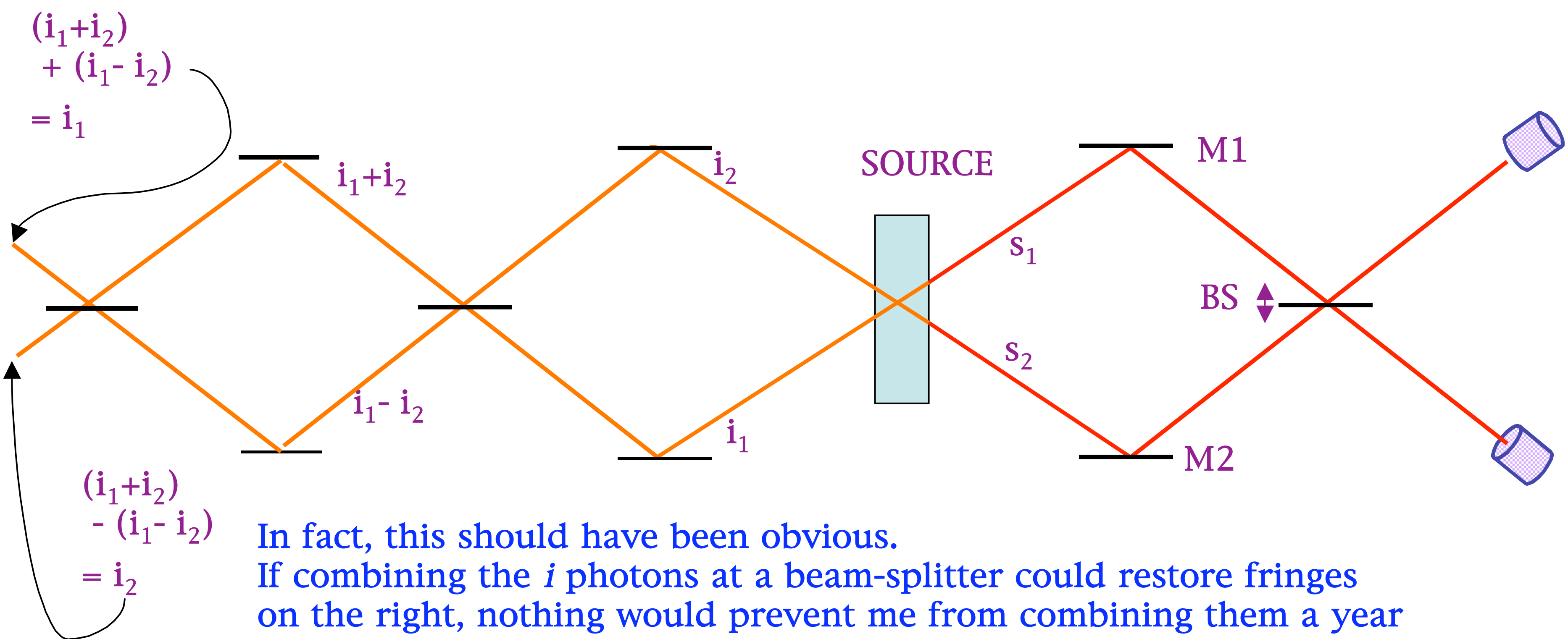
Let “i” photons constitute a which-path measurement on s...




The "i" photons provide which-path information, and destroy the interference. Can this information be "erased"?

If it's no longer possible to tell whether the photon came from $s_1$ or $s_2$, then interference is restored!

# But it *is* still possible...




In fact, this should have been obvious.
If combining the *i* photons at a beam-splitter could restore fringes on the right, nothing would prevent me from combining them a year after you looked at your detectors. Could I change whether or not you had seen fringes ?!

UNITARY EVOLUTION CANNOT DESTROY INFORMATION!
ORTHOGONAL STATES REMAIN ORTHOGONAL FOR ALL TIME.

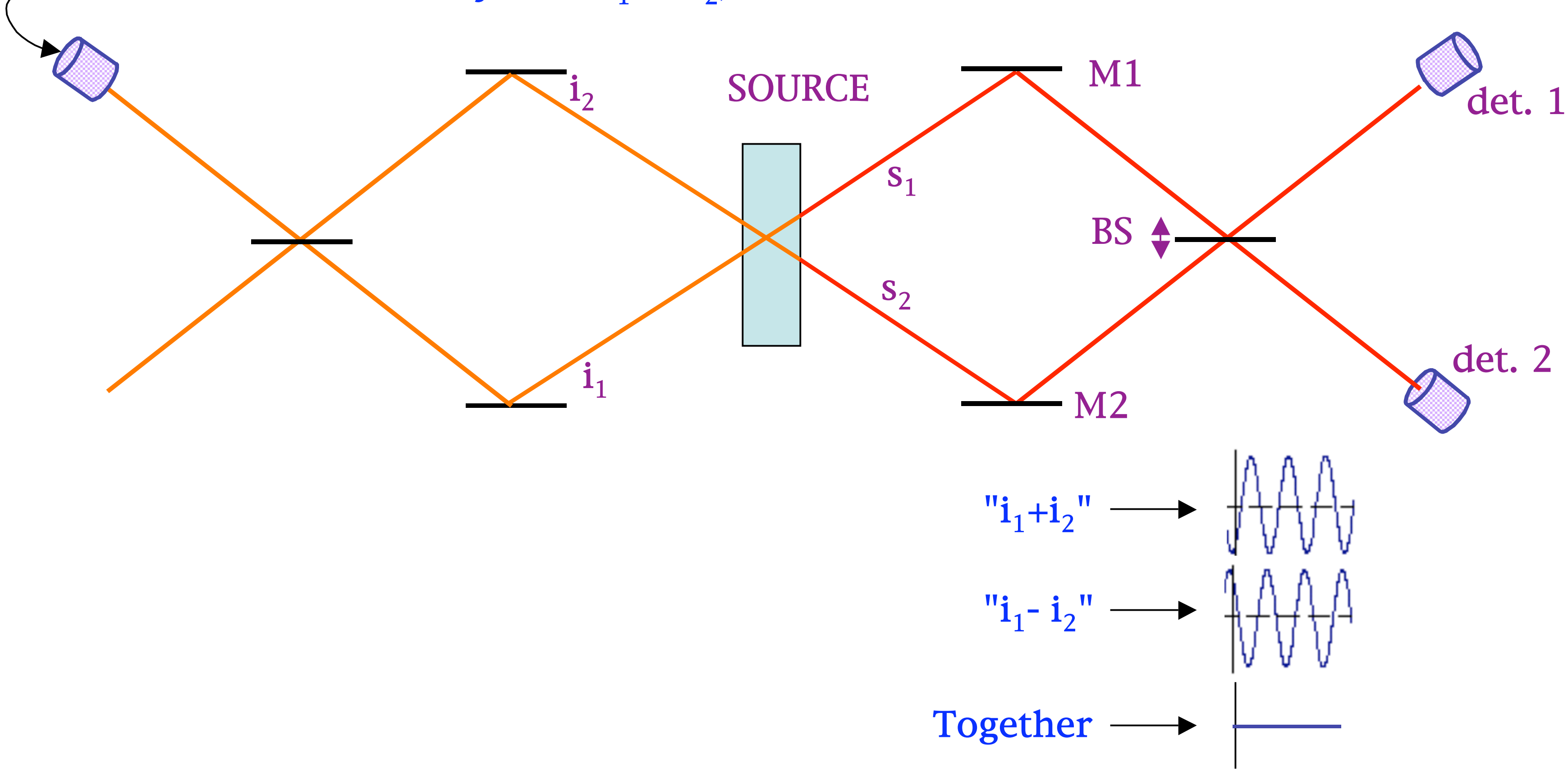
Obviously, nothing you do to the *idlers* can affect the *signals*.

# Sorry, that was another lie.

Nothing *unitary* I do to the idlers affects the signals.

Measurement is not unitary – in other words, if I only keep some events and throw out others, perhaps I *can* restore your interference.

Trigger on "$i_1$+$i_2$" events – no longer any way to tell whether they were $i_1$ or $i_2$, no matter what!



# Don't overlook the symmetry...

Detectors 1 and 2 are equally likely to fire, regardless of the phase setting.

When the "i1-i2" detector fires, this may tell me that detector 1 will fire instead of detector 2.

Of course, half the time, the "i1+i2" detector fires, telling me that detector 2 will fire instead of detector 1.

...or is it that half the time, detector 1 fires, collapsing the "i" photon into "i1-i2"...

...and that half the time, detector 2 fires, collapsing the "i" into "i1+i2"...?

Which is the system and which is the measuring apparatus?

# Some mathematics...

A superposition state: $\psi_s = \psi_a + e^{i\phi}\psi_b$

Probabilities: $|\psi_s|^2 = |\psi_a|^2 + |\psi_b|^2 + e^{i\phi}\psi_a^*\psi_b + e^{-i\phi}\psi_a\psi_b^*$

Interference terms →

Now consider a larger Hilbert space, including a Measuring Apparatus:

$\psi_S$ becomes $\psi_s|\mathrm{MA}\rangle$

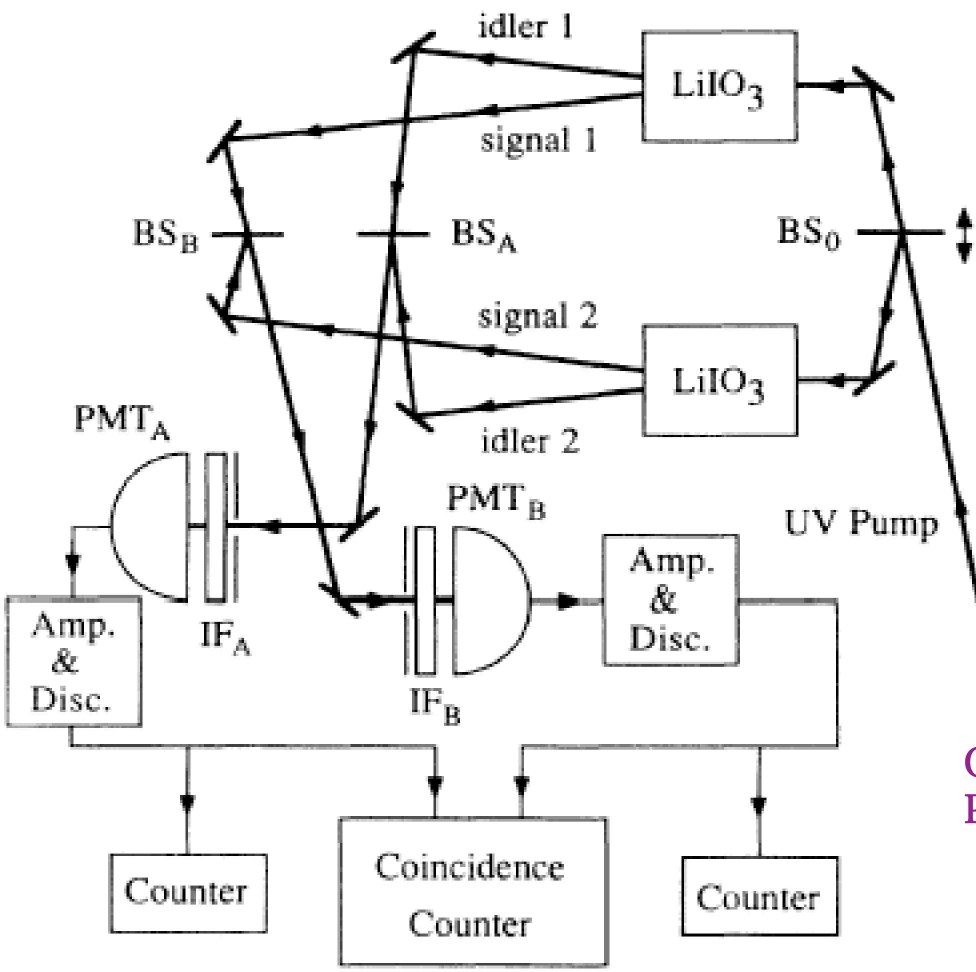
replace $\psi_a$ by $\psi_a|A\rangle$ and $\psi_b$ by $\psi_b|B\rangle$

New probabilities: $$|\psi_s|^2 = |\psi_a|^2\langle A|A\rangle + |\psi_b|^2\langle B|B\rangle + e^{i\phi}\psi_a^*\psi_b\langle A|B\rangle + e^{-i\phi}\psi_a\psi_b^*\langle B|A\rangle$$
$$\rightarrow |\psi_a|^2 + |\psi_b|^2 .$$ *NO INTERFERENCE!*

But what if we select (project) out, not A, and not B, but an equal superposition?

$$\left|\left(\frac{\langle A| + \langle B|}{\sqrt{2}}\right)\psi_s|\mathrm{MA}\rangle\right|^2 = \tfrac{1}{2}|\psi_a + e^{i\phi}\psi_b|^2$$
$$= \tfrac{1}{2}(|\psi_a|^2 + |\psi_b|^2 + e^{i\phi}\psi_a^*\psi_b$$
$$+ e^{-i\phi}\psi_a\psi_b^*) ,$$ *INTERFERENCE RETURNS!*

# Making it look more complicated...




Ou, Wang, Zou, & Mandel,
Phys Rev A 41, 566 (1990).

# Plus ça change...

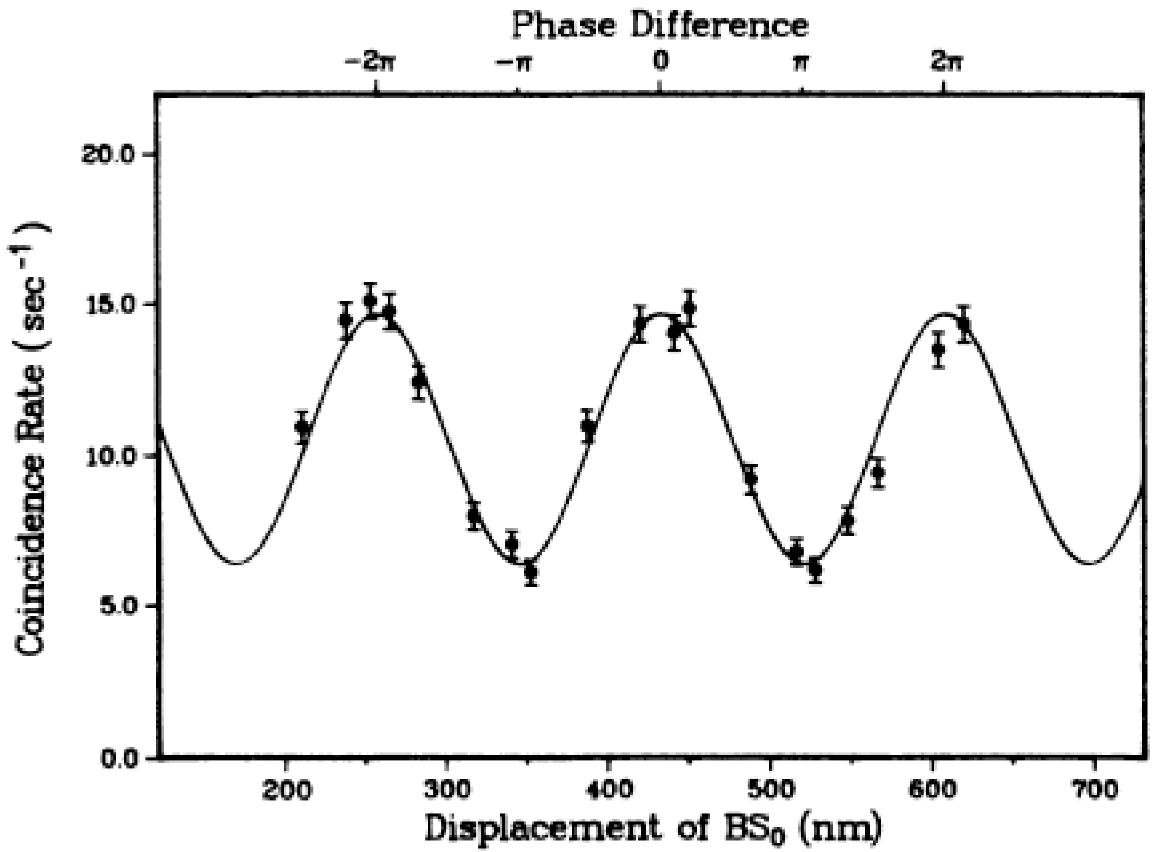
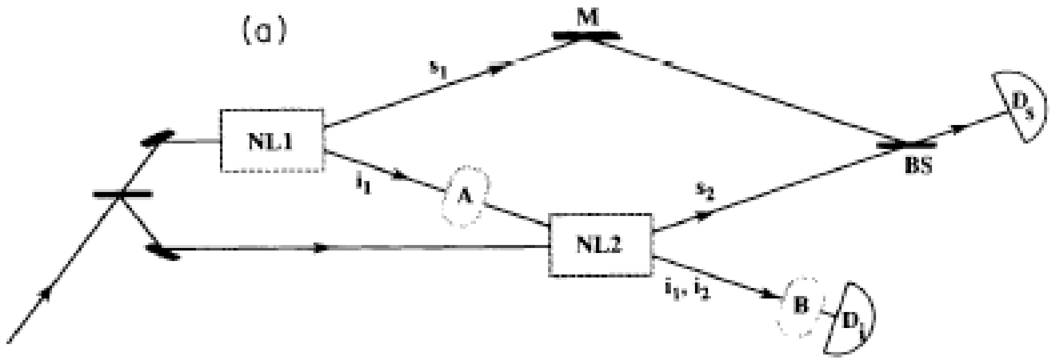
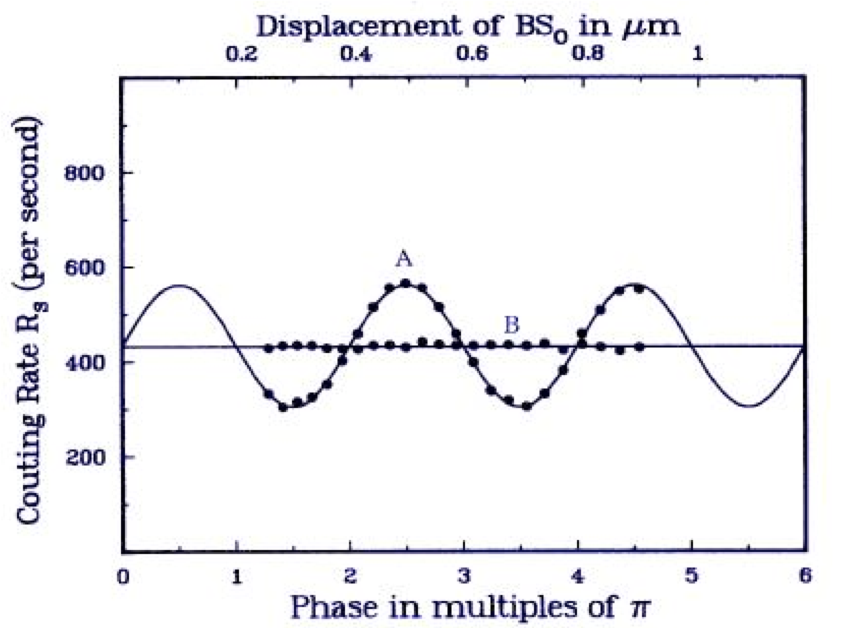



FIG. 3. Observed two-photon coincidence rate as a function of $BS_0$ displacement. The upper scale shows the phase difference between the two pump waves.

# What if you combine the idlers so they've got nowhere else to go?




Zou, Wang, Mandel, PRL 67, 318 (1991).



# More Bohr-Einstein debates

**Einstein:**
**I can't believe God plays dice with the universe.**

**Bohr:**
**Albert, stop telling God what to do.**

# Einstein, Podolsky, & Rosen (1935)

Source

Alice Bob

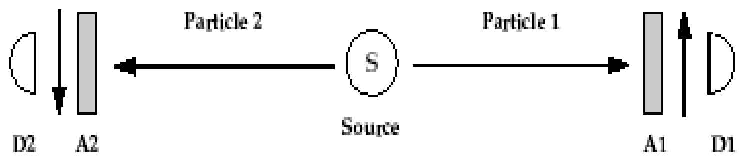



FIG. 1. Bohm's version of the EPR Gedankenexperiment

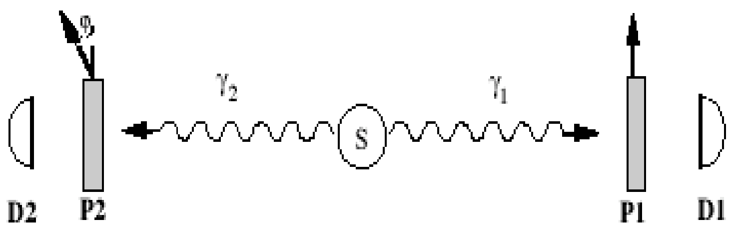



FIG. 2. Optical version of EPR experiment

2 particles emitted together at the same time with opposite speeds.

If Alice measures her particle's position, she knows Bob's. But if she measures her particle's momentum, she knows Bob's.

Did her measurement "affect" Bob's particle instantaneously?

Spooky action at a distance

Or did Bob's particle already have both?

Hidden variables (QM "incomplete")

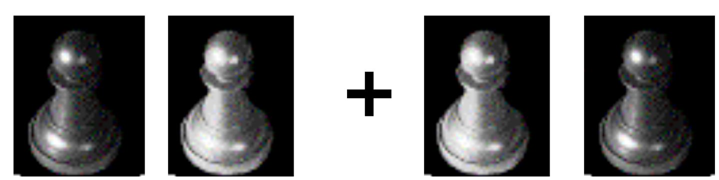
Schrödinger 1935:

"entanglement"

"Verschrenkung" (SP?!)

$$|\psi\rangle = |B\rangle_L |W\rangle_R + |W\rangle_L |B\rangle_R$$

=



# Hidden variables?

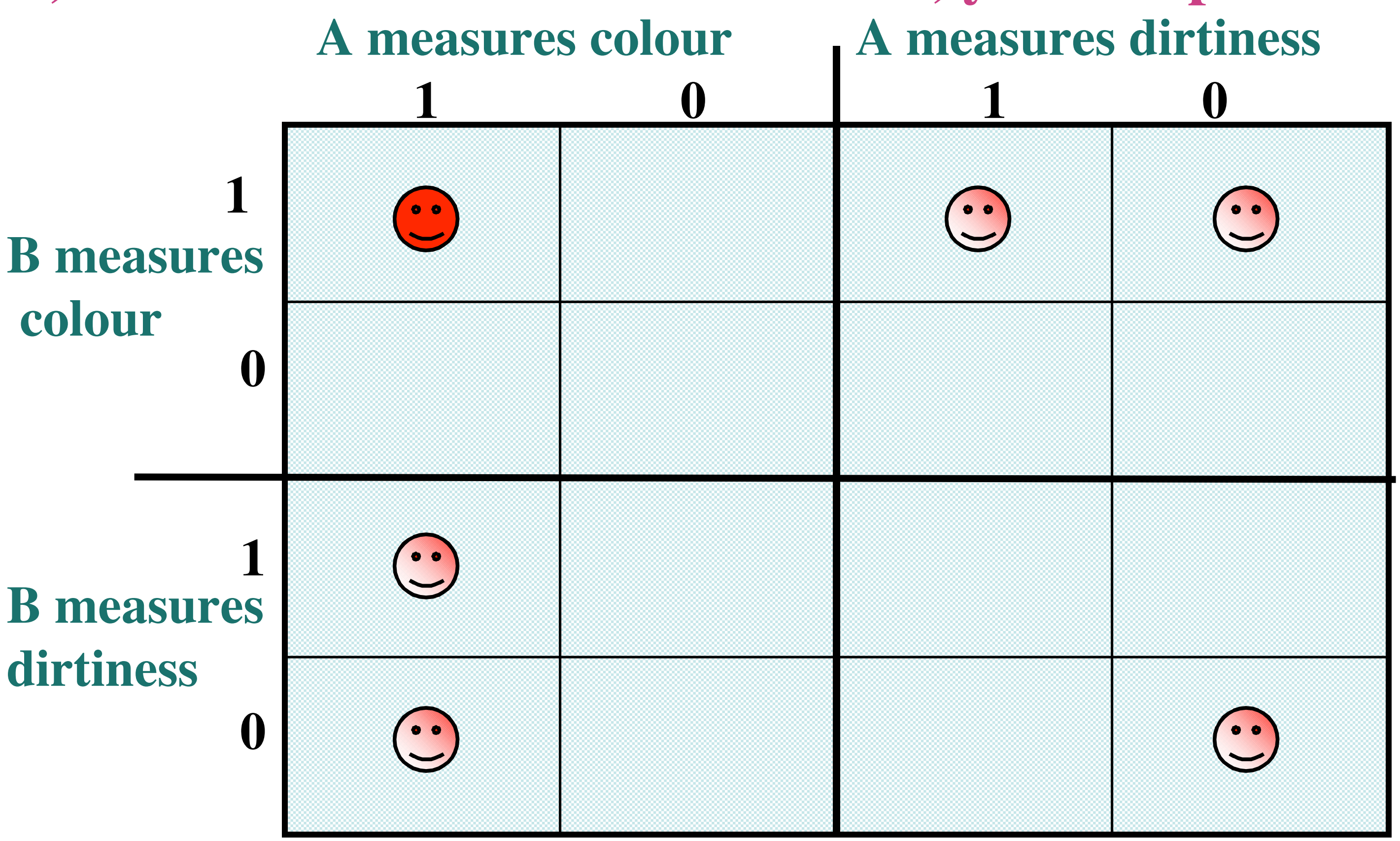
Einstein seems to have thought the particles "knew" what they were going to do, even if we didn't: QM not wrong but "incomplete".

John Bell's example, "Bertlmann's socks":


# Bell's Theorem

**Forget Quantum Mechanics.**

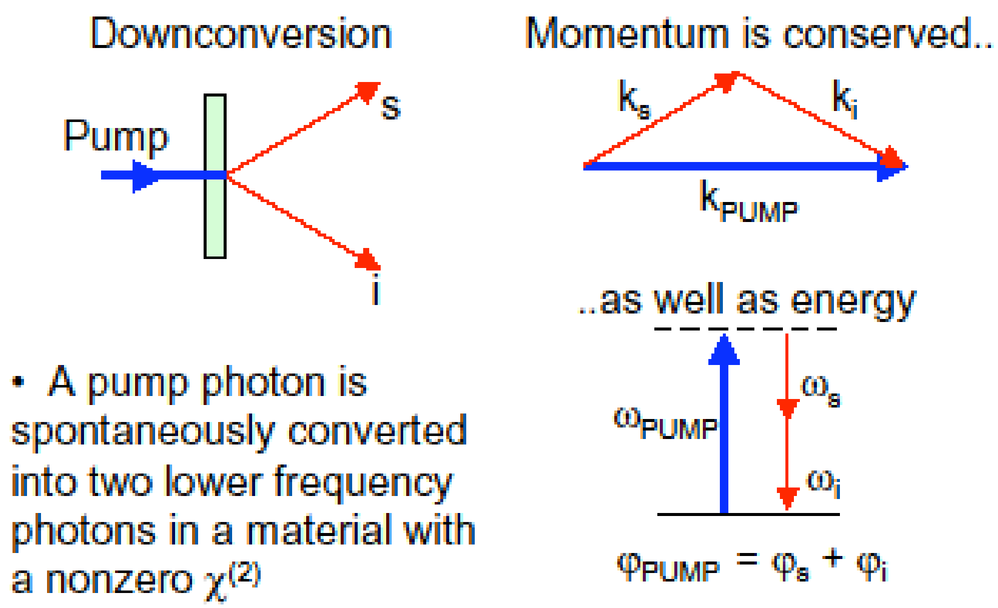
**Suppose you've got two particles, and A & B can choose what to measure on each of them – "color" or "dirtiness", for example. For each measurement, they either get "1" or "0". If there are "hidden variables," then A's choice doesn't affect B, and vice versa – from this alone, you can prove something.**




**The HVs must tell me what would happen for any choice of measure-ment: i.e., which box of *each quadrant* the particle is "in."**

$$P(cc \Rightarrow 11) \leq P(cd \Rightarrow 11) + P(dc \Rightarrow 11) + P(dd \Rightarrow 00)$$

# Entangled photon pairs
## (spontaneous parametric down-conversion)




The time-reverse of second-harmonic generation.

A purely quantum process (cf. parametric amplification)

Each energy is uncertain, yet their sum is precisely defined.

Each emission time is uncertain, yet they are simultaneous.

# "Spontaneous parametric down-conversion"

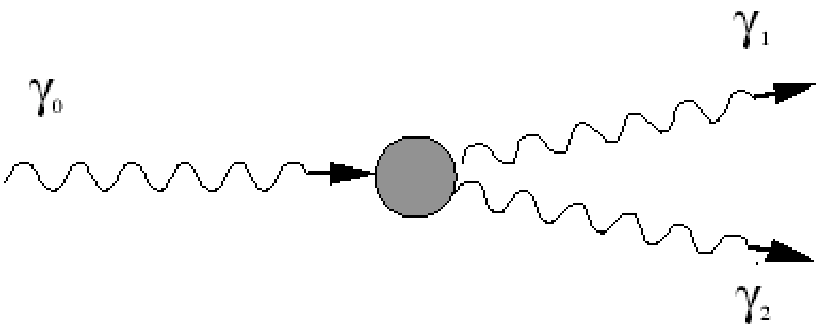



FIG. 3. Two-photon decay from one photon

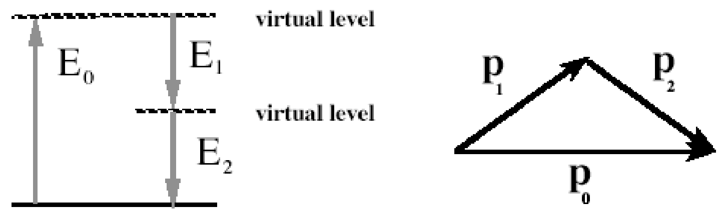
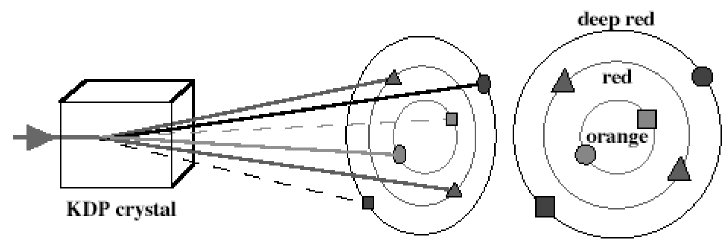
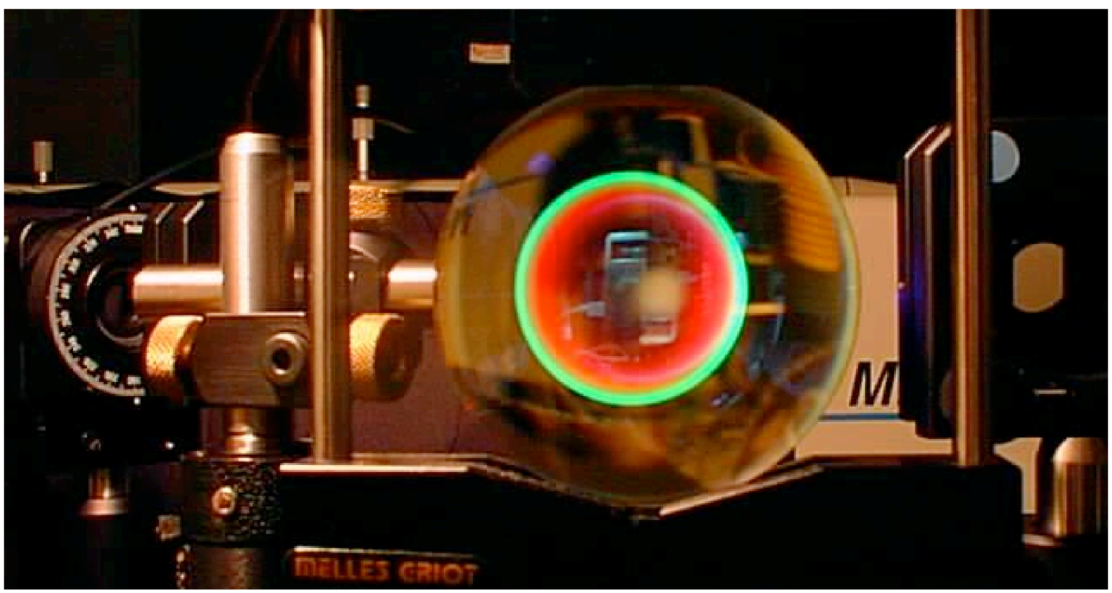


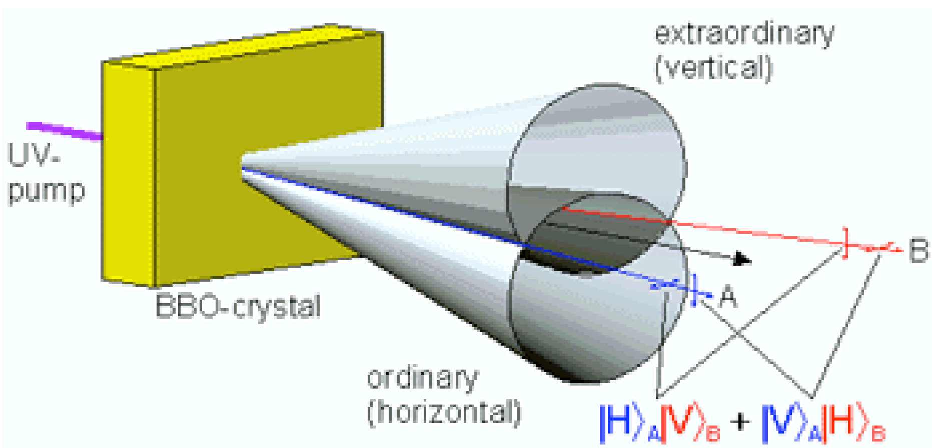

FIG. 5. Energy level diagram; momentum conservation triangle

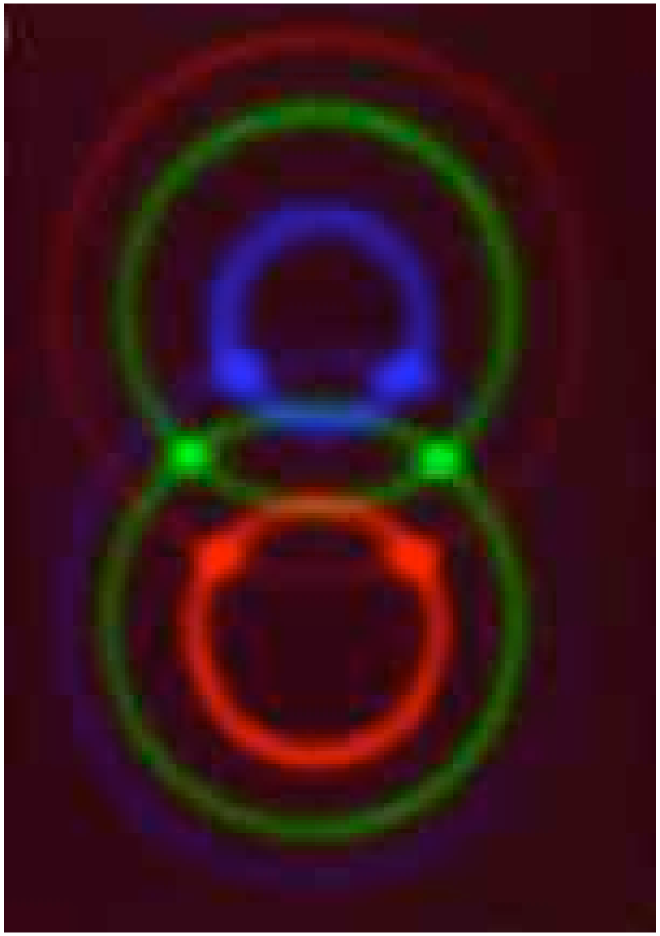


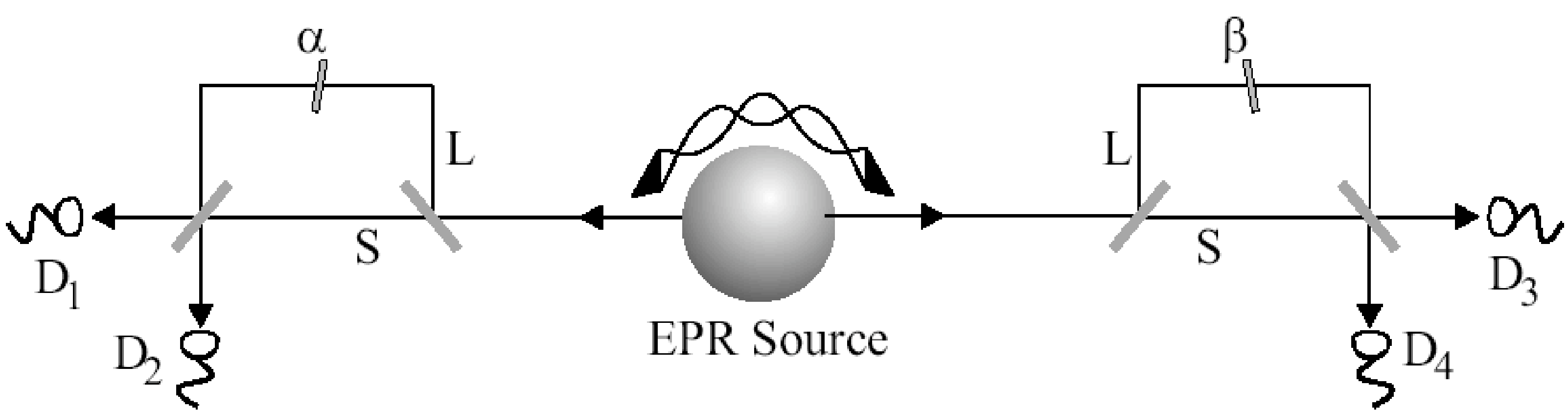
# More sophisticated "sources of entangled photons"

**Type-I: signal & idler have same pol's**
**Type-II: signal & idler have opposite pol's**



# The “Franson experiment” – an example of an optical EPR exp’t




$$\alpha|\text{short}\rangle_1|\text{short}\rangle_2 + \beta|\text{long}\rangle_1|\text{long}\rangle_2$$

# An example of an EPR ("Bell inequality") experiment

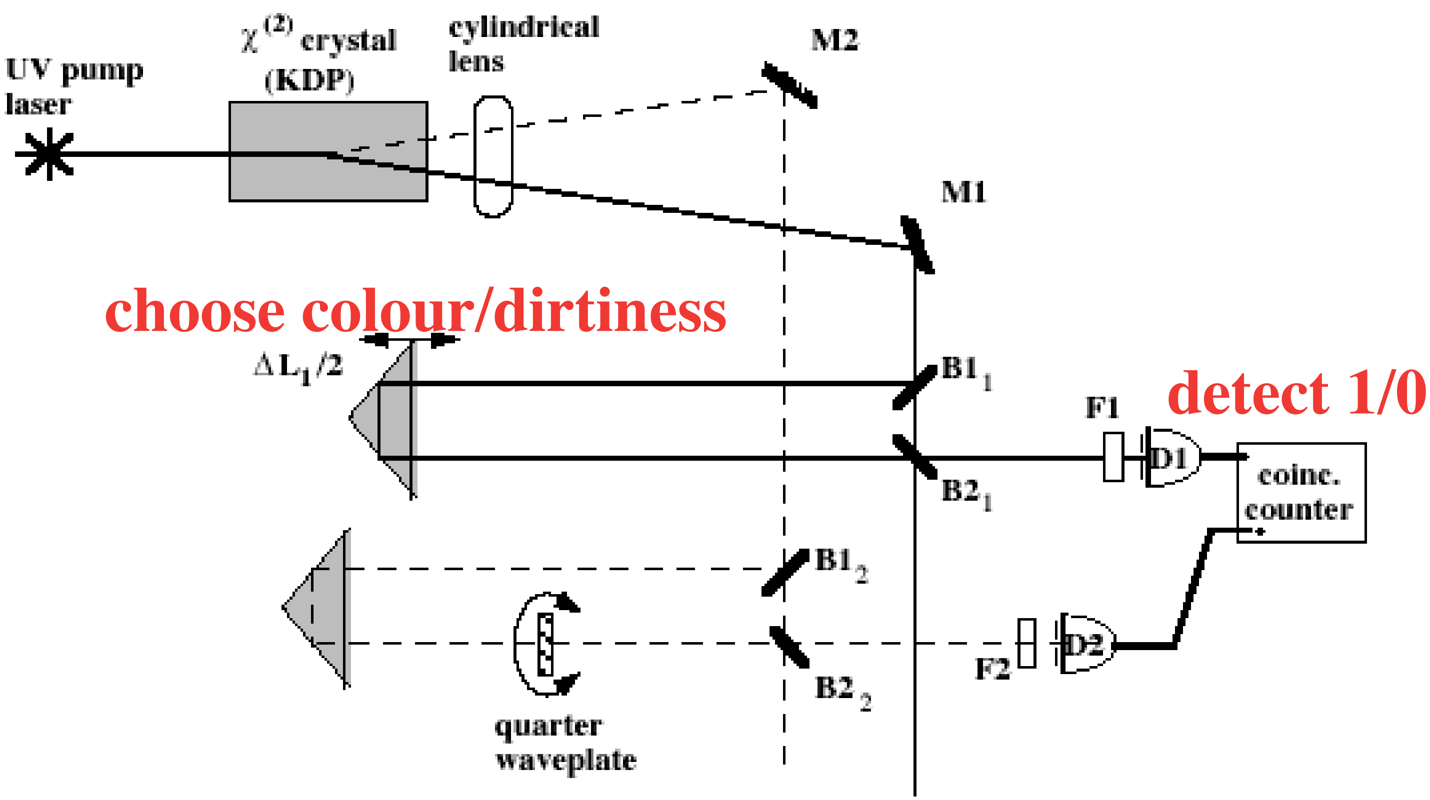
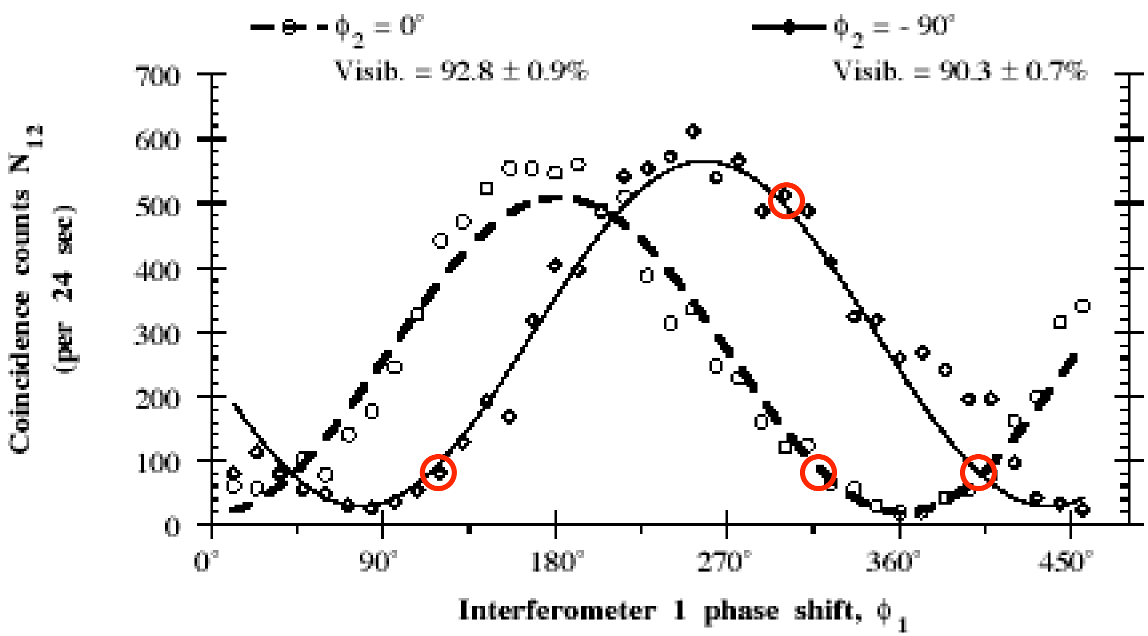



FIG. 7. Apparatus used at Berkeley to perform the Franson experiment

# The "colour/dirtiness" curve for a photon pair




Bell's inequality is violated – in other words, whether or not quantum mechanics is right, this experiment can't be explained by "local hidden variables."

Somehow, we know that the particles don't know what they're doing!

# "FLASH" !?

**So, does Bob immediately know what Alice chose to measure?**

**NO! If she chose "dirtiness," she already knows whether his is clean or dirty – but the answer was random.**
**If she chose "colour," then she knows whether his is pink or not pink – so its "dirtiness" is undetermined.**

**Bob gets a random answer no matter what... but was the random answer known before he made his measurement?**

**Nick Herbert: if he made 100 copies ("clones") of his photon before measuring, then he could see whether they all have the same dirtiness (because Alice already knew it), or whether each one was random (because Alice measured "colour").**

**They could communicate faster than light!**