Michigan Quantum Summer School

Ann Arbor, June 16-27, 2008.

Lecture2: Quantum Decoherence and Maxwell Angels

L. J. Sham, University of California San Diego

1. Motivation: Quantum superiority in superposition states
2. Problem: How does a quantum coin turn into a mundane one?
   a. Study the decoherence of a spin in the model of an electron in a dot with a million of mutually interacting nuclear spins
3. A knowledge of the decoherence process leads to the existence of the Maxwell angel who restores coherence.
   a. How?

Decoherence time and dipole strength

Order of magnitude estimates from measurements and theory

Stronger interaction with macroscopic systems: shorter decoherence time and faster operation

System	Size	Dipole (eA)	T1	T2	Op time
Exciton in FQD	30 nm	20	50ps	100ps	1ps
Spin in SAQD	10 nm	5	10ms	10μs	op 10ps μw 1ns
Supercond. phase gate	10 μm	--	100ns	200ns	10ns
Trapped ions	0.1 nm	0.1	1 s	10 ms	10 ns

Nielsen & Chuang: 0.1 fs but phonon limited

Relaxation & Decoherence of Spin in a Bath

Prepare state **Quantum system** $\beta|+\rangle + \alpha|-\rangle$

Initial state **Spin + Bath** $(\beta|+\rangle + \alpha|-\rangle)|\,J\rangle$

Time evolution **Entanglement** $\beta|+\rangle\,|J^{+}\rangle + \alpha|-\rangle|J^{-}\rangle$

Query on spin **Reduced density matrix**

$$\hat{\rho} = \begin{bmatrix} \beta\beta^* & \beta\alpha^* \\ \alpha\beta^* & \alpha\alpha^* \end{bmatrix} \rightarrow \begin{bmatrix} |\beta|^2\langle J^+|J^+\rangle & \beta\alpha^*\boxed{\langle J^+|J^-\rangle} \\ \alpha\beta^*\langle J^-|J^+\rangle & |\alpha|^2\langle J^-|J^-\rangle \end{bmatrix}$$

Ensemble average over bath state J

population decay from ρ_{++} to ρ_{--} : longitude relax time T_1 **Long!**

decoherence: coherence ρ_{+-} decay transverse relaxation time T_2

Zurek, Physics Today (1991), revised in Los Alamos Science (1992).

Existence of a Maxwell Angel

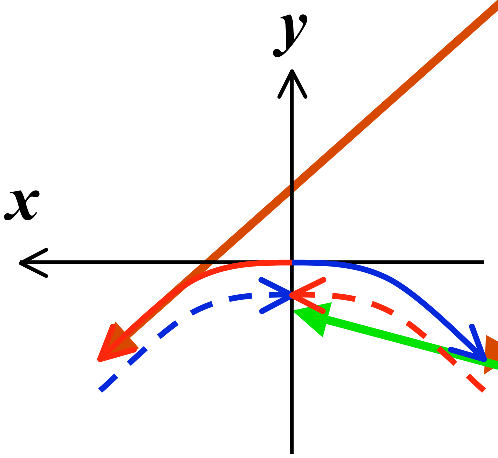
Coherence: $\langle J^+ | J^- \rangle$

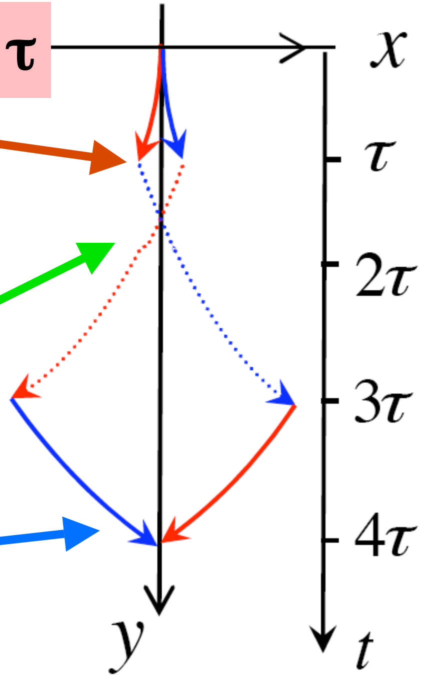
Trajectories of bath states:
blue lines $| J^+ \rangle$
red lines $| J^- \rangle$

Spin echo recovery of coherence

Pure coherence recovery

The angel changes $| + \rangle$ to $| - \rangle$ at τ





At (√2)τ
dis-entanglement
not original state

echo at 4τ

echo at 2τ

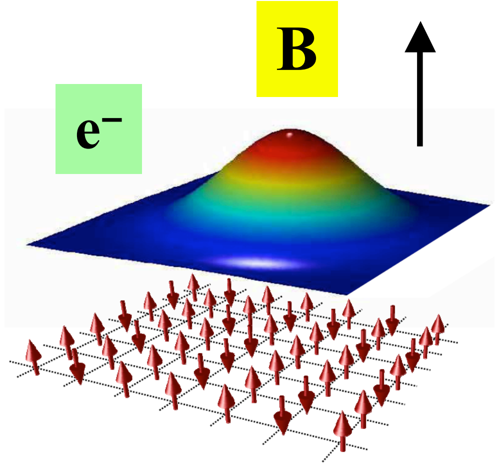
Bath interaction A

Bath interaction B

Spin decoherence of an electron in a III-V quantum dot

Mechanisms of decoherence:

- Spin-orbit + electron-phonon: negligible below 1 K
  - Semenov & Kim (2004)
  - Golovach, Khaetskii & Loss (2004)
  - Experiments (Kroutvar et al., Steel's group) at 4K
- Spontaneous emission: negligible
- Ultimate decoherence source
  - **electron interaction with nuclear spins (hyperfine)**
  - **nuclear spin-nuclear spin interaction**



Measured for SQAD dots

B ~ 4 T

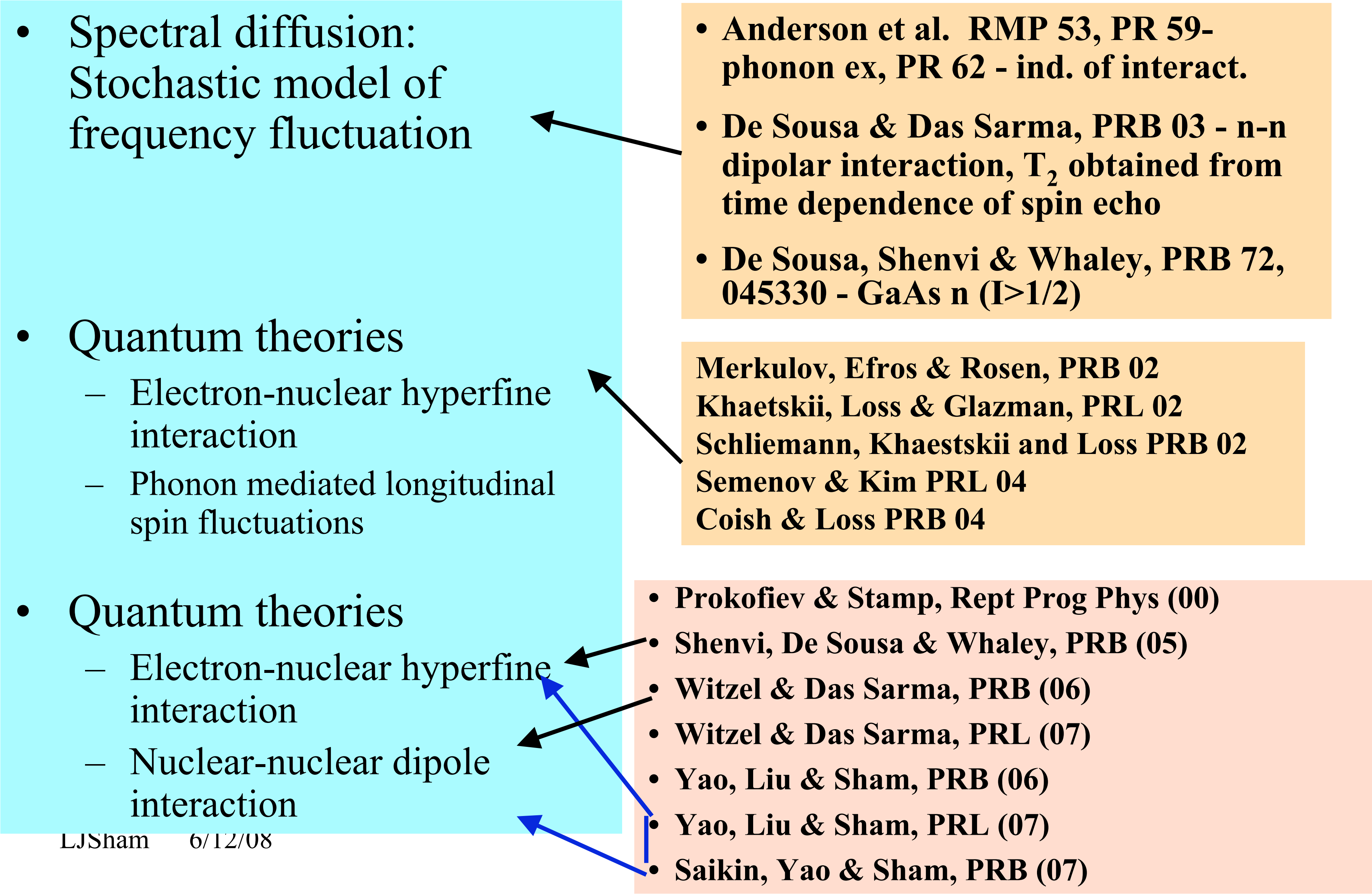
T_1 ~ 20 μs **T_2^* ~ 10 ns**

Calc **T_2 > 1 μs**

Expt **T_2 > 3 μs**

Op **t ~ 10 ps**

Antecedents of Spin Decoherence Theories



- Spectral diffusion: Stochastic model of frequency fluctuation

- Quantum theories
  - Electron-nuclear hyperfine interaction
  - Phonon mediated longitudinal spin fluctuations

- Quantum theories
  - Electron-nuclear hyperfine interaction
  - Nuclear-nuclear dipole interaction

- **Anderson et al. RMP 53, PR 59-phonon ex, PR 62 - ind. of interact.**
- **De Sousa & Das Sarma, PRB 03 - n-n dipolar interaction, T_2 obtained from time dependence of spin echo**
- **De Sousa, Shenvi & Whaley, PRB 72, 045330 - GaAs n (I>1/2)**

Merkulov, Efros & Rosen, PRB 02
Khaetskii, Loss & Glazman, PRL 02
Schliemann, Khaestskii and Loss PRB 02
Semenov & Kim PRL 04
Coish & Loss PRB 04

- **Prokofiev & Stamp, Rept Prog Phys (00)**
- **Shenvi, De Sousa & Whaley, PRB (05)**
- **Witzel & Das Sarma, PRB (06)**
- **Witzel & Das Sarma, PRL (07)**
- **Yao, Liu & Sham, PRB (06)**
- **Yao, Liu & Sham, PRL (07)**
- **Saikin, Yao & Sham, PRB (07)**

Decoherence process when T_1 is long

Hamiltonian $\hat{H} = \sum_{\pm} |\pm\rangle \hat{H}_{\pm} \langle \pm|$ **Terms leading to T_1 removed**

Quantum system $\beta|+\rangle + \alpha|-\rangle$ **Initial state**

spin bath $(\beta|+\rangle + \alpha|-\rangle)|J\rangle \xrightarrow{t} \beta|+\rangle|J^+\rangle + \alpha|-\rangle|J^-\rangle$

Decoherence by spin query (tracing over bath states)

Coherence

$$\mathcal{L}(t) = \langle J^+|J^-\rangle = \langle J|e^{iH_+t}e^{-iH_-t}|J\rangle$$

decoherence: **Coherence $\mathcal{L}(t)$ decays with time**

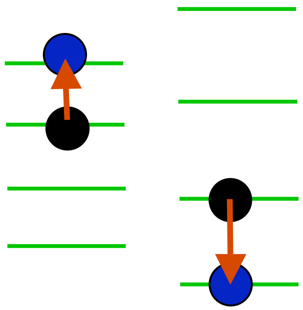
Ensemble average over bath state J

No a priori stochastic assumption

Independent nuclear spin-pair correlation approximation

Dominant interaction effect: Nuclear pair flip excitations

$|m\rangle_1\ |m'\rangle_2 \Longleftrightarrow |m-1\rangle_1\ |m'+1\rangle_2$ mapped to a pseudo-spin 1/2



Each pseudospin moves in an effective magnetic field which is calculated from H to second order in hf spin flip

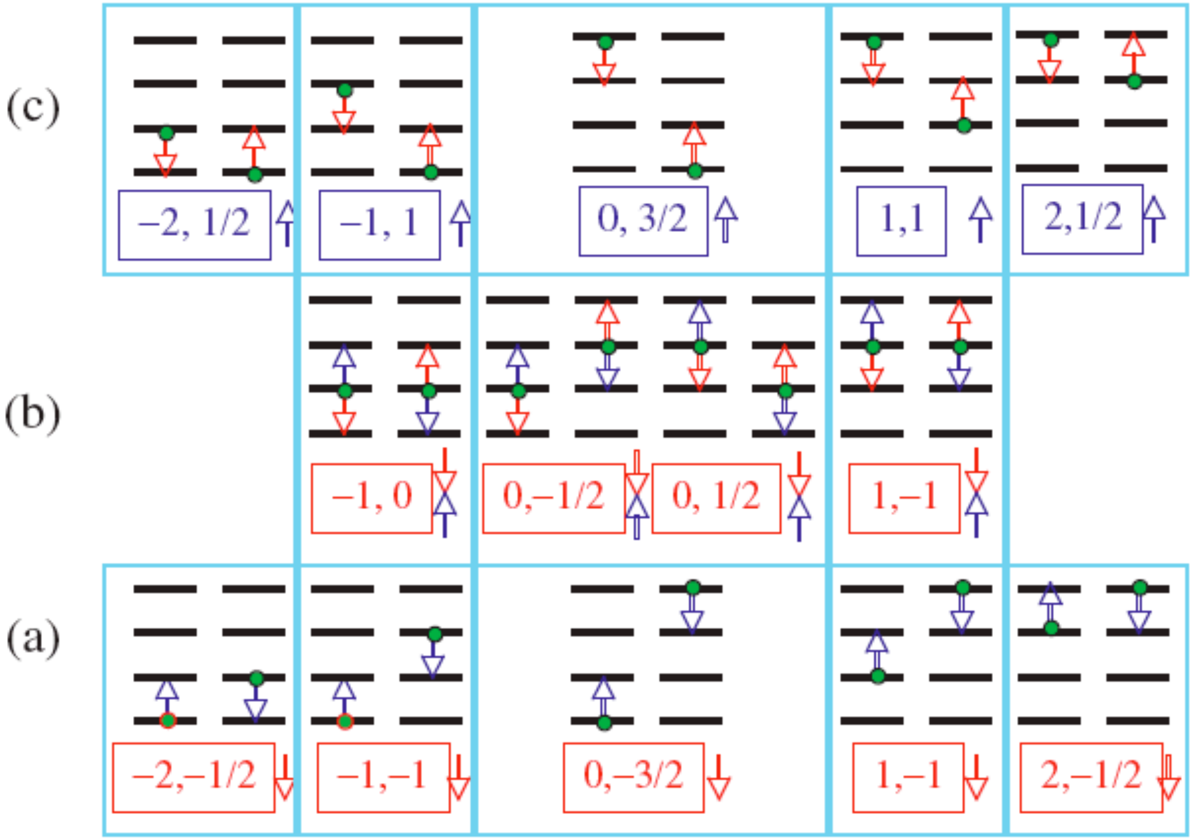
Entanglement overlap

$$\langle J_-(t)|J_+(t)\rangle = \langle J| \prod_k \left[e^{i\mathcal{H}_k^- t} e^{-i\mathcal{H}_k^+ t} \right] |J\rangle$$

Bath spin pair excitation states

$$
\begin{aligned}
k_{nm} &= j_n + j_m, \qquad -2I+1 \le k_{nm} \le 2I-1; \\
\ell_{nm} &= \frac{1}{2}(j_n - j_m), \qquad -I + \frac{1}{2}|k| \le \ell_{nm} \le I - \frac{1}{2}|k| - 1.
\end{aligned}
$$

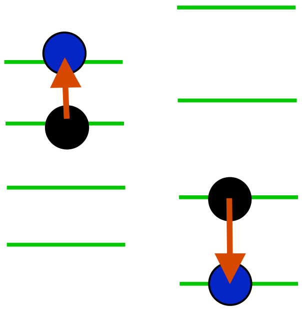
$$
\begin{array}{c|ccccccc}
k = & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
 & & & & \frac{3}{2} & & & \\
 & & & 1 & & 1 & & \\
 & & \frac{1}{2} & & \frac{1}{2} & & \frac{1}{2} & \\
\ell = & 0 & & 0 & & 0 & & 0 \\
 & & -\frac{1}{2} & & -\frac{1}{2} & & -\frac{1}{2} & \\
 & & & -1 & & -1 & & \\
 & & & & -\frac{3}{2} & & &
\end{array}
$$



Validity of the independent pair correlation approximation

Error estimate of higher order correlations negligible if

Dot is mesoscopic: # nuclear spins N ~ 10^4 -- 10^8



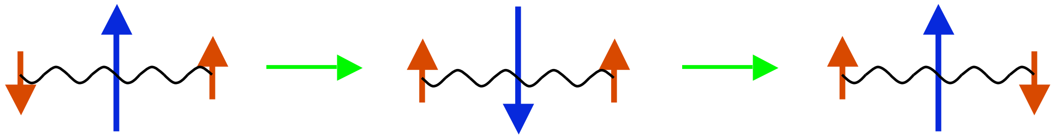
Time scale t ~ $T_2 \ll 1/b$

b = intrinsic n-n interaction

$$\max\left[\sqrt{N}, a/(b^2\omega_e^4)^{1/6}\right] \ll N \ll a/b$$

statistics

extrinsic n-n interaction



e spin

n spin

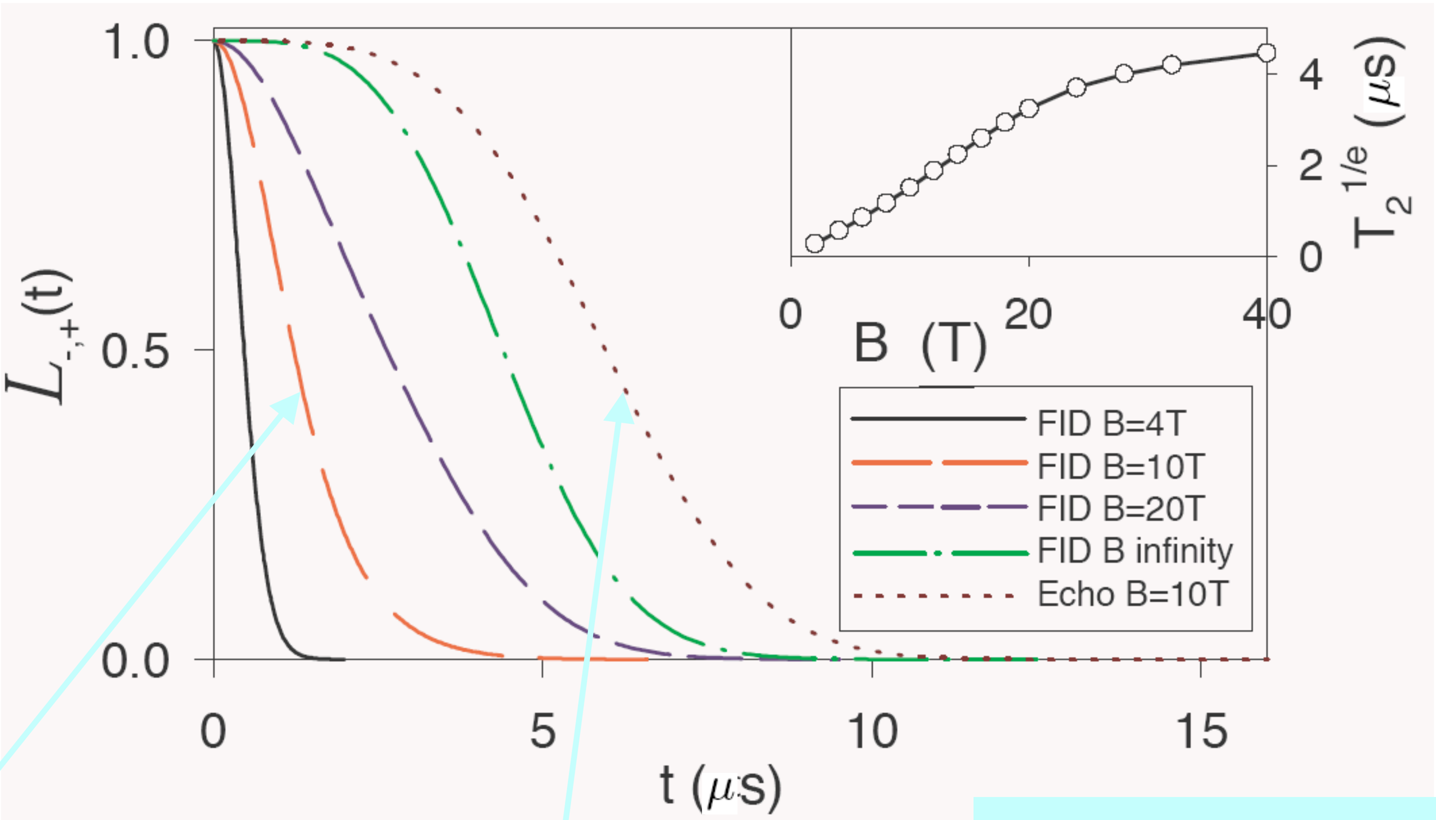
ground state

virtual excited state

n-pair flip state

Free induction decay for a single bath state

GaAs dot (h=3 nm x r=15 nm)



FID decoherence time ≠ Spin echo decay time

Partial recovery of pure coherence known in NMR

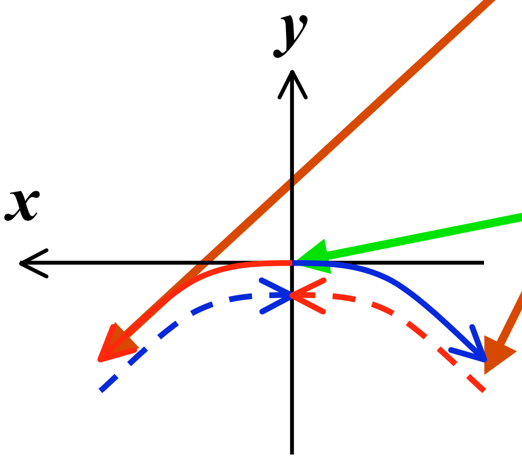
Yao, Liu & Sham, PRB (06)

Physical picture of nuclear spin dynamics

Extrinsic n-n interaction

Intrinsic n-n interaction

No N state prep!

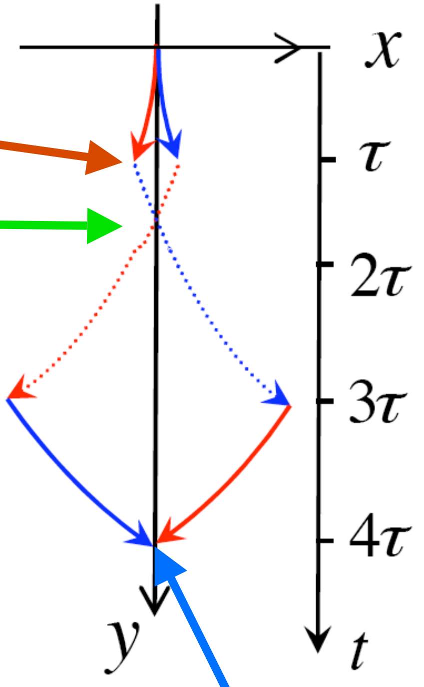


π pulse at τ

dis-entanglement at (√2)τ

echo at 2τ

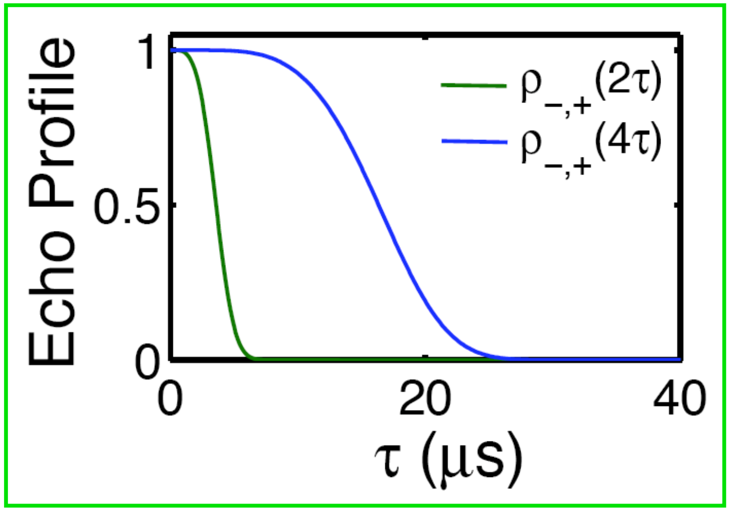
CP: π pulse at 3τ



echo at 4τ

GaAs dot
h = 3 nm
r = 15 nm

B = 10 T
T_N high



Decoherence evolution under sequence of pulses

InAs dot

33nm x 33 nm x 3nm

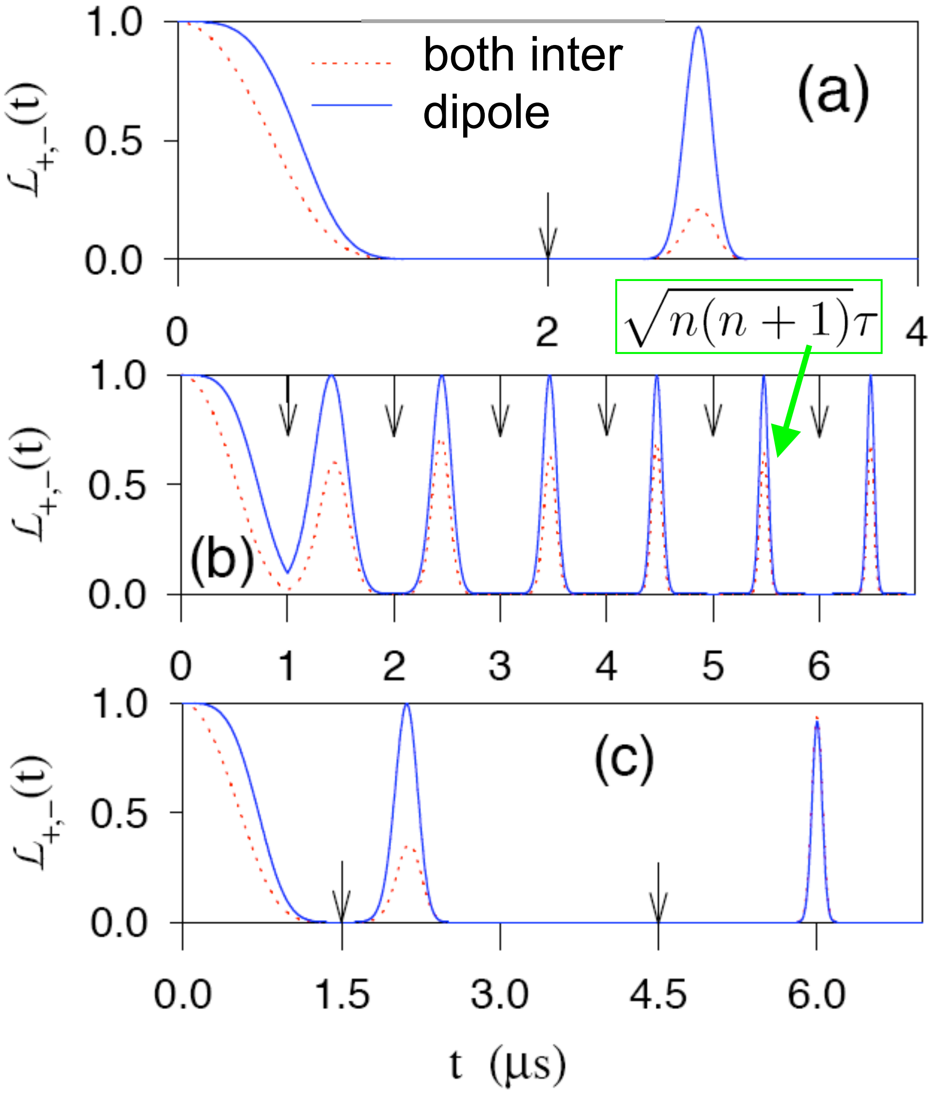
B = 12 Tesla

T = 1K

Single pulse

Periodic pulses

Carr-Purcell pulses



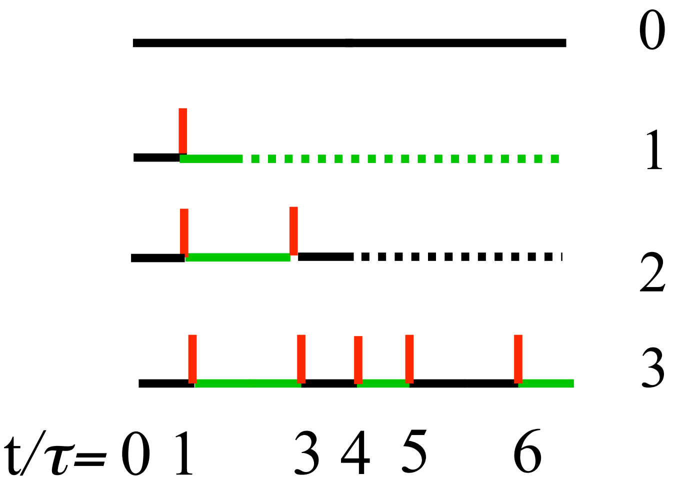
Liu, Yao, Sham, New J Phys. 9, 226 (2007)

Concatenated pulses for control of entanglement

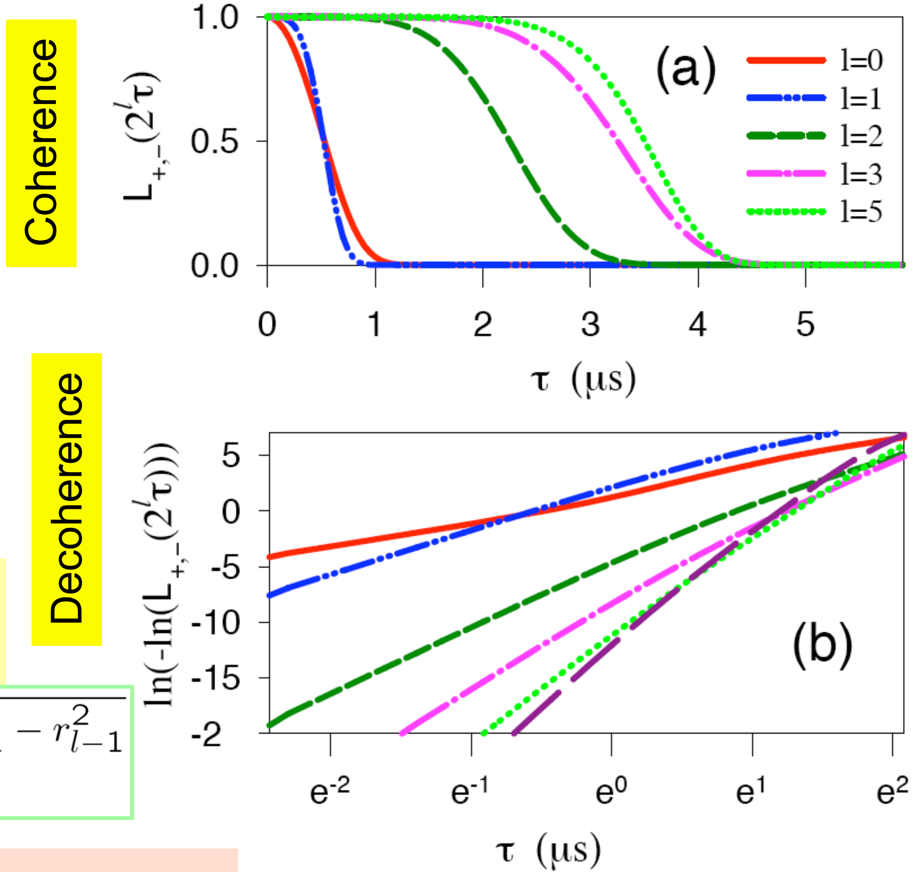
Concatenated dynamical decoupling Khojasteh & Lidar, PRL 95, 180501 (2005)

Adopt concatenation idea but not to decouple the e-n interaction

π-pulse sequences for concatenation level l =



$L_{[001]} = 5.7$ nm, $r_0 = 20$ nm, $B_{ext} = 10$ T



$$U_0^+ \equiv \exp\left(-iH^+\tau\right), \quad U_{l+1}^+ = U_l^- U_l^+$$

Rot $\sin\theta_l^{\pm}\hat{\boldsymbol{n}}_l^{\pm} \equiv \mathbf{R}_l \pm \mathbf{r}_l$

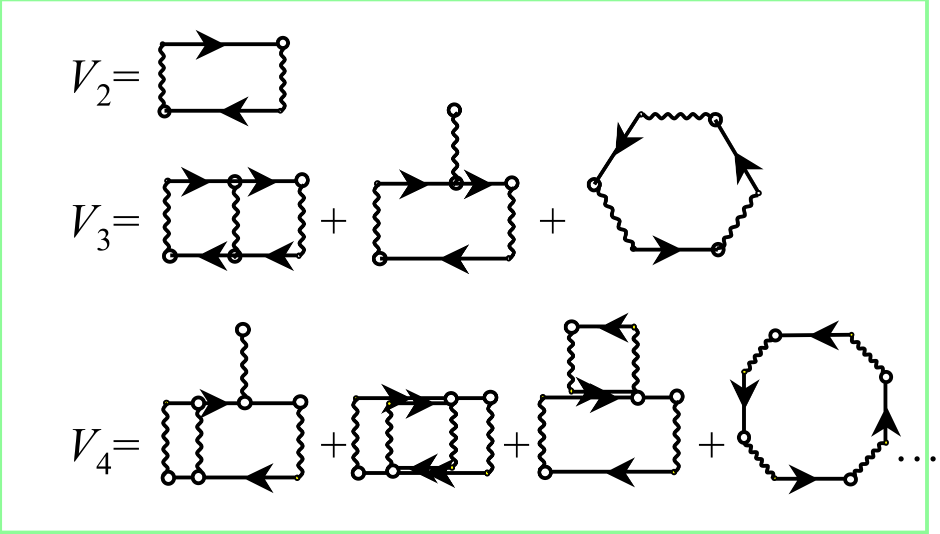
$$\begin{aligned}\mathbf{R}_l &= 2\mathbf{R}_{l-1}\sqrt{1-R_{l-1}^2-r_{l-1}^2}\\ \mathbf{r}_l &= 2\mathbf{R}_{l-1}\times\mathbf{r}_{l-1}\end{aligned}$$

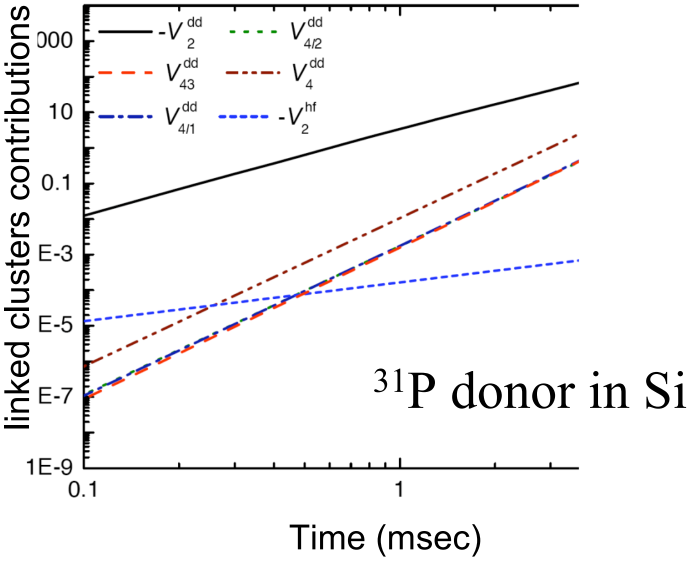
LJSham 6/12/08

Yao, Liu, Sham, PRL 07

Linked cluster expansion of qubit decoherence

$$\rho_{\uparrow\downarrow}(t) = \rho_{\uparrow\downarrow}(0)e^{-i\omega_{en}t+V_2(t)+V_3(t)+V_4(t)+\dots}$$





Results:

- Expansion in exponent provide faster convergence (partial sum of perturb series).
- Up to V2 terms in agreement with pair approx (pseudo-spin model)
- Accounts for bath excitations beyond the spin pair approximation.
- Contributions of 2, 3 and 4 nuclear spin excitations to decoherence of a single ^{31}P donor electron spin in Si are evaluated for FID and 2-pulse echo setups.
- For magnetic fields >100G contributions of hyperfine-mediated interaction are negligible.

Saikin, Yao & Sham, PRB 07

Summary

A two-level system (TLS) in an interacting spin bath

- **Conditions:**
  - Magnetic field
    - High "field" regime: after query of (or non-unitary super-operation on) the small quantum system, its state is irreversible, except at points of separable state of the quantum system and bath. Complexity of reversal control.
    - Low "field" regime: q chaos + decoherence → c chaos (Zurek); outside the scope of our theory; exotic use of q machines?
  - Bath mesoscopic
  - TLS-bath interaction >> bath spin-spin interaction
- **Decoherence in TLS by entanglement with bath**
- **Restoration of its coherence by state flip of TLS**
- **Not treated: decoherence of entangled states, entanglement in many-particle states, QPT, ….**