Quantum Simulation using Optical Lattices
B. DeMarco, University of Illinois at Urbana-Champaign

Agenda:

• Hubbard model loose ends
• Thermometry and Cooling

• Characteristic density; universal phase diagrams
• Challenges in cooling (and thermometry)
• Spin-dependent lattices: something we are doing about it (that won’t work)
High temperature superconductivity / Hubbard model

$H = -t \sum_{\langle ij \rangle, \sigma} c^\dagger_{i,\sigma} c_{j,\sigma} + U \sum_i n_{i,\uparrow} n_{i,\downarrow}$

tunneling interactions

- Can Hubbard model produce d-wave SC?
- Nature of pseudo-gap phase?
Effective chemical potential:

$$
\tilde{\mu} = \mu - m\omega^2 r_i^2 / 2
$$
Universal phase diagram: bosons

Characteristic density

$$\tilde{\rho} = N \left( \frac{m_\omega^2 d^2}{2 j t} \right)^{j/2}$$

$j$: dimensionality
Deviation from LDA

Spielman (JQI/NIST)  
arXiv:1003.1541
Universal phase diagram: fermions

De Leo, Kollath, Georges, Ferrero, and Olivier Parcollet

\[ H = -t \sum_{\langle ij \rangle, \sigma} c_{i,\sigma}^{\dagger} c_{j,\sigma} + U \sum_i n_{i,\uparrow} n_{i,\downarrow} + \sum_{i, \sigma} n_{i, \sigma} \varepsilon_i \]
Fermi Liquid

(Zweirlein)
Universal phase diagram: fermions

De Leo, Kollath, Georges, Ferrero, and Olivier Parcollet

\[ H = -t \sum_{\langle ij \rangle, \sigma} c_{i,\sigma}^\dagger c_{j,\sigma} + U \sum_i n_{i,\uparrow} n_{i,\downarrow} + \sum_{i,\sigma} n_{i,\sigma} \varepsilon_i \]
Anti-ferromagnetic (AFM) order

Effective $t/U \ll 1$ Hamiltonian:

$$H = J \sum_{\langle ij \rangle} \hat{S}_i \cdot \hat{S}_j \quad J = 4t^2/U$$

AFM achieved for $S/N = \frac{1}{2} k_B \ln 2 \approx 0.4 k_B$

So far out of reach!

Requires $T/T_F = 0.03$ (ideal gas)...
...state-of-the-art in lattice: $T/T_F = 0.1$ (?)
A solution?

Species / spin-dependent lattice
- Two spin states
- Two atomic species ($^{40}\text{K}/^{87}\text{Rb},^{133}\text{Cs}/^{40}\text{K}$)
Real Atoms

Example: $^{87}\text{Rb}$

$F = 2, g_F = 1/2$

$F = 1, g_F = -1/2$

$U_{dip} = \frac{\pi c^2 \Gamma}{2\omega_0} \left( \frac{2 + P g_F m_F}{\delta_{3/2}} + \frac{1 - P g_F m_F}{\delta_{1/2}} \right) I(\vec{r})$

$P = -1 (\sigma^-), 0 (\pi), 1 (\sigma^+)$  Polarization *in the atomic basis*

$\Gamma = \frac{\omega_0^3}{3\pi \epsilon_0 \hbar c^3} |\langle e|e\vec{r}|g\rangle|^2 = 1/\tau \approx 2\pi \times 6 \text{ MHz}$
A note about polarization

Polarization in the atomic basis

\[ \hat{\pi} = \hat{z} \]

\[ \hat{\sigma}^+ = -\frac{\hat{x} + i \hat{y}}{\sqrt{2}} \]

\[ \hat{\sigma}^- = \frac{\hat{x} - i \hat{y}}{\sqrt{2}} \]
Polarization Gradient Lattices

Easiest to understand in 1D

\[ \mathbf{E} \quad \mathbf{B} \quad \text{Intensity} \quad \mathbf{E} \]

\[ I_{\sigma^+}, I_{\sigma^-} \]

\[ V_{\text{lattice}} \quad m_F = 0, \ m_F = -1 \]
3D Spin-Dependent Lattices

- $^{87}\text{Rb}$
- $5S_{1/2}$
- $5P_{3/2}$
- $5P_{1/2}$
- 780 nm
- 795 nm
- $\lambda_{\text{laser}}$
- 6.8 GHz
- $F=2$
- $F=1$

Standard ("Scalar") Lattice

- $V_{\text{lattice}} = \frac{2\pi c^2 \Gamma^2 I_0}{2\omega_0^3} \left( \left( \frac{2}{\Delta_{3/2}} + \frac{1}{\Delta_{1/2}} \right) (1 + \cos(\theta) \cos(2kr)) + \right.$
  \left. g_F m_F \left( \frac{1}{\Delta_{3/2}} - \frac{1}{\Delta_{1/2}} \right) \kappa \cdot \vec{B} \sin(\theta) \sin(2kr) \right)$

Spin-Dependent ("Tensor") Lattice
Spin-Dependent Lattices

- Requires detuning comparable to fine structure splitting
- Lattice potential depth proportional to $g_F m_F$
- Lattice wavevector can not be perpendicular to magnetic field along any lattice direction
- Scalar lattice vanishes for “magic” wavelength (790 nm)

$$V_{\text{lattice}} = \frac{2 \pi c^2 \Gamma^2 I_0}{2 \omega_0^3} \left( \frac{2}{\Delta_{3/2}} + \frac{1}{\Delta_{1/2}} \right) (1 + \cos(\theta) \cos(2kr)) + g_F m_F \left( \frac{1}{\Delta_{3/2}} - \frac{1}{\Delta_{1/2}} \right) \mathbf{k} \cdot \mathbf{B} \sin(\theta) \sin(2kr)$$
Spin-Dependent Lattice Thermometry

$\lambda / 4$

$E$

$B$

1064 nm

$\theta = 90^\circ$: “lin-perp-lin”
Lattice Technical Issues

Polarization Impurities

Create strong lattice for $m_F=0$ atoms

Solution: work at 790 nm

Diagram showing energy levels $5S_{1/2}$, $5P_{1/2}$, and $5P_{3/2}$ with transitions at 780 nm and 795 nm. The graph plots diffracted fraction against wavelength (nm) with a peak at 790 nm.
3D Spin-Dependent Lattice

\[ \theta = 90^\circ: \text{lin-perp-lin} \]

- \( |2,0\rangle \)
- \( |1,-1\rangle \)
- \( |2,-2\rangle \)

SF

SF+MI
3D Spin-Dependent Lattice

\[ \theta = 90^\circ: \text{lin-perp-lin} \]

\[ |2,0\rangle \]
\[ |1,-1\rangle \]
\[ |2,-2\rangle \]
$\theta=90^\circ$: lin-perp-lin

|2,0\rangle

|1,-1\rangle

|2,-2\rangle
Hexagonal spin-dependent lattice
Hexagonal spin-dependent lattices

![Diagram showing potential, polarization, and time-of-flight images for different spin states.](image)
Controlled Heating

Thermalization?

$|1, -1\rangle$

$|1, 0\rangle$

Invisible to $|1, 0\rangle$
Controlled Heating

Parametric Oscillation of the Lattice

Ground Band
1st Excited Band
2nd Excited Band

Energy

Quasimomentum

Ground Band
1st Excited Band
2nd Excited Band
Band decay and thermalization

Energy vs. Quasimomentum

0, 3, 5, 7, 10 ms

How fast is the Decay?

*thermalization timescale for lattice atoms = 5–7 ms from 6–11 ER
Thermalization after controlled heating

- In the harmonic trap (before loading the lattice), mixtures completely thermalize within 50 ms.
- In the lattice, we cannot even observe relaxation!
Controlled heating – raw data

![Graph showing data points for 4 $E_R$ lattice depth and 8 $E_R$ lattice depth. The x-axis represents $\Delta(E_R)$, and the y-axis represents $N_0/N$. The graph includes points for different mixtures and states.]
Controlled heating: results

Theory: need to consider effective mass, single-particle localized states, lattice dispersion (Kapitza resistance?), interaction-induced localization...

Possible Explanation

Poor thermalization at higher lattice depth

![Graph showing the relationship between lattice potential depth and an unknown variable A, with a fit function A \propto \exp(-t_{\text{hold}}/\tau).](image)