Coherent excitonic matter

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Momentum distribution of cold atoms

Rb atom condensate, JILA, Colorado

Momentum distribution of cold exciton-polaritons

Exciton condensate ?, Kasprzak et al 2006

Exciton and Polariton Condensation
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Characteristics of Bose-Einstein Condensation

• Macroscopic occupation of the ground state
  – weakly interacting bosons

\[ k_B T_0 \approx \frac{n^2 \hbar^2/3}{2m} \approx \frac{1.3}{r_s^2} \text{Ryd} \]

• Macroscopic quantum coherence
  – Interactions (exchange) give rise to synchronisation of states

\[ \psi \rightarrow \psi e^{i\phi} \]

• Superfluidity
  – Rigidity of wavefunction gives rise to new collective sound mode

Ketterle group, MIT
Christiaan Huygens 1629-95

1656 – Patented the pendulum clock
1663 – Elected to Royal Society
1662-5 With Alexander Bruce, and sponsored by the Royal Society, constructed maritime pendulum clocks – periodically communicating by letter
Exciton and Polariton Condensation

Huygens Clocks

In early 1665, Huygens discovered “..an odd kind of sympathy perceived by him in these watches [two pendulum clocks] suspended by the side of each other.”

He deduced that effect came from “imperceptible movements” of the common frame supporting the clocks.

Two metronomes on a cart
Issues for these lectures

- Characteristics of a Bose condensate
- Excitons, and why they might be candidates for BEC

How do you make a BEC wavefunction based on pairs of fermions?

- BCS (interaction-driven high density limit) to Bose (low density limit) crossover

- Excitons may decay directly into photons

What happens to the photons if the “matter” field is coherent?

- Two level systems interacting via photons

How do you couple to the environment?

- Decoherence phenomena and the relationship to lasers

Excitons are the solid state analogue of positronium

Combined excitation is called a polariton

Stimulated Emission > Absorption

These photons have a fixed phase relationship

Laser
Outline

• General review
• Exciton condensation
  – mean field theory of Keldysh – BCS analogy
  – BCS-BEC crossover
  – broken symmetries, tunnelling, and (absence of) superfluidity
• Polaritons (coherent mixture of exciton and photon)
  – mean field theory
  – BCS-BEC crossover (again) and 2D physics
  – signatures of condensation
  – disorder
  – pairbreaking
  – phase-breaking and decoherence
• Review of Experiment intermingled (though mostly see other lectures)
• Other systems (if there is time)
  – quantum Hall bilayers
  – “triplons” in quantum spin systems
  – ultracold fermions and the Feshbach resonance
Background material and details for the lectures

I will not give detailed derivations in lectures, but they can all be found in these papers

Reviews

D Snake and P B Littlewood, Physics Today 2010

Basic equilibrium models:

Mean field theory (polaritons): P. R. Eastham, P. B. Littlewood , Phys. Rev. B 64, 235101 (2001)

Decoherence and non-equilibrium physics


Physical signatures

Y.Lozovik and V Yudson, JETP Lett. 22, 274 (1975)
Bose-Einstein condensation

- Macroscopic ground state occupation

\[ n = \int d\epsilon \frac{D(\epsilon)}{e^\beta(\epsilon - \mu) - 1} \sim \int d\epsilon \frac{\epsilon^{(d-2)/2}}{\beta(\epsilon - \mu)} \]

Density of states

\[ D(\epsilon) \]

Thermal occupation

\[ n(\epsilon) \]

- Macroscopic phase coherence

Condensate described by macroscopic wave function \( \psi e^{i\phi} \) which arises from interactions between particles

\[ \psi \rightarrow \psi e^{i\phi} \]

Genuine symmetry breaking, distinct from BEC
Couple to internal degrees of freedom - e.g. dipoles, spins

- Superfluidity

Implies linear Goldstone mode in an infinite system with dispersion \( \omega = v_s k \)
and hence a superfluid stiffness \( \alpha v_s \)
Excitons in semiconductors

At high density - an electron-hole plasma

At low density - excitons

Exciton - bound electron-hole pair (analogue of hydrogen, positronium)
In GaAs, $m^* \sim 0.1 \, m_e$, $\varepsilon = 13$
Rydberg = 5 meV (13.6 eV for Hydrogen)
Bohr radius = 7 nm (0.05 nm for Hydrogen)
Measure density in terms of a dimensionless parameter $r_s$ - average spacing between excitons in units of $a_{\text{Bohr}}$

$$1/n = \frac{4\pi}{3}(a_{\text{Bohr}}r_s)^3$$
Speculative phase diagram of electron-hole system (T=0)

Radius containing one electron and one hole in units of Bohr radius

1CP metal

Molecular solid

Phase separation

2CP metal

Exciton liquid

Mott insulator

Hydrogen

Biexcitons - analogue of H₂ very weakly bound for equal mass of electron and hole

Positronium

PBLittlewood and XJZhu Physica Scripta T68, 56 (1996)
Interacting electrons and holes in double quantum well

Two parabolic bands, direct gap, equal masses
Layers of electrons and holes in quantum wells spaced a distance $d$ apart

$$H = \sum_i [T_i^e + T_i^h] + \sum_{i,j} [V_{ij}^{ee} + V_{ij}^{hh} - V_{ij}^{eh}]$$

$$T_i^\alpha = \frac{p_{i\alpha}^2}{2m_\alpha}$$

$$V_{ij}^{\alpha\beta} = \frac{e^2}{\varepsilon |r_{i\alpha} - r_{j\beta}|}$$

Units: density- energy- Rydberg
$n = \frac{1}{\pi} (r_s a_B)^2$
$a_B = \varepsilon \hbar^2 / me^2$

$e^2 / 2\varepsilon a_B$

Ignore interband exchange - spinless problem
Ignore biexcitons - disfavoured by dipole-dipole repulsion
Neutral bosons with repulsive dipolar interaction in 2D

Binding energy few meV in GaAs
Bohr radius ~ 10 nm

Long lifetime up to 100 nsec – recombination by tunnelling through barrier
Exciton and Polariton Condensation

**Excitons**

\[ \Phi^\dagger |0\rangle = \sum_k \phi(k) a^\dagger_{ck} a_{vk} |0\rangle \]

\[ \phi(k) = \frac{1}{1 + k^2 a^2_B} \]

Bohr radius \( a_B \sim \text{few nm} \)
Mean field theory of excitonic insulator

\[ \Phi^\dagger |0\rangle = \sum_k \phi(k) a^\dagger_{ck} a_{vk} |0\rangle \]

Wavepacket of bound e-h pair
Composite boson

A coherent state – like a laser
Bose condensation of excitons

\[ e^{\lambda \Phi^\dagger} |0\rangle = \prod_k \left[ u(k) + v(k) a^\dagger_{ck} a_{vk} \right] |0\rangle \]

BCS-like instability
of Fermi surfaces

Special features: order parameter; gap

\[ \langle a^\dagger_{ck} a_{vk} \rangle = u_k v_k = (\Delta_k / 2E_k); \quad E_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta_k^2} \]

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Exciton and Polariton Condensation
Mean Field Solution

\[ H_0 = \sum_k \left[ \varepsilon_{ck} a_{ck}^\dagger a_{ck} + \varepsilon_{vk} a_{vk}^\dagger a_{vk} \right] \]

Single particle band energies

\[ H_c = \frac{1}{2} \sum_q \left[ V_{qee} \rho_q e^{-\rho_q} + V_{qhh} \rho_q h^{-\rho_q} + 2V_{qeh} \rho_q e^{-\rho_q} \right] \]

Coulomb interaction between layers separated by distance \(d\)

\[ \rho_q = \sum_k a_{ck+q}^\dagger a_{ck} \]

Variational “BCS” mean field solution

\[ V^{ee} = V^{hh} = \frac{2\pi}{q} \]

\[ V^{eh} = \frac{2\pi e^{-qd}}{q} \]

\[ \Psi_0 = e^{\lambda} \sum_k a_{ck}^\dagger a_{vk} |0\rangle = \prod_k \left[ u_k + v_k a_{ck}^\dagger a_{vk} \right] |0\rangle \]

Self-energy

\[ \xi_k = \varepsilon_k - \mu - \sum_{k'} V^{ee}_{k-k'} n_{k'} \]

\[ \Delta_k = 2 \sum_{k'} V^{eh}_{k-k'} \langle a_{ck'}^\dagger a_{vk'} \rangle = \sum_{k'} V^{eh}_{k-k'} \Delta_{k'}/E_{k'} \]

Order parameter equation

\[ E_{k}^2 = \xi_k^2 + \Delta_k^2 \]

Spectrum with a gap

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Exciton and Polariton Condensation
Excitation spectra

$E_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta_k^2}$

- Band energy
- Chemical potential (<0 for bound exciton)
- Correlation energy

Low density $\mu < 0$
Chemical potential below band edge

High density $\mu > 0$
No bound exciton below band edge

Absorption
Emission
2D exciton condensate: Mean field solution

\[ m = 2.7 \]
\[ r_s = 5.9 \]
\[ r_s = 2.7 \]
Crossover from BCS to BEC

Smooth crossover between BCS-like fermi surface instability and exciton BEC

Model: 2D quantum wells separated by distance = 1 Bohr radius Zhu et al PRL 74, 1633 (1995)
Improved solution: Variational Monte Carlo

\[ \psi = \exp \left[ \sum_{i \neq j} u(r_{i,e} - r_{j,e}) \right] \exp \left[ \sum_{i \neq j} u(r_{i,h} - r_{j,h}) \right] \exp \left[ \sum_{i,j} v(r_{i,e} - r_{j,h}) \right] \times \psi_0 \]

Jastrow factors - smooth functions with variational parameters to add extra correlations to wavefunction

Either 2 component plasma (Slater determinant of plane waves) or BCS (Slater determinant of e-h pairs)

Zhu et al, PRB 54, 13575 (1996)

Search through variational parameter space by Monte Carlo
Output: Better energies, also pair correlation functions

Further improvements possible:
- Diffusion Monte Carlo (fixed node)
- Path Integral Monte Carlo (finite T)
Include biexcitons, Wigner crystal phases etc...
3D exciton condensate - mean field vs VMC

Energy

Correlation functions

Zhu et al PRB 54, 13575 (1996)
2D BEC - no confining potential

Interpolation – by hand – between two limits

\[ k_B T_0 = R_y^* \exp \left( -\frac{1}{2r_s} \right) \]

\[ R_y^* = \frac{m^*/m}{\epsilon^2} \times \text{Rydberg} \]

\[ a_o^* = \frac{\epsilon}{m^*/m} \times a_{Bohr} \]

\[ r_s^2 = \frac{1}{\pi n a_o^*} \]

Mean field - should be K-T transition, but OK to estimate energy scales

GaAs CQW
\[ T = 4 \text{ K} \]
\[ n = 3 \times 10^{11} \text{ cm}^{-2} \]
2D solid phases

- At low density, single layer will become a Wigner crystal
- Paired electron-hole Wigner crystal is an excitonic solid
- In 2D biexciton formation is disfavored by dipole repulsion

Effect of crystal lattice
- Stabilise the insulating phases to higher densities – Wigner transition turns into a Mott transition
- Induce commensurability effects
- One component plasma (e.g. Metallic H) is believed to be a superconductor
  - In the context of excitons with localised holes and delocalised electrons this would be called excitonic superconductivity ...
Conclusions from numerics

- Condensation is a robust process
  - energy scale is fraction of exciton Rydberg (few meV in GaAs)
- No evidence for droplet formation
  - positive compressibility
  - bi-excitons ignored here, but X-X interaction repulsive in bilayers
  - contrast to multivalley bulk semiconductors like Ge, Si
- BCS-like wavefunction captures smoothly the crossover from high to low densities
- Solid phases also competitive in energy (but higher for moderate $r_s$)
- So it should be easy to make experimentally ….
Experimental signatures

• Phase-coherent luminescence - order parameter is a macroscopic dipole
  – Should couple photons and excitons right from the start – polaritons

• Gap in absorption/luminescence spectrum
  – small and low intensity in BEC regime

• Momentum and energy-dependence of luminescence spectrum \( I(k,\omega) \) gives direct measure of occupancy
  \[
  I(k) \propto n_k = \left[ e^{\beta(\varepsilon_k - \mu)} - 1 \right]^{-1}
  \]
  – 2D Kosterlitz-Thouless transition
  – confined in unknown trap potential
  – only excitons within light cone are radiative
Angular profile of light emission

- Emitted photon carries momentum of electron-hole pair
- Condensation (to $k_{\parallel} \sim 0$) then has signature in sharp peak for emission perpendicular to 2D trap.
- In 2D the phase transition is of Kosterlitz-Thouless type – no long range order below $T_c$
- Peak suppressed once thermally excited phase fluctuations reach size of droplet

$$\xi_T = \left( \frac{\lambda \rho}{4m} \right)^{1/2} / k_B T$$

$$T \approx T_* = T_{\text{BEC}} / \ln(R/\xi_T),$$

Parameters estimated for coupled quantum wells of separation $\sim 5$ nm; trap size $\sim 10 \mu m$; $T_{\text{BEC}} \sim 1 K$
Vortices

Angular emission into $\theta_x$, $\theta_y$

Vortex position (x,y) inside droplet
**Dipolar superfluid**

- What could be the superfluid response?
  - exciton transport carries no charge or mass
  - in a bilayer have a static dipole

\[ B(t) = B_0 e^{i\omega t} \hat{y} \]
\[ \nabla \times E = -\frac{\partial B}{\partial t} \]
\[ F = i\omega B_0 dq \hat{x} \]

\[ j_{dipole} = \sigma(\omega)F = i\omega \chi(\omega)F \]

\[ \sigma(\omega) = \sigma_0 [\delta(\omega) + i/\omega] \]

“Pinning” of the phase by interlayer tunnelling shifts response to nonzero frequency

Lozovik & Yudson 1975
Joglekar, Balatsky, PBL, 2004
Recap

Exciton liquid in semiconductors

Interacting electrons and holes
Characteristic energy scale is the exciton Rydberg

\[
H = \sum_k [\epsilon_{ck} a_{ck}^\dagger a_{ck} + \epsilon_{vk} a_{vk}^\dagger a_{vk}] + \frac{1}{2} \sum_q \left[ V_{qee} \rho_q^e \rho_{-q}^e + V_{qh} \rho_q^h \rho_{-q}^h + 2 V_{qeh} \rho_q^e \rho_{-q}^h \right]
\]

A very good wavefunction to capture the crossover from low to high density is BCS

\[
\psi_0 = e^{\lambda} \sum_k a_{ck}^\dagger a_{vk} |0\rangle = \prod_k \left[ u_k + v_k a_{ck}^\dagger a_{vk} \right] |0\rangle
\]

Just like a BCS superconductor, this has an order parameter, and a gap

\[
\langle a_{ck}^\dagger a_{vk} \rangle = u_k v_k = (\Delta_k / 2 E_k) \; ; \; \; E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta_k^2}
\]

The order parameter has an undetermined phase -> superfluid.

Unfortunately, there are some terms in H that have been left out.
**Digression: tunnelling and recombination**

- Our Hamiltonian has only included interaction between electron and hole densities, and no e-h recombination.
- In a semimetal tunnelling between electron and hole pockets is allowed.

If pockets related by symmetry, generates single particle terms $t a_c^{\dagger} a_v k$

Rediagonalise $(\alpha, \beta)$ linear combinations of $(a_{ck}, a_{vk})$

Introduces single particle gap

New Coulomb coupling terms $V_1 t a_c^{\dagger} a_v^{\dagger} \alpha \beta$, $V_2 t^2 a_c^{\dagger} a_v^{\dagger} \alpha \beta \beta$

If pockets are unrelated by symmetry, still the eigenstates are Bloch states

\[ \hat{V} = \sum_{n_1 \ldots n_4, k_1 \ldots k_4} \langle n_1 k_1, n_2 k_2 | V | n_3 k_3 n_4 k_4 \rangle \times a_{n_1 k_1}^{\dagger} a_{n_2 k_2}^{\dagger} a_{n_3 k_3} a_{n_4 k_4}, \]

In general, terms of the form $V_1 t a_c^{\dagger} a_v^{\dagger} \alpha \beta$, $V_2 t^2 a_c^{\dagger} a_v^{\dagger} \alpha \beta \beta$

Most general Hamiltonian does not separately conserve particles and holes.
Tunnelling and recombination - 2

- Single particle gap - trivial physics, no extra symmetry to break….

E.g. Artificial 2D semimetal - GaSb/InAs interface
electron-hole mixing introduces gap [Lakrimi et al 1997]
In QH bilayers: tunnelling between layers -> S/AS splitting

- Consider the effect of general Coulomb matrix elements at first order

\[ \psi_k = \langle a^+_c a_v \rangle = |\Delta_k| e^{i\phi} \]
\[ \langle V_2 \rangle = V_2 |\Delta_k|^2 \cos 2\phi \]
\[ \langle V_1 \rangle = V_1 |\Delta_k| \cos(\phi - \phi_0) \]

Complex order parameter in mean field approximation
Josephson-like term; fixes phase; gapped Goldstone mode
Symmetry broken at all T; just like band-structure gap

- No properties to distinguish this phase from a normal dielectric, except in that
these symmetry breaking effects may be small
- In that case, better referred to as a commensurate charge density wave
- Indeed an *excitonic insulator*

Not unfamiliar or exotic at all (but not a superfluid either)
Tunnelling and recombination - 3

- If electron and hole not degenerate, recombination accompanied by emission of a photon

\[ H_{dipole} = \sum_q \omega_q \phi_q^\dagger \phi_q + \sum_{k,q} g_{q,k} \left[ \phi_q^\dagger a_{q-k}^\dagger + \phi_q a_{q+k}^\dagger \right] \]

- Evaluate at zeroth order

\[ \langle H_{dipole} \rangle = \sum_k g_0 |\Delta_k| \phi_0^\dagger e^{i\mu t - i\phi} + h.c. \]

- Phase of order parameter couples to phase of electric field
- Resonant radiation emitted/absorbed at frequency = chemical potential
- Behaves just like an antenna (coherent emission, not incoherent luminescence)

Must include light and matter on an equal footing from the start - POLARITONS
Quantum Well Excitons

Weakly bound electron-hole pair
EXCITON
Rydberg – few meV
Bohr Radius – few nm

Excitation spectrum

Particle-hole continuum

QW exciton

In-plane center of mass momentum
Excitons + Cavity Photons

Excitons + Cavity Photons

mirror QW mirror

photon

energy

photon

QW exciton

in-plane momentum
Excitons + Cavity Photons

Excitons + Cavity Photons

energy

photon

QW exciton

mirror QW mirror

Exciton and Polariton Condensation

QW exciton

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Excitons + Cavity Photons

Excitons + Cavity Photons

| mirror | QW | mirror |

Energy diagram:

- Exciton
- Cavity Photon

Photon transitions:

- e-
- h+

Momentum diagram:

- Exciton and Polariton Condensation
Polaritons: Matter-Light Composite Bosons

\[ |\text{pol}\rangle = c_1 |\text{exc}\rangle + c_2 |\text{ph}\rangle \]

[C. Weisbuch et al., PRL 69 3314 (1992)]
Resonantly pumped microcavity

Address in plane momentum by measurement or excitation as function of angle

Photoluminescence from non-resonantly pumped microcavity

PL normalised to pump intensity

Excitation at ~ 1.7 eV

Senellart & Bloch, PRL 82, 1233 (1999)
Non-resonant(?) pumping in Lower Polariton Branch

Deng et al 2002

Substantial blue shift appears at threshold
Polariton dispersion seen above threshold

Exciton and Polariton Condensation
Microcavity polaritons

Experiments: Kasprzak et al 2006 CdTe microcavities
II-VI quantum well microcavities

Increasing pumping

Kasprzak, Dang, unpublished
Occupancy as a function of power

T=5K

a)

b)
Distribution at varying density

Coherent(?) peak

Maxwellian tail

\[ T_{\text{eff}} = (16 \pm 1) \text{ K} \]
Distribution at varying density

Blue shift used to estimate density
High energy tail of distribution used to fix temperature
Onset of non-linearity gives estimate of critical density
Linewidth well above transition is inhomogeneous
Measurement of first order coherence

Temperature and density estimates predict a phase coherence length ~ 5 μm

Experiment also shows broken polarisation symmetry
Coherence

Temporal coherence and multimode behavior

Love et al., PRL 2008
Other recent experiments

- Stress traps for polariton condensates
- Coherence and line narrowing
  - Love et al. PRL 101 067404 (2008)
- Changes in the excitation spectrum
- Superflow in driven condensates
- Vortices and half-vortices

Photon superfluid is an electromagnetic cloaking device
What’s new about a polariton condensate?

- Composite particle – mixture of electron-hole pair and photon
  - How does this affect the ground state?
- Extremely light mass ($\sim 10^{-5} m_e$) means that polaritons are large, and overlap strongly even at low density
  - BEC – “BCS” crossover?
- Two-dimensional physics
  - Berezhinski-Kosterlitz-Thouless transition?
- Polariton lifetime is short
  - Non-equilibrium, pumped dynamics
  - Decoherence?
  - Relationship to the laser?
- Can prepare out-of-equilibrium condensates
  - Quantum dynamics of many body system
Bose Condensation of Composite Bosons

Review old picture of excitonic insulator
Interacting polaritons and the Dicke model
Analogs to other systems
Excitons

\[ \Phi^\dagger |0\rangle = \sum_k \phi(k) a^\dagger_{ck} a_{vk} |0\rangle \]

\[ \phi(k) = \frac{1}{1 + k^2 a_B^2} \]

Bohr radius \( a_B \sim \text{few nm} \)
Mean field theory of excitonic insulator

\[ \Phi^\dagger |0\rangle = \sum_k \phi(k) a^\dagger_{ck} a_{vk} |0\rangle \]

Wavepacket of bound e-h pair
Composite boson

\[ e^{\lambda \Phi^\dagger} |0\rangle = \prod_k \left[ u(k) + v(k) a^\dagger_{ck} a_{vk} \right] |0\rangle \]

A coherent state – like a laser
Bose condensation of excitons

Boson condensation of excitons

A coherent state – like a laser
Bose condensation of excitons

Special features: order parameter; gap

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Exciton and Polariton Condensation
Bose Condensation of Composite Bosons

Interacting polaritons and the Dicke model
Because excitons are “heavy”, it's a good enough approximation to treat them as localized two-level systems (i.e., bosons with a repulsive interaction)
Photon component is “light” and mediates long range coupling
Polaritons and the Dicke Model – a.k.a Jaynes-Tavis-Cummings model

Localised excitons behave like spins

Spins are flipped by absorption/emission of photon

\[ H = \omega \psi^\dagger \psi + \sum_i \epsilon_i S_i^z + \frac{g}{\sqrt{N}} \sum_i \left[ S_i^+ \psi + \psi^\dagger S_i^- \right] \]

\[ N \sim \left[ \frac{\text{photon wavelength}}{\text{exciton radius}} \right]^d \gg 1 \]

Mean field theory – i.e. BCS coherent state – expected to be good approximation

\[ |\lambda, w_i\rangle = \exp \left[ \lambda \psi^\dagger + \sum_i w_i S_i^+ \right] |0\rangle \quad T_c \approx g \exp(-1/gN(0)) \]

Transition temperature depends on coupling constant
Two metronomes on a cart

- Modelocking – the Huygens experiment of 1665
Localized excitons in a microcavity - the Dicke model

- Simplifications
  - Single cavity mode
  - Equilibrium enforced by not allowing excitations to escape
  - Thermal equilibrium assumed (at finite excitation)
  - No exciton collisions or ionisation (OK for dilute, disordered systems)
    Work in k-space, with Coulomb added - then solution is extension of Keldysh mean field theory (used by Schmitt-Rink and Chemla for driven systems)
    Important issues are not to do with localisation/delocalisation or binding/unbinding of e-h pairs but with decoherence

- Important physics
  - Fermionic structure for excitons (saturation; phase-space filling)
  - Strong coupling limit of excitons with light

- To be added later
  - Decoherence (phase-breaking, pairbreaking) processes
  - Non-equilibrium (pumping and decay)
Mean field solution: 1. Variational method

\[ H = \omega \psi^\dagger \psi + \sum_i \epsilon_i S_i^z + \frac{g}{\sqrt{N}} \sum_i \left[ S_i^+ \psi + \psi^\dagger S_i^- \right] \]

Variational wavefunction

\[ |\lambda, w_i\rangle = \exp[\lambda \psi^\dagger + \sum_i w_i S_i^+] |0\rangle \]

Minimise Free Energy

\[ \frac{\partial}{d\lambda}, \frac{\partial}{dw_i} \langle \lambda, w_i | \hat{H} - \mu \hat{N} | \lambda, w_i \rangle = 0 \]

Order parameter equation

\[ (\omega - \mu) \lambda = \frac{g^2 \lambda}{2N} \sum_i \frac{1}{|E_i|} \]

Gap in spectrum

\[ E_i = \left[ (\epsilon_i - \mu)^2 + g^2 \lambda^2 \right]^{1/2} \]

Excitation density from both photons and excitons

\[ \rho_{ex} = \lambda^2 - \frac{1}{2N} \sum_i \frac{\epsilon_i - \mu}{|E_i|} \]
Mean field solution: 2. Equation of motion

Conventional 2-level system notation

Semiclassical approximation to Heisenberg equations of motion for two level system in time dependent field

Rotating frame \( \psi(t) = \psi e^{-i\mu t}, p(t) = pe^{-i\mu t} \)

Conservation: solution lies on Bloch sphere

\[
\begin{align*}
\frac{d\psi}{dt} &= -i\omega \psi - igp \\
\frac{dp}{dt} &= -i\epsilon p - ig\psi n
\end{align*}
\]

\[
\psi = \frac{\text{sign}(\epsilon - \mu)}{\omega - \mu} \frac{g^2\psi}{\sqrt{(\mu - \epsilon)^2 + 4g^2|\psi|^2}}
\]

\[
p = g\psi/\sqrt{(\mu - \epsilon)^2 + 4g^2\psi^2}
\]

\[
\psi = \frac{g^2\psi}{\omega - \mu} \sum_j \frac{\text{sign}(\epsilon_j - \mu)}{\sqrt{(\mu - \epsilon_j)^2 + 4g^2|\psi|^2}}
\]
Mean field solution: 3. Functional field theory

\[ H = \sum_{j=1}^{j=nA} 2\epsilon_j S_j^2 + \sum_{k=2\pi l/\sqrt{A}} \hbar \omega_k \psi_k^\dagger \psi_k + \frac{g}{\sqrt{A}} \sum_{j,k} \left( e^{2\pi i k \cdot r_j} \psi_k^\dagger S_j^+ + e^{-2\pi i k \cdot r_j} \psi_k^\dagger S_j^- \right) \]

Generalise to multiple modes in area A

Construct coherent state path integral and integrate out spins

\[ S[\psi] = \int_0^\beta d\tau \sum_k \psi_k^\dagger (\partial_\tau + \hbar \tilde{\omega}_k) \psi_k + N \text{Tr} \ln(\mathcal{M}) \]

\[ \mathcal{M}^{-1} = \begin{pmatrix} \partial_\tau + \tilde{\epsilon} & \frac{g}{\sqrt{A}} \sum_k e^{2\pi i k \cdot r_j} \psi_k^\dagger \\ \frac{g}{\sqrt{A}} \sum_k e^{2\pi i k \cdot r_j} \psi_k & \partial_\tau + \tilde{\epsilon} \end{pmatrix} \]

Minimise action around stationary uniform saddle point

\[ \hbar \tilde{\omega}_0 \psi_0 = g^2 n \frac{\tanh(\beta E)}{2E} \psi_0, \quad E = \sqrt{\tilde{\epsilon}^2 + g^2 |\psi_0|^2} / A. \]

\[ \rho_{\text{M.F.}} = \frac{|\psi_0|^2}{A} + \frac{n}{2} \left[ 1 - \frac{\tilde{\epsilon}}{E} \tanh(\beta E) \right] \]
Dictionary of broken symmetries

• Connection to excitonic insulator generalises the BEC concept – different guises

\[ e^\lambda \sum_k \phi_k a_{ck}^\dagger a_{vk} = \prod_k \left[ 1 + \lambda \phi_k a_{ck}^\dagger a_{vk} \right] \]

• Rewrite as spin model

\[ S_i^+ = a_{ci}^\dagger a_{vi} \quad ; \quad S_i^z = a_{ci}^\dagger a_{ci} - a_{vi}^\dagger a_{vi} \]

• XY Ferromagnet / Quantum Hall bilayer / BaCuSiO

\[ |w_i\rangle = e^{\sum_i w_i S_i^+} |0\rangle \]

• Couple to an additional Boson mode:
  photons -> polaritons;
  molecules -> cold fermionic atoms near Feshbach resonance

\[ |\lambda, w_i\rangle = \exp[\lambda \psi^\dagger + \sum_i w_i S_i^+] |0\rangle \]

Dynamics – precession in self-consistent field
Condensation in the Dicke model (g/T = 2)

Increasing excitation density

No inhomogeneous broadening

\[ \Delta = \omega - \varepsilon = 0 \]

Excitation energies (condensed state)

Chemical potential (normal state)

Coherent light

Upper polariton

Lower polariton

Chemical potential (normal state)
Phase diagram

Detuning $\Delta = (\omega - \varepsilon)/g$

Photon occupation

Exciton occupation

Inversion and Photon Density vs $\rho_{ex}$

Inversion and Photon Density vs $\rho_{ex}$
Excitation spectrum for different detunings

\[ \Delta = 0 \]
\[ \Delta = 1 \]
\[ \Delta = 3 \]

\[ g\beta = 1 \]
Excitation spectrum with inhomogeneous broadening

Zero detuning: $\omega = \varepsilon$
Gaussian broadening of exciton energies $\sigma = 0.5 \ g$

 photon spectral function

$A(\omega)$

$(\omega - E)/g$

$\rho_{\text{ex}} = -0.50$

$\rho_{\text{ex}} = -0.37$

$\rho_{\text{ex}} = -0.28$

$\rho_{\text{ex}} = -0.22$

$\rho_{\text{ex}} = -0.19$

$\rho_{\text{ex}} = -0.08$
Interaction dominated physics or dilute Bose gas?

Mean field theory – coherent state – is BCS

“BCS” to Bose crossover
Beyond mean field: Interaction driven or dilute gas?

• Conventional “BEC of polaritons” will give high transition temperature because of light mass \( m^* \)
• Single mode Dicke model gives transition temperature \( \sim g \)

Which is correct?

\[
k_B T_0 \approx \frac{\hbar^2}{2 m} n
\]

0.1 \( g \)  

\[
\left( \frac{g}{\hbar^2 / 2 m a_0^2} \right) \times \left( \frac{m^*}{m} \right) \approx 10^{-4}
\]

\( k_{\parallel} \)

\( g \)

Upper polariton

Lower polariton

\( a_o = \) characteristic separation of excitons
\( a_o > \) Bohr radius

Dilute gas BEC only for excitation levels \( < 10^9 \) cm\(^{-2} \) or so

A further crossover to the plasma regime when \( na_B^2 \sim 1 \)
Fluctuation spectrum

Expand action in quadratic fluctuations around mean field solutions, and diagonalise to determine the new collective modes.

\[ E_{\pm} = \frac{1}{2} \left[ (\hbar \omega_k + 2\bar{\varepsilon}) \pm \sqrt{(\hbar \omega_k - 2\bar{\varepsilon})^2 + 4g^2n \tanh(\beta \bar{\varepsilon})} \right]. \]

Conventional upper and lower polaritons

Bogoliubov sound mode

\[ \xi_1 = \pm \hbar ck \]

\[ c = \sqrt{\frac{1}{2m} \left( \frac{4\hbar \bar{\omega}_0 g^2 n}{\xi_2(0)^2} \right) \left( \left| \psi_0 \right|^2 \right) \approx \sqrt{\frac{\lambda \rho_0}{2m n}}}. \]
2D polariton spectrum

- Excitation spectrum calculated at mean field level
- Thermally populate this spectrum to estimate suppression of superfluid density (one loop)
- Estimate new $T_c$

Keeling et al. PRL 93, 226403 (2004)
Phase diagram

- \( T_c \) suppressed in low density (polariton BEC) regime and high density (renormalised photon BEC) regimes
- For typical experimental polariton mass \( \sim 10^{-5} \) deviation from mean field is small

2D physics

No long range order at finite temperatures
Berezhinskii-Kosterlitz-Thouless transition
Angular dependence of luminescence becomes sharply peaked at small angles
(No long-range order because a 2D system)
Decoherence

Despite large Q of cavity, lifetime is only a few psec
Even if a thermal distribution can be obtained, the system is non-equilibrium
Particle fluxes produce decoherence
Conventional theory of the laser

\[ H = H_0 + H_{SB} + H_B \]

\[ H_0 = \sum_i \epsilon_i (b_i^\dagger b_i - a_i^\dagger a_i) + \omega_c \psi^\dagger \psi + \frac{g}{\sqrt{N}} \sum_i \left[ \psi^\dagger a_i^\dagger b_i + h.c. \right] \] system

\[ H_B = \sum_k \left[ \omega_{k+} c_{k+}^\dagger c_{k+} + \omega_{k-} c_{k-}^\dagger c_{k-} + \omega_{1,k} c_{1,k}^\dagger c_{1,k} + \omega_{2,k} c_{2,k}^\dagger c_{2,k} \right] \] bosonic “baths”

\[ H_{SB} = \sum_k g_k (\psi^\dagger d_k + d_k^\dagger \psi) \] decay of cavity mode

\[ + \sum_{jk} \left[ b_j^\dagger a_j (g_{jk}^+ c_{+,k}^\dagger + g_{jk}^- c_{-,k}^\dagger) + h.c. \right] \] phase-breaking

\[ + \sum_{jk} \Gamma_{jk}^{(1)} (b_j^\dagger b_j + a_j^\dagger a_j) (c_{1,k}^\dagger + c_{1,k}) \] pair-breaking

\[ + \sum_{jk} \Gamma_{jk}^{(2)} (b_j^\dagger b_j - a_j^\dagger a_j) (c_{2,k}^\dagger + c_{2,k}) \] non-pair-breaking

\[ b^\dagger b - a^\dagger a = S^z \]
\[ b^\dagger a = S^+ \]
\[ a^\dagger b = S^- \]
\[ b^\dagger b + a^\dagger a = 1 \]
From Heisenberg to Langevin equations of motion

\[ \frac{d}{dt} \psi = -i \omega_c \psi - ig \sum_i a_i^\dagger b_i - i \sum_k g_k d_k \]

\[ \frac{d}{dt} d_k = -i \omega_k d_k - ig_k \psi \]

\[ d_k(t) = d_k(t_0) e^{-i \omega_k(t-t_0)} - g_k \int_{t_0}^{t} dt' \psi(t') e^{-i \omega_k(t-t')} \]

\[ \frac{d}{dt} \psi = -i \omega_c \psi - ig \sum_i a_i^\dagger b_i - i \sum_k g_k d_k(t_0) e^{-i \omega_k(t-t_0)} - \sum_k g_k^2 \int_{t_0}^{t} dt' \psi(t') e^{-i \omega_k(t-t')} \]

Markov approximation

\[ \frac{d}{dt} \psi = (-i \omega_c - \kappa) \psi - ig \sum_i a_i^\dagger b_i + F(t) \]

\[ \frac{d}{dt} a_j^\dagger b_j = (-i \epsilon_j - \gamma_{\perp}) a_j^\dagger b_j + ig \psi(b_j^\dagger b_j - a_j^\dagger a_j) + \Gamma_{j-} \]

Polarisation \( S^{+-} \)

\[ \frac{d}{dt}(b_j^\dagger b_j - a_j^\dagger a_j) = \gamma_{\parallel}(d_0 - b_j^\dagger b_j + a_j^\dagger a_j) + 2ig(\psi^\dagger a_j^\dagger b_j - b_j^\dagger a_j \psi) + \Gamma_{j,d} \]

Inversion \( S^Z \)
From Langevin equations to mean field

Bloch equations in a self-consistent field

\[ \frac{d}{dt} \langle \psi \rangle = (-i \omega_c - \kappa) \langle \psi \rangle - ig \sum_j \langle S_j^- \rangle \]

\[ \frac{d}{dt} \langle S_j^- \rangle = (-i \epsilon_j - \gamma_\perp) \langle S_j^- \rangle + ig \langle \psi \rangle \langle S_j^z \rangle \]

\[ \frac{d}{dt} \langle S_j^z \rangle = \gamma_\parallel (d_o - \langle S_j^z \rangle) + 2ig(\langle \psi^\dagger \rangle \langle S_j^- \rangle - \langle S_j^+ \rangle \langle \psi \rangle) \]

If decay processes are turned off, solutions are identical to BCS mean field equations – but these are unstable to infinitesimal damping
Apparently arbitrarily weak decoherence destroys coherent ground state unless system is inverted, so the only solution is that of a regular laser

... but the assumption was that the noise is Markovian – uncorrelated – whereas the noise is coupled via the spectrum of the correlated ground state

....which has a gap, and a stiffness
Decoherence

Decay, pumping, and collisions may introduce “decoherence” - loosely, lifetimes for the elementary excitations - include this by coupling to bosonic “baths” of other excitations

in analogy to superconductivity, the external fields may couple in a way that is “pair-breaking” or “non-pair-breaking”

\[
\begin{align*}
\sum_{i,k} g_{i,k}^{(1)} \left[ b_i^\dagger b_i - a_i^\dagger a_i \right] (c_{1,k}^\dagger + c_{1,k}) &= \text{non-pairbreaking (inhomogeneous distribution of levels)} \\
\sum_{i,k} g_{i,k}^{(2)} \left[ b_i^\dagger b_i + a_i^\dagger a_i \right] (c_{2,k}^\dagger + c_{2,k}) &= \text{pairbreaking disorder}
\end{align*}
\]

- Conventional theory of the laser assumes that the external fields give rise to rapid decay of the excitonic polarisation - incorrect if the exciton and photon are strongly coupled
- Correct theory is familiar from superconductivity - Abrikosov-Gorkov theory of superconductors with magnetic impurities

\[
\sum_{i,k} g_{i,k}^{(3)} \left[ b_i^\dagger a_i c_{3,k}^\dagger + a_i^\dagger b_i c_{3,k} \right] \quad \text{symmetry breaking – XY random field destroys LRO}
\]
Detour - Abrikosov-Gorkov theory of gapless superconductivity

- Ordinary impurities that do not break time reversal symmetry are “irrelevant”. Construct pairing between degenerate time-reversed pairs of states (Anderson’s theorem).
- Fields that break time reversal (e.g. magnetic impurities, spin fluctuations) suppress singlet pairing, leading first to gaplessness, then to destruction of superconductivity.

[Abrikosov & Gorkov ZETF 39, 1781 (1960); JETP 12, 12243 (1961)]

Skalski et al, PR136, A1500 (1964)
Exciton and Polariton Condensation

Phase diagram of Dicke model with pairbreaking

Pairbreaking characterised by a single parameter $\gamma = \lambda^2 N(0)$

No inhom. broadening

$\varepsilon_1 = \omega$

Dotted lines - non-pairbreaking disorder
Solid lines - pairbreaking disorder

MH Szymanska et al 2003
Transition to gaplessness and lasing

Model with both pairbreaking and non-pairbreaking (inhom. broadening)

Low density: below inversion

Higher density: above inversion

Increasing pairbreaking
Strong pairbreaking -> semiconductor laser

Large decoherence -- “laser”
- order parameter nearly photon like
- electronic excitations have short lifetime
- excitation spectrum gapless
real lasers are usually far from equilibrium

Small decoherence -- BEC of polaritons
- order parameter mixed exciton/photon
- excitation spectrum has a gap
Model for coupling to external baths

- Couple cavity photons to bulk photon modes (outside the cavity) – damping parameter $\kappa$
- Couple excitons to baths of free fermions (electrons and holes) – damping parameter $\gamma$
- Fermions kept at a chemical potential $\mu_{\text{ext}}$
- For fixed parameters, system reaches steady state equilibrium – increase occupation of the system by changing the external chemical potential
- Mathematically, construct non-equilibrium formulation using Keldysh method

Linewidth and luminescence

• Linewidth vanishes as transition is approached
• Luminescence – N(0) diverges at transition
• Both have mean field exponents $\sim (P_c - P)^{1/2}$
Phase boundary in presence of dissipation
Photoluminescence

- Crossover from propagating Bogoliubov mode to diffusion on ion length scales
- Crossover length determined by dissipation rate
FIG. 10: Spectral weight, photoluminescence and absorption spectra, as a function of emission angle, $\tan^{-1}(c_p/\omega_0)$. For all graphs, $\kappa = 0.02g$ and $T = 0.1g$. Top row: Uncondensed case, $\gamma = 0.2g, \mu_B = -0.5g$. (cf parameters in Fig. 2 and Fig. 3) Middle row: Condensed case, $\gamma = 0.2g, \mu_B = 0.0g$. Bottom row: Condensed case, $\gamma = 0.5g, \mu_B = 0.0g$ (transition to weak

Damped, driven Gross-Pitaevski equation

- Microscopic derivation consistent with simple behavior at long wavelengths for the condensate order parameter $\psi$ and polariton density $n_R$

\[
i \frac{\partial \psi}{\partial t} = \left\{ -\frac{\hbar \nabla^2}{2m_{LP}} + i\left[ R(n_R) - \gamma \right] + g|\psi|^2 + 2\tilde{g}n_R \right\} \psi.
\]

\[
\frac{\partial n_R}{\partial t} = P - \gamma_R n_R - R(n_R)|\psi(x)|^2 + D\nabla^2 n_R.
\]

\[
\omega_{\pm}(k) = -\frac{i\Gamma}{2} \pm \sqrt{\omega_{Bog}(k)^2 - \frac{\Gamma^2}{4}},
\]

From Wouters and Carusotto 2007
Effect of dissipation on $T_c$ and condensate

Two parameters:

- $\kappa$ – photon linewidth (measured)
- $\gamma$ – pumping rate – unknown but bounded

![Diagram depicting the effect of dissipation on $T_c$ and condensate.](image)
Quantum dynamics

On time scales $< \text{few psec}$, not in thermal equilibrium
Coupling to light allows driven dynamics
Controlled pumping of a many-particle state


Distribution of energy levels in, e.g. Quantum dots

- Direct creation of a *many-exciton* state
- .. equivalent to excitons in equilibrium at 0.6K
Spontaneous dynamical coherence

\[ \langle P_{k=0} \rangle \]

are macroscopic (scaling with the number of dots \( \sqrt{N} \))

\[ \langle \psi_{k=0} \rangle \]

\( \Rightarrow \) A quantum condensate of both photons and \( k=0 \) excitons
Conclusions

- Excitonic insulator is a broad concept that logically includes CDW’s, ferromagnets, quantum Hall bilayers as well as excitonic BEC
- Polariton condensates
  - Mean field like (long range interactions)
  - Strong coupling (not in BEC limit)
- Excitonic coherence – oscillator phase-locking
  - enemy of condensation is decoherence
  - excitons are not conserved so all exciton condensates are expected to show coherence for short enough times only
  - condensates will either be diffusive (polaritons) or have a gap (CDW)
- Mean field+ pairbreaking or phasebreaking fluctuations gives a robust model that connects exciton/polariton BEC continuously to
  - semiconductor plasma laser (pairbreaking) or
  - solid state laser (phase breaking)
  - is a laser a condensate? – largely semantic

- Now good evidence for polariton condensation in recent experiments