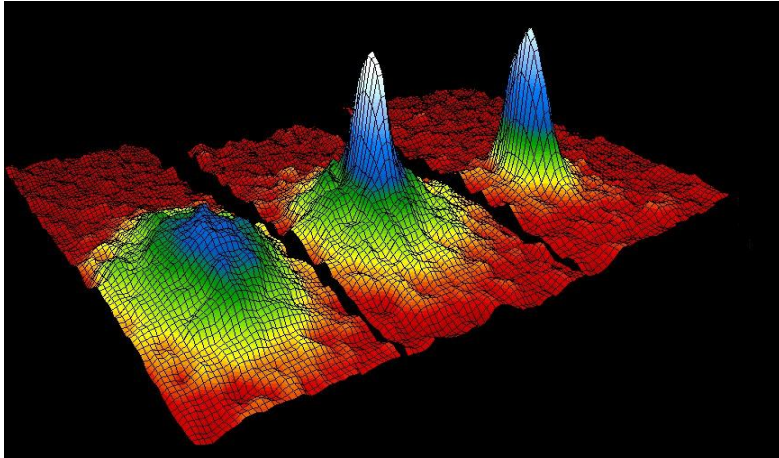




# Coherent excitonic matter

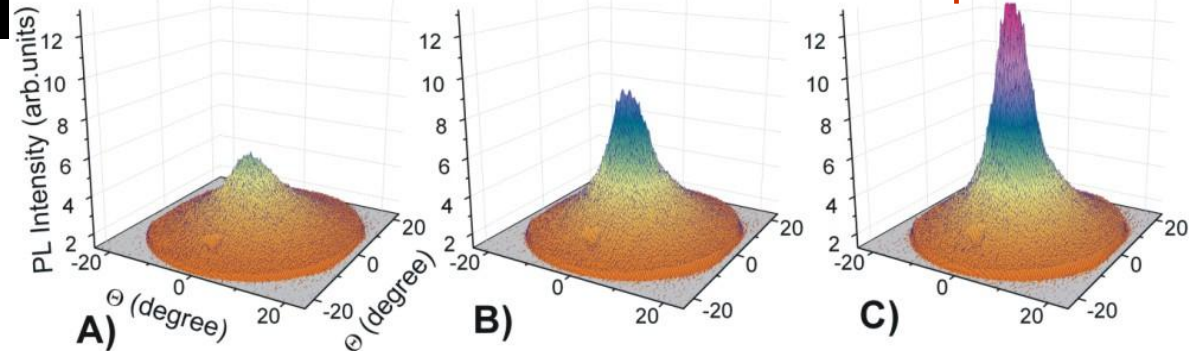
Peter Littlewood, University of Cambridge  
pbl21@cam.ac.uk



Rb atom condensate, JILA, Colorado

Momentum distribution of cold atoms

Momentum distribution of cold exciton-polaritons



Exciton condensate ?, Kasprzak et al 2006

# Acknowledgements

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Grenoble

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# Characteristics of Bose-Einstein Condensation

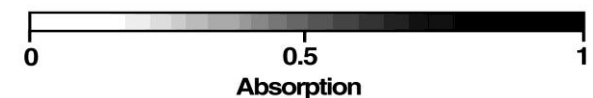
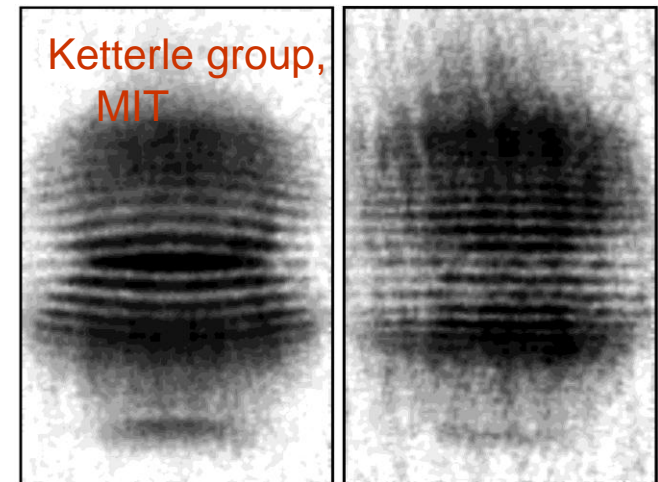
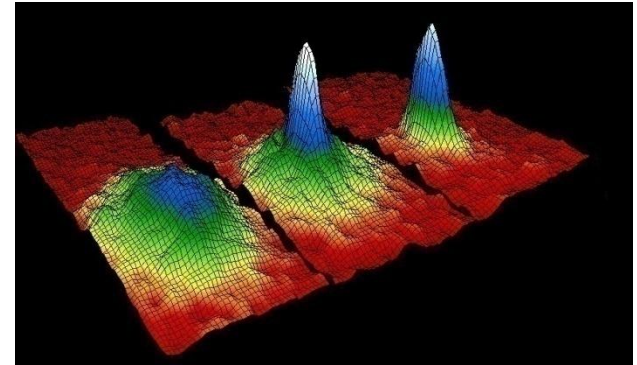
- Macroscopic occupation of the ground state
  - weakly interacting bosons

$$k_B T_0 \approx \frac{\hbar^2 n^{2/3}}{2m} \approx \frac{1.3}{r_s^2} \text{ Ryd}$$

- Macroscopic quantum coherence
  - Interactions (exchange) give rise to synchronisation of states

$$\psi \rightarrow \psi e^{i\phi}$$

- Superfluidity
  - Rigidity of wavefunction gives rise to new collective sound mode





## Christiaan Huygens 1629-95

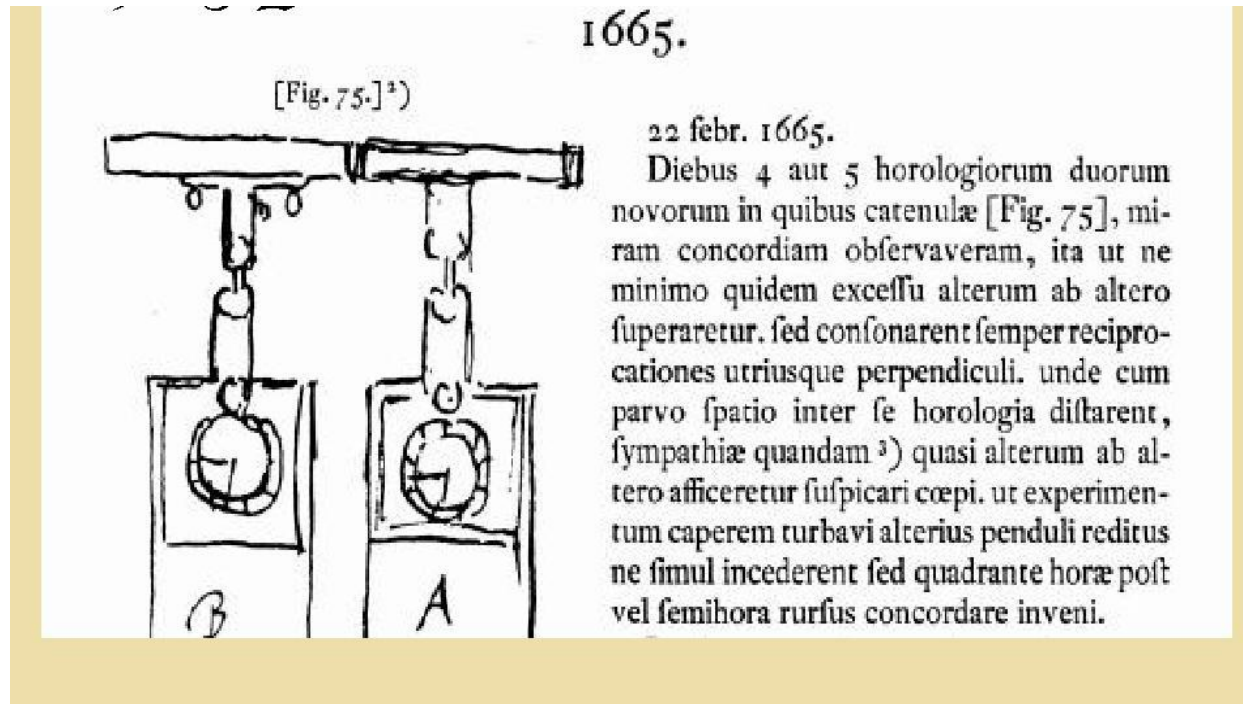
1656 – Patented the pendulum clock

1663 – Elected to Royal Society

1662-5 With Alexander Bruce, and sponsored by the Royal Society, constructed maritime pendulum clocks – periodically communicating by letter

# Huygens Clocks

In early 1665, Huygens discovered ``..an odd kind of sympathy perceived by him in these watches [two pendulum clocks] suspended by the side of each other."



He deduced that effect came from “imperceptible movements” of the common frame supporting the clocks

## Two metronomes on a cart



# Issues for these lectures

- Characteristics of a Bose condensate
- Excitons, and why they might be candidates for BEC

How do you make a BEC wavefunction based on pairs of fermions?

- BCS (interaction-driven high density limit) to Bose (low density limit) crossover

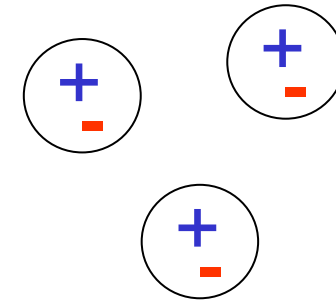
- Excitons may decay directly into photons

What happens to the photons if the “matter” field is coherent?

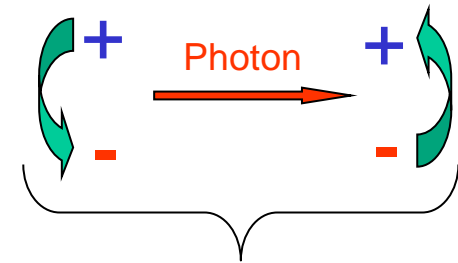
- Two level systems interacting via photons

How do you couple to the environment ?

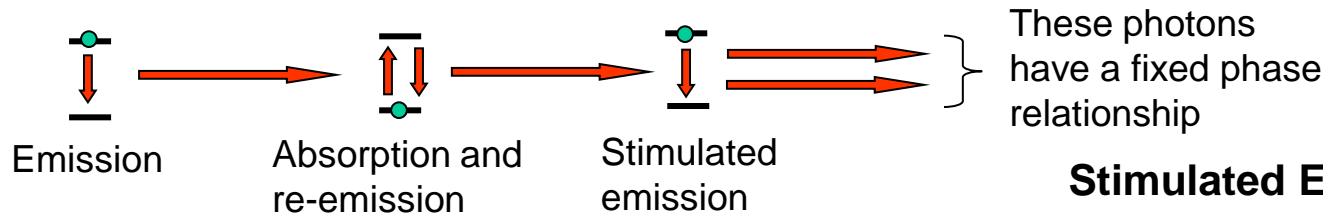
- Decoherence phenomena and the relationship to lasers



**Excitons** are the solid state analogue of positronium



Combined excitation is called a **polariton**



These photons have a fixed phase relationship

**Stimulated Emission > Absorption**  
**( Laser**

# Outline

- General review
- Exciton condensation
  - mean field theory of Keldysh – BCS analogy
  - BCS-BEC crossover
  - broken symmetries, tunnelling, and (absence of) superfluidity
- Polaritons (coherent mixture of exciton and photon)
  - mean field theory
  - BCS-BEC crossover (again) and 2D physics
  - signatures of condensation
  - disorder
  - pairbreaking
  - phase-breaking and decoherence
- Review of Experiment intermingled (though mostly see other lectures)
- Other systems (if there is time)
  - quantum Hall bilayers
  - “triplons” in quantum spin systems
  - ultracold fermions and the Feshbach resonance



# Background material and details for the lectures

I will not give detailed derivations in lectures, but they can all be found in these papers

## Reviews

Bose-Einstein Condensation, ed Griffin, Snoke, and Stringari, CUP, (1995)  
PB Littlewood and XJ Zhu, *Physica Scripta* T68, 56 (1996)  
P. B. Littlewood, P. R. Eastham, J. M. J. Keeling, F. M. Marchetti, B. D. Simons, M. H. Szymanska. *JPCM* 16 (2004) S3597-S3620.  
J. Keeling, F. M. Marchetti, M. H. Szymanska, P. B. Littlewood, *Semiconductor Science and Technology*, 22,R1-26, 2007.  
D Snoke and P B Littlewood, *Physics Today* 2010  
H. Deng, H, Haug and Y Yamamoto, *Rev. Mod. Phys.* 82:1489 (2010)

## Basic equilibrium models:

Mean field theory (excitons): C. Comte and P. Nozieres, *J. Phys. (Paris)*,43, 1069 (1982); P. Nozieres and C. Comte, *ibid.*, 1083 (1982); P. Nozieres, *Physica* 117B/118B, 16 (1983). Y.Lofovik and V Yudson, *JETP Lett.* 22, 274 (1975)  
Mean field theory (polaritons): P. R. Eastham, P. B. Littlewood , *Phys. Rev. B* 64, 235101 (2001)  
BCS-BEC crossover (polaritons): Jonathan Keeling, P. R. Eastham, M. H. Szymanska, P. B. Littlewood, *Phys. Rev. Lett.* 93, 226403 (2004) *cond-mat/0407076*; *Phys. Rev. B* 72, 115320 (2005)  
Effects of disorder: F. M. Marchetti, B. D. Simons, P. B. Littlewood, *Phys. Rev. B* 70, 155327 (2004)

## Decoherence and non-equilibrium physics

M. H. Szymanska, P. B. Littlewood, B. D. Simons, *Phys. Rev. A* 68, 013818 (2003)  
M. H. Szymanska, J. Keeling, P. B. Littlewood *Phys. Rev. Lett.* 96 230602 (2006)  
F. M. Marchetti, J. Keeling, M. H. Szymanska, P. B. Littlewood, *Phys. Rev. Lett.* 96, 066405 (2006)  
M. H. Szymanska, J. Keeling, P. B. Littlewood, *Physical Review B* 75, 195331 (2007)  
M Wouters and I. Carusotto, *PRL* 99, 140402 (2007).

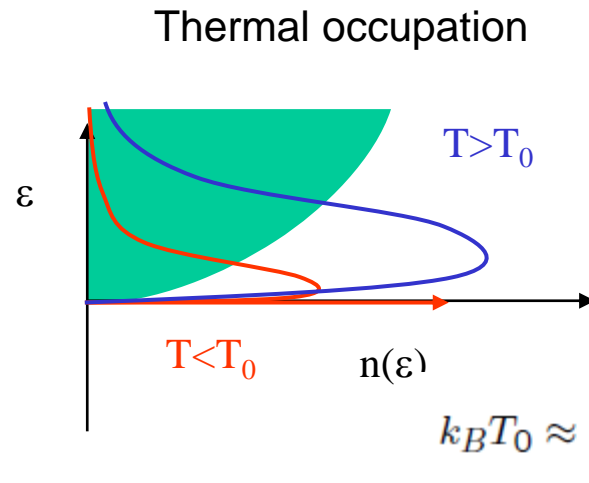
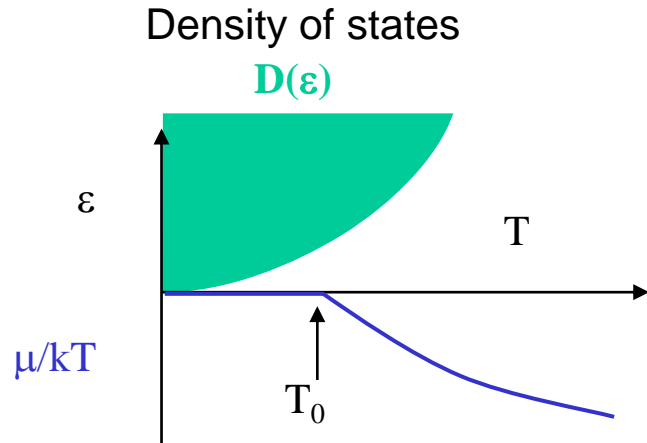
## Physical signatures

Y.Lofovik and V Yudson, *JETP Lett.* 22, 274 (1975)  
Fernandez-Rossier et al. , *Solid State Commun* 108, 473 (1998)  
A V. Balatsky, Y N. Joglekar, PB Littlewood, *Phys. Rev. Lett.* 93, 266801 (2004).  
JMJ Keeling, L. S. Levitov, P. B. Littlewood, *Phys. Rev. Lett.* 92, 176402 (2004)  
I. Shelykh, F.P. Laussy, A. V. Kavokin, G. Malpuech *Phys. Rev. B* 73, 035315 (2006)  
K. G. Lagoudakis, T. Ostatnický, A.V. Kavokin, Y. G. Rubo, R. Andre, B. Deveaud-Pledran ,*Science* 326, 974 - 976 (2009)

# Bose-Einstein condensation

- Macroscopic ground state occupation

$$n = \int d\epsilon \frac{D(\epsilon)}{e^{\beta(\epsilon-\mu)} - 1} \sim \int d\epsilon \frac{\epsilon^{(d-2)/2}}{\beta(\epsilon-\mu)}$$



- Macroscopic phase coherence

Condensate described by macroscopic wave function  $\psi e^{i\phi}$  which arises from **interactions** between particles

$$\psi \rightarrow \psi e^{i\phi}$$

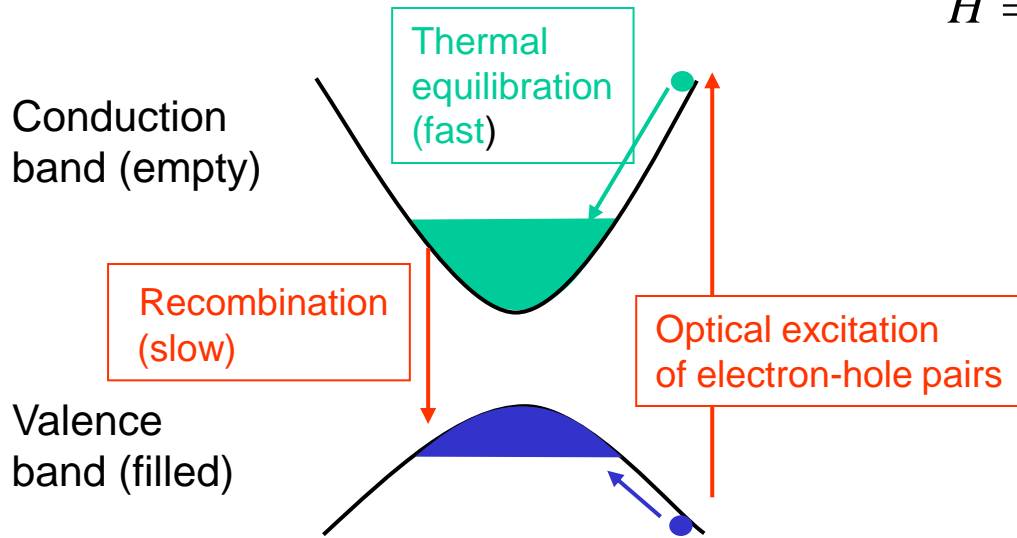
Genuine symmetry breaking, distinct from BEC

Couple to internal degrees of freedom - e.g. dipoles, spins

- Superfluidity

Implies linear Goldstone mode in an infinite system with dispersion  $\omega = v_s k$  and hence a superfluid stiffness  $\propto v_s$

# Excitons in semiconductors



$$H = \sum_i [T_i^e + T_i^h] + \sum_{i,j} [V_{ij}^{ee} + V_{ij}^{hh} - V_{ij}^{eh}]$$

$$T_i^\alpha = \frac{p_{i\alpha}^2}{2m_\alpha}$$

$$V_{ij}^{\alpha\beta} = \frac{e^2}{\epsilon |r_{i\alpha} - r_{j\beta}|}$$

At high density - an electron-hole plasma

At low density - excitons

Exciton - bound electron-hole pair (analogue of hydrogen, positronium)

In GaAs,  $m^* \sim 0.1 m_e$ ,  $\epsilon = 13$

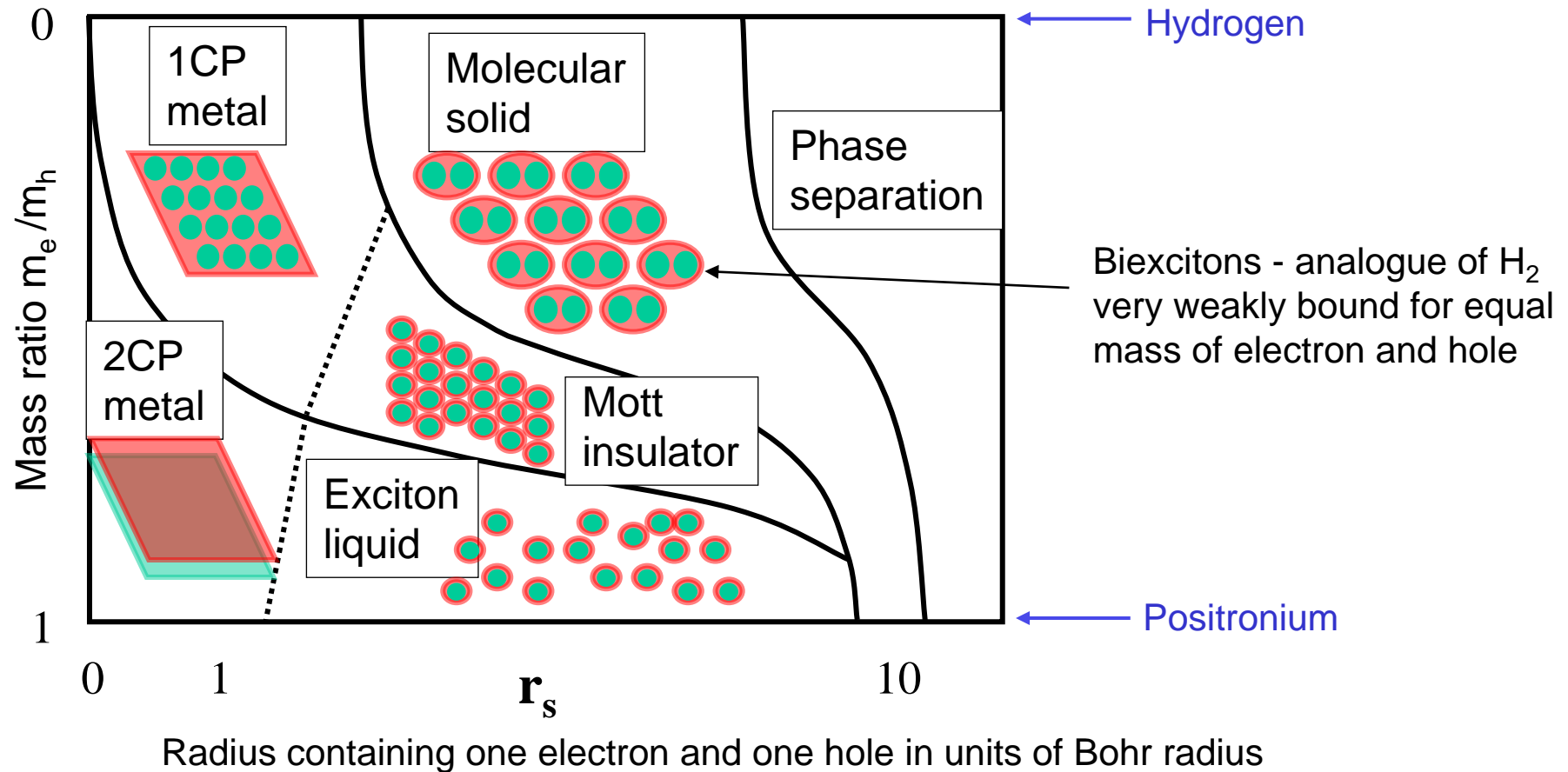
Rydberg = 5 meV (13.6 eV for Hydrogen)

Bohr radius = 7 nm (0.05 nm for Hydrogen)

Measure density in terms of a dimensionless parameter  $r_s$  - average spacing between excitons in units of  $a_{\text{Bohr}}$

$$1/n = \frac{4\pi}{3} (a_{\text{Bohr}} r_s)^3$$

# Speculative phase diagram of electron-hole system ( $T=0$ )

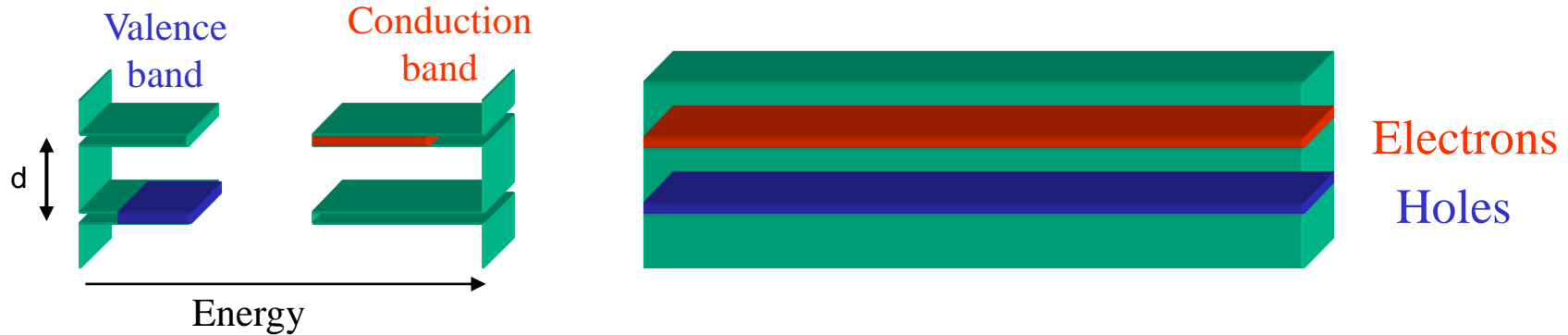


PBLittlewood and XJZhu Physica Scripta T68, 56 (1996)

# Interacting electrons and holes in double quantum well

Two parabolic bands, direct gap, equal masses

Layers of electrons and holes in quantum wells spaced a distance  $d$  apart



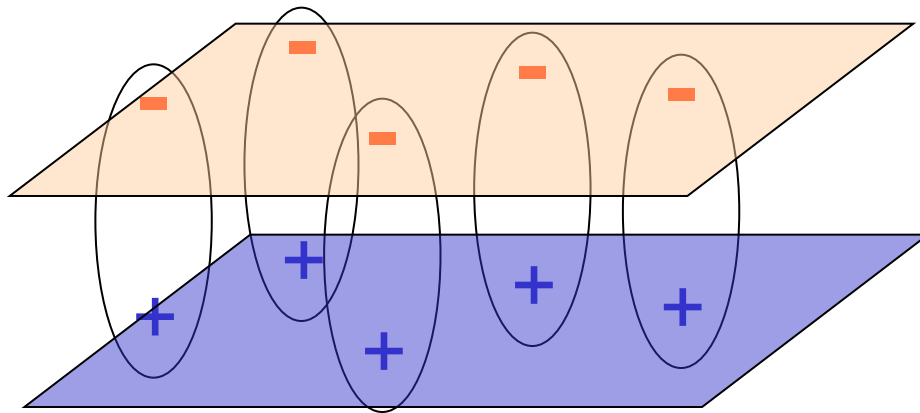
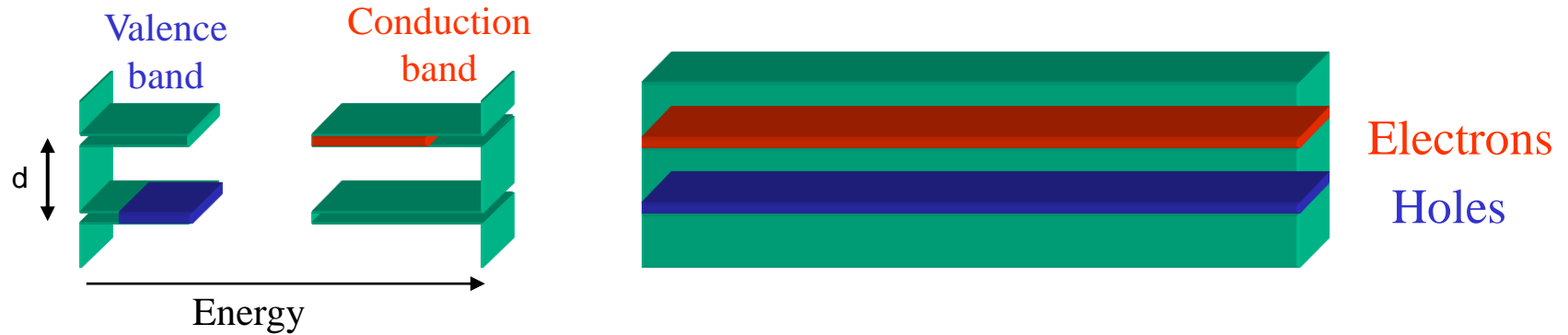
$$H = \sum_i [T_i^e + T_i^h] + \sum_{i,j} [V_{ij}^{ee} + V_{ij}^{hh} - V_{ij}^{eh}] \quad T_i^\alpha = \frac{p_{i\alpha}^2}{2m_\alpha} \quad V_{ij}^{\alpha\beta} = \frac{e^2}{\epsilon |r_{i\alpha} - r_{j\beta}|}$$

Units: density-  $n = 1/\pi(r_s a_B)^2$   $a_B = \epsilon \hbar^2 / m e^2$   
 energy- Rydberg  $e^2 / 2\epsilon a_B$

Ignore interband exchange - spinless problem

Ignore biexcitons - disfavoured by dipole-dipole repulsion

# Coupled Quantum Wells

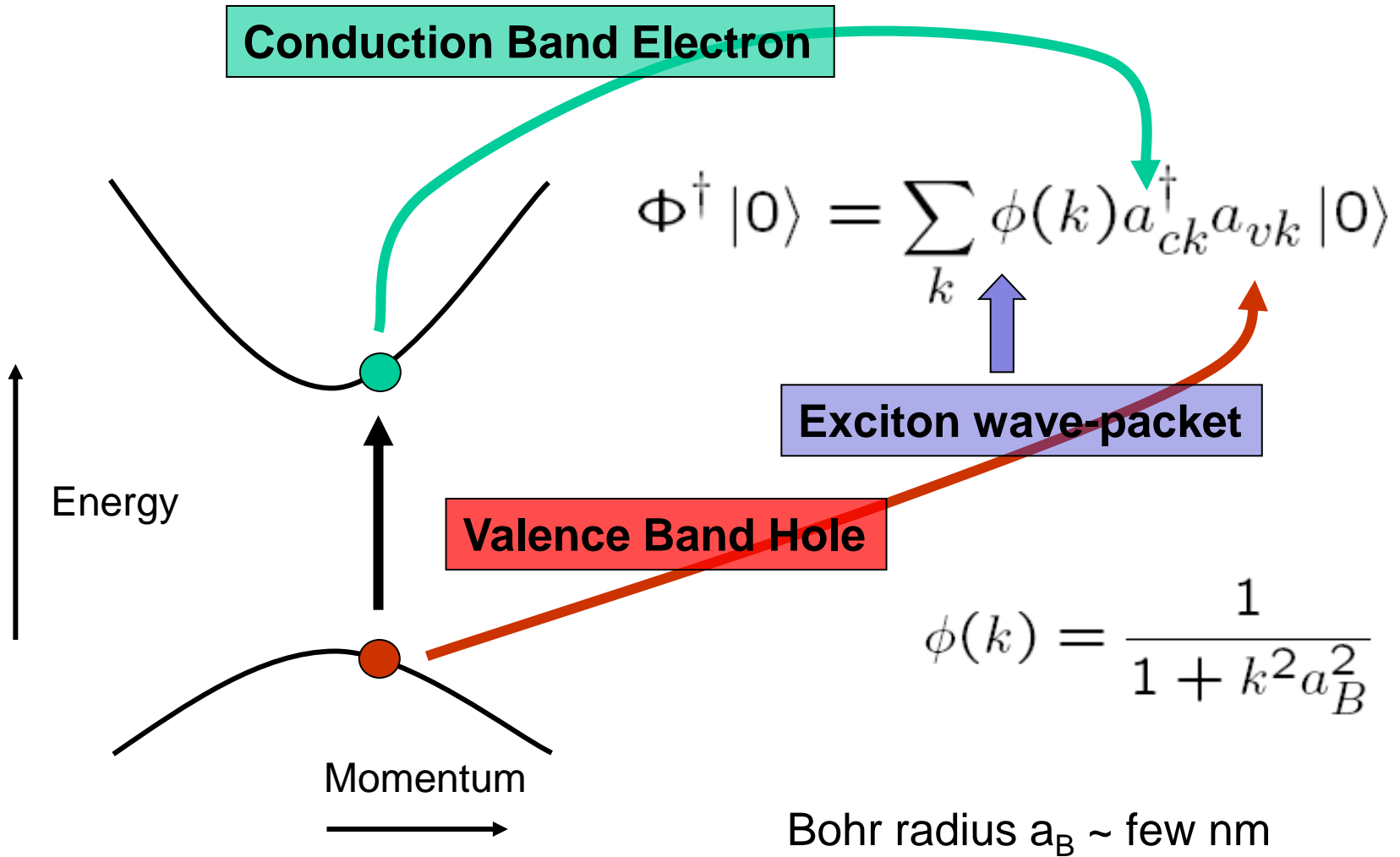


Neutral bosons with repulsive dipolar interaction in 2D

Binding energy few meV in GaAs  
Bohr radius  $\sim 10$  nm

Long lifetime up to 100 nsec –  
recombination by tunnelling  
through barrier

# Excitons



# Mean field theory of excitonic insulator

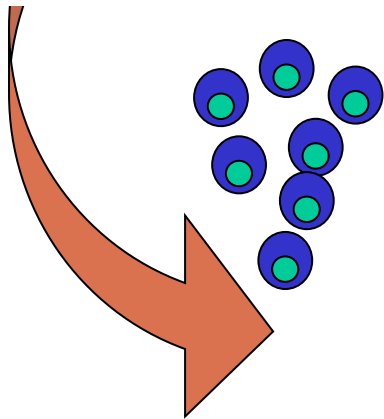
$$\Phi^\dagger |0\rangle = \sum_k \phi(k) a_{ck}^\dagger a_{vk} |0\rangle$$

Wavepacket of bound e-h pair  
Composite boson

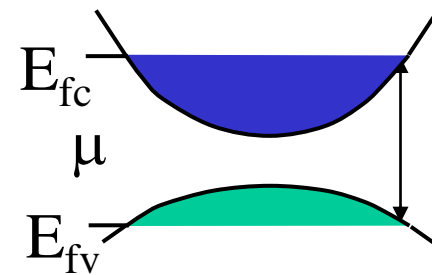
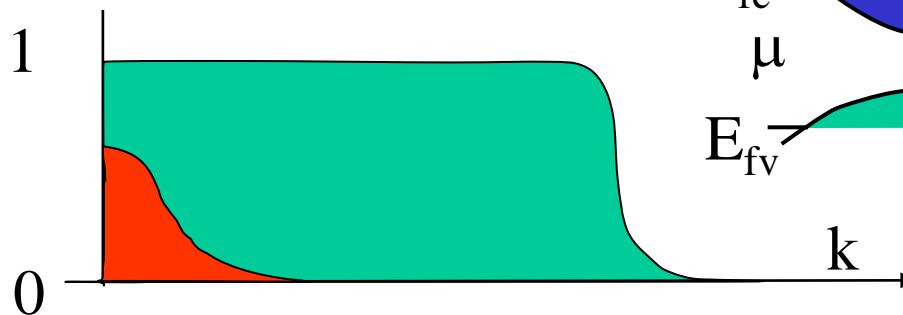
$$e^{\lambda\Phi^\dagger} |0\rangle = \prod_k [u(k) + v(k) a_{ck}^\dagger a_{vk}] |0\rangle$$

A coherent state – like a laser  
Bose condensation of excitons

BCS-like instability  
of Fermi surfaces



$$v(k) = \lambda\phi(k)$$



Special features: order parameter; gap

$$\langle a_{ck}^\dagger a_{vk} \rangle = u_k v_k = (\Delta_k / 2E_k); \quad E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta_k^2}$$

Exciton and Polariton Condensation



# Mean Field Solution

$$H_o = \sum_k [\epsilon_{ck} a_{ck}^\dagger a_{ck} + \epsilon_{vk} a_{vk}^\dagger a_{vk}] \quad \text{Single particle band energies}$$

$$H_c = \frac{1}{2} \sum_q [V_q^{ee} \rho_q^e \rho_{-q}^e + V_q^{hh} \rho_q^h \rho_{-q}^h + 2V_q^{eh} \rho_q^e \rho_{-q}^h] \quad \text{Coulomb interaction between layers separated by distance } d$$

$$\rho_q^e = \sum_k a_{ck+q}^\dagger a_{ck} \quad V^{ee} = V^{hh} = 2\pi/q \quad V^{eh} = 2\pi e^{-qd}/q$$

$$\Psi_0 = e^{\lambda \sum_k a_{ck}^\dagger a_{vk}} |0\rangle = \prod_k [u_k + v_k a_{ck}^\dagger a_{vk}] |0\rangle \quad \text{Variational "BCS" mean field solution}$$

$$\xi_k = \epsilon_k - \mu - \sum_{k'} V_{k-k'}^{ee} n_{k'} \quad \text{Self-energy}$$

$$\Delta_k = 2 \sum_{k'} V_{k-k'}^{eh} \langle a_{ck'}^\dagger a_{vk'} \rangle = \sum_{k'} V_{k-k'}^{eh} \Delta_{k'} / E_{k'} \quad \text{Order parameter equation}$$

$$E_k^2 = \xi_k^2 + \Delta_k^2 \quad \text{Spectrum with a gap}$$

# Excitation spectra

$\pm E_k$  is energy to add (remove) particle-hole pair from condensate (total momentum zero)

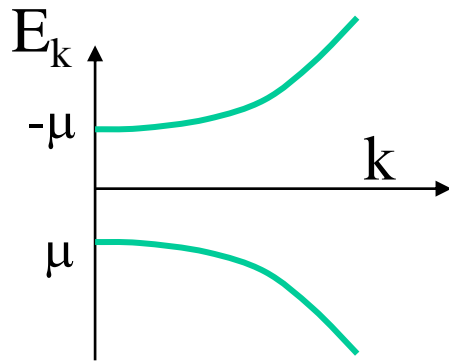
$$E_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta_k^2}$$

Band energy

Chemical potential  
( $<0$  for bound exciton)

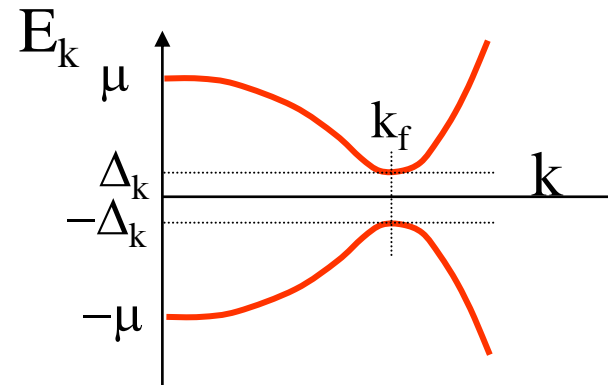
Correlation energy

Low density  $\mu < 0$   
Chemical potential  
below band edge

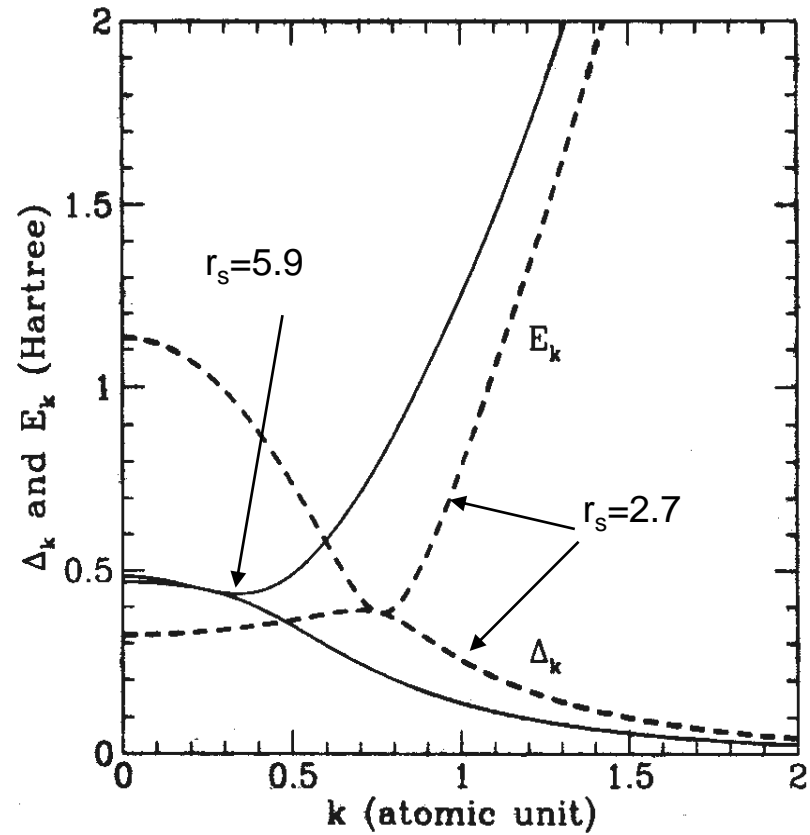
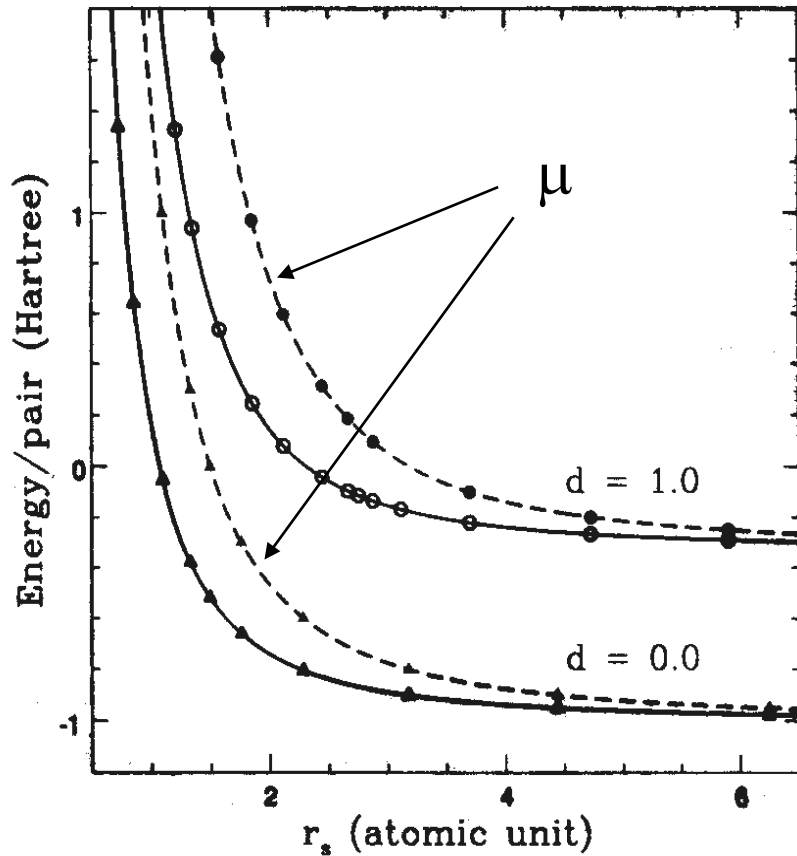


Absorption  
Emission

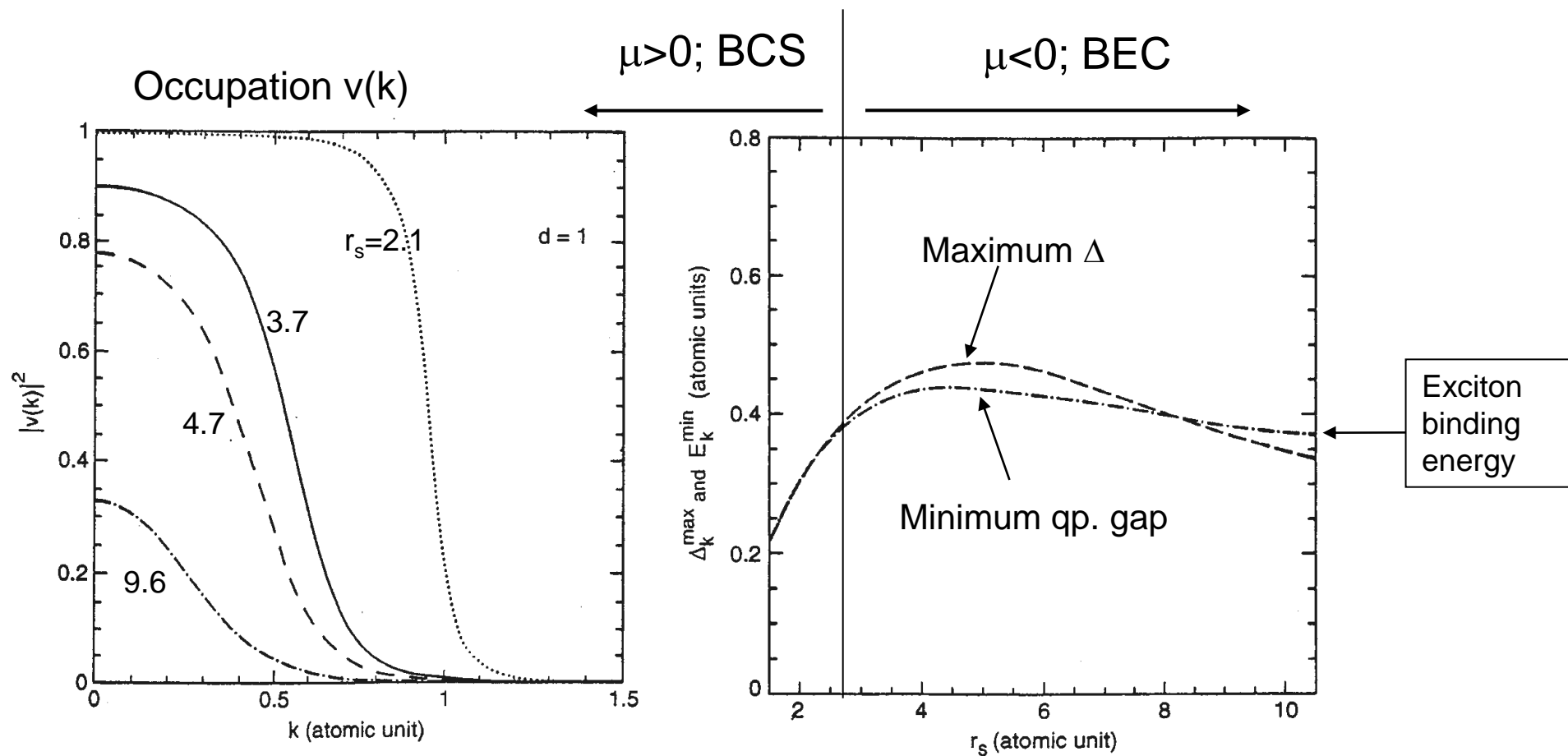
High density  $\mu > 0$   
No bound exciton  
below band edge



# 2D exciton condensate: Mean field solution



# Crossover from BCS to BEC



Smooth crossover between BCS-like fermi surface instability and exciton BEC

Model: 2D quantum wells separated by distance = 1 Bohr radius Zhu et al PRL 74, 1633 (1995)

# Improved solution: Variational Monte Carlo

$$\Psi = \exp \left[ \sum_{i \neq j} u(r_{i,e} - r_{j,e}) \right] \exp \left[ \sum_{i \neq j} u(r_{i,h} - r_{j,h}) \right] \exp \left[ \sum_{i,j} v(r_{i,e} - r_{j,h}) \right] \times \Psi_0$$

Jastrow factors - smooth functions with variational parameters to add extra correlations to wavefunction

Either 2 component plasma (Slater determinant of plane waves) or BCS (Slater determinant of e-h pairs)

Zhu et al, PRB 54, 13575 (1996)

Search through variational parameter space by Monte Carlo  
Output: Better energies, also pair correlation functions

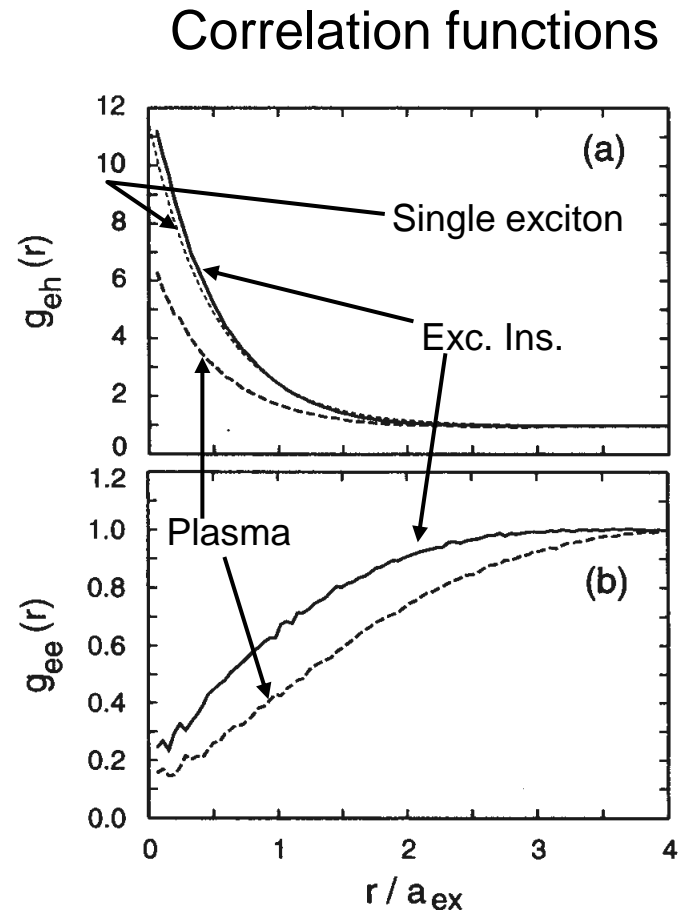
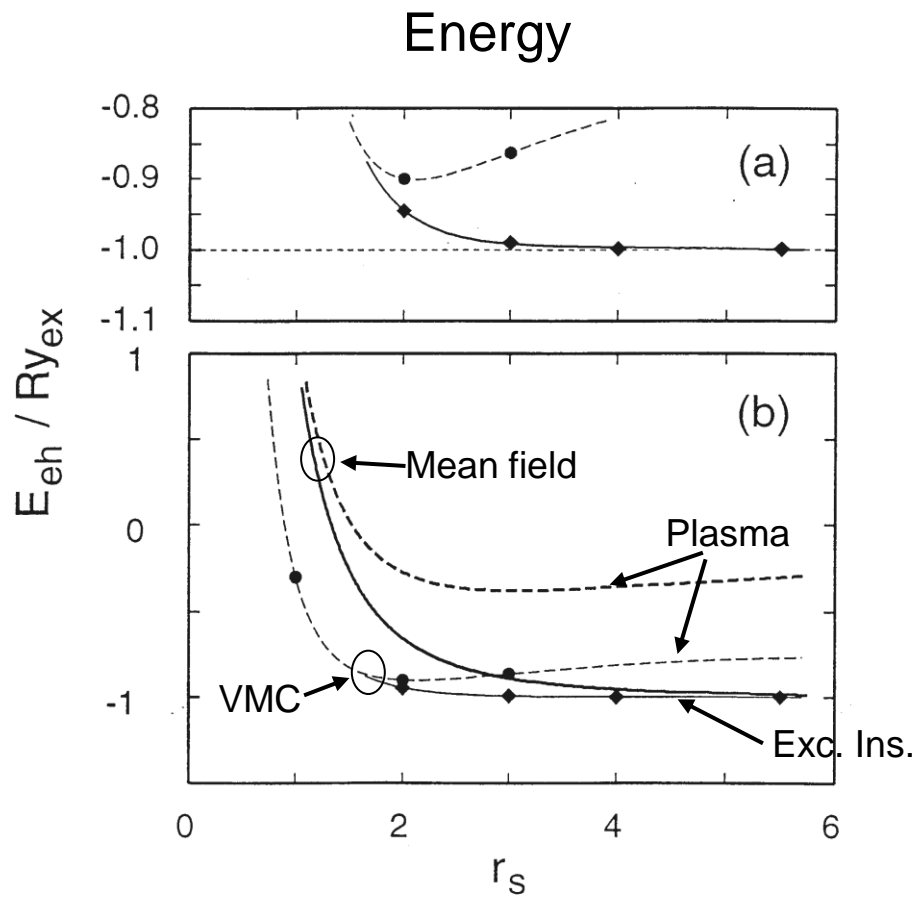
Further improvements possible :

Diffusion Monte Carlo (fixed node)

Path Integral Monte Carlo (finite T)

Include biexcitons, Wigner crystal phases etc...

# 3D exciton condensate - mean field vs VMC

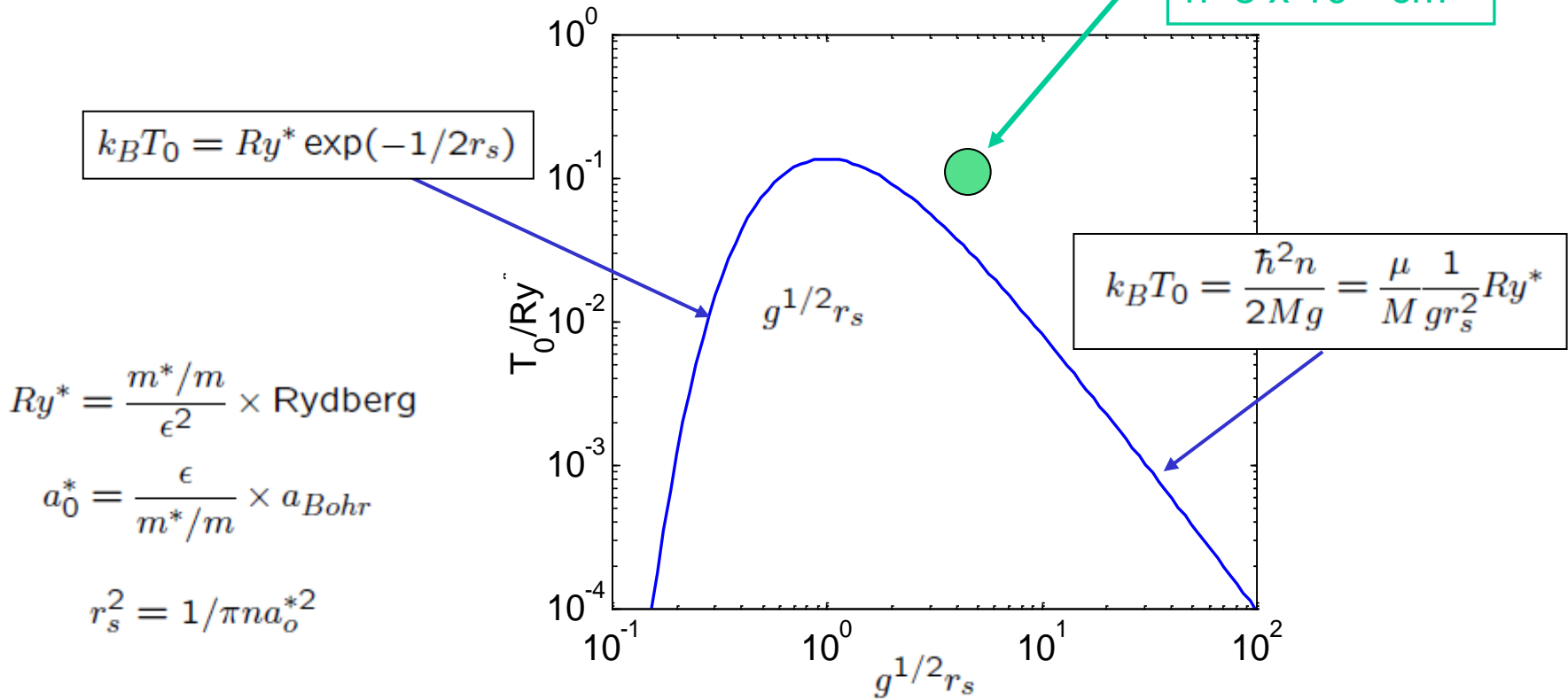


Zhu et al PRB 54, 13575 (1996)

# 2D BEC - no confining potential

Interpolation – by hand – between two limits

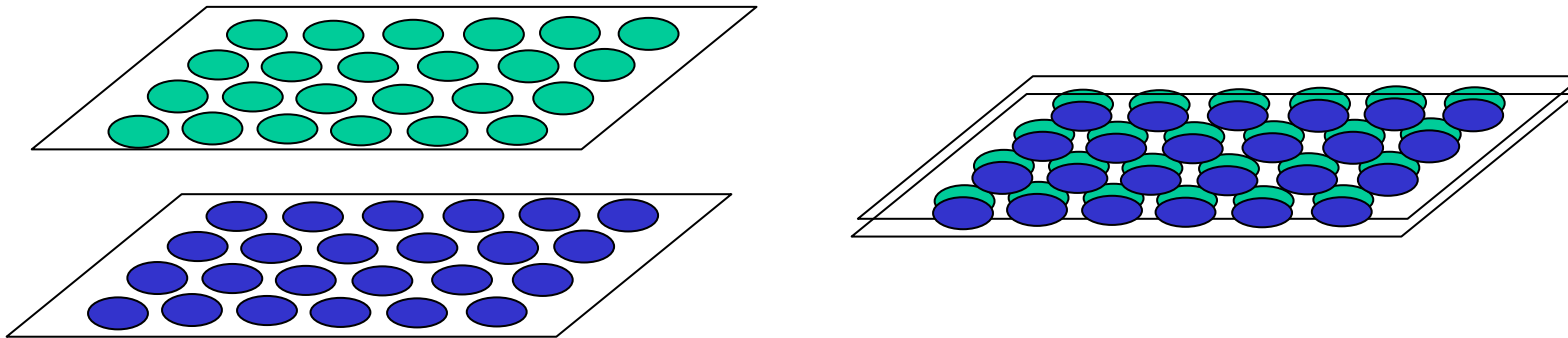
GaAs CQW  
 T = 4 K  
 n = 3 x 10<sup>11</sup> cm<sup>-2</sup>



Mean field - should be K-T transition, but OK to estimate energy scales

## 2D solid phases

- At low density, single layer will become a Wigner crystal
- Paired electron-hole Wigner crystal is an excitonic solid
- In 2D biexciton formation is disfavored by dipole repulsion



- Effect of crystal lattice
  - Stabilise the insulating phases to higher densities – Wigner transition turns into a Mott transition
  - Induce commensurability effects
- One component plasma (e.g. Metallic H) is believed to be a superconductor
  - In the context of excitons with localised holes and delocalised electrons this would be called excitonic superconductivity .....



## Conclusions from numerics

- Condensation is a robust process
  - energy scale is fraction of exciton Rydberg (few meV in GaAs)
- No evidence for droplet formation
  - positive compressibility
  - bi-excitons ignored here, but X-X interaction repulsive in bilayers
  - contrast to multivalley bulk semiconductors like Ge, Si
- BCS-like wavefunction captures smoothly the crossover from high to low densities
- Solid phases also competitive in energy (but higher for moderate  $r_s$ )
- So it should be easy to make experimentally .....

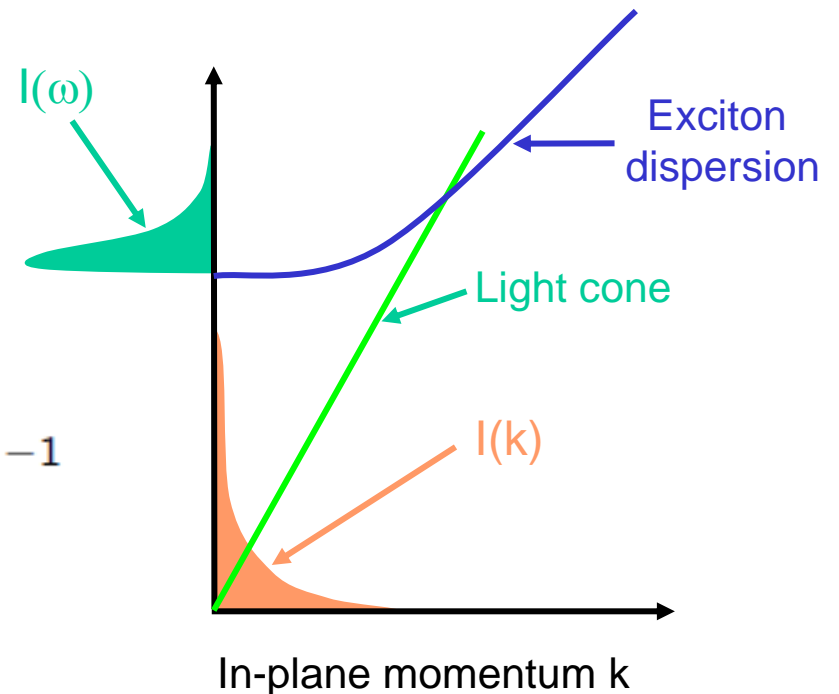
# Experimental signatures

- Phase-coherent luminescence - order parameter is a macroscopic dipole
  - Should couple photons and excitons right from the start – **polaritons**
- Gap in absorption/luminescence spectrum
  - small and low intensity in BEC regime
- Momentum and energy-dependence of luminescence spectrum  $I(k, \omega)$  gives direct measure of occupancy

$$P(t) = \sum_k \langle a_{ck}^\dagger a_{vk} \rangle e^{i\mu t}$$

$$I(k) \propto n_k = \left[ e^{\beta(\epsilon_k - \mu)} - 1 \right]^{-1}$$

- 2D Kosterlitz-Thouless transition
- confined in unknown trap potential
- only excitons within light cone are radiative

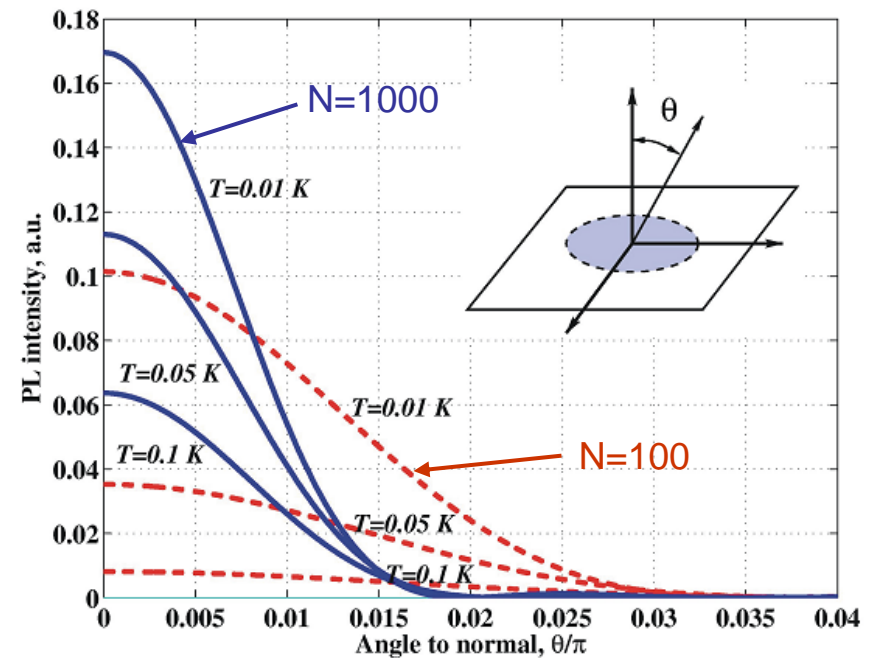


# Angular profile of light emission

Keeling et al, PRL **92**, 176402 (2004)

- Emitted photon carries momentum of electron-hole pair
- Condensation (to  $k_{\parallel} \sim 0$ ) then has signature in sharp peak for emission perpendicular to 2D trap.
- In 2D the phase transition is of Kosterlitz-Thouless type – no long range order below  $T_c$
- Peak suppressed once thermally excited phase fluctuations reach size of droplet

Parameters estimated for coupled quantum wells of separation  $\sim 5$  nm; trap size  $\sim 10 \mu\text{m}$ ;  $T_{\text{BEC}} \sim 1\text{K}$

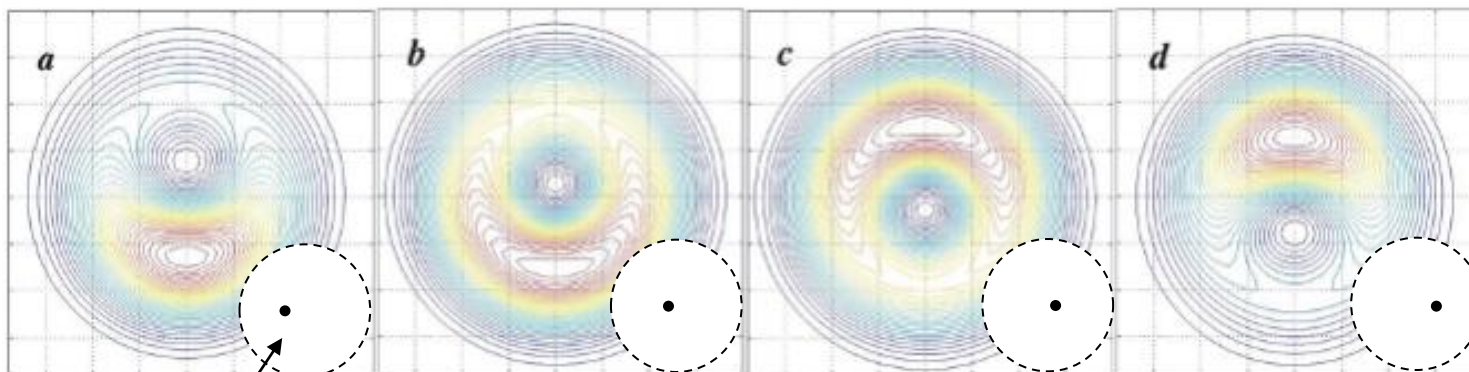


$$\xi_T = (\lambda\rho/4m)^{1/2}/k_B T$$

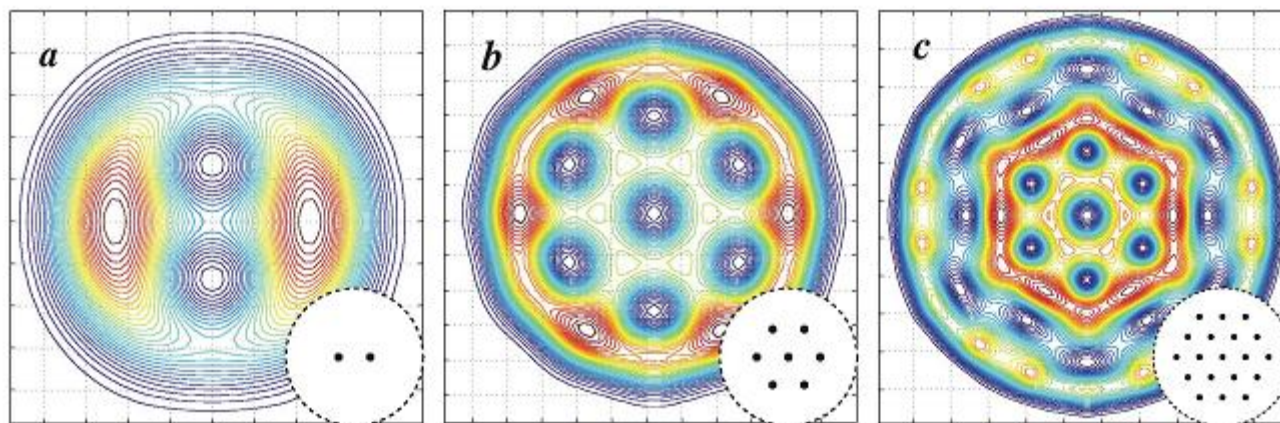
$$T \simeq T_* = T_{\text{BEC}}/\ln(R/\xi_T),$$

# Vortices

Angular emission into  $\theta_x, \theta_y$



Vortex position (x,y) inside droplet

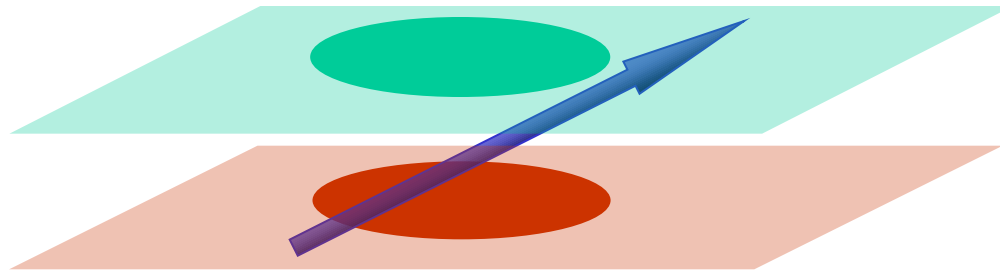


## Dipolar superfluid

- What could be the superfluid response?
  - exciton transport carries no charge or mass
  - in a bilayer have a static dipole

Lozovik & Yudson 1975

Joglekar, Balatsky, PBL, 2004



$$B(t) = B_0 e^{i\omega t} \hat{y}$$

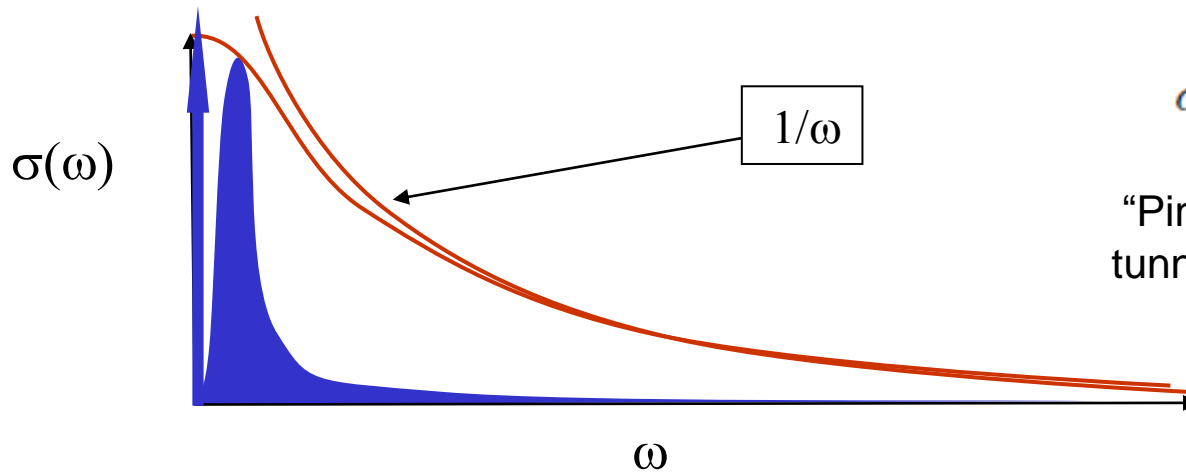
$$\nabla \times \mathbf{E} = -\frac{\partial B}{\partial t}$$

$$F = i\omega B_0 dq \hat{x}$$

$$\mathbf{j}_{dipole} = \sigma(\omega) F = i\omega \chi(\omega) F$$

$$\sigma(\omega) = \sigma_0 [\delta(\omega) + i/\omega]$$

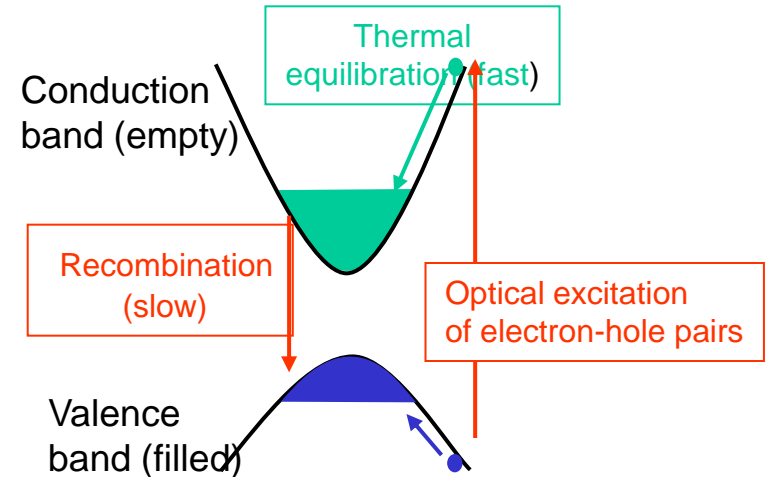
“Pinning” of the phase by interlayer tunnelling shifts response to nonzero frequency



# Recap

## Exciton liquid in semiconductors

Interacting electrons and holes  
 Characteristic energy scale is the exciton Rydberg



$$H = \sum_k [\epsilon_{ck} a_{ck}^\dagger a_{ck} + \epsilon_{vk} a_{vk}^\dagger a_{vk}] + \frac{1}{2} \sum_q [V_q^{ee} \rho_q^e \rho_{-q}^e + V_q^{hh} \rho_q^h \rho_{-q}^h + 2V_q^{eh} \rho_q^e \rho_{-q}^h]$$

A very good wavefunction to capture the crossover from low to high density is BCS

$$\Psi_0 = e^{\lambda \sum_k a_{ck}^\dagger a_{vk}} |0\rangle = \prod_k [u_k + v_k a_{ck}^\dagger a_{vk}] |0\rangle$$

Just like a BCS superconductor, this has an order parameter, and a gap

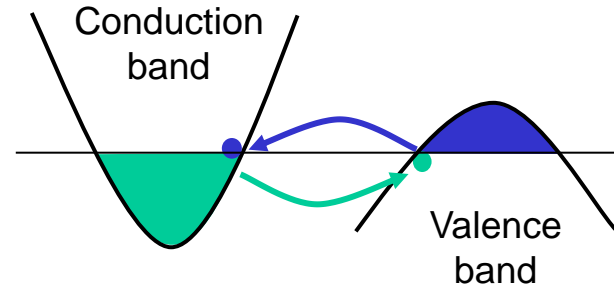
$$\langle a_{ck}^+ a_{vk} \rangle = u_k v_k = (\Delta_k / 2E_k); \quad E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta_k^2}$$

The order parameter has an undetermined phase -> superfluid.

Unfortunately, there are some terms in H that have been left out

## Digression: tunnelling and recombination

- Our Hamiltonian has only included interaction between electron and hole densities, and no e-h recombination
- In a semimetal tunnelling between electron and hole pockets is allowed

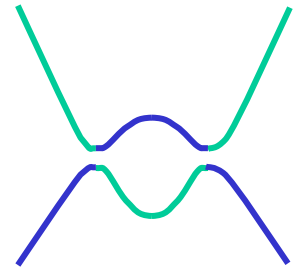


If pockets related by symmetry, generates single particle terms  $ta_{ck}^\dagger a_{vk}$

Rediagonalise  $(\alpha, \beta)$  linear combinations of  $(a_{ck}, a_{vk})$

Introduces single particle gap

New Coulomb coupling terms  $V_1 t a_{ck}^\dagger a_{vk}^\dagger a_{ck} a_{vk}$   $V_2 t^2 a_{ck}^\dagger a_{vk}^\dagger a_{ck} a_{vk}$



If pockets are unrelated by symmetry, still the eigenstates are Bloch states

$$\hat{V} = \sum_{n_1 \dots n_4, k_1 \dots k_4} \langle n_1 k_1, n_2 k_2 | V | n_3 k_3, n_4 k_4 \rangle \times a_{n_1 k_1}^\dagger a_{n_2 k_2}^\dagger a_{n_3 k_3} a_{n_4 k_4},$$

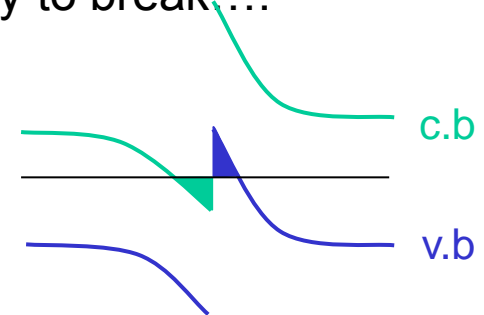
In general, terms of the form  $V_1 t a_{ck}^\dagger a_{vk}^\dagger a_{ck} a_{vk}$   $V_2 t^2 a_{ck}^\dagger a_{vk}^\dagger a_{ck} a_{vk}$

**Most general Hamiltonian does not separately conserve particles and holes**

## Tunnelling and recombination - 2

- Single particle gap - trivial physics, no extra symmetry to break...

E.g. Artificial 2D semimetal - GaSb/InAs interface  
 electron-hole mixing introduces gap [Lakrimi et al 1997]  
 In QH bilayers: tunnelling between layers -> S/AS splitting



- Consider the effect of general Coulomb matrix elements at first order

$$\psi_k = \langle a_{ck}^\dagger a_{vk} \rangle = |\Delta_k| e^{i\phi}$$

Complex order parameter in mean field approximation

$$\langle V_2 \rangle = V_2 |\Delta_k|^2 \cos 2\phi$$



Josephson-like term; fixes phase;  
 gapped Goldstone mode

$$\langle V_1 \rangle = V_1 |\Delta_k| \cos(\phi - \phi_0)$$



Symmetry broken at all T; just like band-  
 structure gap

- No properties to distinguish this phase from a normal dielectric, except in that these symmetry breaking effects may be small
- In that case, better referred to as a commensurate charge density wave
- Indeed an *excitonic insulator*

Not unfamiliar or exotic at all (but not a superfluid either)



## Tunnelling and recombination - 3

- If electron and hole **not** degenerate, recombination accompanied by emission of a photon

$$H_{dipole} = \sum_q \omega_q \phi_q^\dagger \phi_q + \sum_{k,q} g_{q,k} \left[ \phi_q^\dagger a_{vk-q}^\dagger a_{ck} + \phi_q a_{ck+q}^\dagger a_{vk} \right]$$

- Evaluate at zeroth order

$$\langle H_{dipole} \rangle = \sum_k g_0 |\Delta_k| \phi_0^\dagger e^{i\mu t - i\phi} + h.c.$$

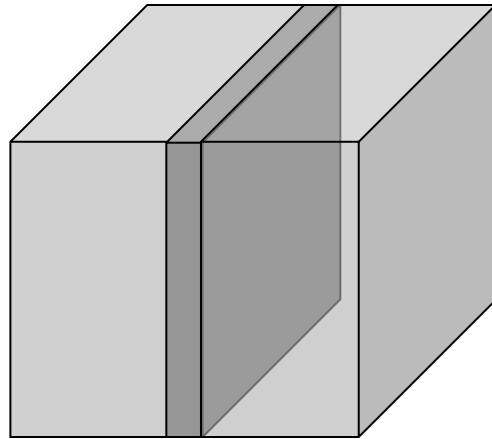
- Phase of order parameter couples to phase of electric field
- Resonant radiation emitted/absorbed at frequency = chemical potential
- Behaves just like an antenna (coherent emission, not incoherent luminescence)

**Must include light and matter on an equal footing from the start - POLARITONS**

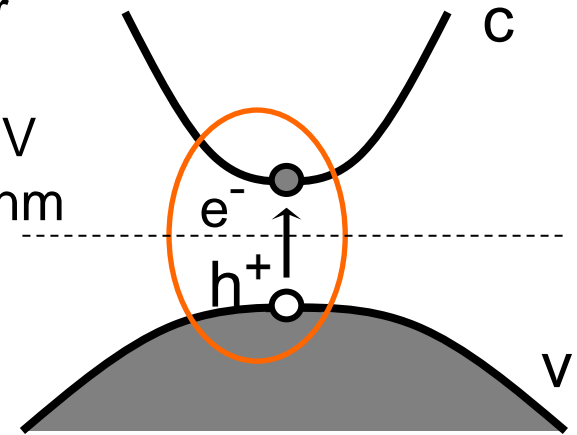
# Quantum Well Excitons

Weakly bound  
electron-hole pair  
EXCITON

Rydberg – few meV  
Bohr Radius – few nm



QW



energy

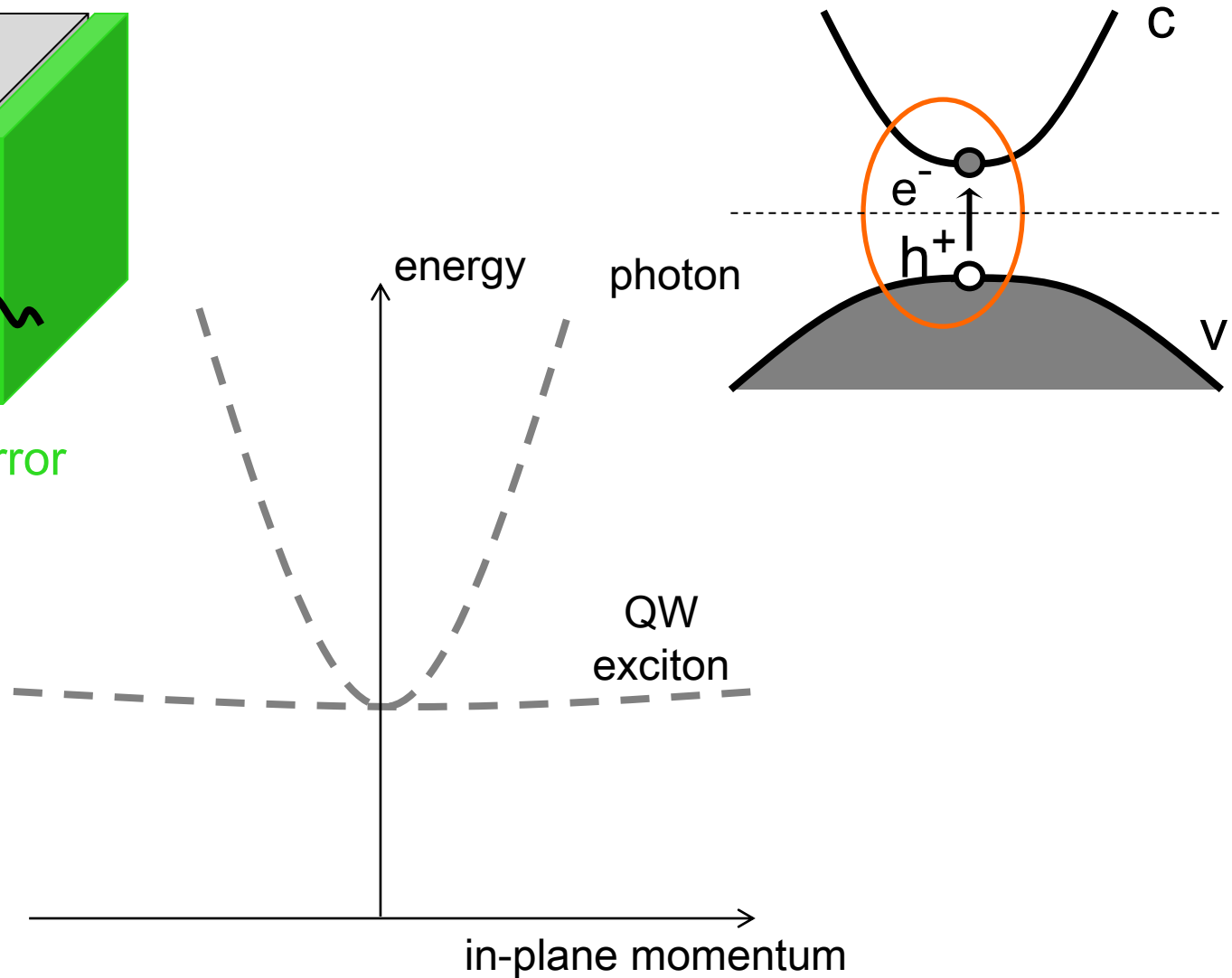
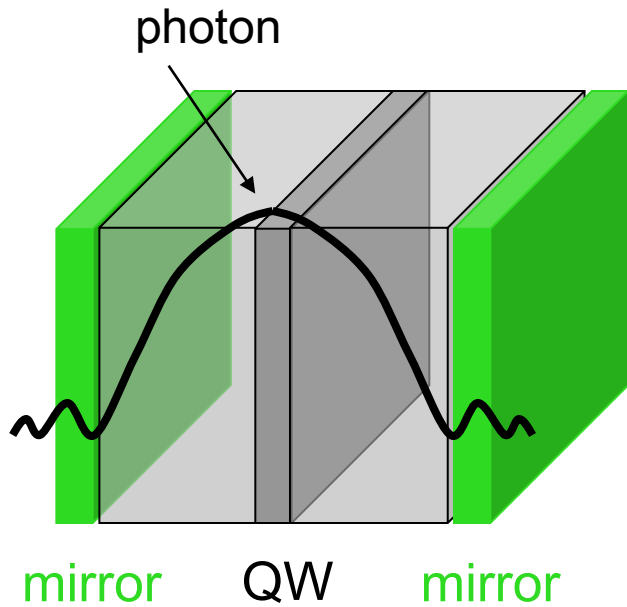
particle-hole  
continuum

Excitation spectrum

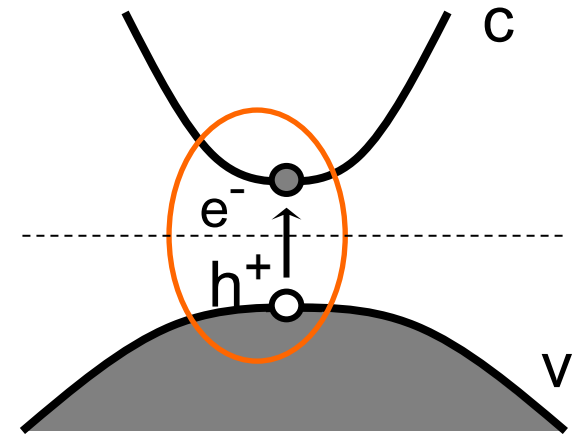
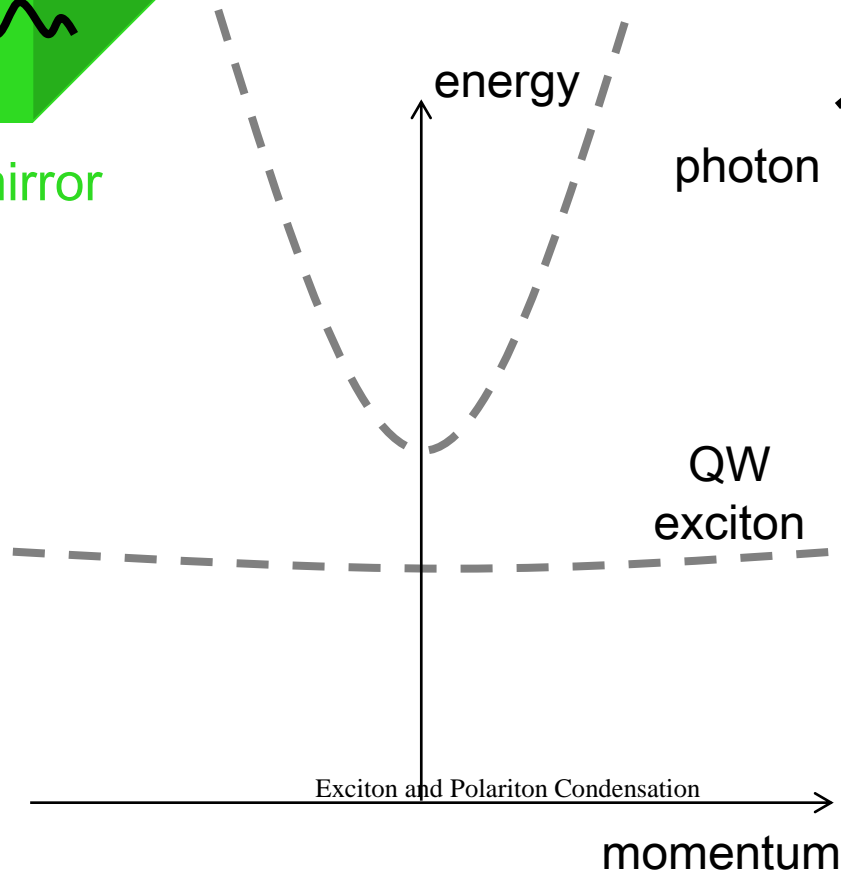
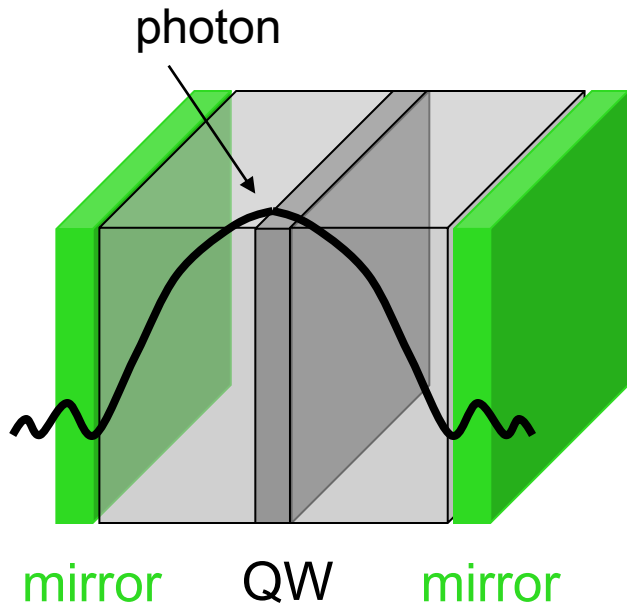
QW  
exciton

in-plane center of mass momentum

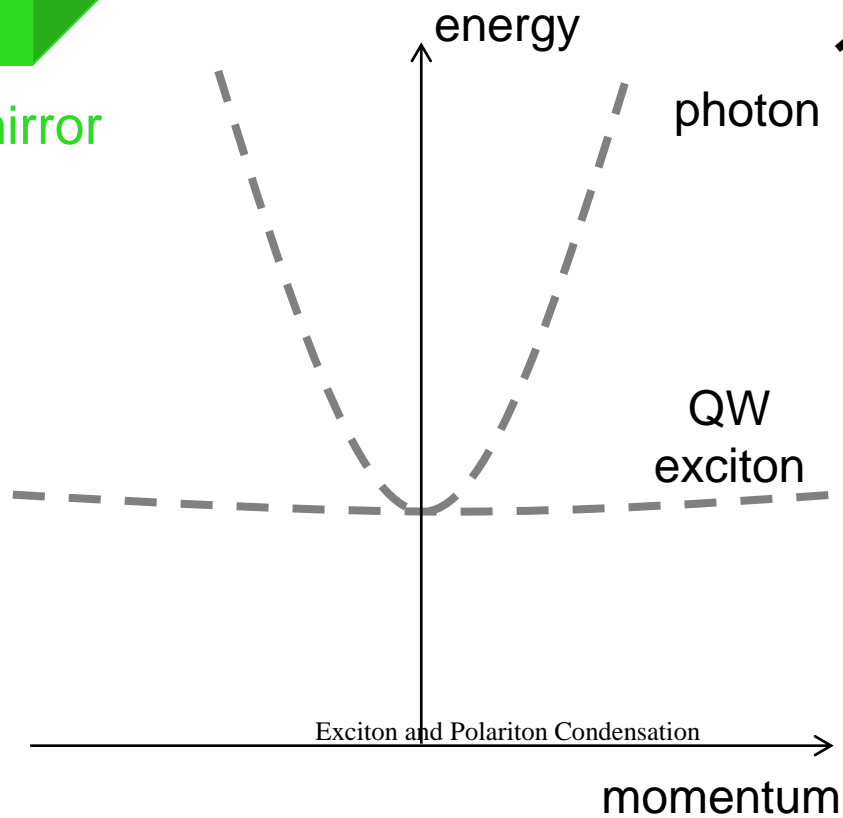
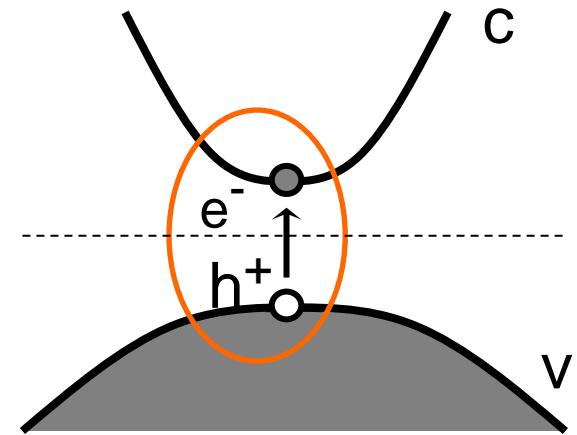
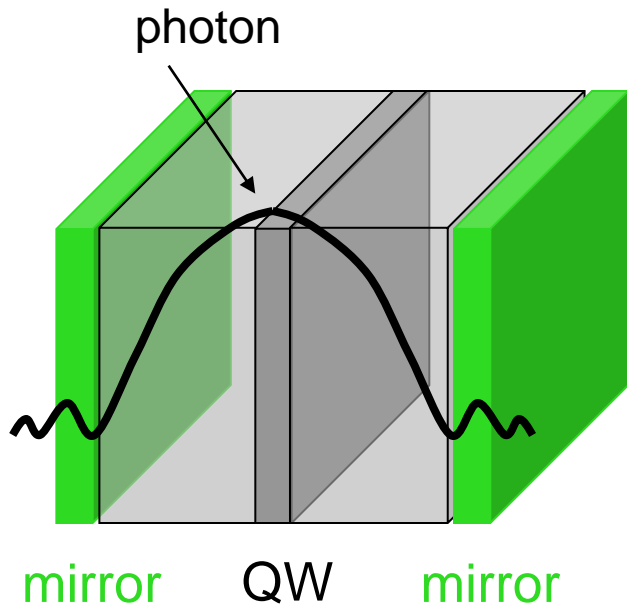
# Excitons + Cavity Photons



# Excitons + Cavity Photons



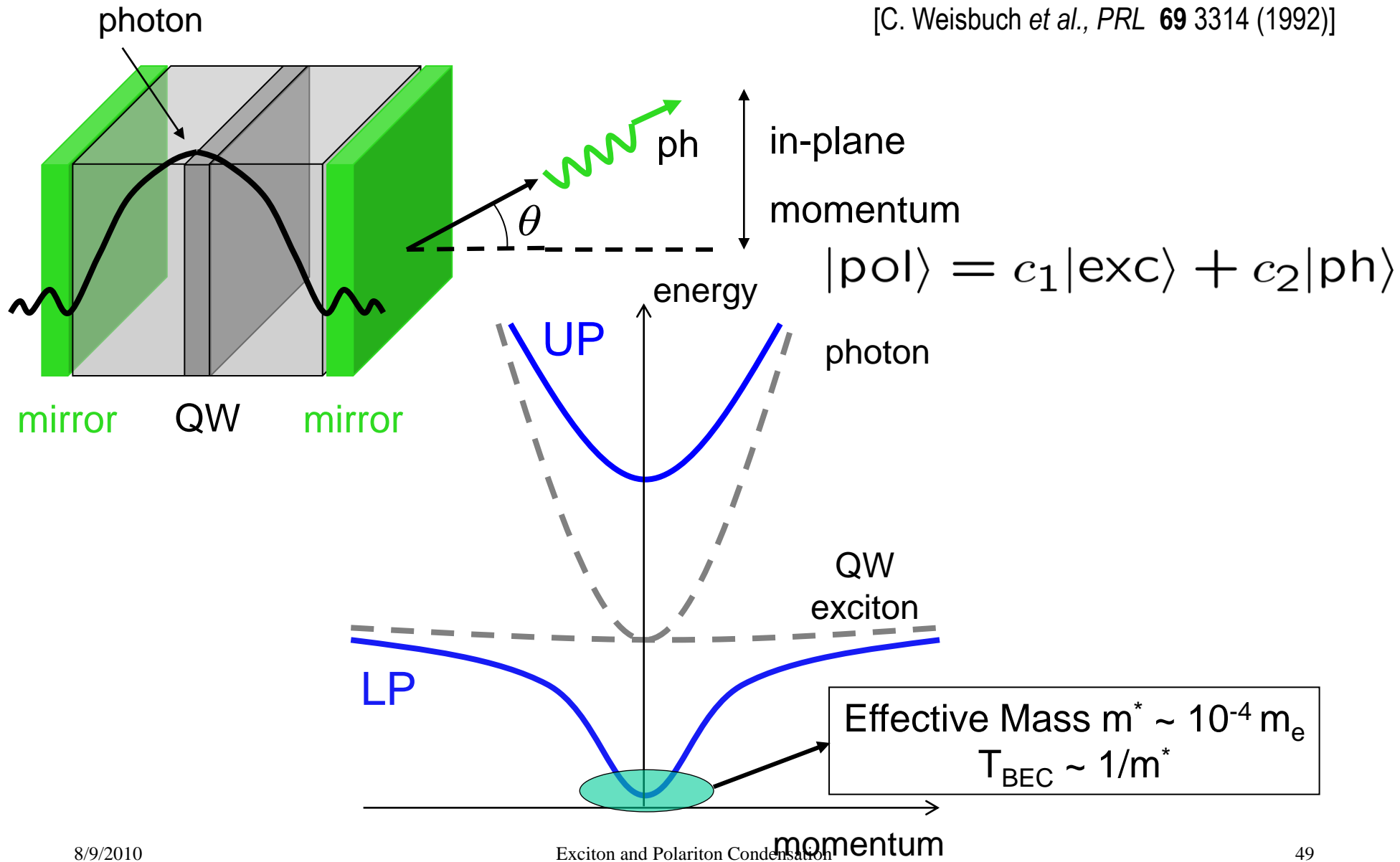
# Excitons + Cavity Photons



Exciton and Polariton Condensation

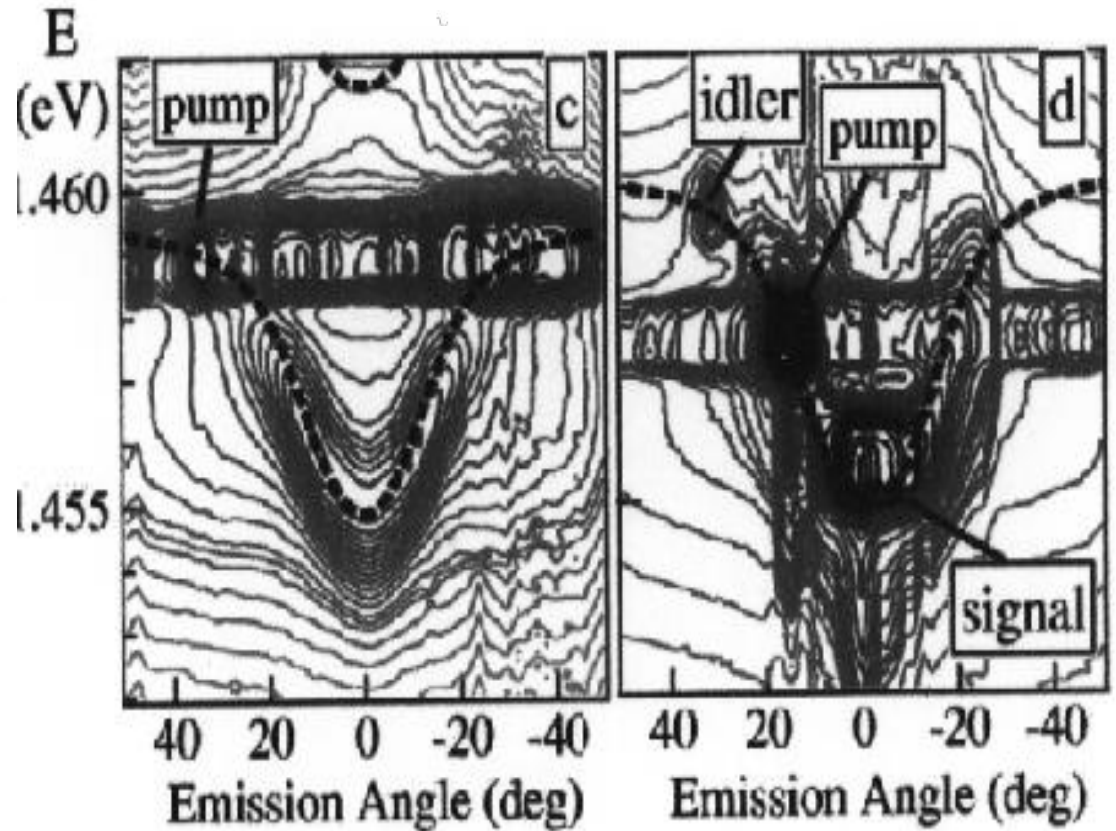
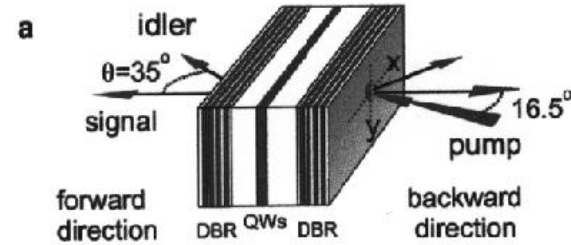
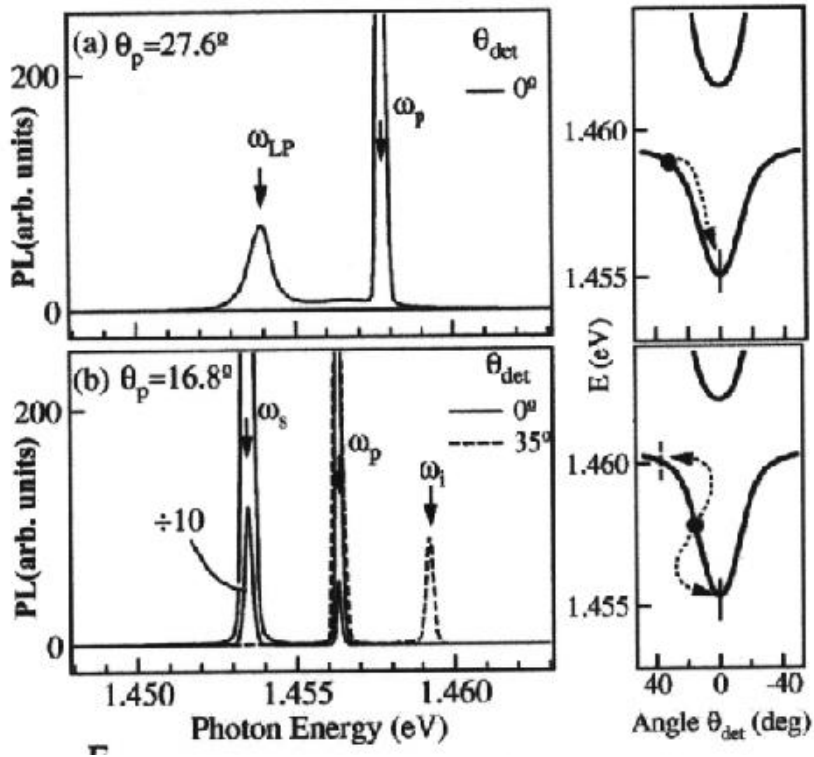
# Polaritons: Matter-Light Composite Bosons

[C. Weisbuch *et al.*, *PRL* **69** 3314 (1992)]



# Resonantly pumped microcavity

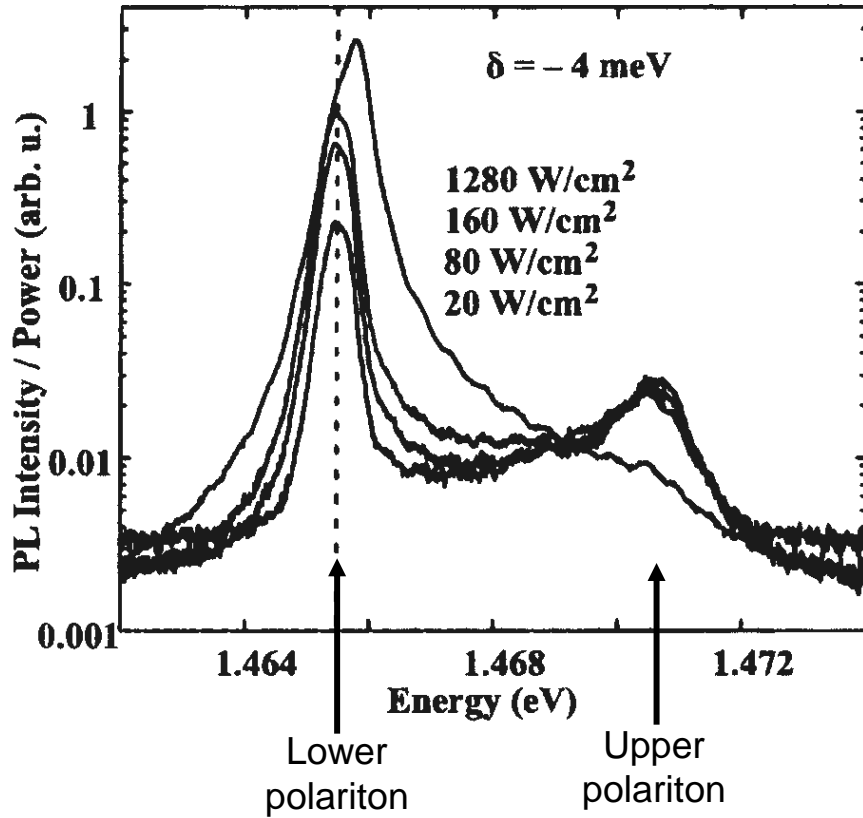
Address in plane momentum by measurement or excitation as function of angle



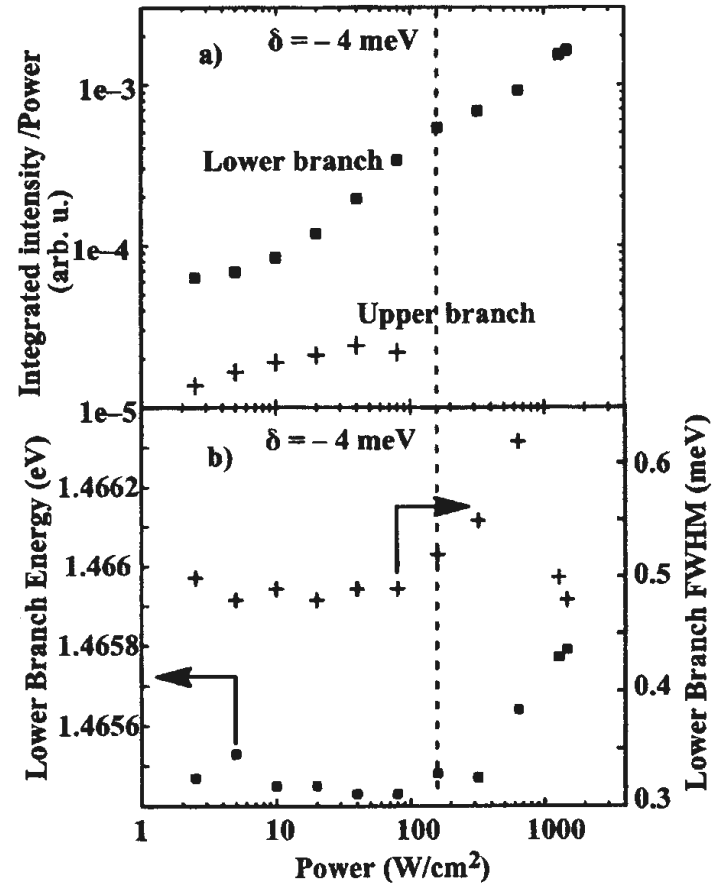
Baumberg et al Phys Rev B 62, 16247 (2000)

# Photoluminescence from non-resonantly pumped microcavity

PL normalised to pump intensity



Excitation at  $\sim 1.7 \text{ eV}$

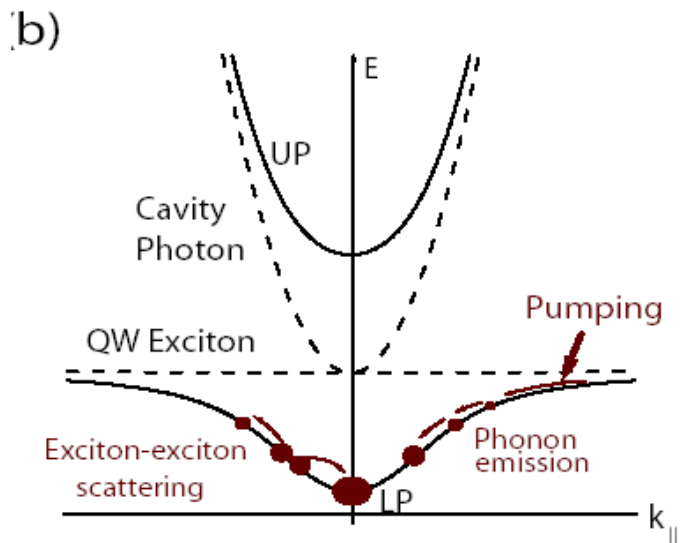


Senellart & Bloch, PRL 82, 1233 (1999)



# Non-resonant(?) pumping in Lower Polariton Branch

Deng et al 2002



Substantial blue shift appears at threshold  
Polariton dispersion seen above threshold

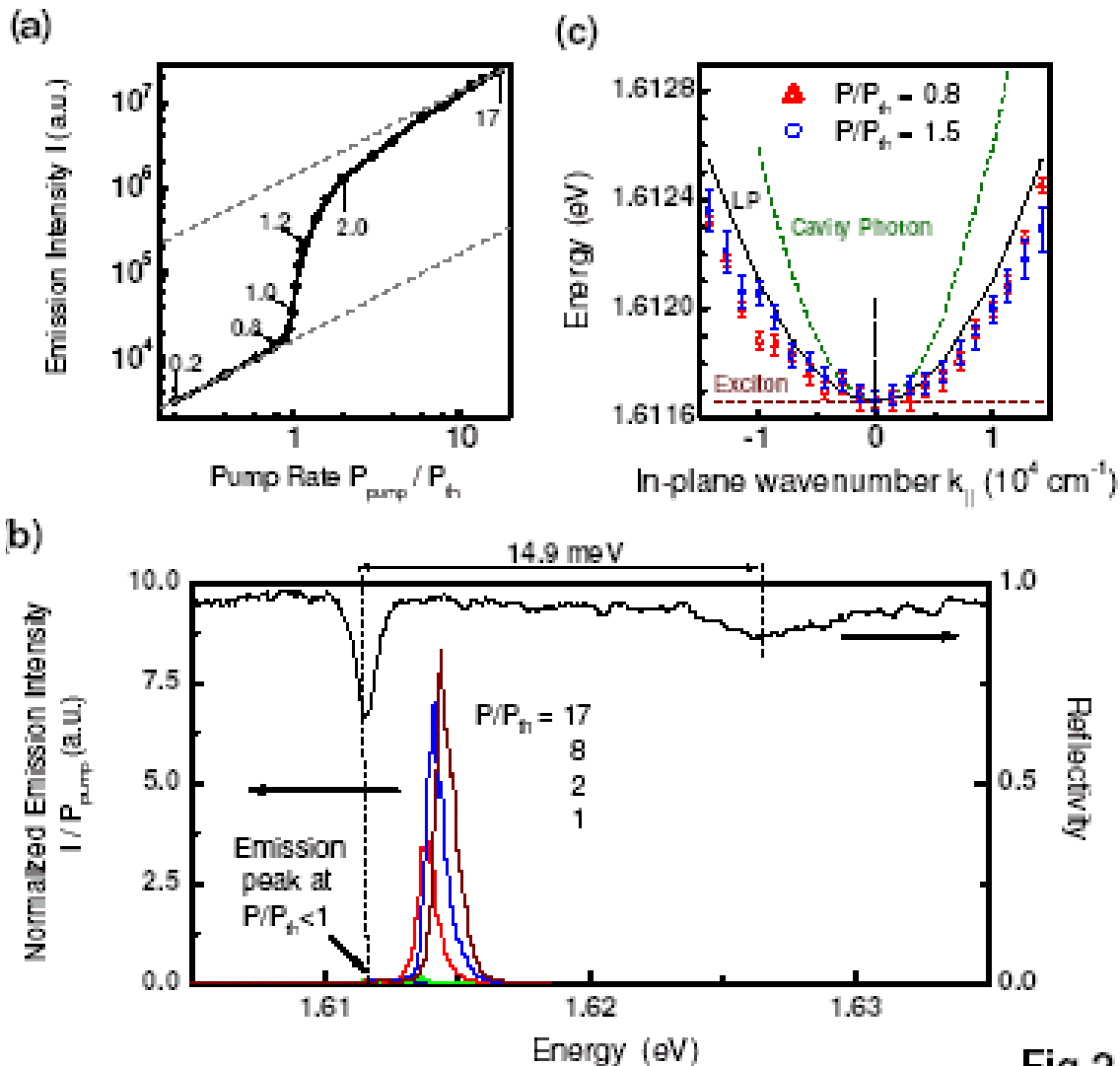
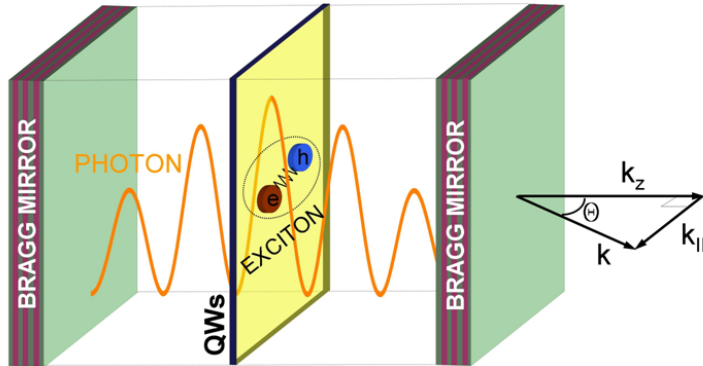
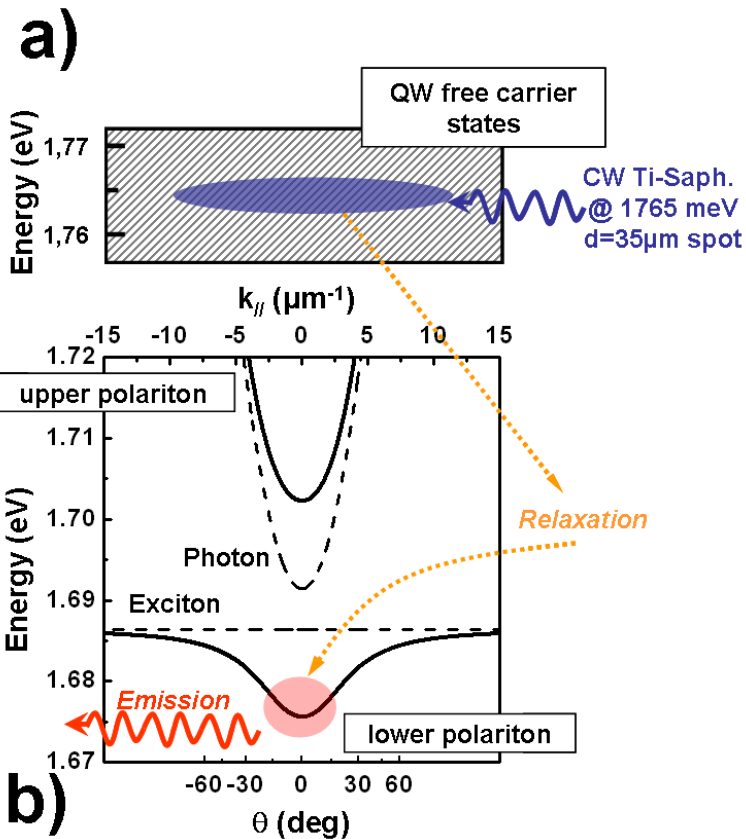


Fig.2



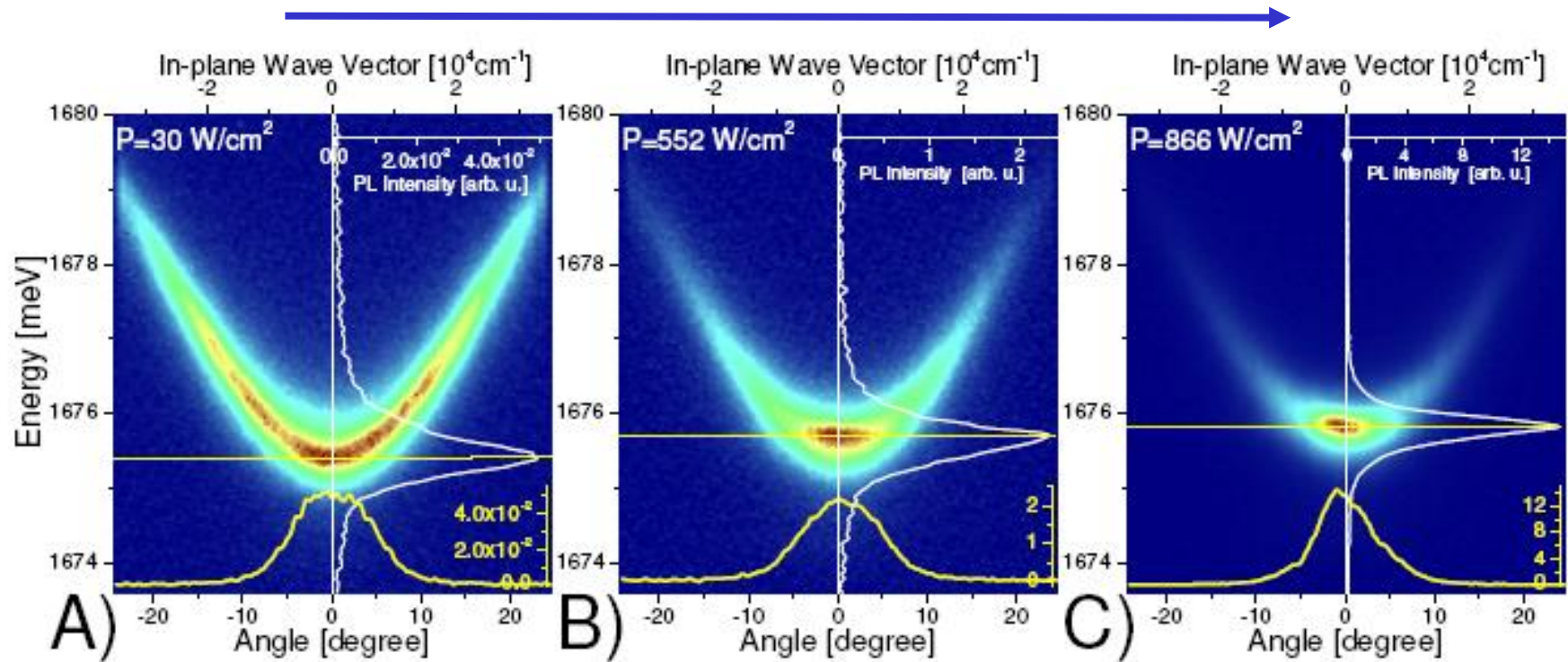
# Microcavity polaritons



Experiments:  
Kasprzak et al 2006  
CdTe microcavities

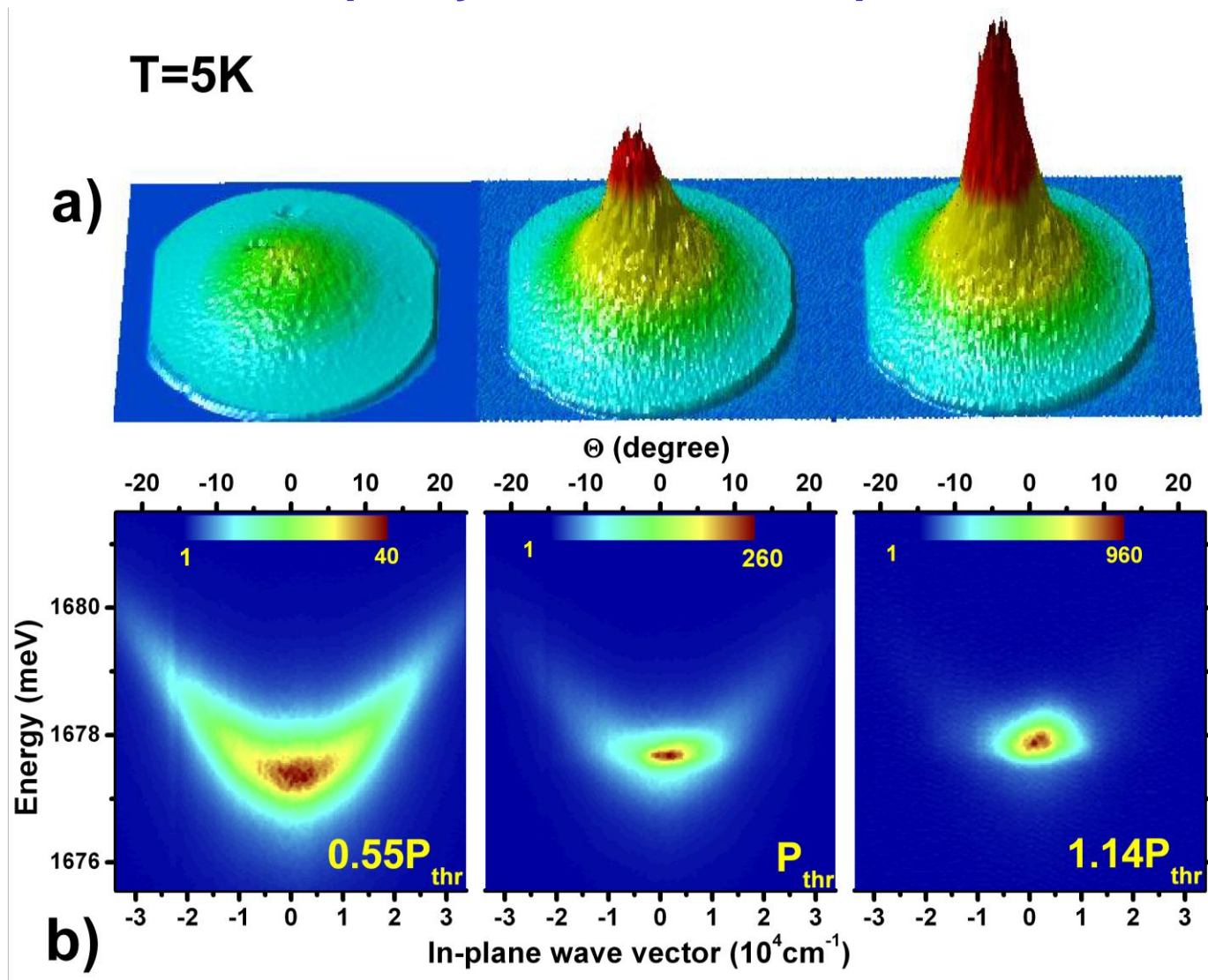
## II-VI quantum well microcavities

Increasing pumping



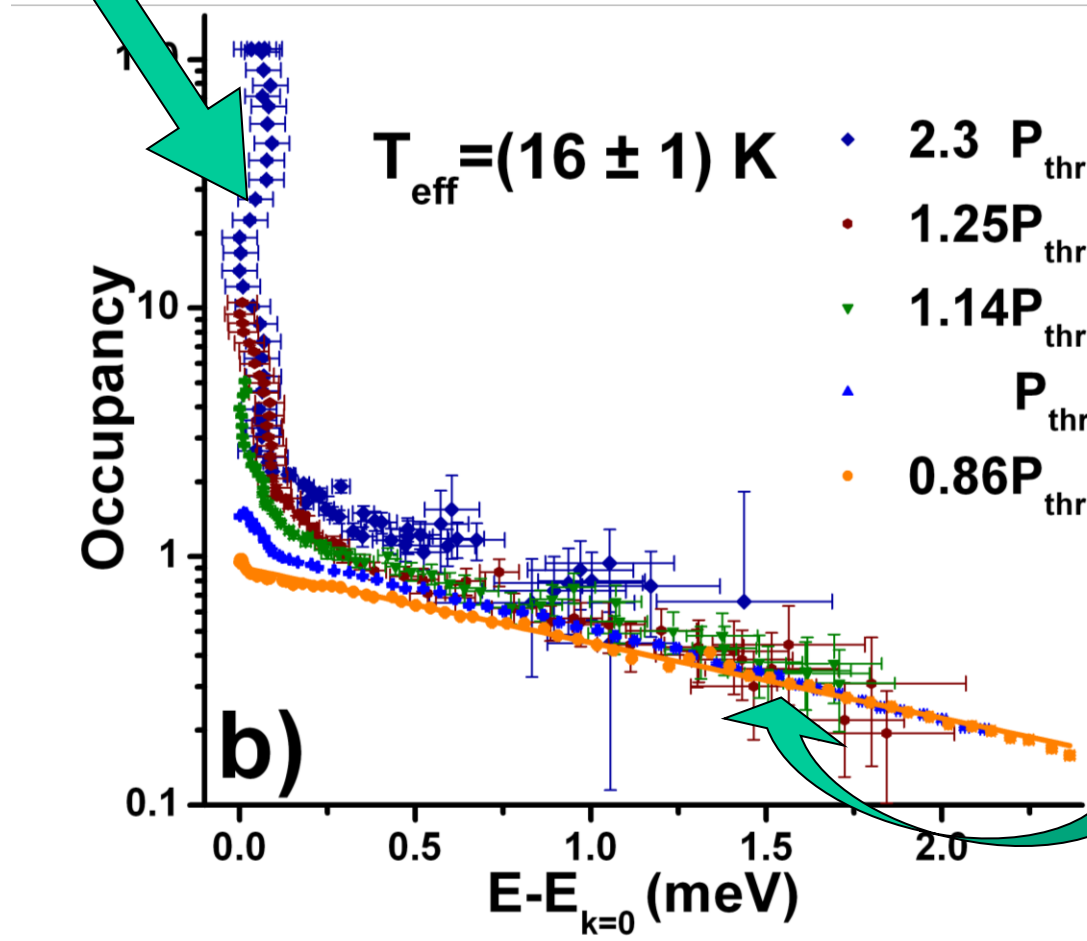
Kasprzak, Dang, unpublished

# Occupancy as a function of power



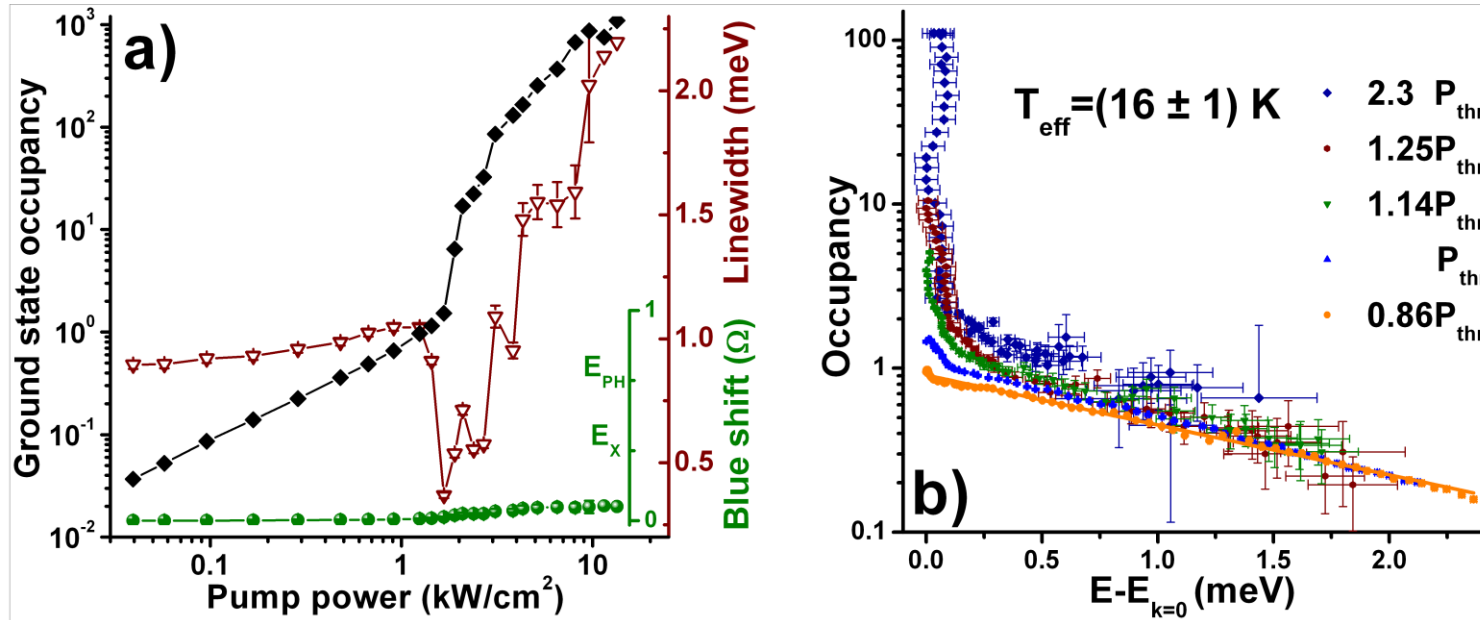
# Distribution at varying density

Coherent(?) peak



Maxwellian tail

## Distribution at varying density



Blue shift used to estimate density

High energy tail of distribution used to fix temperature

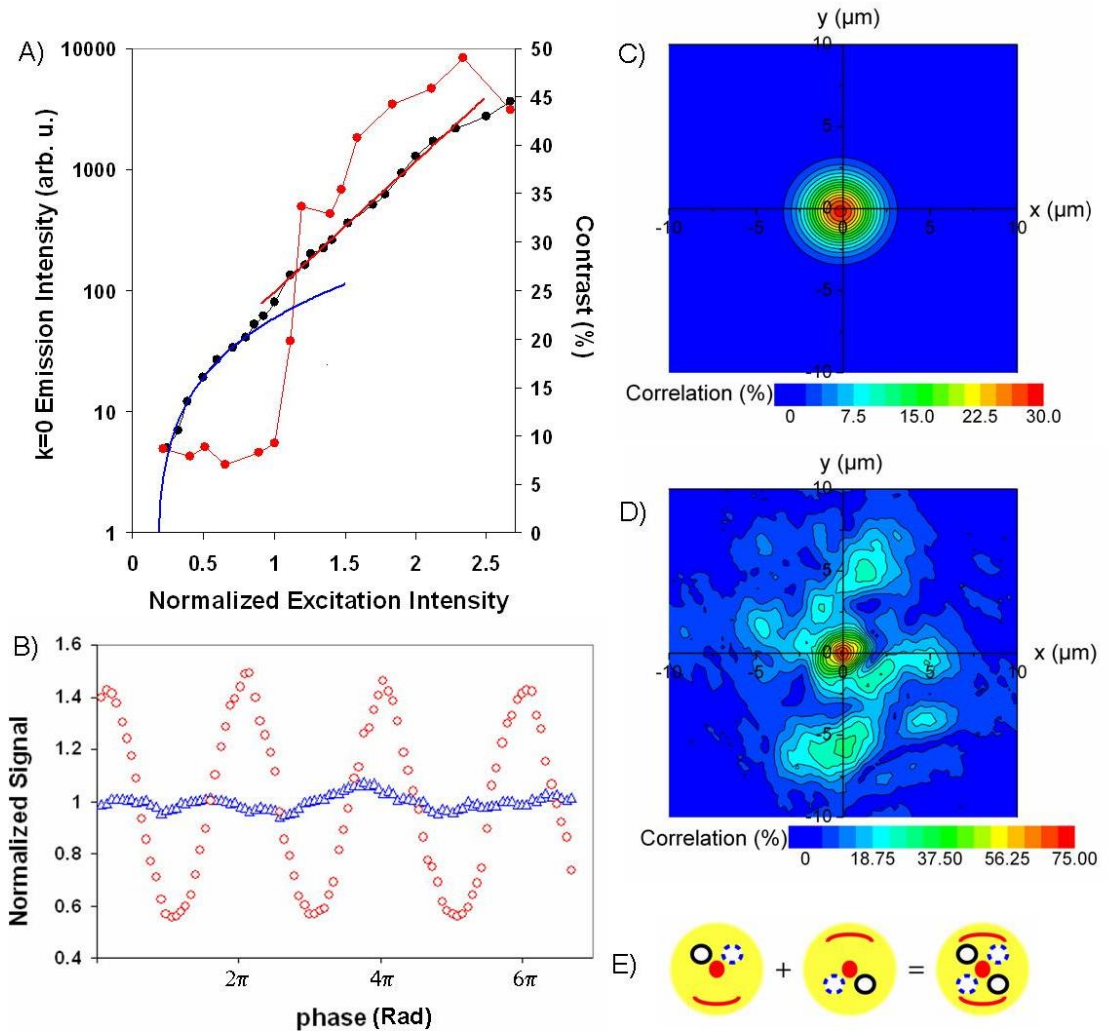
Onset of non-linearity gives estimate of critical density

Linewidth well above transition is *inhomogeneous*

# Measurement of first order coherence

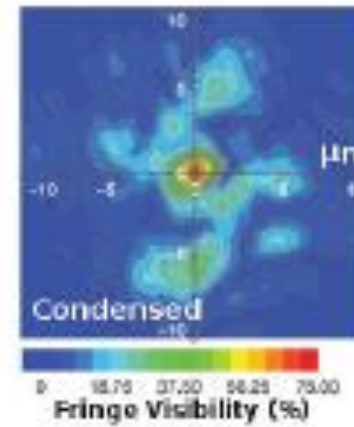
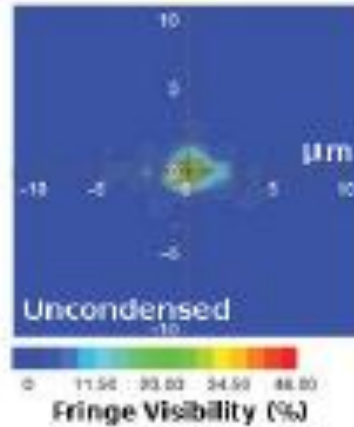
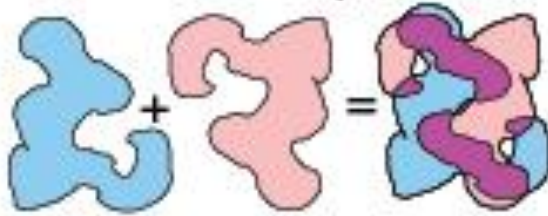
Temperature and density estimates predict a phase coherence length  $\sim 5 \mu\text{m}$

Experiment also shows broken polarisation symmetry



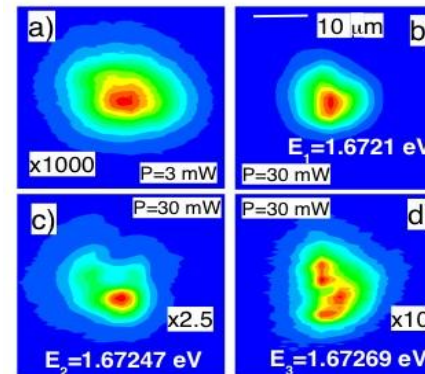
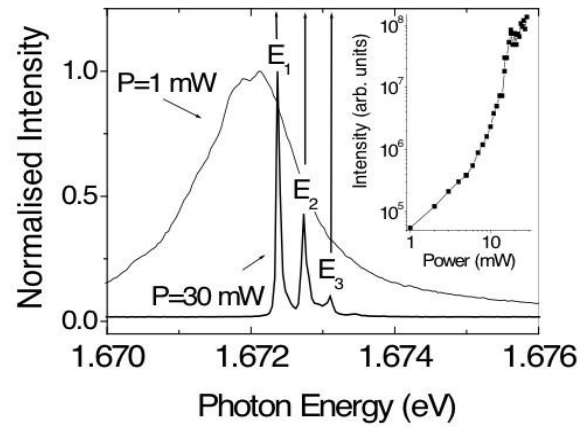
# Coherence

Coherence map:



[Kasprzak, et al., Nature, 2006]

## Temporal coherence and multimode behavior

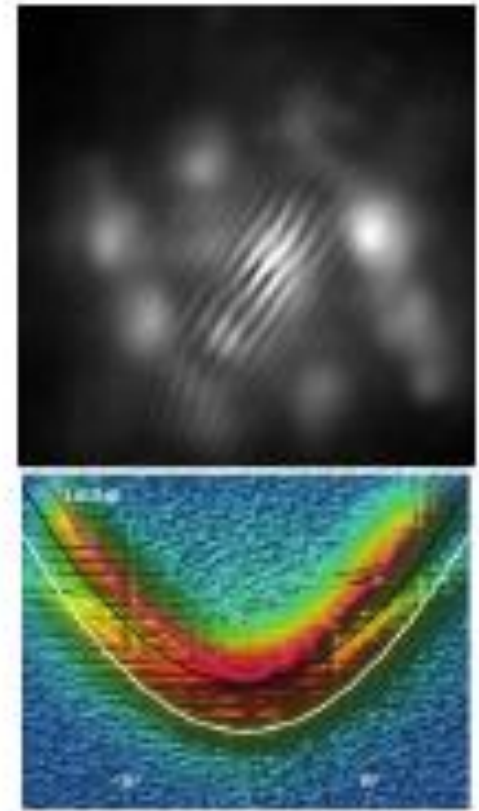


Love et al., PRL 2008

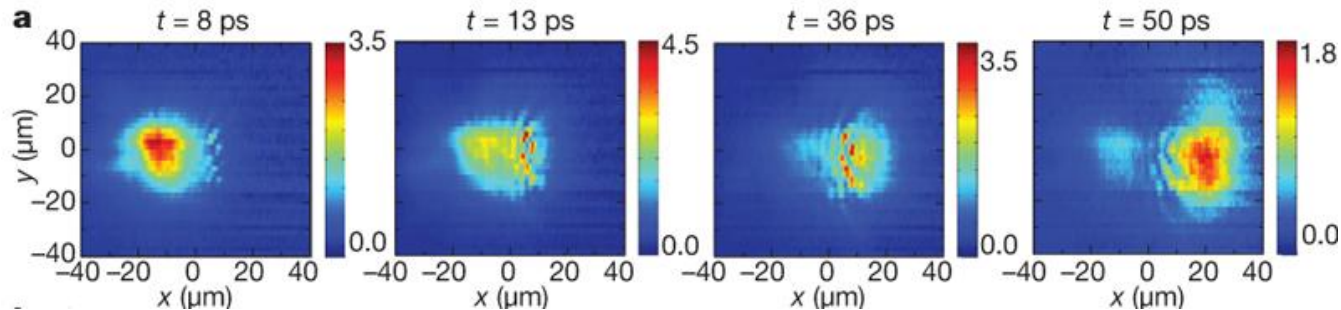


## Other recent experiments

- Stress traps for polariton condensates
  - Balili et al Science 316 1007 (2007)
- Coherence and line narrowing
  - Love et al PRL 101 067404 (2008)
- Changes in the excitation spectrum
  - Utsonomiya et al Nature Physics 4 700 (2008)
- Superflow in driven condensates
  - Amo. *et al.* Nature 457, 291–295 (2009).
- Vortices and half-vortices
  - Lagoudakis et al Nature Physics 4 706 (2008)
  - Lagoudakis et al Science 326 974 (2009)



### Photon superfluid is an electromagnetic cloaking device



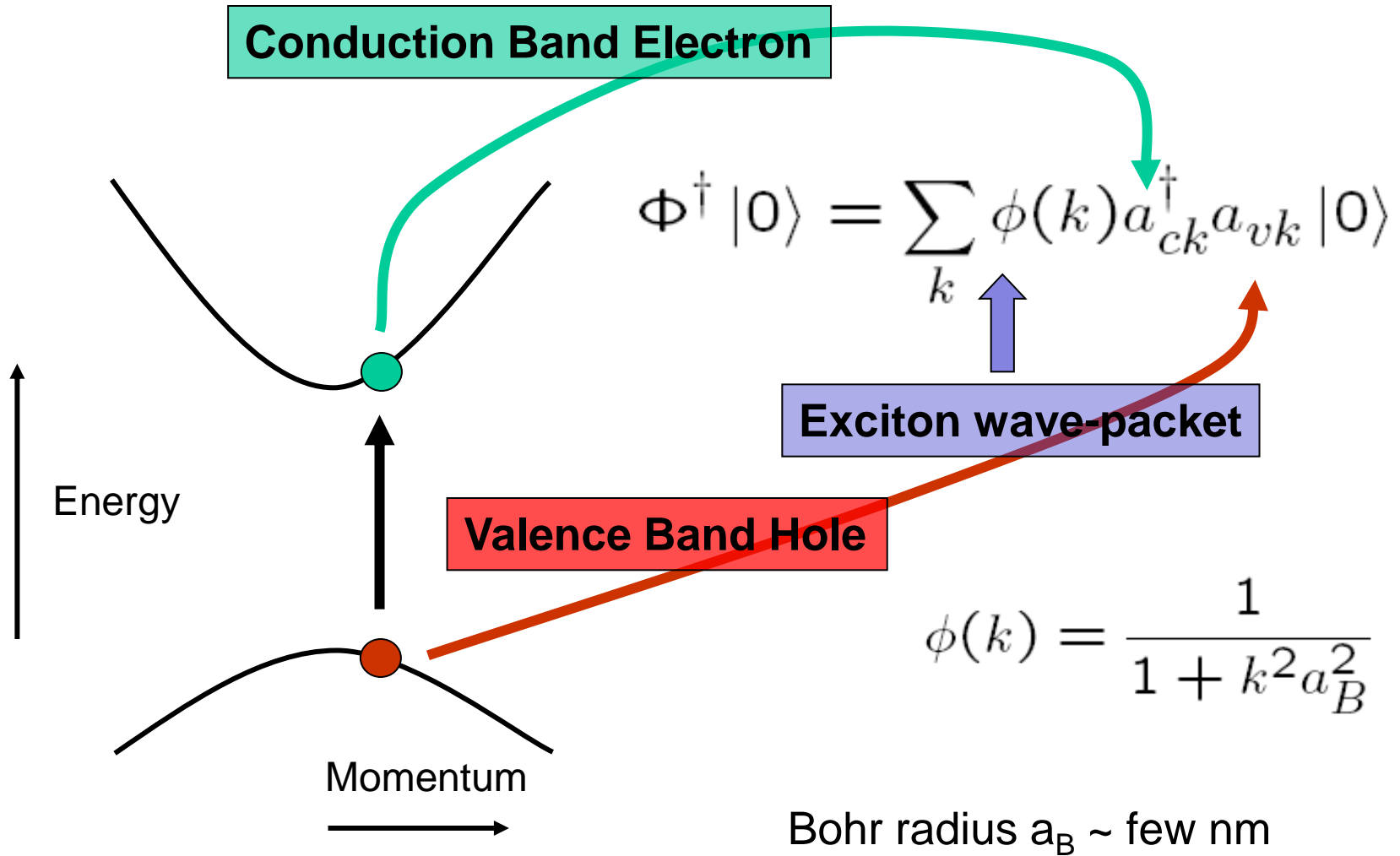
# What's new about a polariton condensate ?

- Composite particle – mixture of electron-hole pair and photon
  - How does this affect the ground state ?
- Extremely light mass ( $\sim 10^{-5} m_e$ ) means that polaritons are large, and overlap strongly even at low density
  - BEC – “BCS” crossover ?
- Two-dimensional physics
  - Berezinski-Kosterlitz-Thouless transition ?
- Polariton lifetime is short
  - Non-equilibrium, pumped dynamics
  - Decoherence ?
  - Relationship to the laser ?
- Can prepare out-of-equilibrium condensates
  - Quantum dynamics of many body system

# Bose Condensation of Composite Bosons

Review old picture of excitonic insulator  
Interacting polaritons and the Dicke model  
Analogues to other systems

# Excitons



# Mean field theory of excitonic insulator

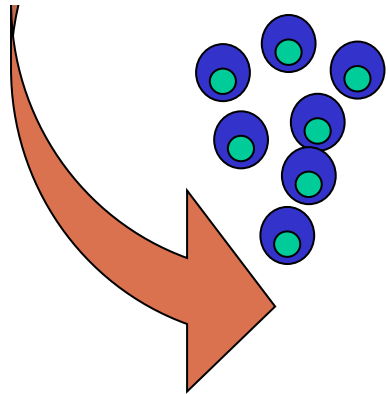
$$\Phi^\dagger |0\rangle = \sum_k \phi(k) a_{ck}^\dagger a_{vk} |0\rangle$$

Wavepacket of bound e-h pair  
Composite boson

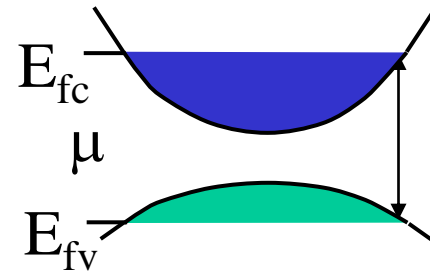
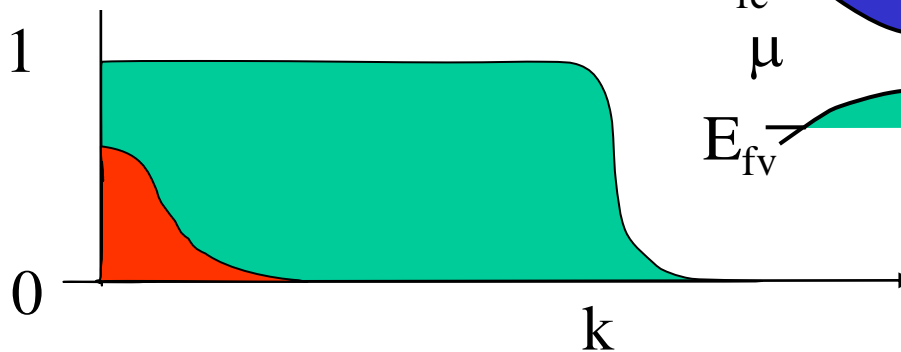
$$e^{\lambda\Phi^\dagger} |0\rangle = \prod_k [u(k) + v(k) a_{ck}^\dagger a_{vk}] |0\rangle$$

A coherent state – like a laser  
Bose condensation of excitons

BCS-like instability  
of Fermi surfaces



$$v(k) = \lambda\phi(k)$$



Special features: order parameter; gap

# Bose Condensation of Composite Bosons

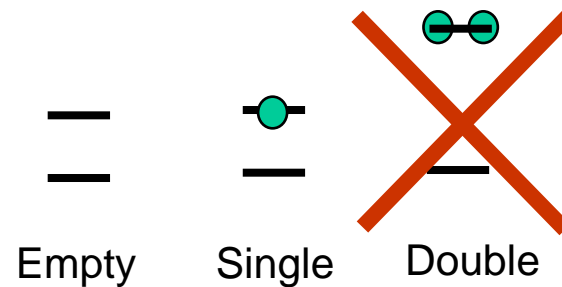
Interacting polaritons and the Dicke model

Because excitons are “heavy”, its a good enough approximation to treat them as localised two-level systems (i.e bosons with a repulsive interaction)

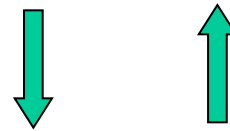
Photon component is “light” and mediates long range coupling

# Polaritons and the Dicke Model – a.k.a Jaynes-Tavis-Cummings model

Localised excitons behave like spins



Spins are flipped by absorption/emission of photon



$$H = \omega \psi^\dagger \psi + \sum_i \epsilon_i S_i^z + \frac{g}{\sqrt{N}} \sum_i [S_i^+ \psi + \psi^\dagger S_i^-]$$

$$N \sim [(\text{photon wavelength})/(\text{exciton radius})]^d \gg 1$$

Mean field theory – i.e. BCS coherent state – expected to be good approximation

$$|\lambda, w_i\rangle = \exp \left[ \lambda \psi^\dagger + \sum_i w_i S_i^+ \right] |0\rangle \quad T_c \approx g \exp(-1/gN(0))$$

Transition temperature depends on coupling constant

## Two metronomes on a cart

- Modelocking – the Huygens experiment of 1665





# Localized excitons in a microcavity - the Dicke model

- Simplifications
  - Single cavity mode
  - Equilibrium enforced by not allowing excitations to escape
  - Thermal equilibrium assumed (at finite excitation)
  - No exciton collisions or ionisation (OK for dilute, disordered systems)
    - Work in k-space, with Coulomb added - then solution is extension of Keldysh mean field theory (used by Schmitt-Rink and Chemla for driven systems)
    - Important issues are not to do with localisation/delocalisation or binding/unbinding of e-h pairs but with **decoherence**
- Important physics
  - Fermionic structure for excitons (saturation; phase-space filling)
  - Strong coupling limit of excitons with light
- To be added later
  - Decoherence (phase-breaking, pairbreaking) processes
  - Non-equilibrium (pumping and decay)

# Mean field solution: 1. Variational method

$$H = \omega \psi^\dagger \psi + \sum_i \epsilon_i S_i^z + \frac{g}{\sqrt{N}} \sum_i [S_i^+ \psi + \psi^\dagger S_i^-]$$

Variational wavefunction  $|\lambda, w_i\rangle = \exp[\lambda \psi^\dagger + \sum_i w_i S_i^+] |0\rangle$

Minimise Free Energy  $\frac{\partial}{d\lambda}, \frac{\partial}{dw_i} \langle \lambda, w_i | \hat{H} - \mu \hat{N} | \lambda, w_i \rangle = 0$

Order parameter equation  $(\omega - \mu)\lambda = \frac{g^2 \lambda}{2N} \sum_i \frac{1}{|E_i|}$

Gap in spectrum  $E_i = [(\epsilon_i - \mu)^2 + g^2 \lambda^2]^{1/2}$

Excitation density from both photons and excitons  $\rho_{ex} = \lambda^2 - \frac{1}{2N} \sum_i \frac{\epsilon_i - \mu}{|E_i|}$

## Mean field solution: 2. Equation of motion

Conventional 2-level system notation

$$\langle S_z \rangle = n, \langle S^+ \rangle = p, \langle \psi \rangle = \psi.$$

Semiclassical approximation to  
Heisenberg equations of motion for two  
level system in time dependent field

$$\begin{aligned} \frac{dp}{dt} &= -i\epsilon p - ig\psi n \\ \frac{dn}{dt} &= 2ig(\psi^* p - p^* \psi) \\ \frac{d\psi}{dt} &= -i\omega\psi - igp \end{aligned}$$

Rotating frame  $\psi(t) = \psi e^{-i\mu t}, p(t) = p e^{-i\mu t}$

Conservation: solution lies on Bloch sphere  $n^2 + 4|p|^2 = 1$

$$\left. \begin{aligned} \frac{d\psi}{dt} &= -i\omega\psi - igp \rightarrow \psi = p/(\mu - \omega) \\ \frac{dp}{dt} &= -i\epsilon p - ig\psi n \rightarrow (\mu - \epsilon)p = g\psi n \end{aligned} \right\} \rightarrow p = g\psi / \sqrt{(\mu - \epsilon)^2 + 4g^2\psi^2}$$

$$\psi = \frac{\text{sign}(\epsilon - \mu)}{\omega - \mu} \frac{g^2\psi}{\sqrt{(\mu - \epsilon)^2 + 4g^2|\psi|^2}}$$

$$\psi = \frac{g^2\psi}{\omega - \mu} \sum_j \frac{\text{sign}(\epsilon_j - \mu)}{\sqrt{(\mu - \epsilon_j)^2 + 4g^2|\psi|^2}}$$

## Mean field solution: 3. Functional field theory

Generalise to multiple modes in area A

$$H = \sum_{j=1}^{j=nA} 2\epsilon_j S_j^z + \sum_{k=2\pi l/\sqrt{A}} \hbar\omega_k \psi_k^\dagger \psi_k + \frac{g}{\sqrt{A}} \sum_{j,k} (e^{2\pi i k \cdot r_j} \psi_k S_j^+ + e^{-2\pi i k \cdot r_j} \psi_k^\dagger S_j^-)$$

Construct coherent state path integral and integrate out spins

$$S[\psi] = \int_0^\beta d\tau \sum_k \psi_k^\dagger (\partial_\tau + \hbar\tilde{\omega}_k) \psi_k + N \text{Tr} \ln(\mathcal{M}) \quad \mathcal{M}^{-1} = \begin{pmatrix} \partial_\tau + \tilde{\epsilon} & \frac{g}{\sqrt{A}} \sum_k e^{2\pi i k \cdot r_j} \psi_k \\ \frac{g}{\sqrt{A}} \sum_k e^{2\pi i k \cdot r_j} \psi_k^\dagger & \partial_\tau + \tilde{\epsilon} \end{pmatrix}$$

Minimise action around stationary uniform saddle point

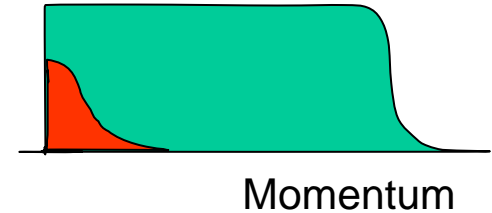
$$\hbar\tilde{\omega}_0 \psi_0 = g^2 n \frac{\tanh(\beta E)}{2E} \psi_0, \quad E = \sqrt{\tilde{\epsilon}^2 + g^2 \frac{|\psi_0|^2}{A}}$$

$$\rho_{\text{M.F.}} = \frac{|\psi_0|^2}{A} + \frac{n}{2} \left[ 1 - \frac{\tilde{\epsilon}}{E} \tanh(\beta E) \right]$$

## Dictionary of broken symmetries

- Connection to excitonic insulator generalises the BEC concept – different guises  
Occ.

$$e^{\lambda \sum_k \phi_k a_{ck}^\dagger a_{vk}} = \prod_k \left[ 1 + \lambda \phi_k a_{ck}^\dagger a_{vk} \right]$$



- Rewrite as spin model

$$S_i^+ = a_{ci}^\dagger a_{vi} \quad ; \quad S_i^z = a_{ci}^\dagger a_{ci} - a_{vi}^\dagger a_{vi}$$

- XY Ferromagnet / Quantum Hall bilayer / BaCuSiO

$$|w_i\rangle = e^{\sum_i w_i S_i^+} |0\rangle$$

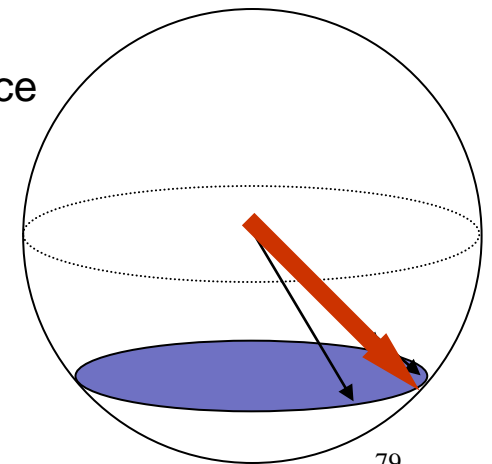
Dynamics – precession in self-consistent field

- Couple to an additional Boson mode:

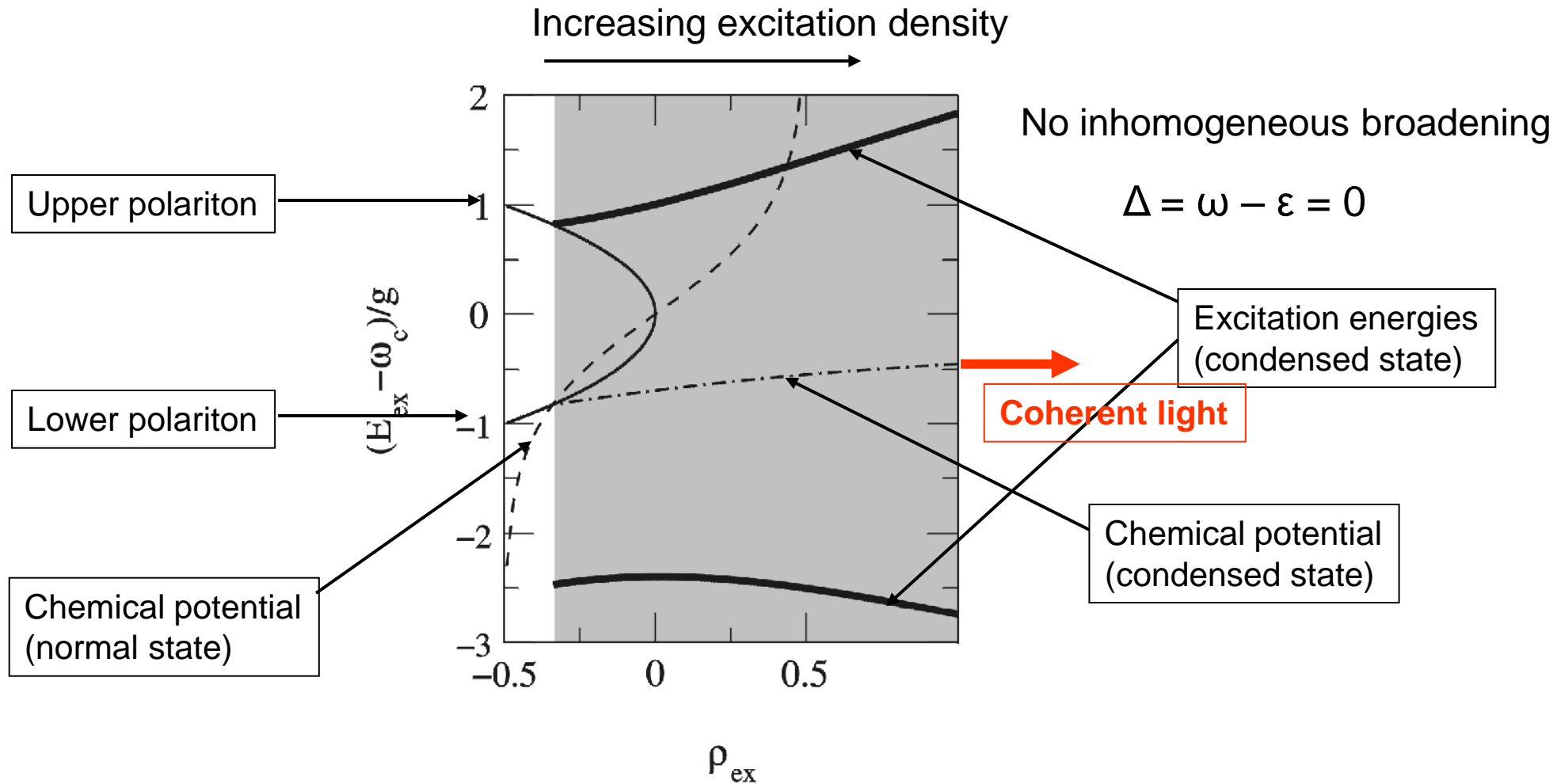
photons -> polaritons;

molecules -> cold fermionic atoms near Feshbach resonance

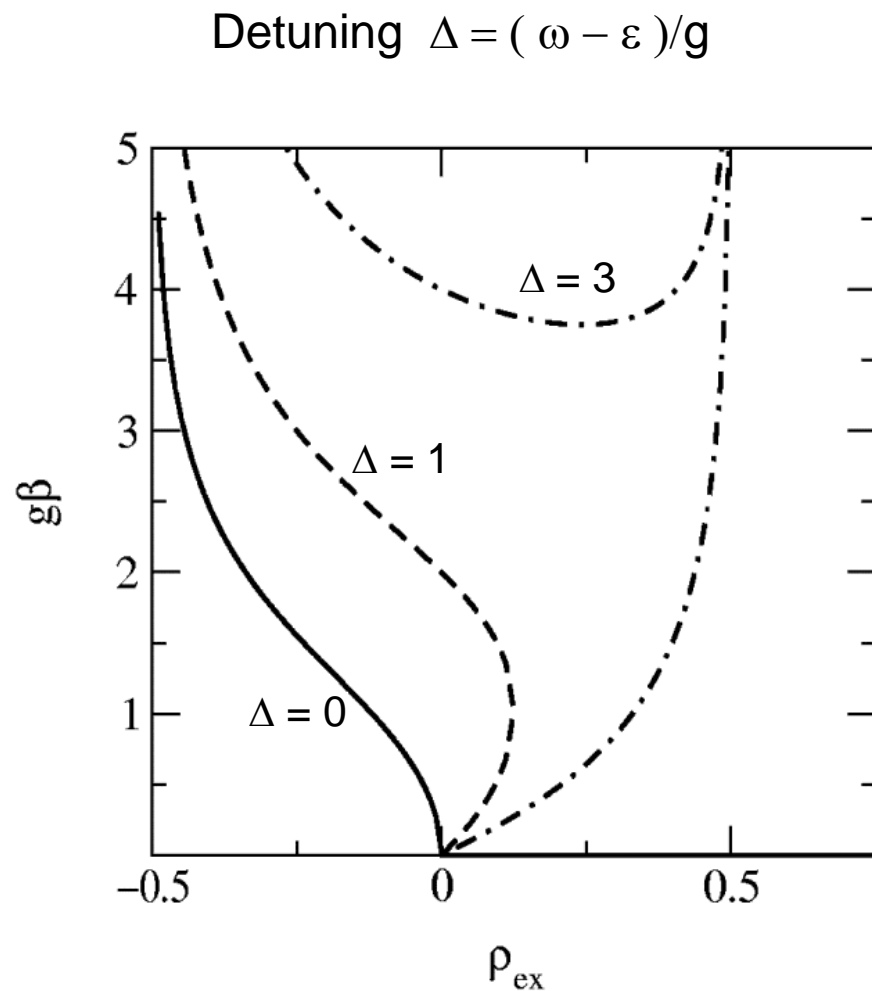
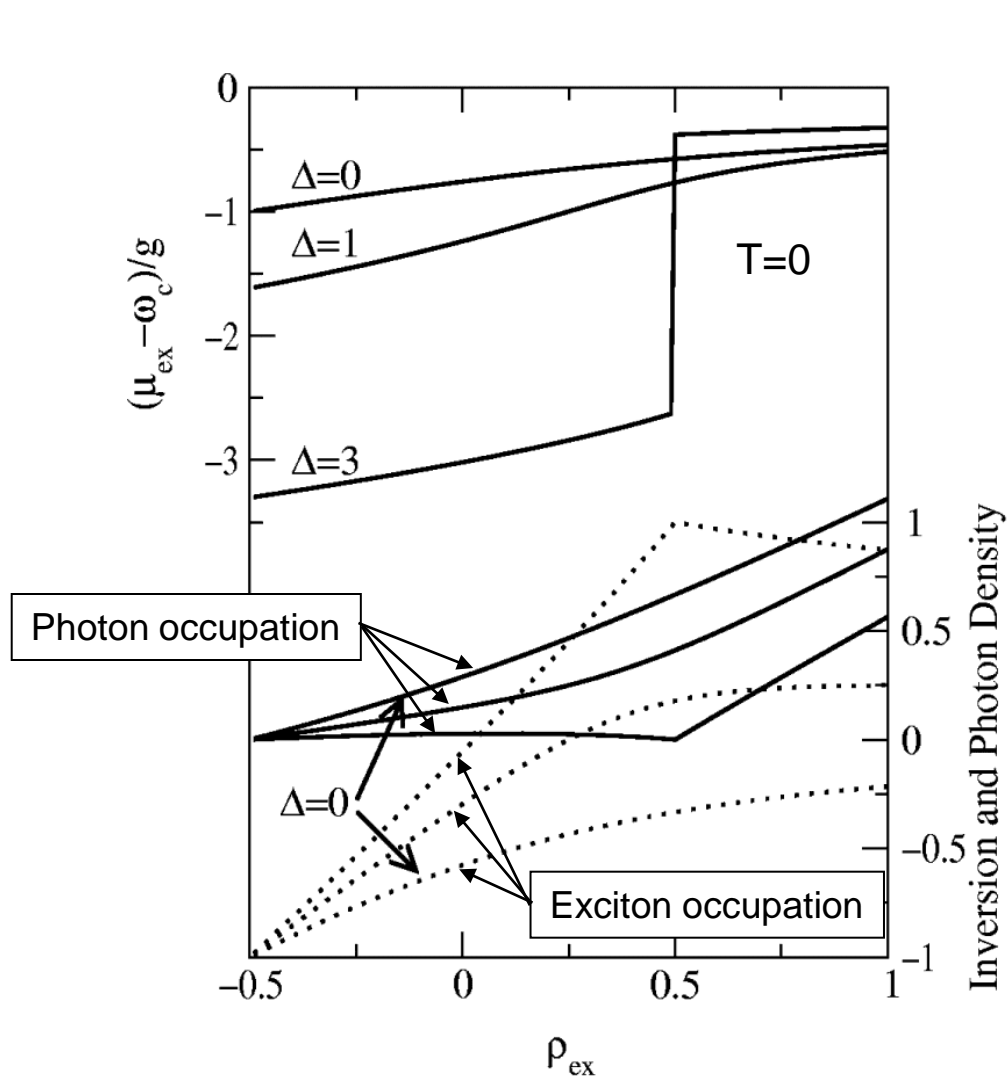
$$|\lambda, w_i\rangle = \exp[\lambda \psi^\dagger + \sum_i w_i S_i^+] |0\rangle$$



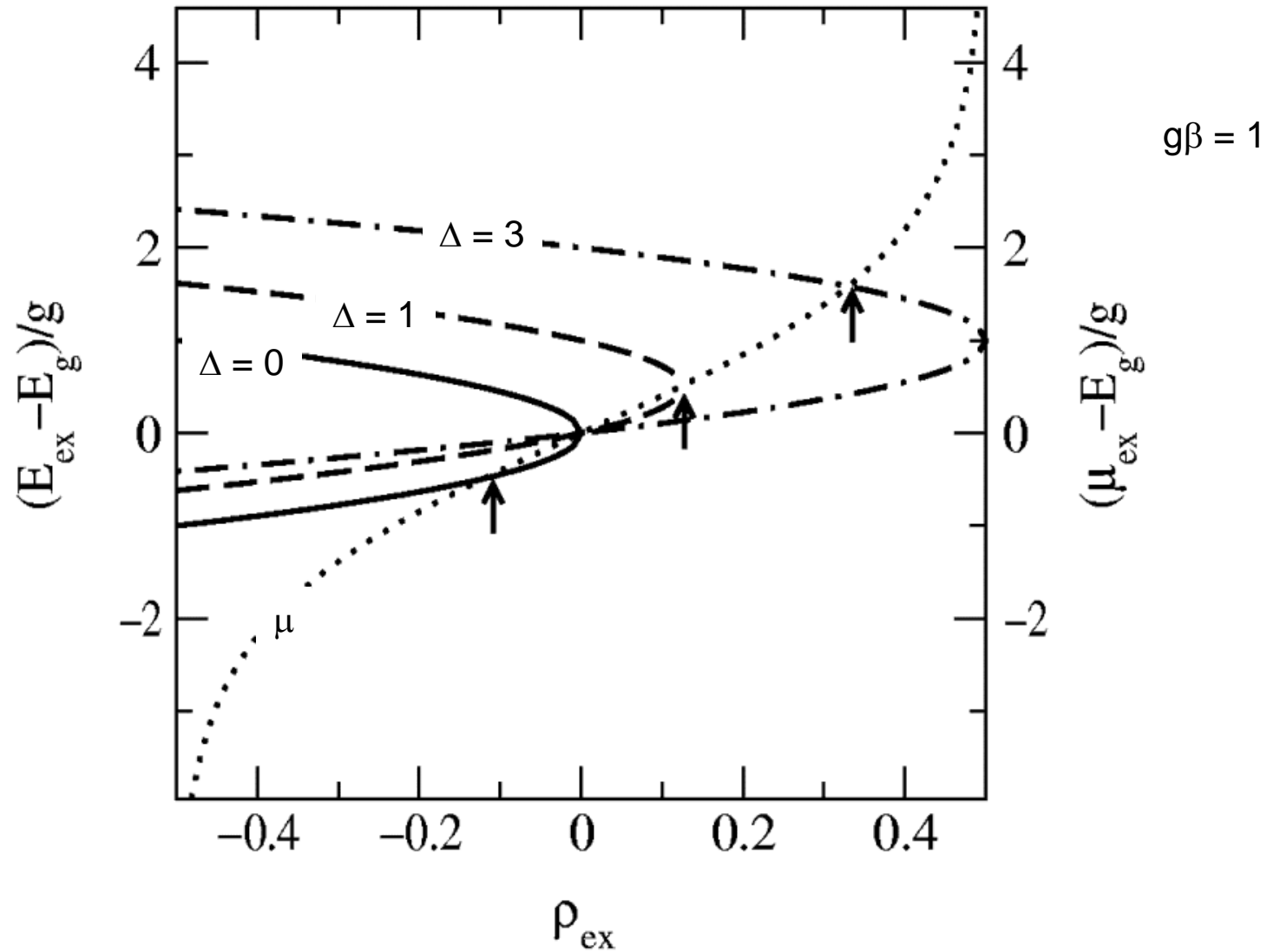
# Condensation in the Dicke model ( $g/T = 2$ )



# Phase diagram



# Excitation spectrum for different detunings

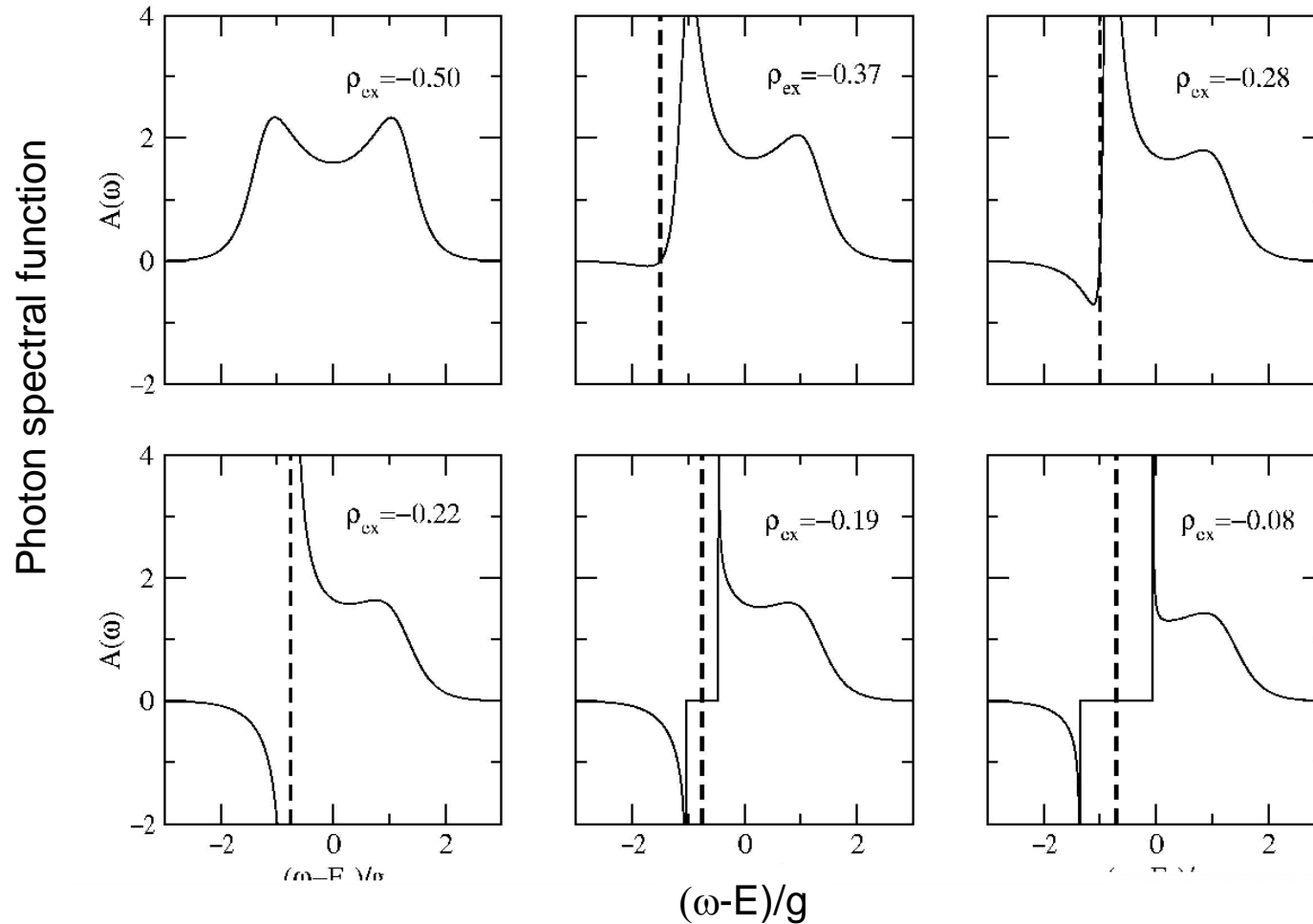




# Excitation spectrum with inhomogeneous broadening

Zero detuning:  $\omega = \varepsilon$

Gaussian broadening of exciton energies  $\sigma = 0.5 g$



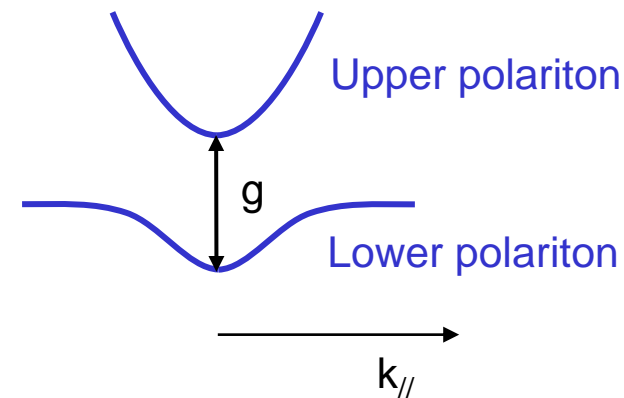
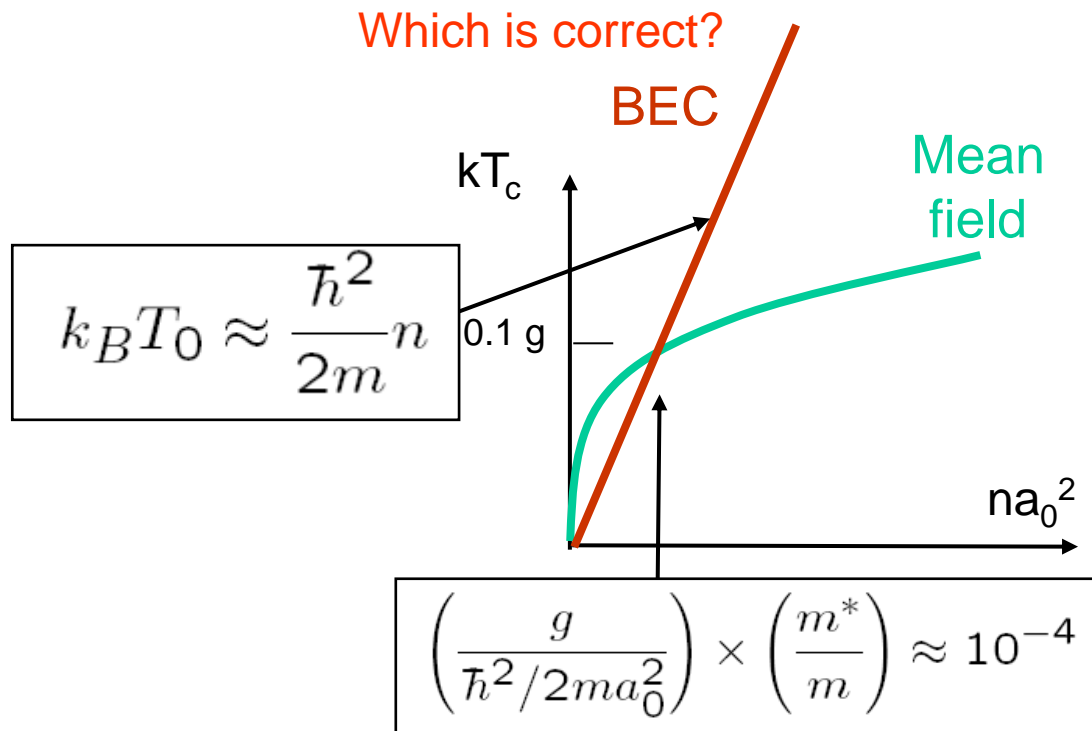
# Interaction dominated physics or dilute Bose gas ?

Mean field theory – coherent state – is BCS  
“BCS” to Bose crossover

# Beyond mean field: Interaction driven or dilute gas?

- Conventional “BEC of polaritons” will give high transition temperature because of light mass  $m^*$
- Single mode Dicke model gives transition temperature  $\sim g$

Which is correct?



$a_0$  = characteristic separation of excitons  
 $a_0 >$  Bohr radius

Dilute gas BEC only for excitation levels  $< 10^9 \text{ cm}^{-2}$  or so

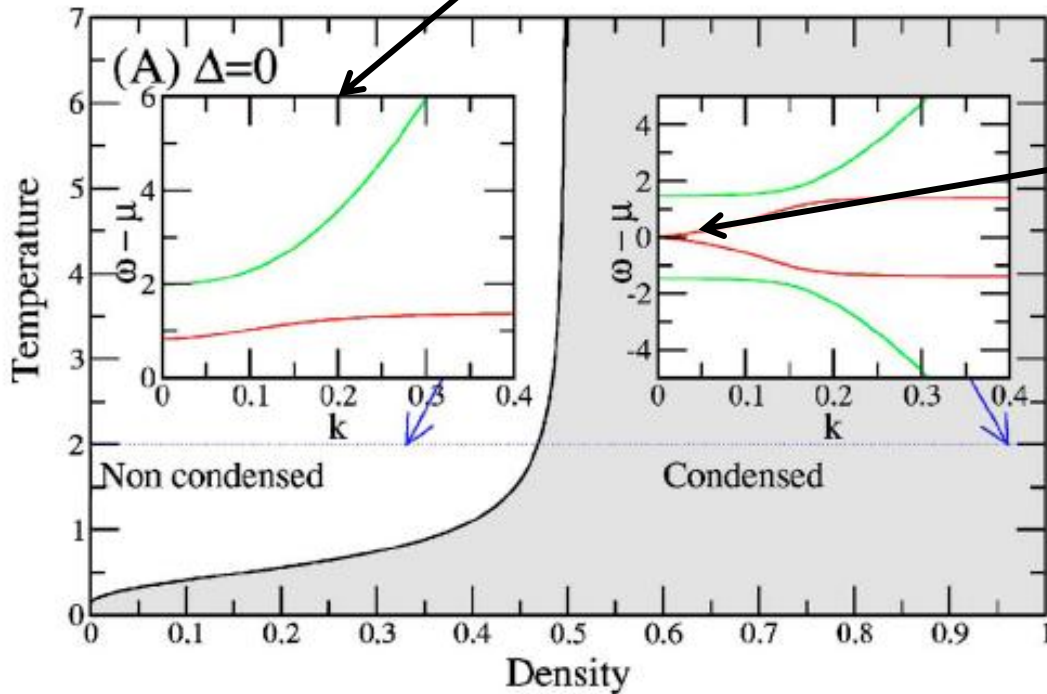
A further crossover to the plasma regime when  $na_B^2 \sim 1$

# Fluctuation spectrum

Expand action in quadratic fluctuations around mean field solutions, and diagonalise to determine the new collective modes.

$$E_{\pm} = \frac{1}{2} [(\hbar\tilde{\omega}_k + 2\tilde{\epsilon}) \pm \sqrt{(\hbar\tilde{\omega}_k - 2\tilde{\epsilon})^2 + 4g^2n \tanh(\beta\tilde{\epsilon})}].$$

Conventional upper and lower polaritons



Bogoliubov sound mode

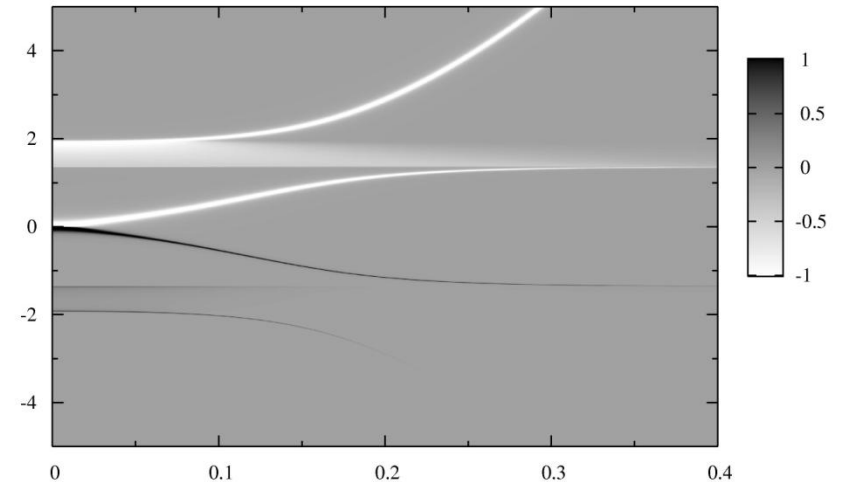
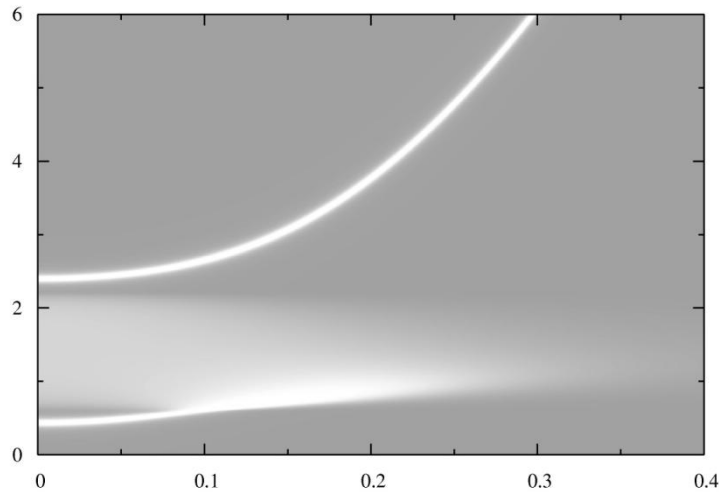
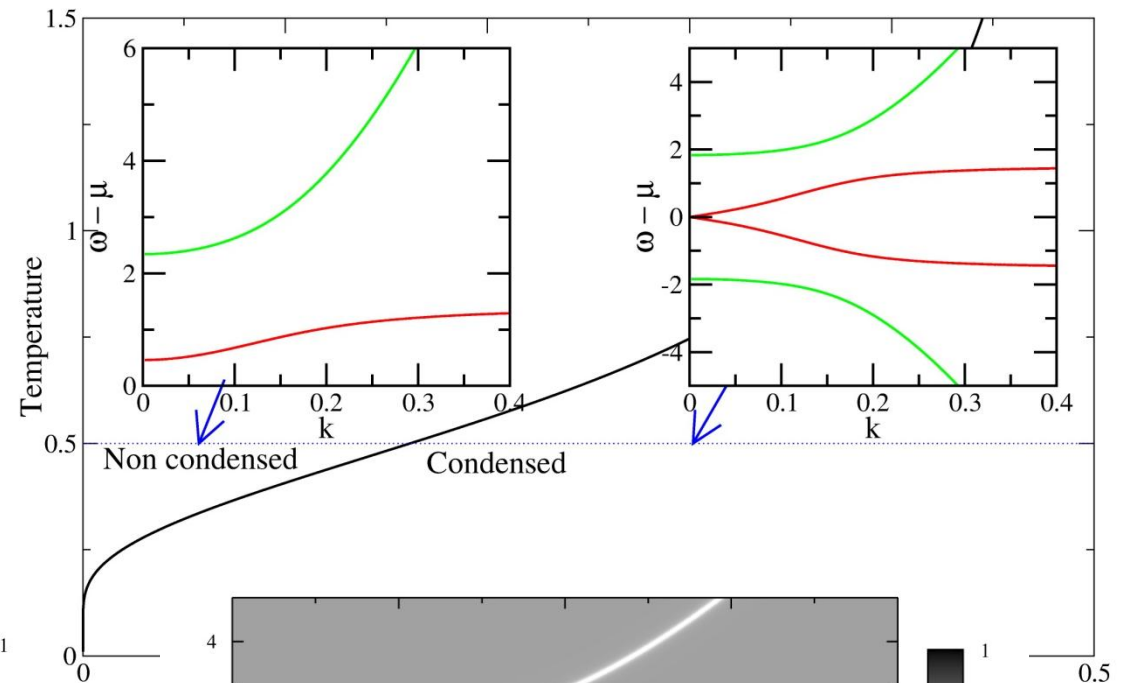
$$\xi_1 = \pm \hbar ck$$

$$c = \sqrt{\frac{1}{2m} \left( \frac{4\hbar\tilde{\omega}_0 g^2 n}{\xi_2(0)^2} \right) \left( \frac{|\psi_0|^2}{N} \right)} \approx \sqrt{\frac{\lambda}{2m} \frac{\rho_0}{n}}$$

## 2D polariton spectrum

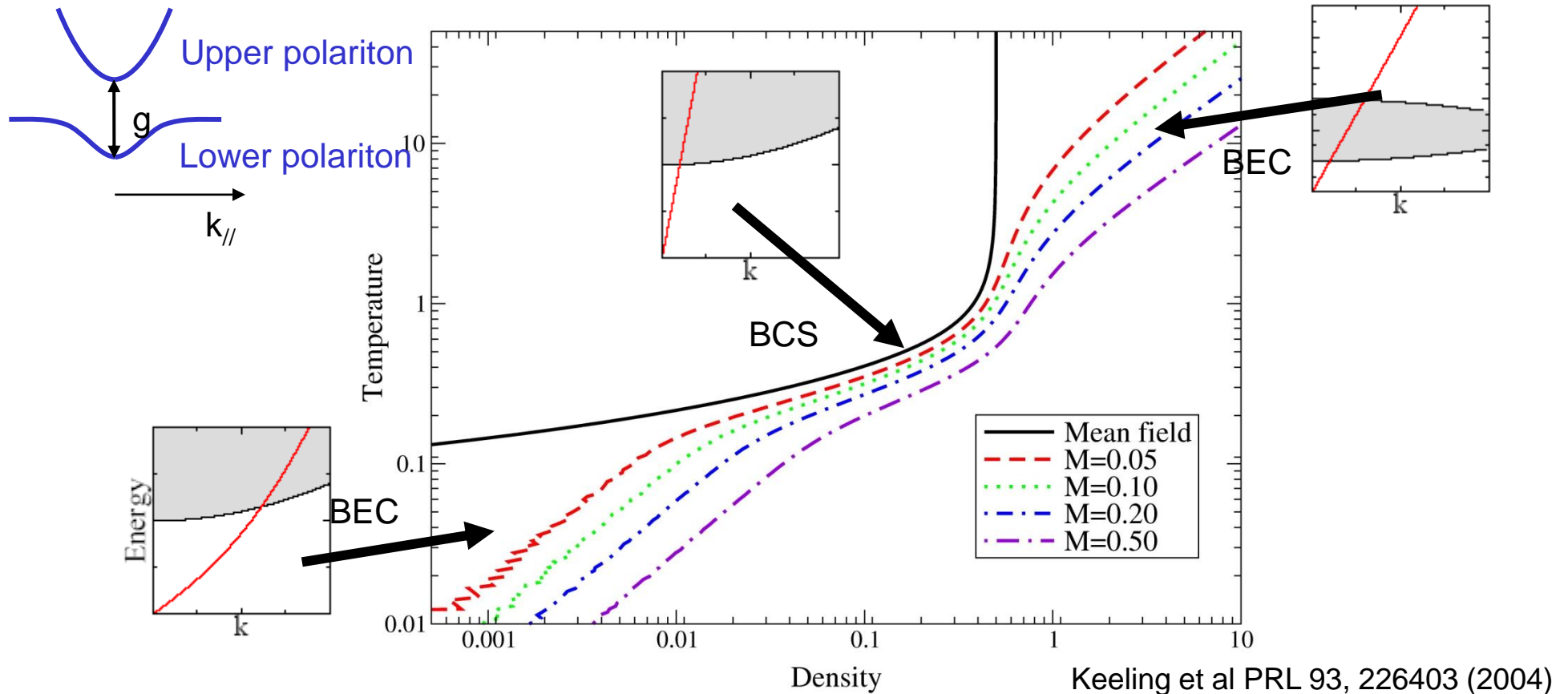
Keeling et al PRL 93, 226403 (2004)

- Excitation spectrum calculated at mean field level
- Thermally populate this spectrum to estimate suppression of superfluid density (one loop)
- Estimate new  $T_c$



## Phase diagram

- $T_c$  suppressed in low density (polariton BEC) regime and high density (renormalised photon BEC) regimes
- For typical experimental polariton mass  $\sim 10^{-5}$  deviation from mean field is small



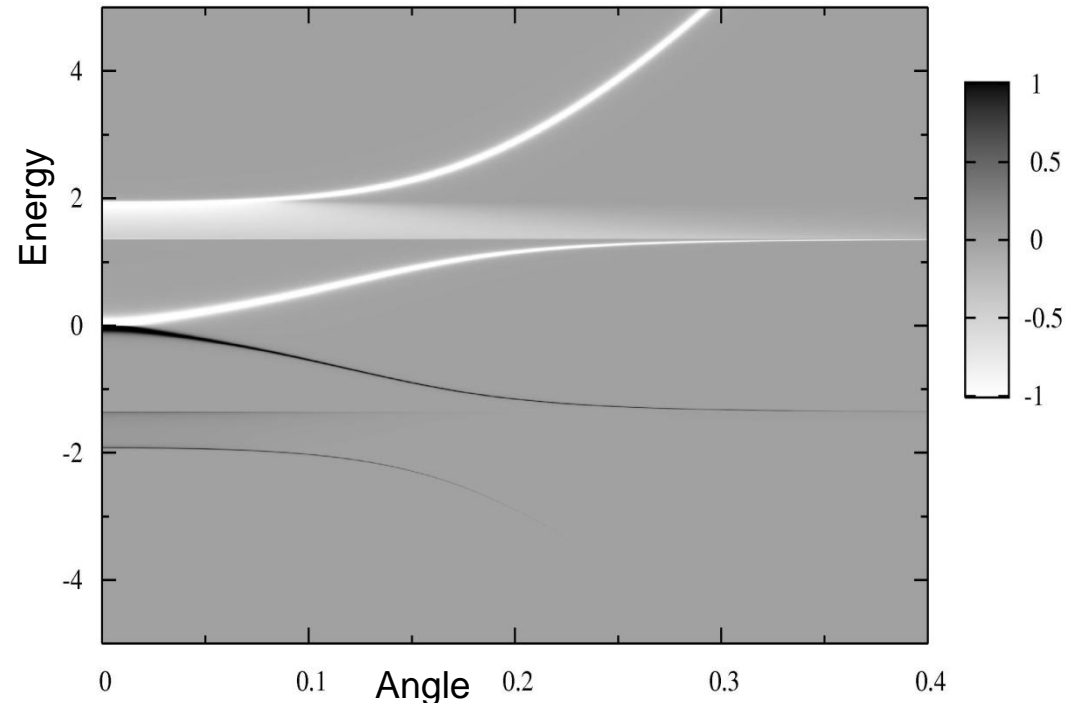
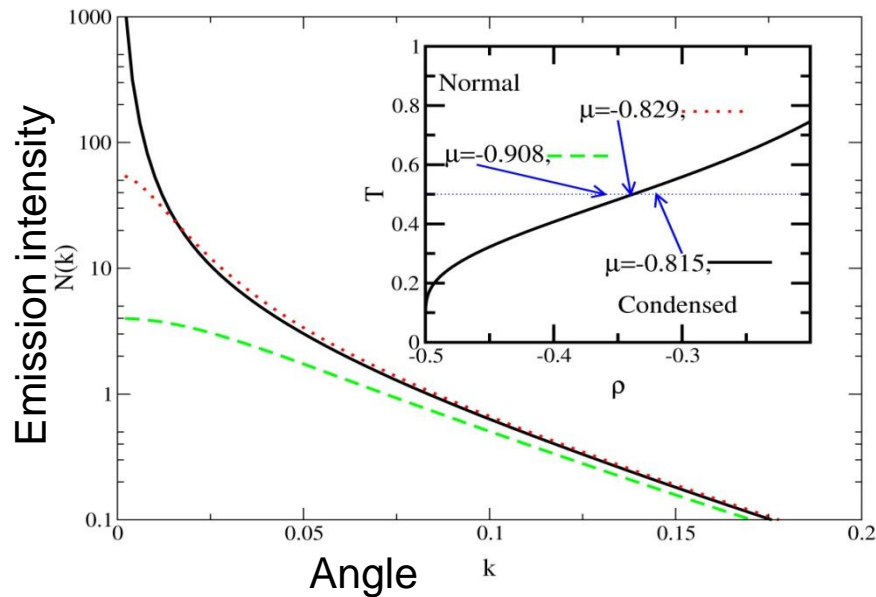
# 2D physics

No long range order at finite temperatures  
Berezinskii-Kosterlitz-Thouless transition

# Excitation spectra in microcavities with coherence

Keeling, Eastham, Szymanska, PBL PRL 2004

Angular dependence of luminescence becomes sharply peaked at small angles  
(No long-range order because a 2D system)



Absorption(white) / Gain(black) spectrum of coherent cavity



# Decoherence

Despite large  $Q$  of cavity, lifetime is only a few psec  
Even if a thermal distribution can be obtained, the system is  
non-equilibrium  
Particle fluxes produce decoherence

## Conventional theory of the laser

$$\begin{aligned} b^\dagger b - a^\dagger a &= S^z \\ b^\dagger a &= S^+ \\ a^\dagger b &= S^- \\ b^\dagger b + a^\dagger a &= 1 \end{aligned}$$

$$H = H_0 + H_{SB} + H_B$$

$$H_0 = \sum_i \epsilon_i (b_i^\dagger b_i - a_i^\dagger a_i) + \omega_c \psi^\dagger \psi + \frac{g}{\sqrt{N}} \sum_i [\psi^\dagger a_i^\dagger b_i + h.c.] \quad \text{system}$$

$$H_B = \sum_k [\omega_k d_k^\dagger d_k + \omega_{+,k} c_{+,k}^\dagger c_{+,k} + \omega_{-,k} c_{-,k}^\dagger c_{-,k} + \omega_{1,k} c_{1,k}^\dagger c_{1,k} + \omega_{2,k} c_{2,k}^\dagger c_{2,k}] \quad \text{bosonic "baths"}$$

$$H_{SB} = \sum_k g_k (\psi^\dagger d_k + d_k^\dagger \psi) \quad \text{decay of cavity mode}$$

$$+ \sum_{jk} [b_j^\dagger a_j (g_{jk}^{\gamma+} c_{+,k}^\dagger + g_{jk}^{\gamma-} c_{-,k}) + h.c.] \quad \text{phase-breaking}$$

$$+ \sum_{jk} \Gamma_{jk}^{(1)} (b_j^\dagger b_j + a_j^\dagger a_j) (c_{1,k}^\dagger + c_{1,k}) \quad \text{pair-breaking}$$

$$+ \sum_{jk} \Gamma_{jk}^{(2)} (b_j^\dagger b_j - a_j^\dagger a_j) (c_{2,k}^\dagger + c_{2,k}) \quad \text{non-pair-breaking}$$

## From Heisenberg to Langevin equations of motion

$$\frac{d}{dt}\psi = -i\omega_c\psi - ig\sum_i a_i^\dagger b_i - i\sum_k g_k d_k$$

$$\frac{d}{dt}d_k = -i\omega_k d_k - ig_k \psi$$

$$d_k(t) = d_k(t_0)e^{-i\omega_k(t-t_0)} - g_k \int_{t_0}^t dt' \psi(t')e^{-i\omega_k(t-t')}$$

$$\frac{d}{dt}\psi = -i\omega_c\psi - ig\sum_i a_i^\dagger b_i - i\sum_k g_k d_k(t_0)e^{-i\omega_k(t-t_0)} - \sum_k g_k^2 \int_{t_0}^t dt' \psi(t')e^{-i\omega_k(t-t')}$$

$$\frac{d}{dt}\psi = (-i\omega_c - \kappa)\psi - ig\sum_i a_i^\dagger b_i + F(t)$$

Markov approximation

$$\frac{d}{dt}a_j^\dagger b_j = (-i\epsilon_j - \gamma_\perp)a_j^\dagger b_j + ig\psi(b_j^\dagger b_j - a_j^\dagger a_j) + \Gamma_{j-}$$

Polarisation  $S^+$

$$\frac{d}{dt}(b_j^\dagger b_j - a_j^\dagger a_j) = \gamma_{||}(d_0 - b_j^\dagger b_j + a_j^\dagger a_j) + 2ig(\psi^\dagger a_j^\dagger b_j - b_j^\dagger a_j \psi) + \Gamma_{j,d}$$

Inversion  $S^Z$

## From Langevin equations to mean field

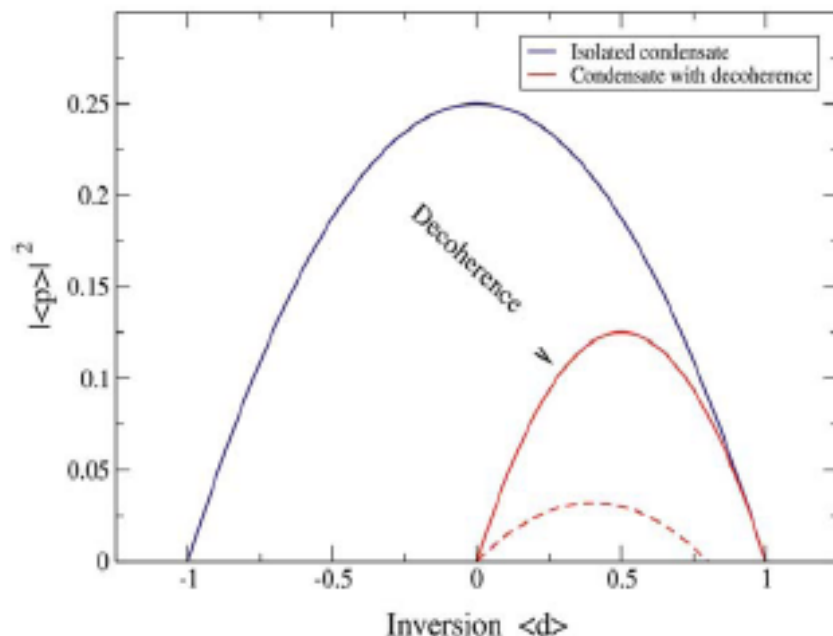
Bloch equations in a self-consistent field

$$\frac{d}{dt} \langle \psi \rangle = (-i\omega_c - \kappa) \langle \psi \rangle - ig \sum_j \langle S_j^- \rangle$$

$$\frac{d}{dt} \langle S_j^- \rangle = (-i\epsilon_j - \gamma_{\perp}) \langle S_j^- \rangle + ig \langle \psi \rangle \langle S_j^z \rangle$$

$$\frac{d}{dt} \langle S_j^z \rangle = \gamma_{\parallel} (d_o - \langle S_j^z \rangle) + 2ig (\langle \psi^\dagger \rangle \langle S_j^- \rangle - \langle S_j^+ \rangle \langle \psi \rangle)$$

If decay processes are turned off, solutions are identical to BCS mean field equations – but these are unstable to infinitesimal damping

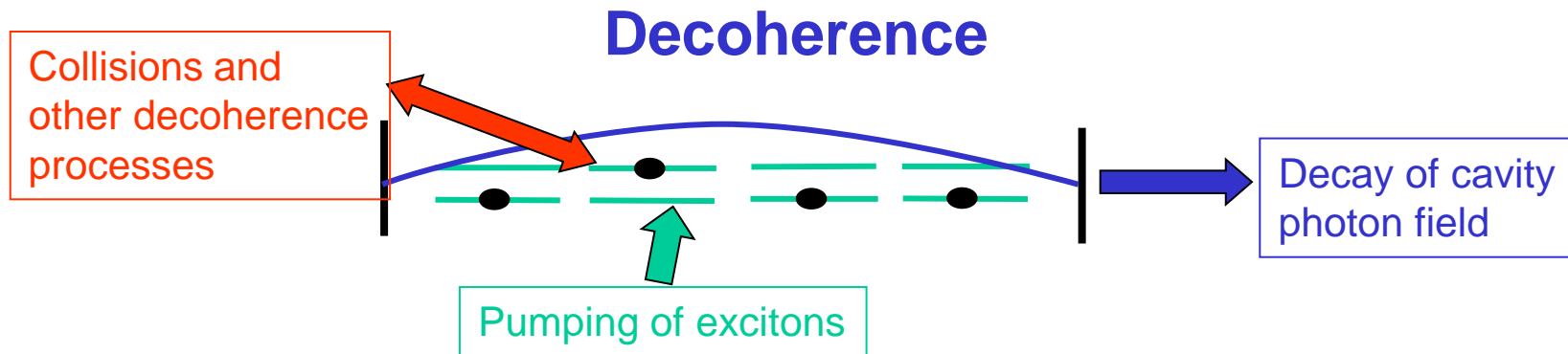


## Disaster ?

Apparently arbitrarily weak decoherence destroys coherent ground state unless system is inverted, so the only solution is that of a regular laser

... but the assumption was that the noise is Markovian – uncorrelated – whereas the noise is coupled via the spectrum of the correlated ground state

....which has a gap, and a stiffness



Decay, pumping, and collisions may introduce “decoherence” - loosely, lifetimes for the elementary excitations - include this by coupling to bosonic “baths” of other excitations

in analogy to superconductivity, the external fields may couple in a way that is “pair-breaking” or “non-pair-breaking”

$$\sum_{i,k} g_{i,k}^{(1)} [b_i^\dagger b_i - a_i^\dagger a_i] (c_{1,k}^\dagger + c_{1,k}) \quad \text{non-pairbreaking (inhomogeneous distribution of levels)}$$

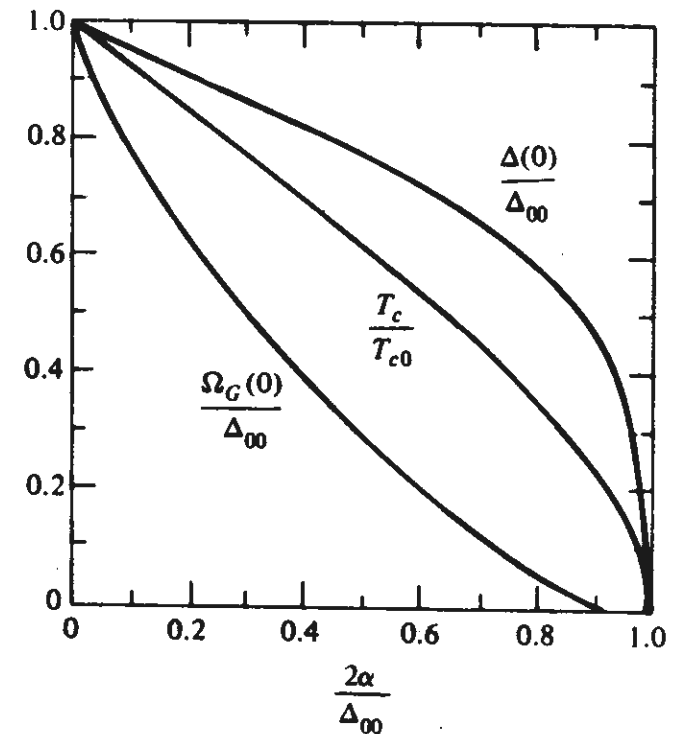
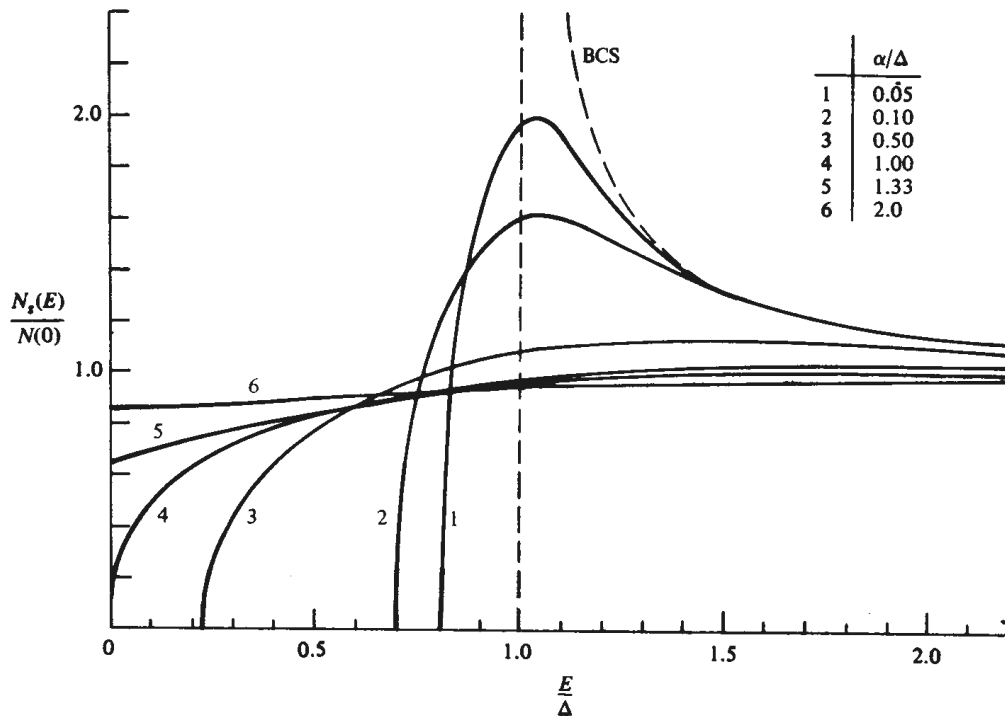
$$\sum_{i,k} g_{i,k}^{(2)} [b_i^\dagger b_i + a_i^\dagger a_i] (c_{2,k}^\dagger + c_{2,k}) \quad \text{pairbreaking disorder}$$

- Conventional theory of the laser assumes that the external fields give rise to rapid decay of the excitonic polarisation - **incorrect if the exciton and photon are strongly coupled**
- Correct theory is familiar from superconductivity - Abrikosov-Gorkov theory of superconductors with magnetic impurities

$$\sum_{i,k} g_{i,k}^{(3)} [b_i^\dagger a_i c_{3,k}^\dagger + a_i^\dagger b_i c_{3,k}] \quad \text{symmetry breaking – XY random field destroys LRO}$$

# Detour - Abrikosov-Gorkov theory of gapless superconductivity

- Ordinary impurities that do not break time reversal symmetry are “irrelevant”. Construct pairing between degenerate time-reversed pairs of states (Anderson’s theorem)
- Fields that break time reversal (e.g. magnetic impurities, spin fluctuations) suppress singlet pairing, leading first to gaplessness, then to destruction of superconductivity  
[Abrikosov & Gorkov ZETF 39, 1781 (1960); JETP 12, 12243 (1961)]



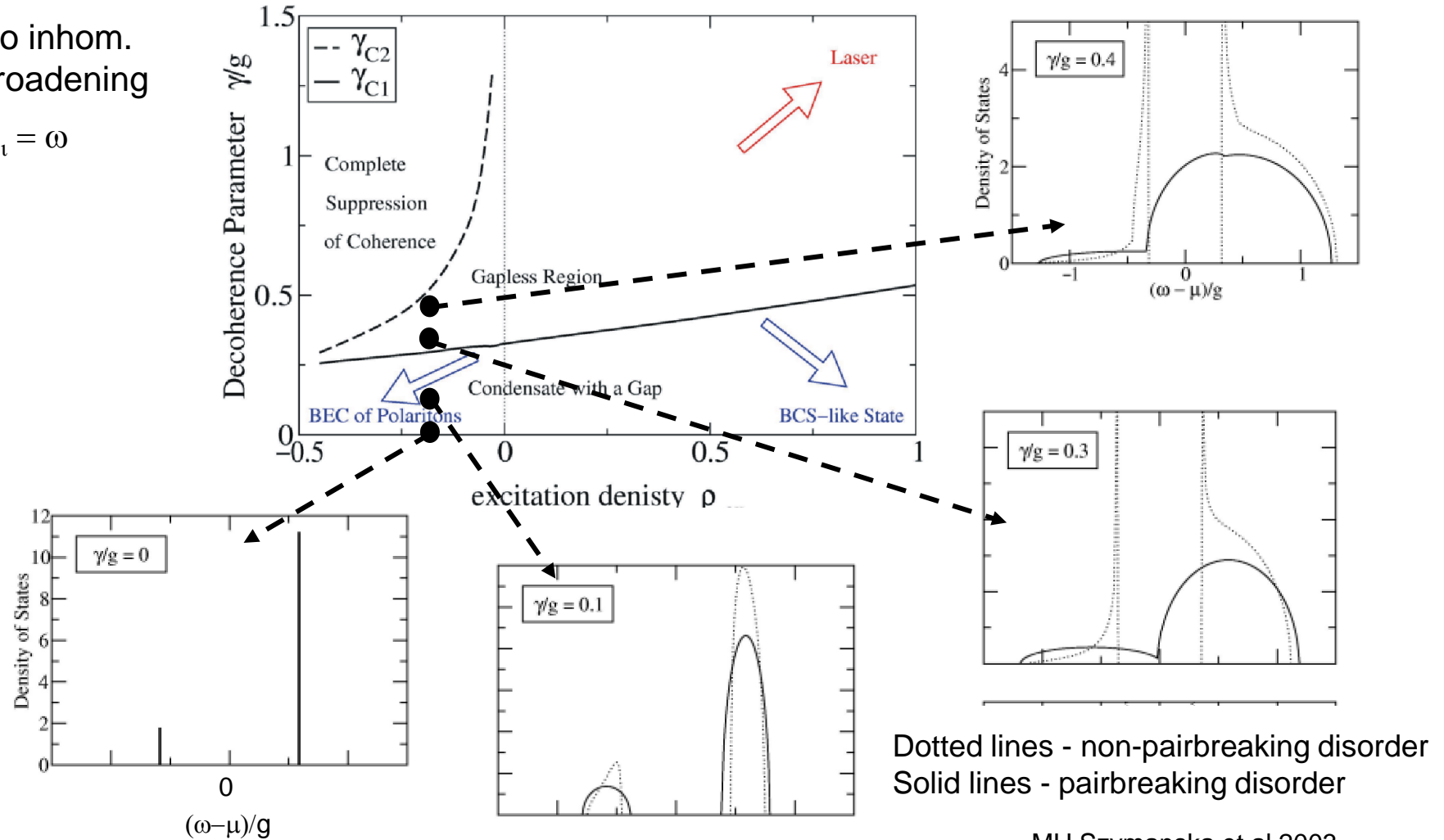
Skalski et al, PR136, A1500 (1964)

# Phase diagram of Dicke model with pairbreaking

Pairbreaking characterised by a single parameter  $\gamma = \lambda^2 N(0)$

No inhom. broadening

$$\varepsilon_l = \omega$$



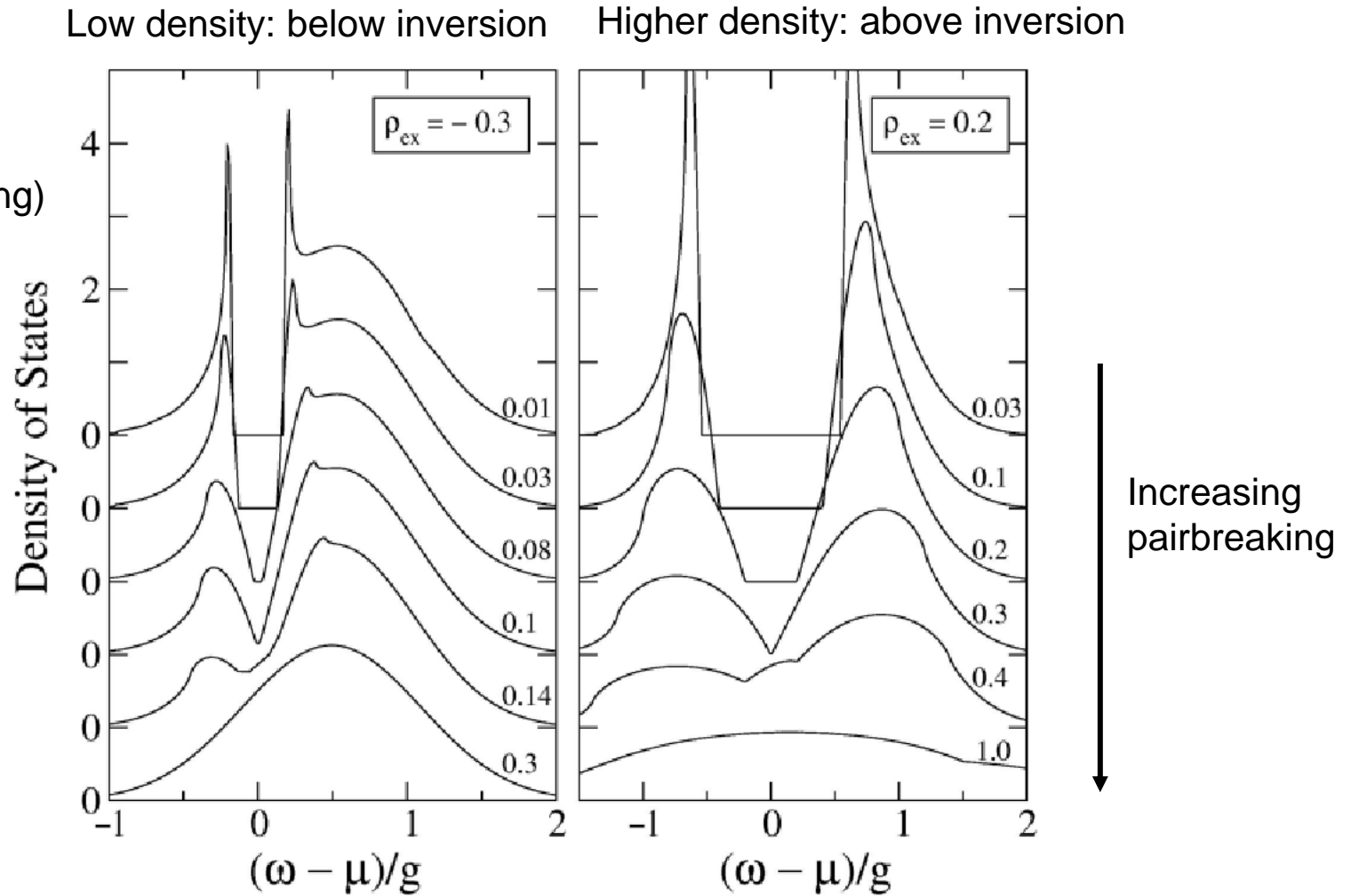
Dotted lines - non-pairbreaking disorder  
Solid lines - pairbreaking disorder

MH Szymanska et al 2003

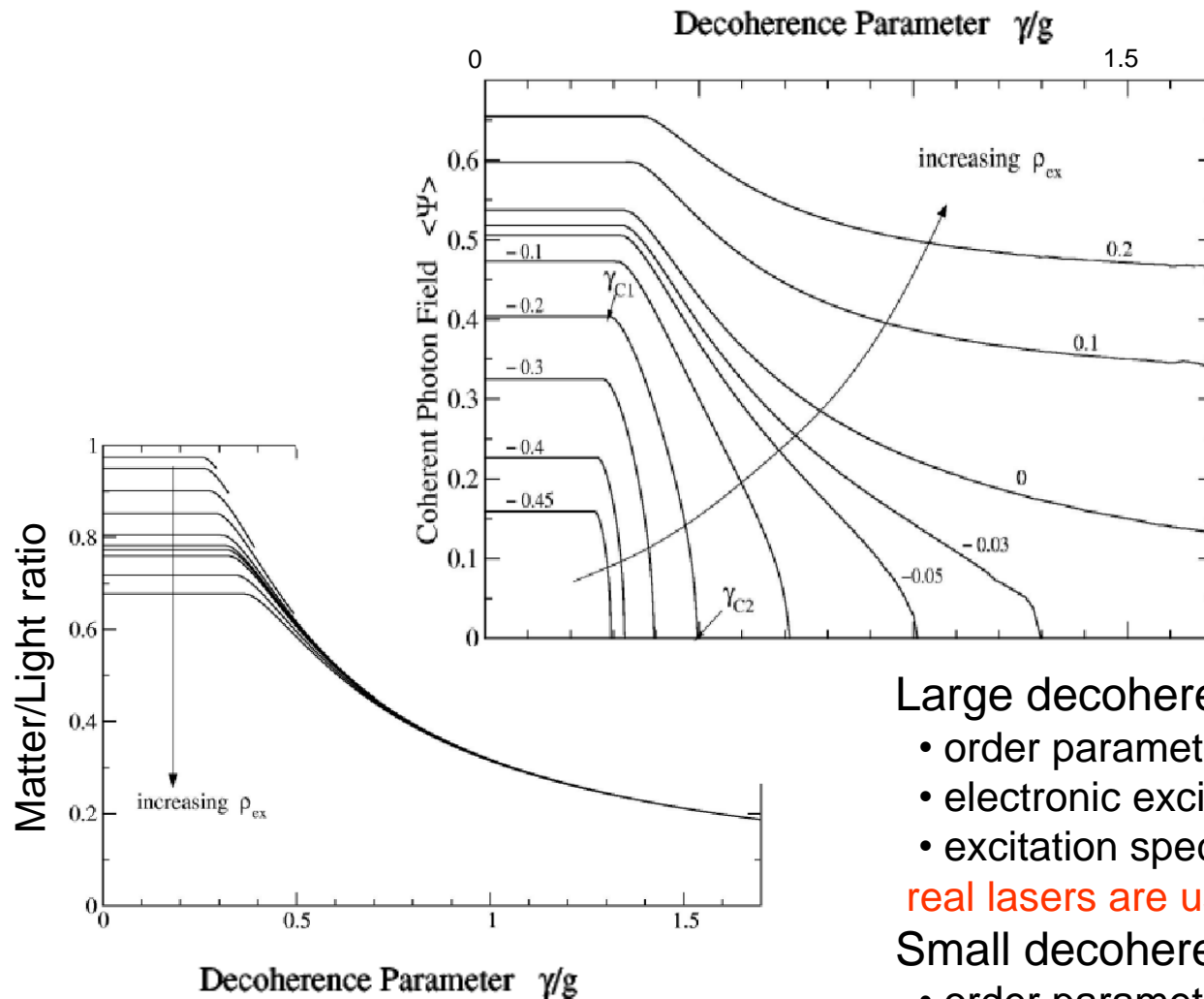


# Transition to gaplessness and lasing

Model with both pairbreaking and non-pairbreaking (inhom. broadening)



# Strong pairbreaking -> semiconductor laser



Large decoherence -- “laser”

- order parameter nearly photon like
- electronic excitations have short lifetime
- excitation spectrum gapless

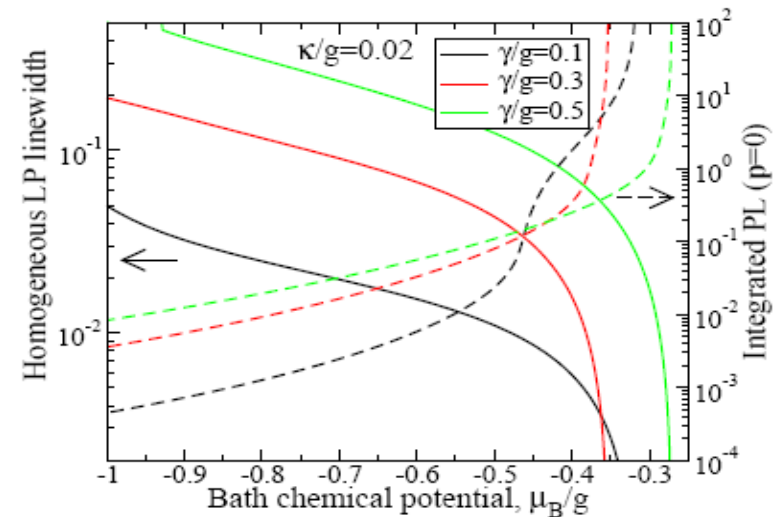
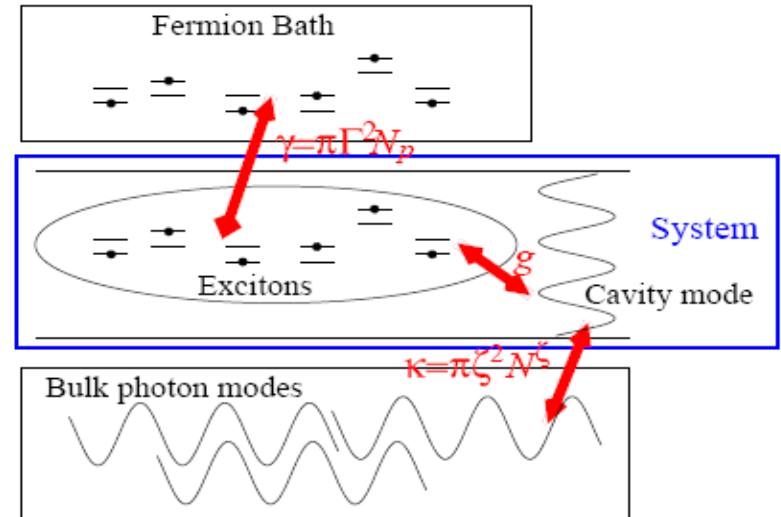
real lasers are usually far from equilibrium

Small decoherence -- BEC of polaritons

- order parameter mixed exciton/photon
- excitation spectrum has a gap

## Model for coupling to external baths

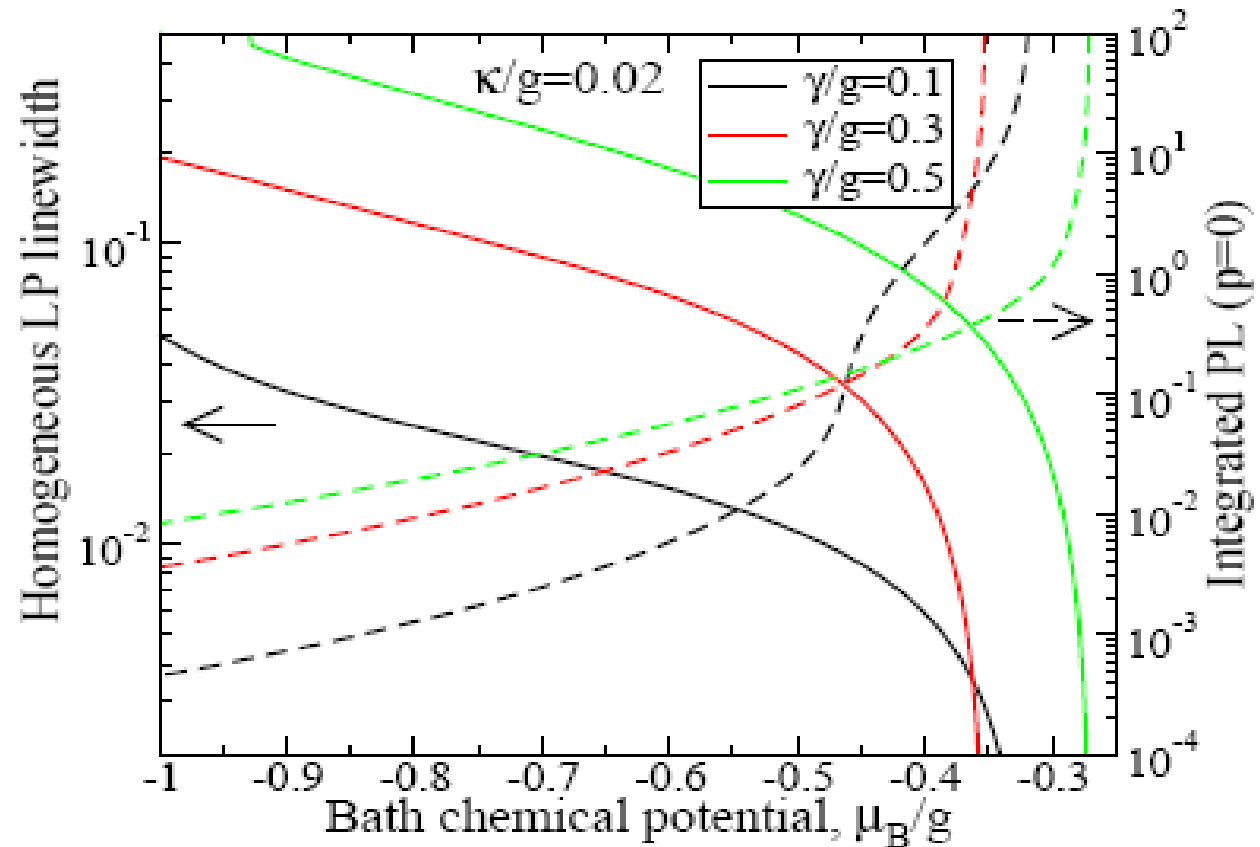
- Couple cavity photons to bulk photon modes (outside the cavity) – damping parameter  $\kappa$
- Couple excitons to baths of free fermions (electrons and holes) – damping parameter  $\gamma$
- Fermions kept at a chemical potential  $\mu_{\text{ext}}$
- For fixed parameters, system reaches steady state equilibrium – increase occupation of the system by changing the external chemical potential
- Mathematically, construct non-equilibrium formulation using Keldysh method



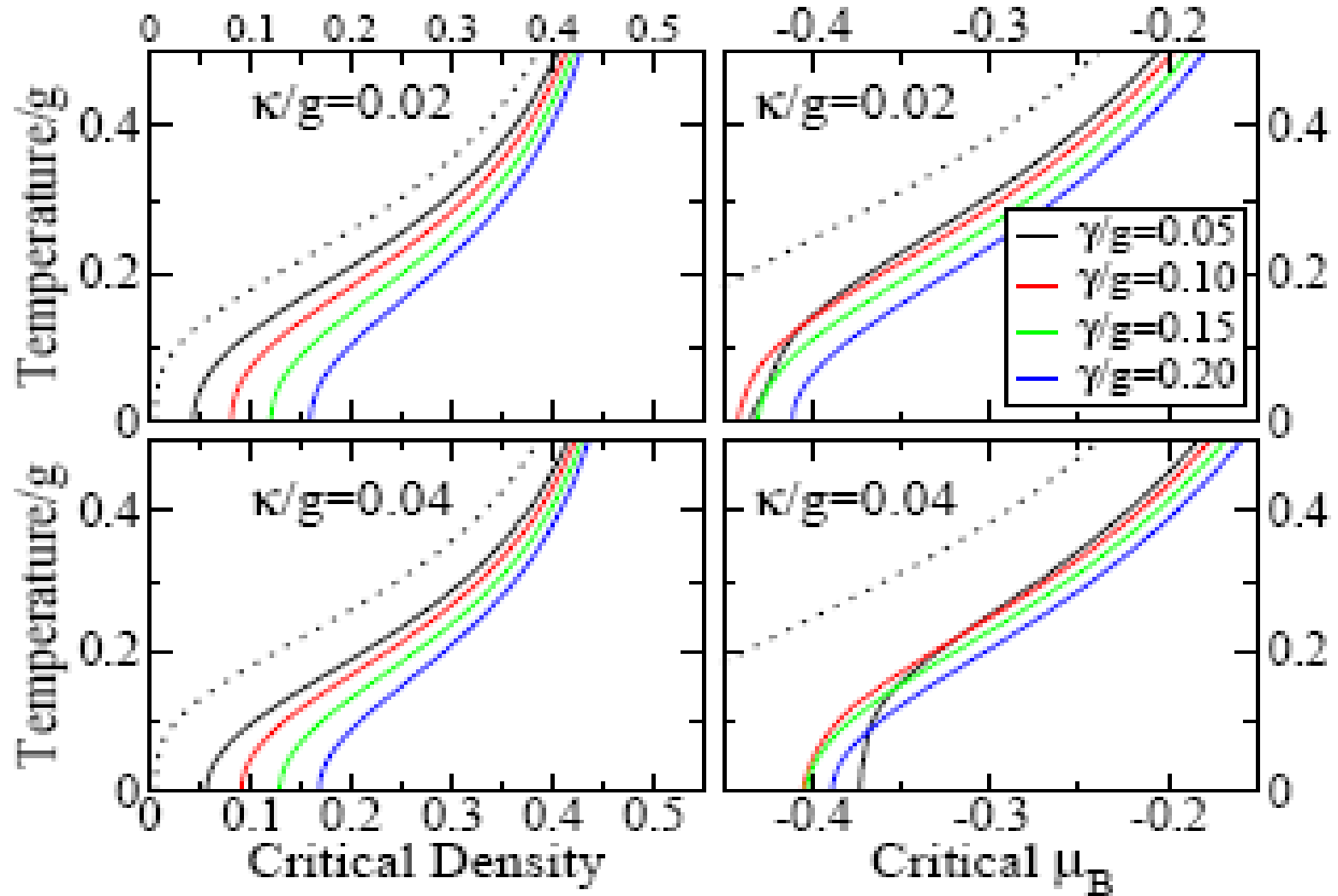
Szymanska, Keeling, Littlewood, *Physical Review B* 75, 195331 (2007)

## Linewidth and luminescence

- Linewidth vanishes as transition is approached
- Luminescence –  $N(0)$  diverges at transition
- Both have mean field exponents  $\sim (P_c - P)^{\pm 1/2}$

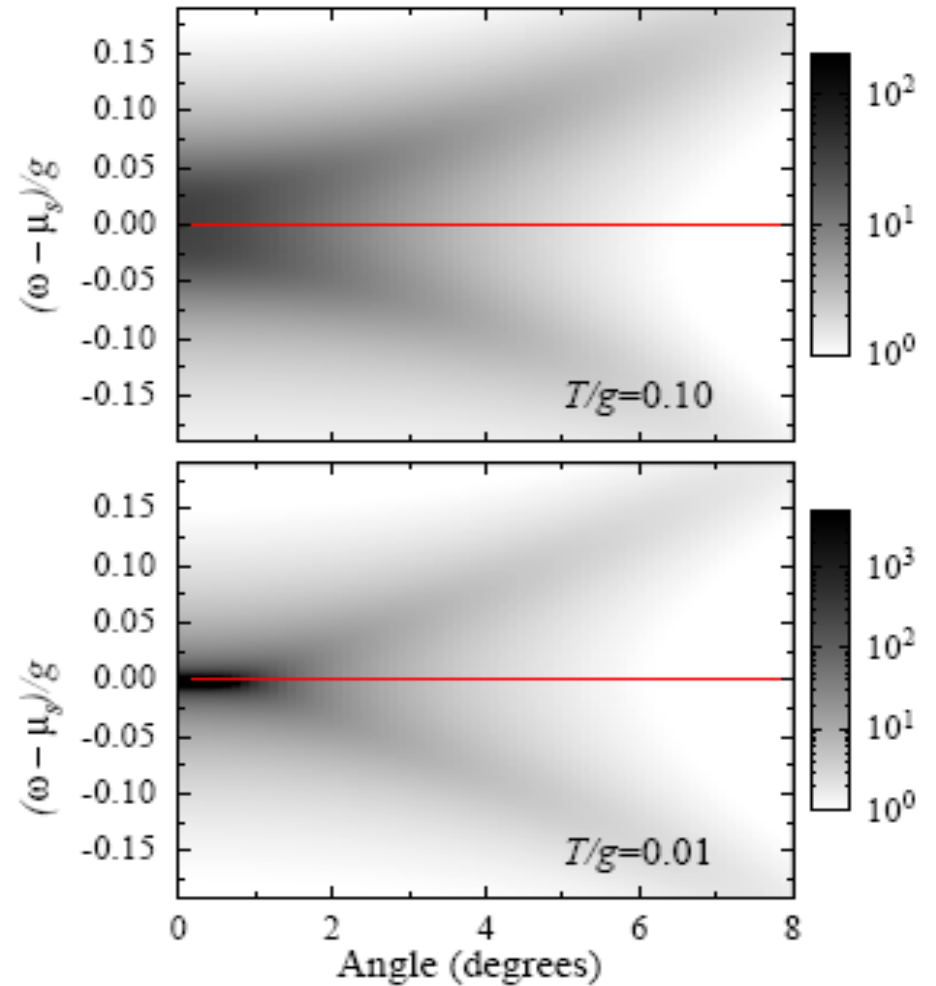


## Phase boundary in presence of dissipation



# Photoluminescence

- Crossover from propagating Bogoliubov mode to diffusion on long length scales
- Crossover length determined by dissipation rate



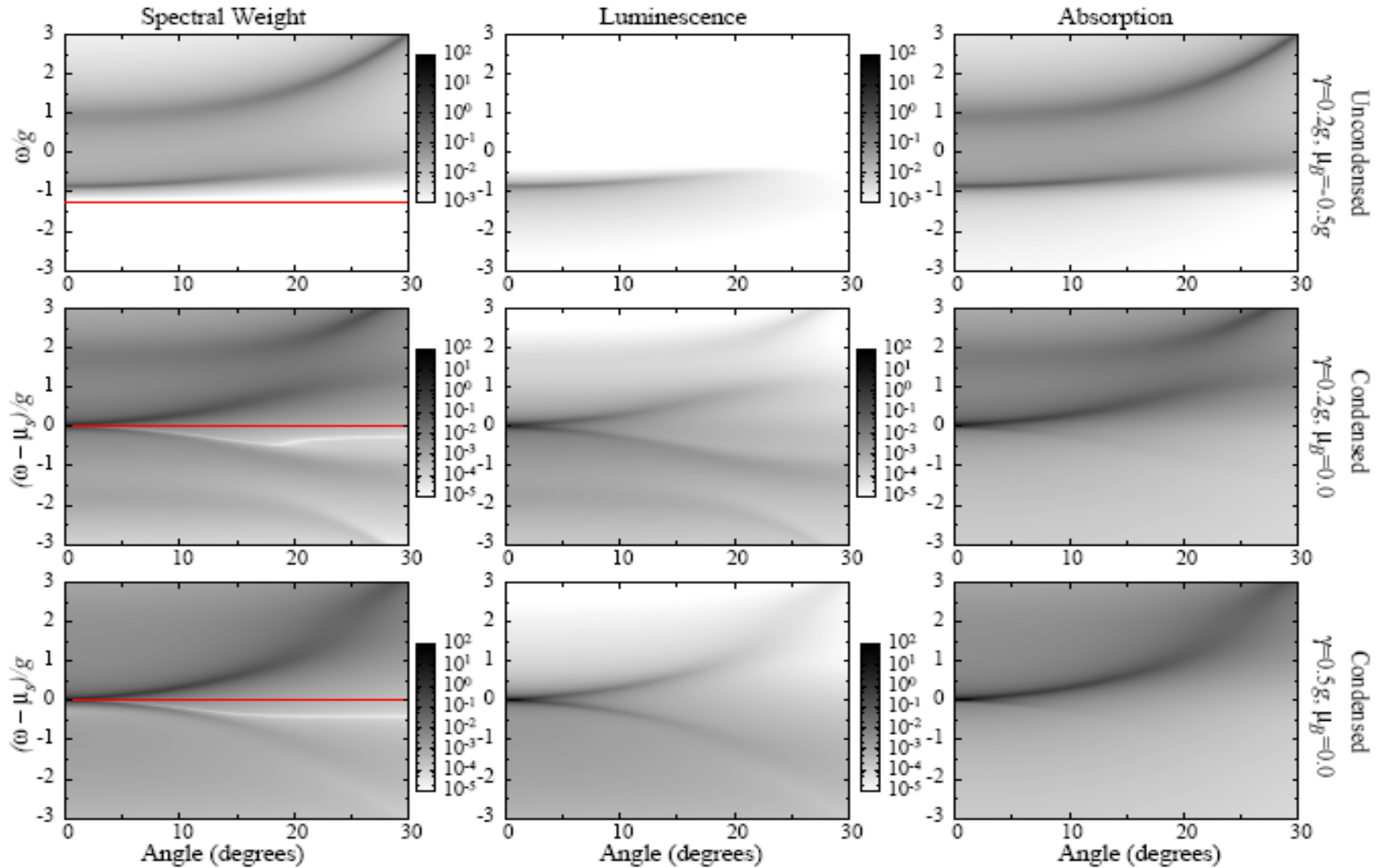


FIG. 10: Spectral weight, photoluminescence and absorption spectra, as a function of emission angle,  $\tan^{-1}(cp/\omega_0)$ . For all graphs,  $\kappa = 0.02g$  and  $T = 0.1g$ . Top row: Uncondensed case,  $\gamma = 0.2g, \mu_B = -0.5g$ . (cf parameters in Fig. 2 and Fig. 3) Middle row: Condensed case,  $\gamma = 0.2g, \mu_B = 0.0g$ . Bottom row: Condensed case,  $\gamma = 0.5g, \mu_B = 0.0g$  (transition to weak

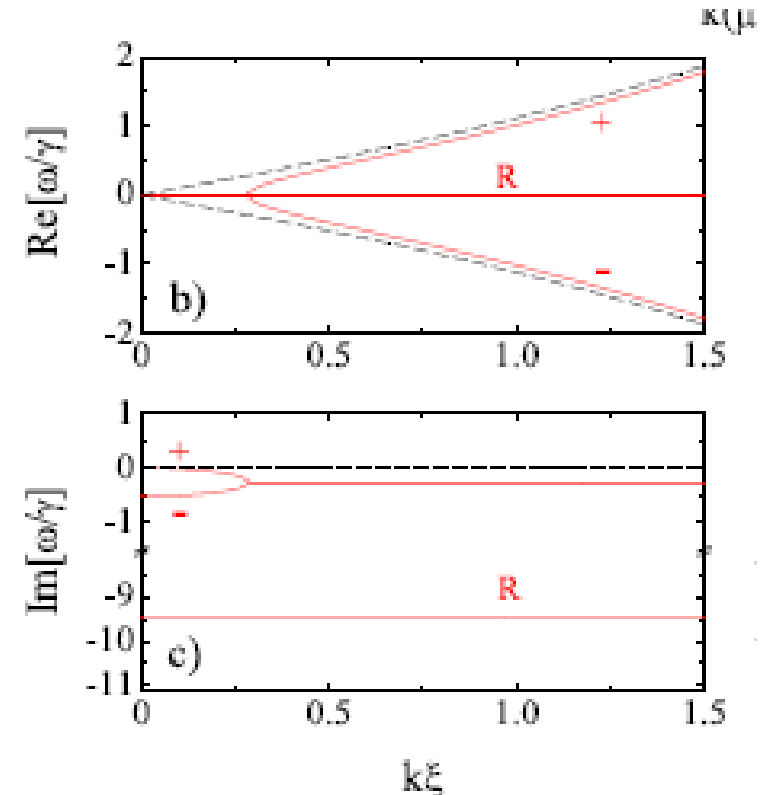
## Damped, driven Gross-Pitaevski equation

- Microscopic derivation consistent with simple behavior at long wavelengths for the condensate order parameter  $\psi$  and polariton density  $n_R$

$$i \frac{\partial \psi}{\partial t} = \left\{ -\frac{\hbar \nabla^2}{2m_{LP}} + \frac{i}{2} [R(n_R) - \gamma] + g |\psi|^2 + 2\tilde{g} n_R \right\} \psi.$$

$$\frac{\partial n_R}{\partial t} = P - \gamma_R n_R - R(n_R) |\psi(x)|^2 + D \nabla^2 n_R.$$

$$\omega_{\pm}(k) = -\frac{i\Gamma}{2} \pm \sqrt{\omega_{Bog}(k)^2 - \frac{\Gamma^2}{4}},$$



From Wouters and Carusotto 2007

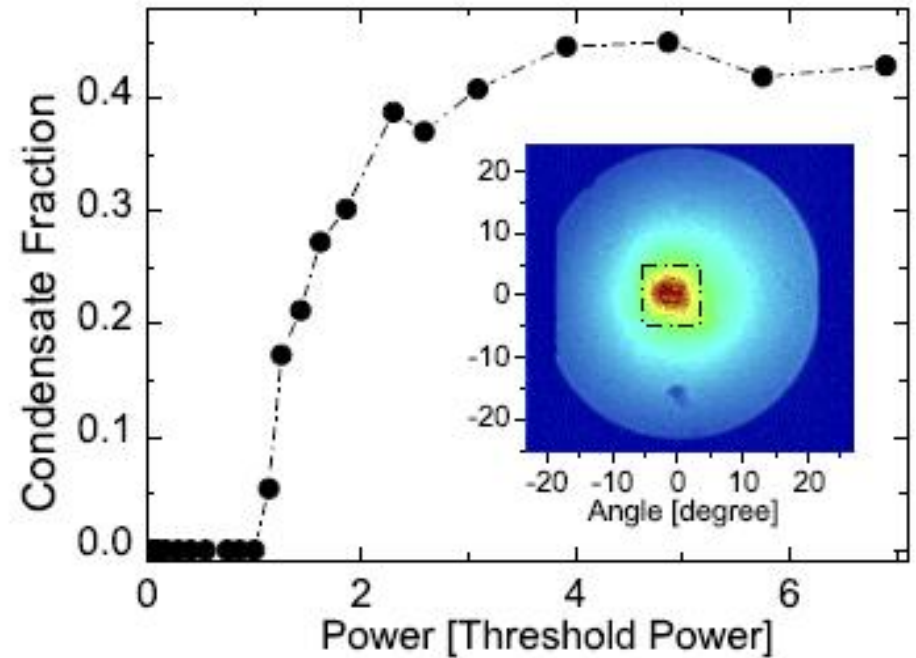
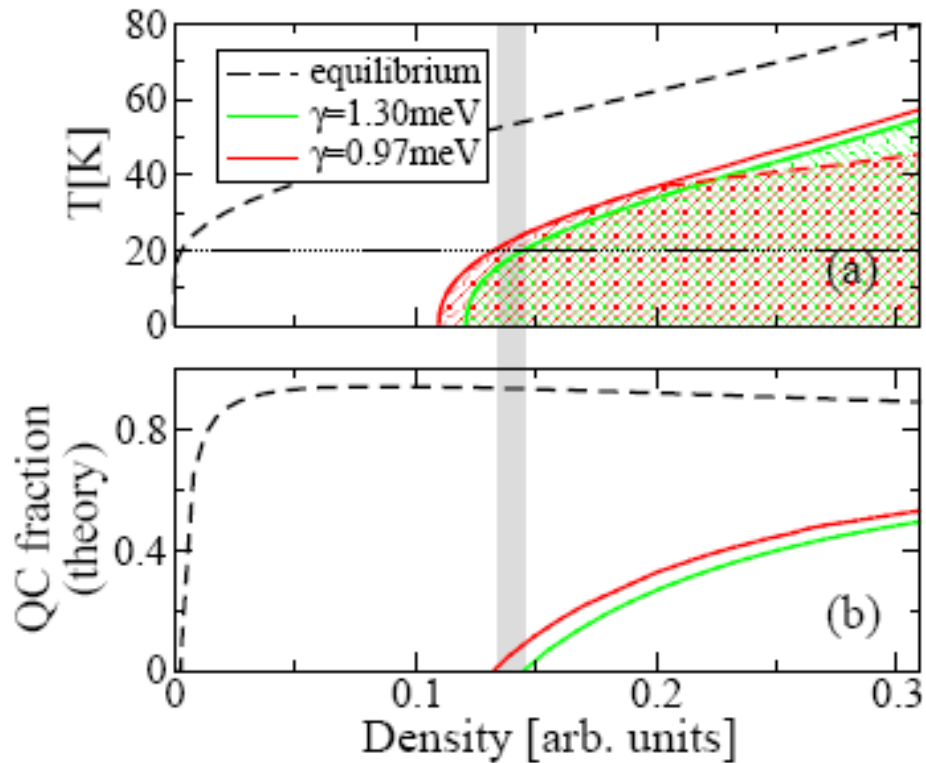


## Effect of dissipation on $T_c$ and condensate

Two parameters:

$\kappa$  – photon linewidth (measured)

$\gamma$  – pumping rate – unknown but bounded



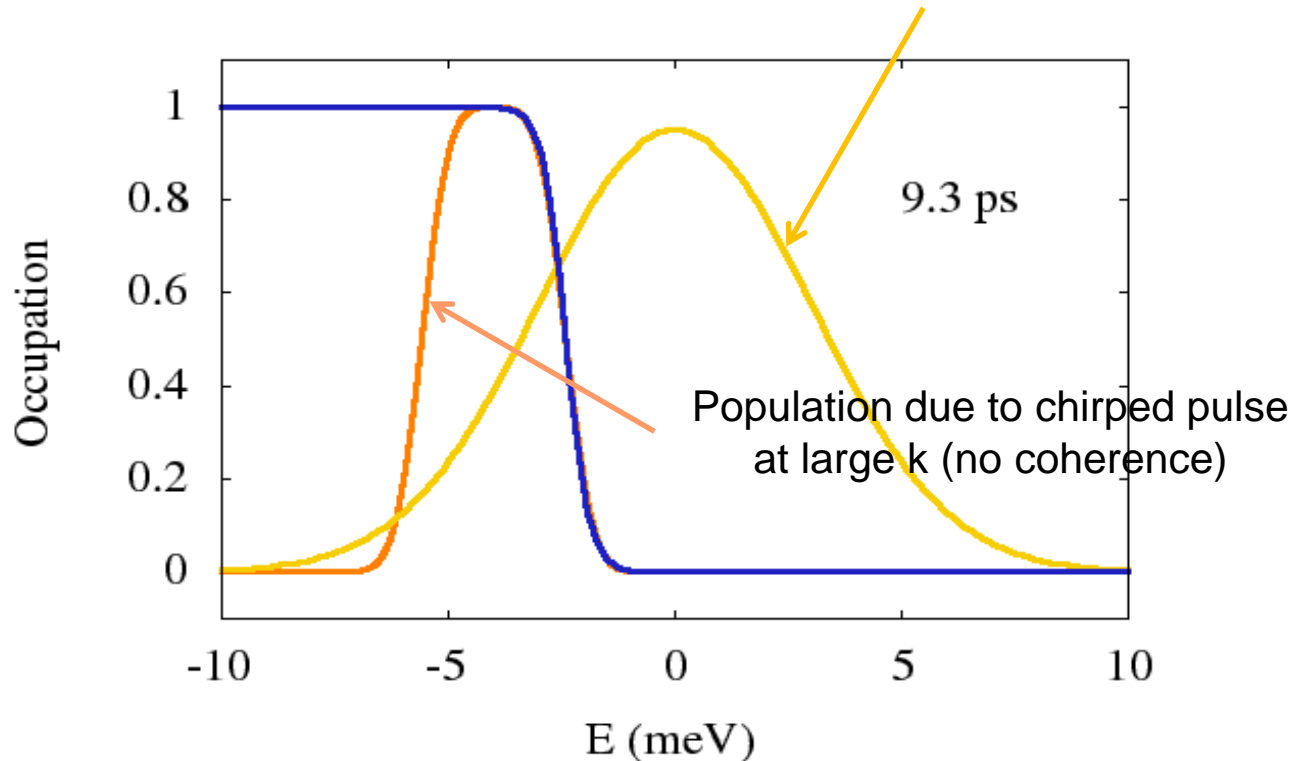
# Quantum dynamics

On time scales  $<$  few psec, not in thermal equilibrium  
Coupling to light allows driven dynamics

# Controlled pumping of a many-particle state

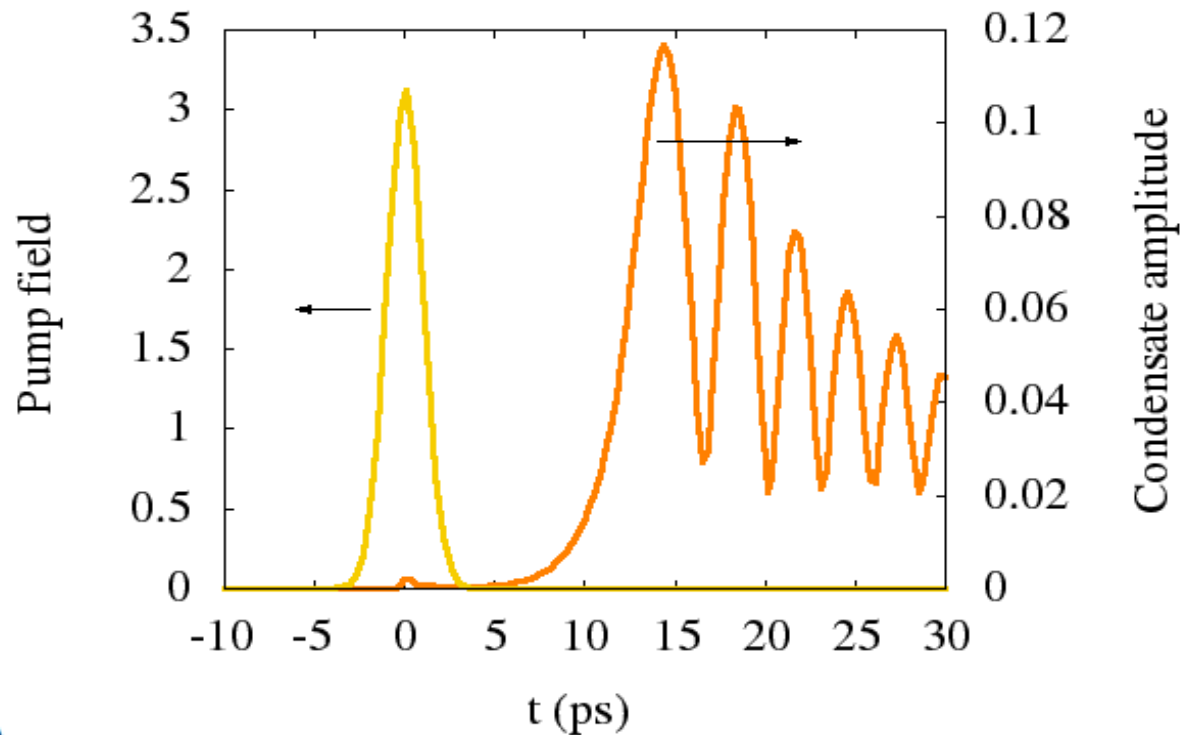
P.R. Eastham and Richard Phillips; arxiv:0708.2009

Distribution of energy levels in, e.g. Quantum dots



- Direct creation of a *many-exciton* state
- .. equivalent to excitons in equilibrium at 0.6K

# Spontaneous dynamical coherence



$$\langle P_{k=0} \rangle$$

are macroscopic (scaling with the number of dots  $\sqrt{N}$ )

$$\langle \psi_{k=0} \rangle$$

$\Rightarrow$  A quantum condensate of both photons and  $k=0$  excitons

## Conclusions

- Excitonic insulator is a broad concept that logically includes CDW's, ferromagnets, quantum Hall bilayers as well as excitonic BEC
- Polariton condensates
  - Mean field like (long range interactions)
  - Strong coupling (not in BEC limit)
- Excitonic coherence – oscillator phase-locking
  - enemy of condensation is decoherence
  - excitons are not conserved so *all* exciton condensates are expected to show coherence for short enough times only
  - condensates will either be diffusive (polaritons) or have a gap (CDW)
- Mean field+ pairbreaking or phasebreaking fluctuations gives a robust model that connects exciton/polariton BEC continuously to
  - semiconductor plasma laser (pairbreaking) or
  - solid state laser (phase breaking)
  - is a laser a condensate? – largely semantic
- Now good evidence for polariton condensation in recent experiments