Ion Crystals and Liquids in Penning Traps

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Outline:

Introduction:
   Penning traps – how do they work, thermal equilibrium, one-component plasmas, strong correlation

- Observation of crystalline structure:
  Bragg scattering, real imaging, structural phase transitions

- Modes

→ No quantum mechanics; everything in this talk uses classical physics

→ Please ask questions!!

→ Ref: Dubin and O’Neil, Rev. Mod. Physics 71, 87 (1999)
Types of traps in atomic physics

Rf or Paul Trap
RF & DC Voltages

(see Monroe lectures)

good for tight confinement and laser cooling smaller numbers of particles

Penning Trap
(or Penning-Malmberg trap)
DC Voltages & Static B-Field

good for laser cooling larger numbers of particles
Penning–Malmberg traps

\[ \text{Be}^+ \]
\[ \nu_c \sim 7.6 \text{ MHz} \]
\[ \nu_z \sim 800 \text{ kHz} \]
\[ \nu_m \sim 40 \text{ kHz} \]

- g-factors, atomic phys.
  - U of Wash., Mainz, Imperial College, NIST, ...

- mass spectroscopy
  - U of Wash., Harvard, ISOLDE/CERN, ...

- cluster studies
  - Mainz/Griefswald

- non-neutral plasmas
  - UCSD, Berkeley, Princeton, NIST, ...

- anti-matter
  - UCSD, Harvard, CERN, Swansea, ...

- quantum information/simulation
  - NIST, Imperial College, Harvard, Mainz/Ulm, ....
Single particle motion in a Penning trap

For $r, z \ll$ trap dimensions,

$$\phi_{\text{trap}}(r, z) \approx \frac{1}{2} m \omega_z^2 \left( z^2 - \frac{r^2}{2} \right)$$

$$z(t) = z_o \sin(2\pi \nu_z t + \phi_z)$$

$$r(t) = r_c \sin(\Phi \pi (\nu_c - \nu_m) t + \phi_c) + r_m \sin(2\pi \nu_m t + \phi_m)$$

$^9\text{Be}^+$

$\nu_c \sim 7.6$ MHz

$\nu_z \sim 800$ kHz

$\nu_m \sim 40$ kHz
Confinement in a Penning trap

Axial confinement $\leftrightarrow$ conservation of energy
Radial confinement $\leftrightarrow$ conservation of angular momentum

$$P_{\theta} = \sum_j \left( mv_{\theta j} + \frac{q}{c} A_{\theta}(r_j) \right) r_j$$

$$\approx \frac{qB}{2c} \sum_j r_j^2 \text{ for } A_{\theta}(r) = \frac{Br}{2} \text{ and } B \text{ large}$$

$\Rightarrow$ axial asymmetries important - magnetic field tilt

Radial confinement due to rotation – ion plasma rotates $v_{\theta} = \omega_r r$ due to $\mathbf{E} \times \mathbf{B}$ fields

In rotating frame, $v_{\theta} \hat{\theta} \times \mathbf{B}$ Lorentz force is directed radially inward
NIST Penning trap

- 4.5 Tesla Super Conducting Solenoid
- Quartz Vacuum envelope $P < 10^{-10}$ Torr
Non-neutral plasmas in traps evolve into bounded thermal equilibrium states

thermal equilibrium, Hamiltonian and total canonical angular momentum conserved

\[ f(r, v) \propto \exp\left[-\left(h + \omega_r p_\theta\right)/k_B T\right] \]
where \( h = \frac{mv^2}{2} + e\phi(r) \) and \( p_\theta = mv_\theta r + \frac{eB}{2c} r^2 \)

\[ f(r, v) \propto n(r, z) \exp\left[-\frac{m}{2k_B T}(v + \omega_r r \hat{\theta})^2\right] \]

density distribution

plasma rotates rigidly at frequency \( \omega_r \)

\[ n(r, z) \propto \exp\left\{-\frac{1}{k_B T}[e\phi_p(r, z) + e\phi_T(r, z) + m\omega_r (\Omega_c - \omega_r) \frac{r^2}{2}]\right\} \]

\( \Omega_c = \frac{eB}{mc} = \) cyclotron frequency

plasma potential

trap potential

Lorentz force potential

centrifugal potential

Lorentz-force potential gives radial confinement !!
Equilibrium plasma properties

thermal equilibrium $\Rightarrow$ rigid rotation $\omega_r$

- $T \sim 0$ $\Rightarrow$ constant plasma density,
  
  $$n_0 = 2\varepsilon_0 m\omega_r (\Omega_c - \omega_r)/e^2,$$
  
  $\Omega_c =$ cyclotron frequency
  
  plasma density $\rightarrow 0$ over a Debye length $\lambda_D = [k_B T/(4\pi n_o e^2)]^{1/2}$

- quadratic trap potential, $e\phi_T \sim m\omega_z^2(z^2 - r^2/2)$ $\Rightarrow$ plasma shape is a spheroid

aspect ratio $\alpha \equiv z_0/r_0$
determined by $\omega_r$

$$\frac{\omega_z^2}{2\omega_r (\Omega_c - \omega_r)} = Q_1^0 \left[ \frac{\alpha}{(\alpha^2 - 1)^{1/2}} \right] / (\alpha^2 - 1)^{1/2}$$

associated Legendre function
Plasma aspect ratio determined by $\omega_r$

$$\frac{\omega_z^2}{2\omega_r(\Omega_c - \omega_r)} = \frac{Q_1^0}{\frac{\alpha}{(\alpha^2 - 1)^{1/2}}} \left(\frac{\alpha}{(\alpha^2 - 1)^{1/2}}\right)$$

Simple equilibrium theory describes the plasma shapes

experimental measurements of plasma shape vs $\omega_r$
Ions in a trap are an example of a one component plasma

one component plasma (OCP) – consists of a single species of charged particles immersed in a neutralizing background charge

ions in a trap are an example of an OCP (Malmberg and O’Neil PRL 39, (77))

\[ n(r, z) \propto \exp\left\{ -\frac{1}{k_BT} \left[ e\phi_p(r, z) + e\phi_T(r, z) + m\omega_r(\Omega_c - \omega_r)\frac{r^2}{2} \right] \right\} \]

looks like neutralizing background

\[ \nabla^2\{\phi_T(r, z) + \frac{1}{e}m\omega_r(\Omega_c - \omega_r)\frac{r^2}{2}\} = -4\pi en_{bkgnd} \]

\[ n_{bkgnd} = -\frac{m\omega_r(\Omega_c - \omega_r)}{2\pi e^2} \]

thermodynamic state of an OCP determined by:

\[ \Gamma \equiv \frac{q^2}{a_{WS}k_BT} , \quad \frac{4}{3}\pi a_{WS}^3n \equiv 1 \]

\[ \Gamma \approx \frac{\text{potential energy between neighboring ions}}{\text{ion thermal energy}} \]

\[ \Gamma > 1 \Rightarrow \text{strongly coupled OCP} \]
Why are strongly coupled OCP’s interesting?

Strongly coupled OCP’s are models of dense astrophysical matter – example: outer crust of a neutron star

For an infinite OCP, \( \Gamma > 2 \Rightarrow \) liquid behavior
\( \Gamma \sim 173 \Rightarrow \) liquid-solid phase transition to bcc lattice

Brush, Salin, Teller (1966) \( \Gamma \sim 125 \)
Hansen (1973) \( \Gamma \sim 155 \)
Slatterly, Doolen, DeWitt (1980) \( \Gamma \sim 168 \)
Ichimaru; DeWitt; Dubin (87-93) \( \Gamma \sim 172-174 \)

Coulomb energies/ion of bulk bcc, fcc, and hcp lattices differ by \( < 10^{-4} \)

with trapped ions, \( n_o \sim 10^9 \text{ cm}^{-3} \)
\( T < 5 \text{ mK} \) \( \Rightarrow \) \( \Gamma > 500 \)
Laser-cooled ion crystals

Increasing Correlation $\Gamma$

$\Gamma = 2$

$\Gamma = 175$

Plasmas vs strongly coupled plasmas
How large must a plasma be to exhibit a bcc lattice?

1989 - Dubin, planar model  **PRA 40, 1140 (89)**
result: plasma dimensions $\geq 60$ interparticle spacings required for bulk behavior
$N > 10^5$ in a spherical plasma $\Rightarrow$ bcc lattice

2001 – Totsji, simulations, spherical plasmas, $N \leq 120$ k
**PRL 88, 125002 (2002)**
result: $N > 15$ k in a spherical plasma $\Rightarrow$ bcc lattice
Experimental techniques:

Doppler laser cooling and resonance fluorescence detection

\[ ^{9}\text{Be}^+ \]

\( P_{3/2} \)
\( S_{1/2} \)

\( \nu_{o} (\lambda=313 \text{ nm}) \)

\( +3/2 \)
\( -1/2 \)
\( -3/2 \)

\( \text{repump} \)
\( +1/2 \)
\( -1/2 \)

\( T_{\text{min}}(^{9}\text{Be}^+) \sim 0.5 \text{ mK} \)

\( T_{\text{measured}} < 1 \text{ mK} \)

Laser torque

The laser beam position and frequency control the torque and \( \omega_r \)

With the laser beam directed as shown, increasing torque ⇒ increasing \( \omega_r \) ⇒ decreasing radius
Evidence for bcc crystals: Bragg scattering

\[ f_{\text{rotation}} = 240 \, \text{kHz} \]
\[ n = 7.2 \times 10^8 /\text{cm}^3 \]

Bragg scattering from spherical plasmas with $N \sim 270$ k ions

Evidence for bcc crystals
Rotating wall control of the plasma rotation frequency

Huang, et al. (UCSD), PRL 78, 875 (97)
Huang, et al. (NIST), PRL 80, 73 (98)
Phase-locked control of the plasma rotation frequency

Huang, *et al.*, Phys. Rev. Lett. 80, 73 (98)

time averaged Bragg scattering

camera strobed by the rotating wall

\[ \text{N} \geq 200,000 \text{ ions} \Rightarrow \text{always observe bcc crystalline patterns} \]

- \[ \text{100,000} \geq \text{N} \geq 20,000 \Rightarrow \text{observe fcc, hcp?}, \text{in addition to bcc} \]
Real space imaging

Mitchell. et al., Science 282, 1290 (98)
Top-view images in a spherical plasma of ~180,000 ions

\[ \omega_r = 2\pi \times 120 \text{ kHz} \]

**bcc (100) plane**
- Predicted spacing: 12.5 µm
- Measured: 12.8 ± 0.3 µm

**bcc (111) plane**
- Predicted spacing: 14.4 µm
- Measured: 14.6 ± 0.3 µm
Real-space images with planar plasmas

with planar plasmas all the ions can reside within the depth of focus
Planar plasmas: observation of structural phase transitions

- **65.70 kHz**
  - **top-views**
  - 1 lattice plane, hexagonal order
  - Used in current quantum information experiments

- **66.50 kHz**
  - **side-views**
  - 2 planes, cubic order
Top- (a,b) and side-view (c) images of crystallized $^9\text{Be}^+$ ions contained in a Penning trap. The energetically favored phase structure can be selected by changing the density or shape of the ion plasma. Examples of the (a) staggered rhombic and (b) hexagonal close packed phases are shown.
Theoretical curve from Dan Dubin, UCSD

not a true phase lock!

- frequency offset \((\omega_r - \omega_{\text{wall}})\) due to creep of 2 -18 mHz
- regions of phase-locked separated by sudden slips in the crystal orientation
- stick-slip motion due to competition between \perp laser and rotating wall torques
- mean time between slips \(~10\) s; what triggers the slips?
Summary of crystal observations

spherical plasmas
- bcc crystals observed with $N > 200 \text{ k ions}$
- other crystal types (fcc, hcp) observed for $20 \text{ k} < N < 200 \text{ k}$
- shell structure observed for $N < 20 \text{ k ions}$

planar plasmas
- structural phase transitions between rhombic planes (bcc-like)
  and hexagonal planes (fcc-like or bcc-like)
- good agreement with the predicted $T=0$ minimum energy lattice
  for plasmas $< 10$ lattice planes thick
Modes of magnetized spheroidal plasmas

Detailed understanding of the plasma modes necessary for quantum information exps

Analytical cold fluid mode theory by Dan Dubin, PRL 66, 2076 (1991); accurate for mode wavelength >> interparticle spacing

Mode geometry characterized by l and m (indices of associated Legendre functions)

m is azimuthal mode number, (l-m) gives number of zeros along boundary

For given (l,m), many different modes exist: 

\[
\begin{align*}
[l-m + 2] & \quad \text{if } l-m \text{ even} \\
[l-m + 1] & \quad \text{if } l-m \text{ odd}
\end{align*}
\]

l=1 modes are the COM excitations

\((l=1, m=0) \leftrightarrow \text{axial COM mode } \omega_z\)

\((1,1) \leftrightarrow \text{cyclotron COM mode } \Omega_c - \omega_m\)

\((1,1) \leftrightarrow \text{magnetron COM mode } \omega_m\)

l=2 modes are quadrupole deformations

\((2,0) \leftrightarrow \text{axial stretch mode}\)
Doppler imaging of plasma mode velocities

Modes can be detected and easily identified at low (linear) amplitudes.

Detectable velocities < 0.5 m/sec corresponding to < 100 nm displacements
(Biercuk et al., arXiv:1004.0780, 18 nm axial COM excitation excited with ~170 yN oscillating force.)
(2,0) mode excitation

\[ \omega_{2,0} = 2\pi \cdot 1.656 \text{ MHz} \]

Good agreement between theory and experiment!
(9,0) mode excitation

ω_{9,0} = 2\pi \cdot 2.952 \text{ MHz}

Mitchell et al., Optics Express 2, 314 (1998)
$l=1$ modes excited by static field errors

Tilt error produces a perturbed electric field $\delta E$ directed down on one side of the trap and up on the other.

Provides alignment to better than 0.01 degrees!!
Plasma waves excited by laser radiation pressure

Push on the top of a rotating ion crystal with a laser beam.

The waves which are excited interfere to form a "wake".
Variations in image intensity (shown with a false color scale) correspond to variations in the axial motion of the ions in the crystal.

A large spectrum of modes are excited, which interfere to form a wake that is stationary in the source (lab) frame.

**Doppler image of wakes**

Image *w/* push beam

Image *w/* push beam

Subtract the two images

δv_z > 0

δv_z < 0
Wakes are Stationary in the frame of the source (ship).

Analogous to wakes in water

\[ \omega = \sqrt{gk} \]

Dispersion curve for gravity waves
Analyze image to obtain $\lambda$ and $\omega$

Analyze wake pattern in an annular region that is directly “behind” the push beam.

Fit to damped sinusoid to get $\lambda$:

$$y = C_0 + C_1 \sin(C_2 x + C_3) e^{-C_4 x}$$

$$C_2 = \omega = \frac{2\pi}{\lambda}$$

Directly behind the beam, the stationary phase condition gives:

$$v_{\text{source}} = \omega_{\text{Rot}} r_{\text{source}} = \omega / k$$

For this case:

$$\lambda = 180 \, \mu m \quad \delta v_z < 1 \, m/s$$

$$\omega / 2\pi = 500 \, kHz \quad \delta z < 0.3 \, \mu m$$
The data agrees with the theoretical dispersion relationship for drumhead waves in a plasma slab of thickness $2Z_p$.

$$\tan \left[ \frac{kZ_p}{\sqrt{\omega_p^2 / \omega^2 - 1}} \right] = \sqrt{\omega_p^2 / \omega^2 - 1}$$
Theory replicates experiment

Side View

Top View

Experiment

Theory - Dubin

$\delta v_z > 0$

$\delta v_z < 0$
Summary

- Penning trap uses static fields for confinement enabling laser cooling of large ion clouds; plasma rotation is required for confinement.

- 2-D and 3-D crystals observed and understood as structures that minimize the Coulomb potential energy

- Long wavelength mode structure can be calculated exactly and precisely understood

Tuesday:

- Quantum information and simulation experiments with 2-D arrays