Quantum Information Experiments in Penning traps

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Trapped Ions

**Rf or Paul Trap**

**RF & DC Voltages**

good for tight confinement and laser cooling
smaller numbers of particles;
quantum computing;
optical and microwave clocks; 1-d ion arrays

**Penning Trap**

(or Penning-Malmberg trap)

**DC Voltages & Static B-Field**

good for laser cooling larger numbers;
microwave clocks; cold plasma studies; 2-d and 3-d ion arrays

quantum simulation experiments:
Quantum simulation or computation?

**quantum computation with trapped ion crystals:**
“Quantum manipulation of trapped ions in 2-d Coulomb crystals,” Porras and Cirac, PRL 96, 250501 (2006)

“Wigner crystals of ions as quantum hard drives,” Taylor and Calarco, PRA 78, 062331 2008


**quantum simulation of magnetic spin systems with trapped ion crystals:**
Effective spin systems with trapped ions
Porras and Cirac, PRL 92, 207901 (2004)

Effective spin quantum phases in systems of trapped ions
Deng, Porras, and Cirac, PRA 72, 063407 (2005)

Another approach: quantum comp/simulation with planar Penning trap arrays:
Ciaramicoli, Galve, Marzoli, and Tombesi, PRA 72, 042323 2005
Crick, Donnellan, Ananthamurthy, Thompson, Segal, RSI 81, 013111 (2010)

...does not require individual addressing...
NIST Penning trap

$^9\text{Be}^+$

$\nu_c \sim 7.6$ MHz

$\nu_z \sim 800$ kHz

$\nu_m \sim 50$ kHz
Outline:

- High magnetic field qubit

\[ 2s \ ^2S_{1/2} \quad (124 \text{ GHz}) \]

- Dynamical decoupling demonstrations using 2-D ion crystals in a Penning trap

- Spin squeezing and quantum simulation (work in progress)

- Decoherence due to elastic Rayleigh scattering

\[
\Gamma_{Rayleigh} = \Omega_R^2 \gamma \left( \sum_J a_{d \rightarrow d}^J - \sum_{J'} a_{u \rightarrow u}^{J'} \right)^2
\]
$^9\text{Be}^+\, ,\, B \sim 4.5 \, \text{T}, \, \omega_o / 2\pi \sim 124.1 \, \text{GHz}$

High magnetic field qubit

$2p\, ^2P_{3/2}$

$F = 0,1,2,3$

$2p\, ^2P_{1/2}$

$F = 1,2$

$2s\, ^2S_{1/2}$

$F = 1$

$F = 2$

our qubit:

$\sim 40 \, \text{GHz}$

$\sim 80 \, \text{GHz}$

cooling

repump

optical dipole force beams
Rabi flopping on 124 GHz electron spin flip

(a) side view
- microwave switch
- horn
- teflon lens
- super conducting magnet
- axil beams
- B

(b) Probability $|\uparrow\rangle$
- Rabi time
- 0 1 2 (ms)
- Probability $|\uparrow\rangle$
- 0 0.2 0.4 0.6 0.8 1.0
- Rabi Time (ms)
- 0 20 40 60 80
Ramsey ($T_2$) coherence on 124 GHz electron spin flip

coherence can be extended to 10’s of ms with spin echo (dynamical decoupling)

Biercuk, Uys, VanDevender, Shiga, Itano, Bollinger, Nature 458, 996 (2009)
Dynamical decoupling - maintaining coherence in the presence of noise

\[ H = \frac{1}{2} \hbar \omega_o \sigma_z + \frac{1}{2} \beta(t) \sigma_z \]

E. L. Hahn, Phys. Rev. 80, 580 (1950)
Dynamical decoupling: evenly vs unevenly spaced $\pi$-pulses

CPMG (Carr-Purcell-Meiboom-Gill) Total sequence time $n(2T)$, $n=$# of $\pi$-pulses

UDD (Uhrig Dynamical Decoupling) – Uhrig, PRL 98, 100504 (2007)

For $n$ $\pi$-pulses, UDD cancels 1st $n$ derivatives of noise

$$H = \frac{1}{2} \sigma_z \left[ \omega_0 + \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 + \ldots \right]$$

1$\pi$-pulse spin echo

2$\pi$-pulse CPMG or UDD

3$\pi$-pulse UDD

UDD Predicted to outperform CPMG by orders of magnitude in suppressing errors, for some noise spectra.
Dynamical decoupling demonstrations with our set-up

1. prepare \(|\uparrow\uparrow\uparrow\ldots\uparrow\uparrow\rangle\) and measure ion fluorescence
2. apply dynamical decoupling sequence
3. adjust phase of last pulse to rotate spins to \(|\downarrow\downarrow\downarrow\ldots\downarrow\rangle\) under full coherence
4. loss of coherence measured from ion fluorescence after sequence

Measure fluorescence

\[
\text{Error} = \frac{\text{Counts after}}{\text{Counts before}}
\]

No dephasing: Error=0
Complete dephasing: Error=0.5
Qubit coherence extended and accurately predicted by dynamical decoupling

- Fits use analytical filter function and measured noise spectrum
- CPMG, UDD perform similarly for $1/f^2$ ambient noise
- UDD performs better for noise spectra with sharp cutoffs
- Ohmic noise with sharp cutoff injected
Locally optimized dynamical decoupling
- improved performance through feedback optimization
- vary inter-pulse delays for fixed precession time (Nelder-Mead simplex method)
  \( \tau \) - fixed
- results shown for injected Ohmic noise
- improved performance with feedback optimization at each total precession time
- no prior knowledge of \( S(\omega) \) required
Spin squeezing:

consider N spin $\frac{1}{2}$ systems $\vec{\sigma}_i, i = 1, \ldots, N$ 
$$\vec{J} = \sum_{i=1}^{N} \vec{\sigma}_i / 2$$

initialize coherent spin state 
$$|\uparrow\uparrow\uparrow\ldots\uparrow\rangle = \left| J = \frac{N}{2}, M_J = \frac{N}{2} \right>$$

rotate about x-axis by $90^\circ$

$$|\Psi\rangle_{CSS} \rightarrow \frac{1}{2^{N/2}} \sum_{M_J = -N/2}^{N/2} \left( \frac{N}{N/2 + M_J} \right)^{1/2} \left| \frac{N}{2}, M_J \right>$$

squeezed spin state – reduction in the variance of some components of $\vec{J} \perp \langle \vec{J} \rangle$

potential applications:
atomic clocks, magnetometry
Implementation of spin squeezing – single axis twisting

1. prepare \( |\uparrow\uparrow\uparrow\cdots\uparrow\rangle = |J = \frac{N}{2}, M_J = \frac{N}{2}\rangle \), \( T_{\text{motional}} \sim 0.5 \text{ mK} \)

2. \( \pi/2 \) pulse of 124 GHz microwaves

\[
|J = \frac{N}{2}, M_J = \frac{N}{2}\rangle \rightarrow \sum_{M_J = -\frac{N}{2}}^{\frac{N}{2}} c(N, M_J) |J, M_J\rangle
\]
3. Apply \( \exp(i\chi \{J_z\}^2 t) \) “push” gate on the axial center-of-mass mode of a single ion plane

\[ v_L + \omega_z + \delta \]

Raman beams


\[ \chi = \frac{(\eta \Omega)^2}{\delta} \]
\[ t = m\frac{2\pi}{\delta}, \; m = 1, 2, 3, ... \]

4. Measure \( \Delta J_z \) as a function of rotation about x-axis (mean spin vector direction)

Microwave with 90° Phase shift
Spin squeezing through single axis twisting

recent spin squeezing demonstrations: Polzik, Vuletic, Treutlein, Oberthaler, ..

features of spin squeezing with trapped ions:
- precise knowledge of ion number N
- enables quantitative analysis of the depth of entanglement

single-axis twisting Hamiltonian $H = \chi J_z^2$ is a uniform Ising model

$$J_z^2 = \left( \sum_i \sigma_z^i / 2 \right) \left( \sum_j \sigma_z^j / 2 \right) = \frac{1}{4} \sum_{i,j} \sigma_z^i \sigma_z^j$$

- interaction strength independent of distance between ion pairs
- evolution exactly calculable; enables test of fidelity of implementation
Optical dipole force implementation of $J_z^2$

$$J_z = \sum_i \sigma_i^z / 2, \quad H = \chi J_z^2 = \chi \left( N/4 + \frac{1}{4} \sum_{i \neq j} \sigma_i^z \sigma_j^z \right)$$

use optical dipole force to near resonantly drive the axial COM mode

$$\nu_L + \omega_z + \delta$$

off-resonance laser beams

with $F_\downarrow = -F_\uparrow$, $| J, M_J \rangle$ acquires a phase $\sim M_J^2$

For $\theta \sim 0.7^\circ$, squeezing will be limited by spontaneous emission
Effective spin systems with trapped ions
Porras and Cirac, PRL 92, 207901 (2004) →
dipolar Ising interaction obtained by tuning beat
note far from resonance with any modes

\[ H = -B_x \sum_i \sigma_i^x + \chi J_z^2 \]

\( B_x = 5 \text{ kHz}; \ > 50 \text{ kHz with new } \mu \text{wave source} \)

present configuration:
\( \theta = 0.72^\circ, 10 \text{ mW/beam} \)
waist \( \sim 500 \mu \text{m} \times 50 \mu \text{m} \)
\( \Delta \sim (2\pi) \cdot 20 \text{ GHz}, N=100 \)

\( \chi \sim 2\pi \cdot 36 \text{ Hz} \)
\( J \sim 2\pi \cdot 7 \text{ Hz} \)

with high power fiber laser based system
10 mW/beam → 100 mW/beam
\( \chi \rightarrow 100 \cdot \chi \)
\( J \rightarrow 100 \cdot J \)
Decoherence due to elastic Rayleigh scattering

\[ \frac{d}{dt} \rho_{ud} = -\frac{\Gamma}{2} \rho_{ud} \]

qubit superposition state described by density matrix

\[ \rho = \begin{pmatrix} \rho_{uu} & \rho_{ud} \\ \rho_{du} & \rho_{dd} \end{pmatrix} \]

non-resonant light (\(\omega_o\)) scattering causes decoherence,

Be\(^+\) energy levels, B=4.5 T
Raman scattering vs Rayleigh scattering

Raman scattering: \(|u\rangle \rightarrow |d\rangle; |d\rangle \rightarrow |u\rangle\)
n-final qubit state entangled with the polarization or frequency of the scattered photon 
\(\Rightarrow\) decoherence after single scattering event

Raman scattering rate given by Kramers-Heisenberg formula

\[
\Gamma_{ij} = \Omega_{R}^2 \gamma \left( \sum_{j} a_{i \rightarrow j}^J \right)^2, \\
a_{i \rightarrow j}^J = \frac{\langle j | \mathbf{d} \cdot \hat{\mathbf{\varepsilon}}^*_{\lambda + (i-j)} | J, \lambda + i \rangle \langle J, \lambda + i | \mathbf{d} \cdot \hat{\mathbf{\varepsilon}}_{\lambda} | i \rangle}{\delta_{i;J,\lambda+i}}
\]

\[
\frac{d\rho_{ud}}{dt} = -\frac{\Gamma_{Raman}}{2} \rho_{ud}, \quad \Gamma_{Raman} = \Gamma_{ud} + \Gamma_{du}
\]

precise test of the above prescription for calculating decoherence due to Raman scattering
Raman scattering vs Rayleigh scattering

elastic Rayleigh scattering: $|u\rangle \rightarrow |u\rangle; |d\rangle \rightarrow |d\rangle$  
$\Gamma_{Rayleigh} = ???

literature indicates that elastic Rayleigh scattering should not produce decoherence when the elastic scatter rates are equal

- “when of equal rate from both qubit levels, off-resonance Rayleigh scattering of photons did not affect the coherence of a hyperfine superposition”, PRA 75, 042329 (2007)

- “In our system, Rayleigh scattering occurs at the same rate for the two clock states, does not reveal the atomic state, and so does not harm the coherence”, PRL 104, 073602 (2010)

estimate based on difference in elastic scatter rates, from Ozeri et al., PRA 75, 042329 (2007)

$$\Gamma_{Rayleigh, \text{diff}} \sim \frac{(\Gamma_{uu} - \Gamma_{dd})^2}{(\Gamma_{uu} + \Gamma_{dd})/2}$$
Master equation treatment of decoherence due to light scattering

- consistent treatment of decoherence due to both Raman and Rayleigh scattering
- credit to: Hermann Uys

\[
\frac{\partial \rho}{\partial t} = \begin{pmatrix}
-\Gamma_{ud} \rho_{uu} + \Gamma_{du} \rho_{dd} & -\frac{1}{2} (\Gamma_{Raman} + \Gamma_{Rayleigh}) \rho_{ud} \\
-\frac{1}{2} (\Gamma_{Raman} + \Gamma_{Rayleigh}) \rho_{du} & -\Gamma_{du} \rho_{dd} + \Gamma_{ud} \rho_{uu}
\end{pmatrix}
\]

\[
\Gamma_{Rayleigh} = \Omega_R^2 \gamma \left( \sum_J a_d^J - \sum_{J'} a_u^{J'} \right)^2
\]

decoherence due to difference in the elastic scattering amplitudes!

- large decoherence expected if scattering amplitudes have opposite sign
- sign of scattering amplitude determined by the detuning
- physically the state of the qubit can be determined from the phase of the scattered photon
Experimental test of decoherence theory

- use single plane arrays of N ~100 ions in a Penning trap

\[ F = 1, 2 \]
\[ F = 0, 1, 2, 3 \]
\[ \sim 40 \text{ GHz} \]
\[ \sim 80 \text{ GHz} \]

- ions Doppler laser cooled to ~0.5 mK

- polarization of off-resonant laser beam adjusted to null light shift
Decoherence measured from decrease in the Bloch vector

Normalized counts = \[ \frac{1}{2} \left[ 1 - e^{-\frac{1}{2}(\Gamma_{\text{Raman}} + \Gamma_{\text{Rayleigh}})\tau} \right] \approx \frac{1}{2} \frac{\Gamma_{\text{Raman}} + \Gamma_{\text{Rayleigh}}}{2} \tau, \tau = \text{total laser on time} \]

de decoherence \( \approx 2 \times \text{(measured slope)} \)
Good agreement between theory and experiment

\[ \frac{1}{2} \left( \Gamma_{\text{Raman}} + \Gamma_{\text{Rayleigh}} \right) \]

\[ \Omega_R^2 \gamma \left( \sum_{J} a_{d \to d}^J - \sum_{J'} a_{u \to u}^{J'} \right)^2 \]

\[ \frac{1}{2} \Gamma_{\text{Raman}} \text{ + theory based on } \Gamma_{uu} - \Gamma_{dd} \]

- Laser electric field calibrated from measured light shift and Raman rates
Summary

- Penning traps appear to provide a platform for the simulation of quantum spin systems with \(N > 100\) ions

- triangular lattice occurs naturally \(\rightarrow\) enables simulation of spin frustration

- initial focus: uniform Ising model \(J_z^2\)

- eventual goal: short range (dipolar) Ising interaction