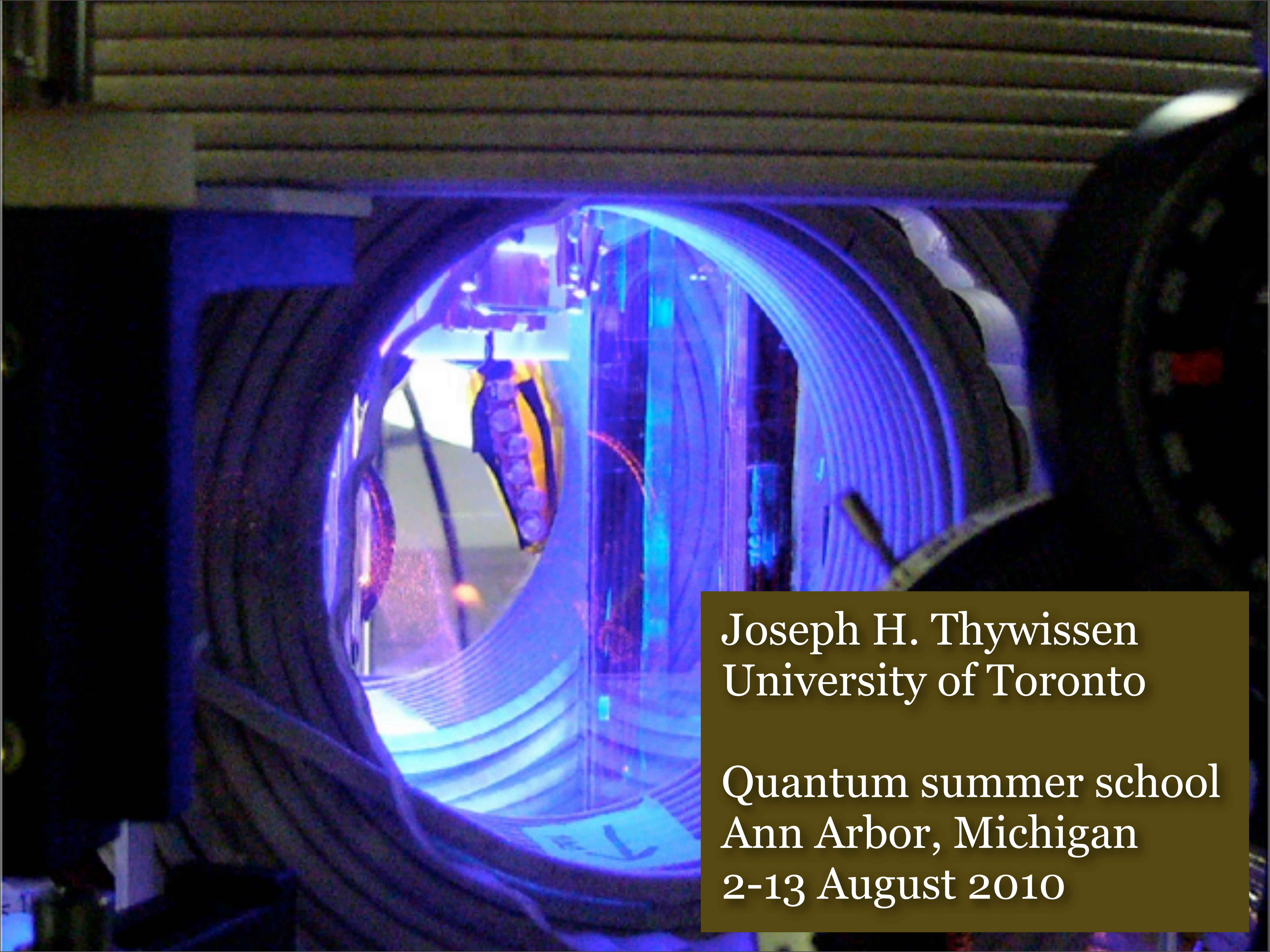


Joseph H. Thywissen
University of Toronto

Quantum summer school
Ann Arbor, Michigan
2-13 August 2010

Introduction to research group

Course on Trapped Quantum Gases:

1. Non-interacting quantum gases

1.1 Thermal DFG; 1.2 T=0 limit; 1.3 Bose gases

Lectures 1&2

2. Interacting bosons

2.1 GPE; 2.2 Gaussian Ansatz; 2.3 TF solution; 2.4 LDA idea & validity; 2.5 Hydrodynamics

Experiment: RF dressed double-well

---

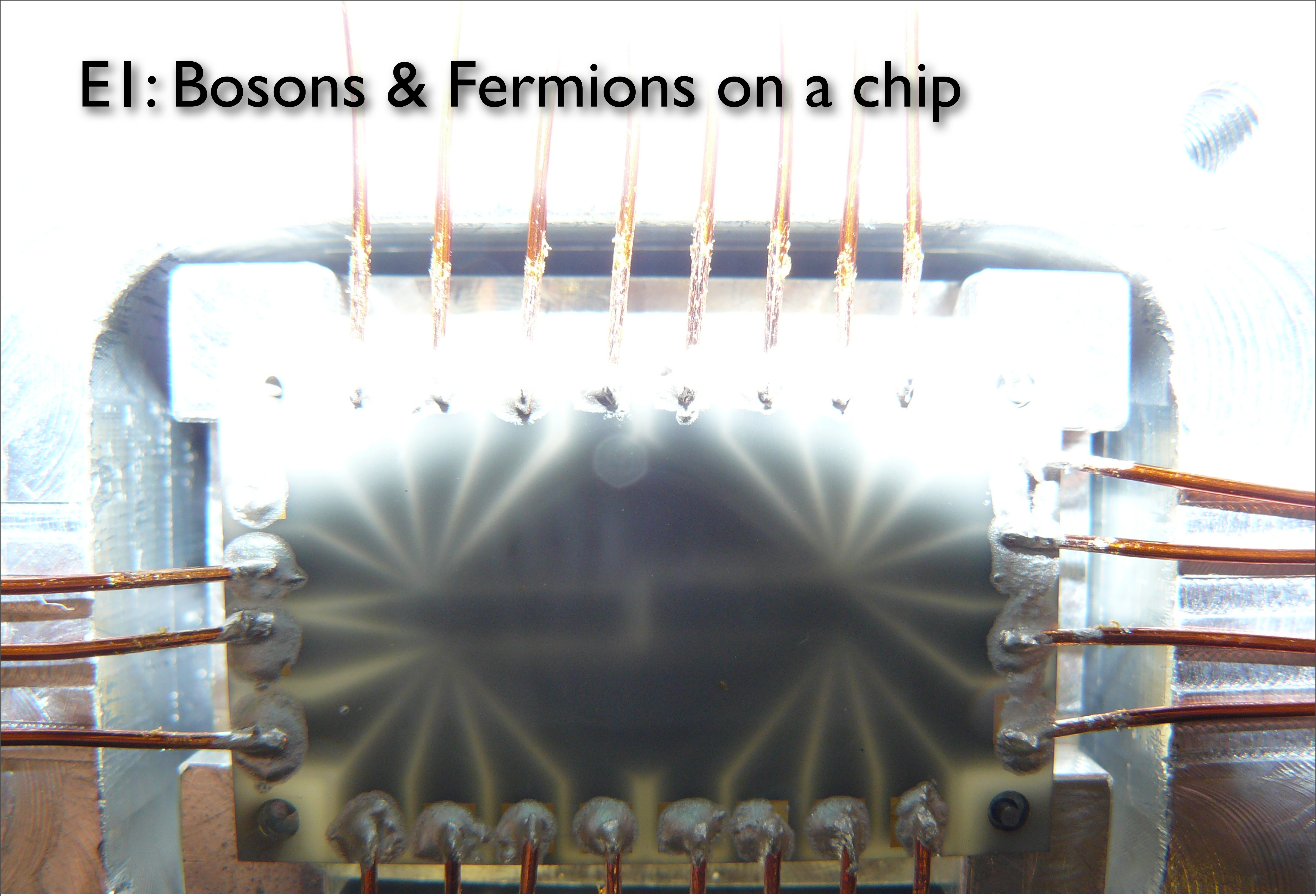
3. Interacting fermions

3.0 Spin degrees of freedom; 3.1 Mean field: variational solution; 3.2 Scattering theory; 3.3 Feshbach resonances & unitarity; 3.4 BEC-BCS crossover; 3.5 Repulsive gases: Ferromagnetic?

Lecture 3

Experiment: Energy of a repulsive DFG

E1: Bosons & Fermions on a chip



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E1: Bosons & Fermions on a chip

Themes:

Bosons in double-well potentials

Hydrodynamic to Josephson regime transition

Breakdown of LDA

Strongly interacting fermions

Ferromagnetism

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E2: Single-site imaging of fermions in an opical lattice

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E2: Single-site imaging of fermions in an opical lattice

<u>Goal</u>:

Local probe of fermion lattice physics

Thermometry

Prospects for cooling?

Quantum simulation of the Fermi hubbard model

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Basic question:

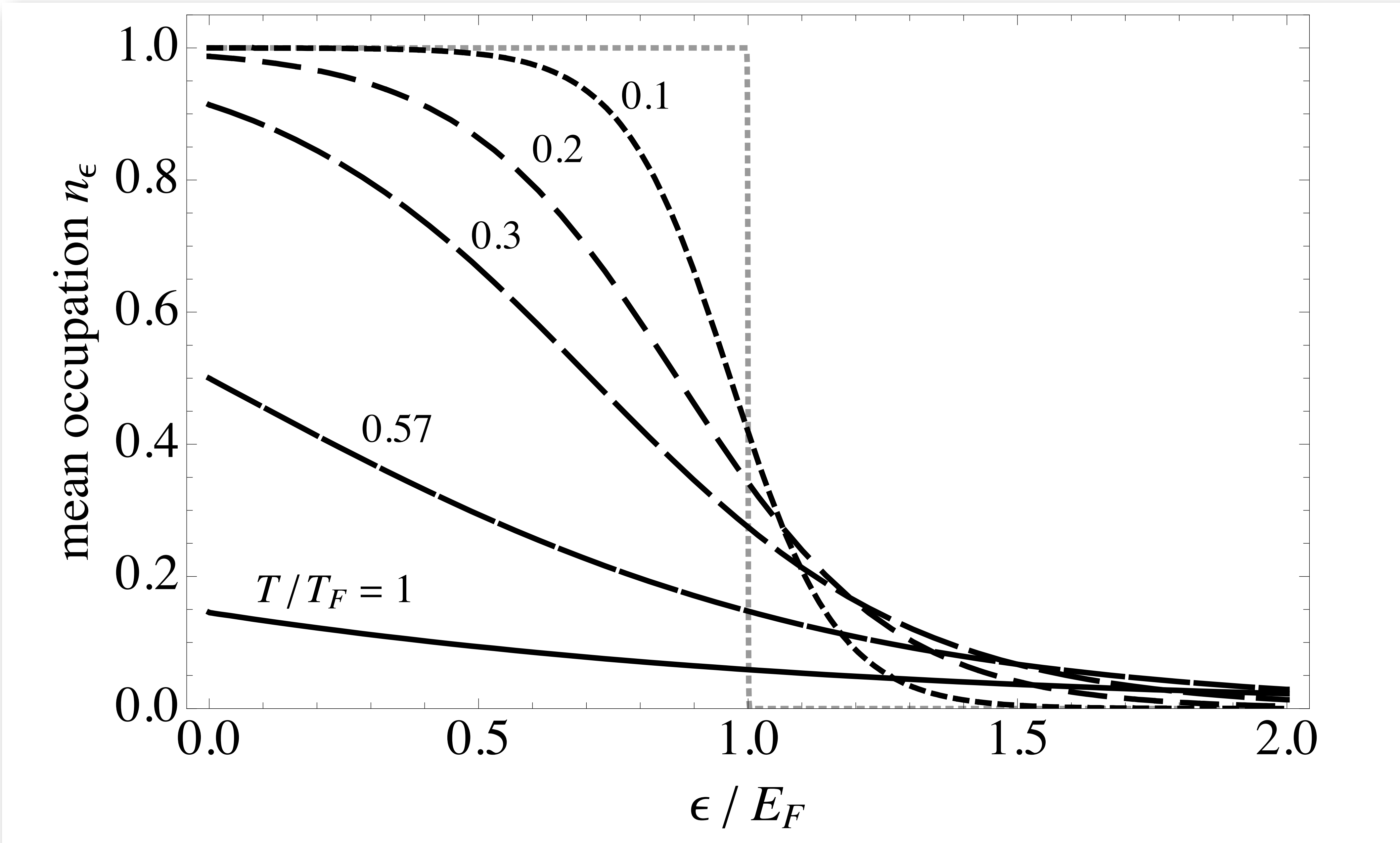
"What do trapped quantum gases look like?"

Basic question:

"What do trapped quantum gases look like?"

Unlike textbook statistical mechanics, we have a trapping potential. Atoms need to be held somehow...! But how do we think about non-uniform gases?

Fermi Dirac distribution

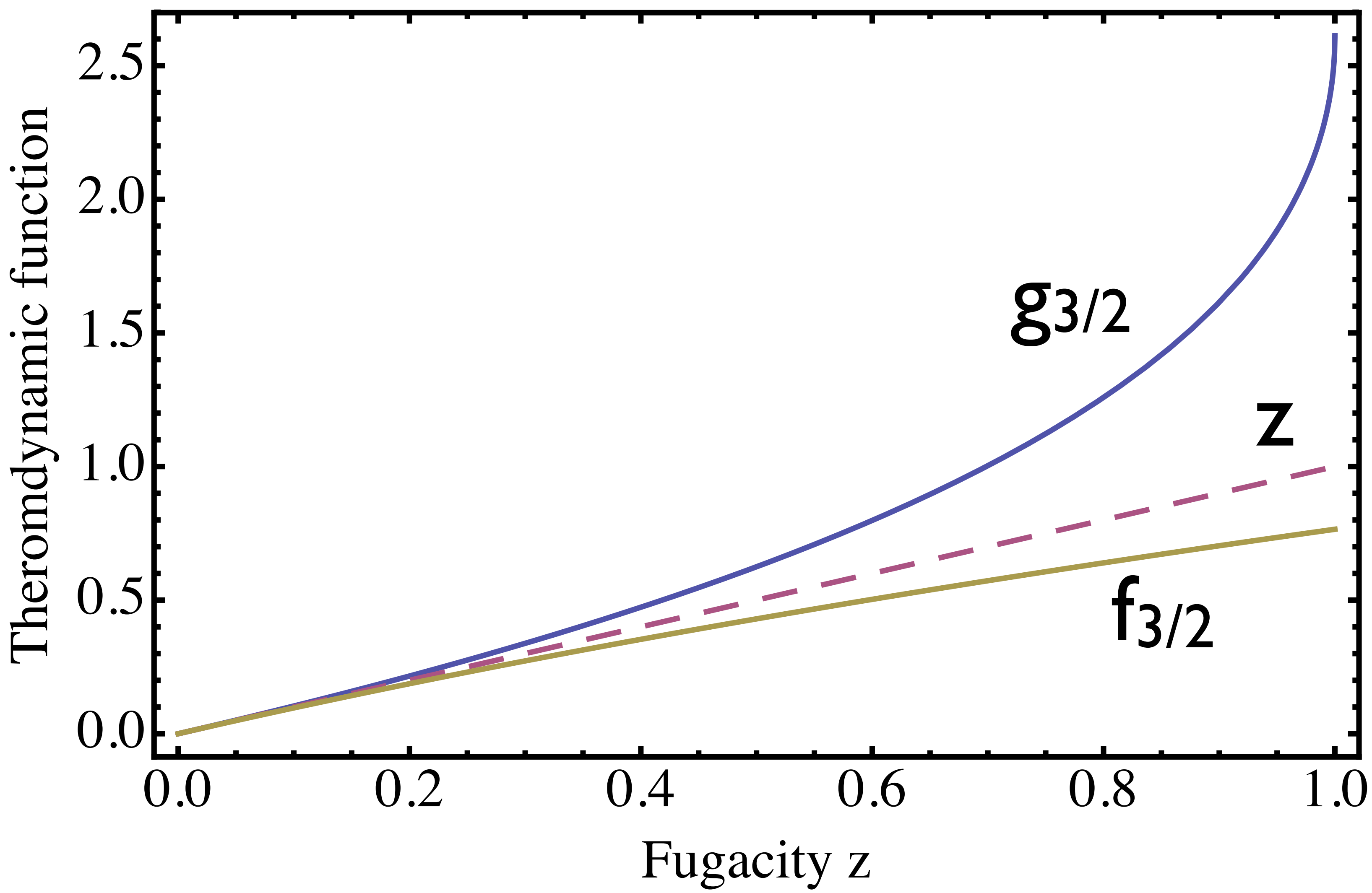


Fermi-Dirac integrals

$$f_n(C) \equiv \frac{1}{\Gamma(C)} \int_0^\infty \frac{a^{\,n-1} da}{C^{-1} e^{\,a} + 1} = -\mathrm{Li}_n(-C)$$

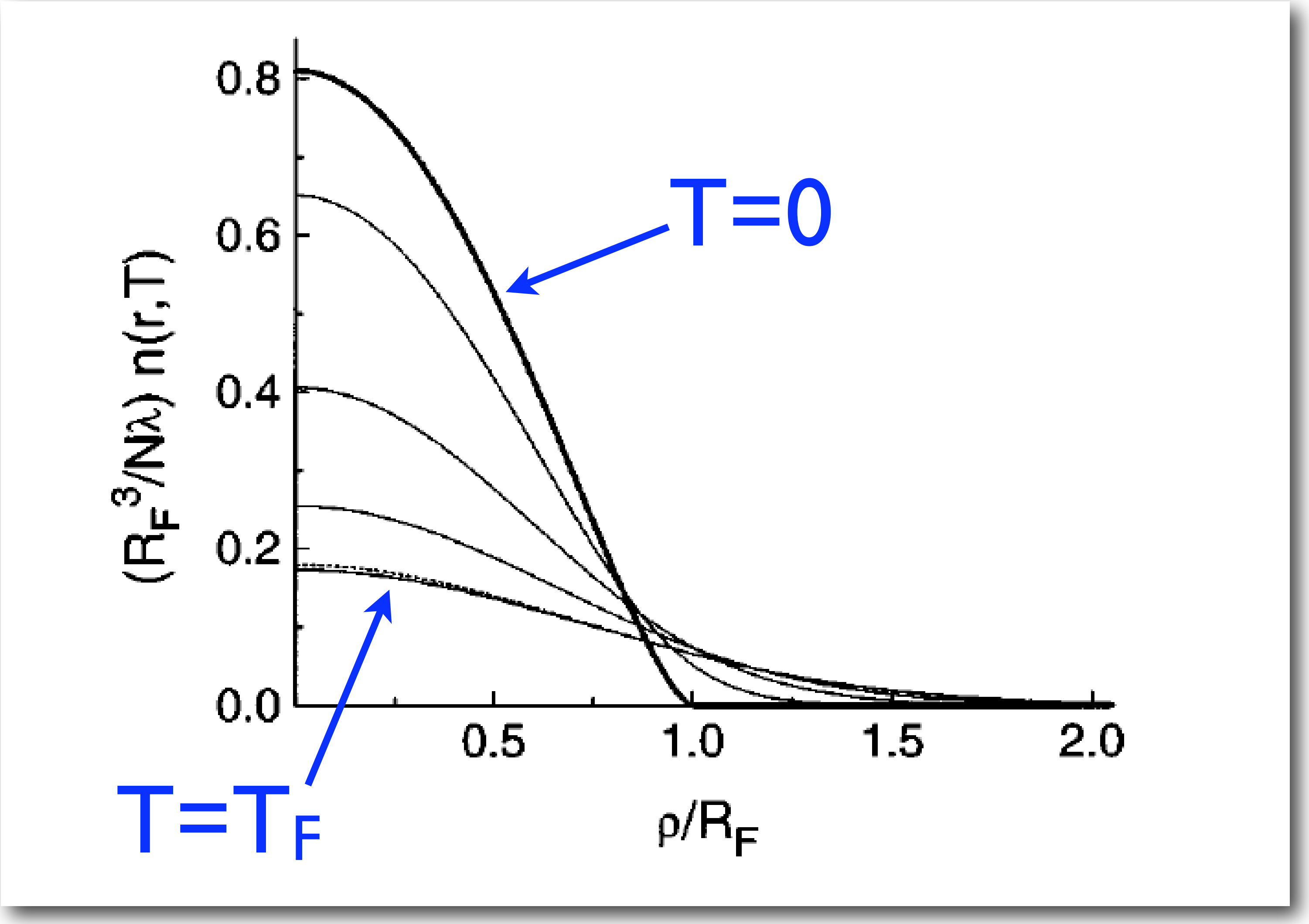
where Li is the "polylog" function: $\mathrm{Li}_n(C) = \sum_{j=1}^{\infty} C^j / j^n$

relation to normal log: $f_1 = \ln(1 + C)$

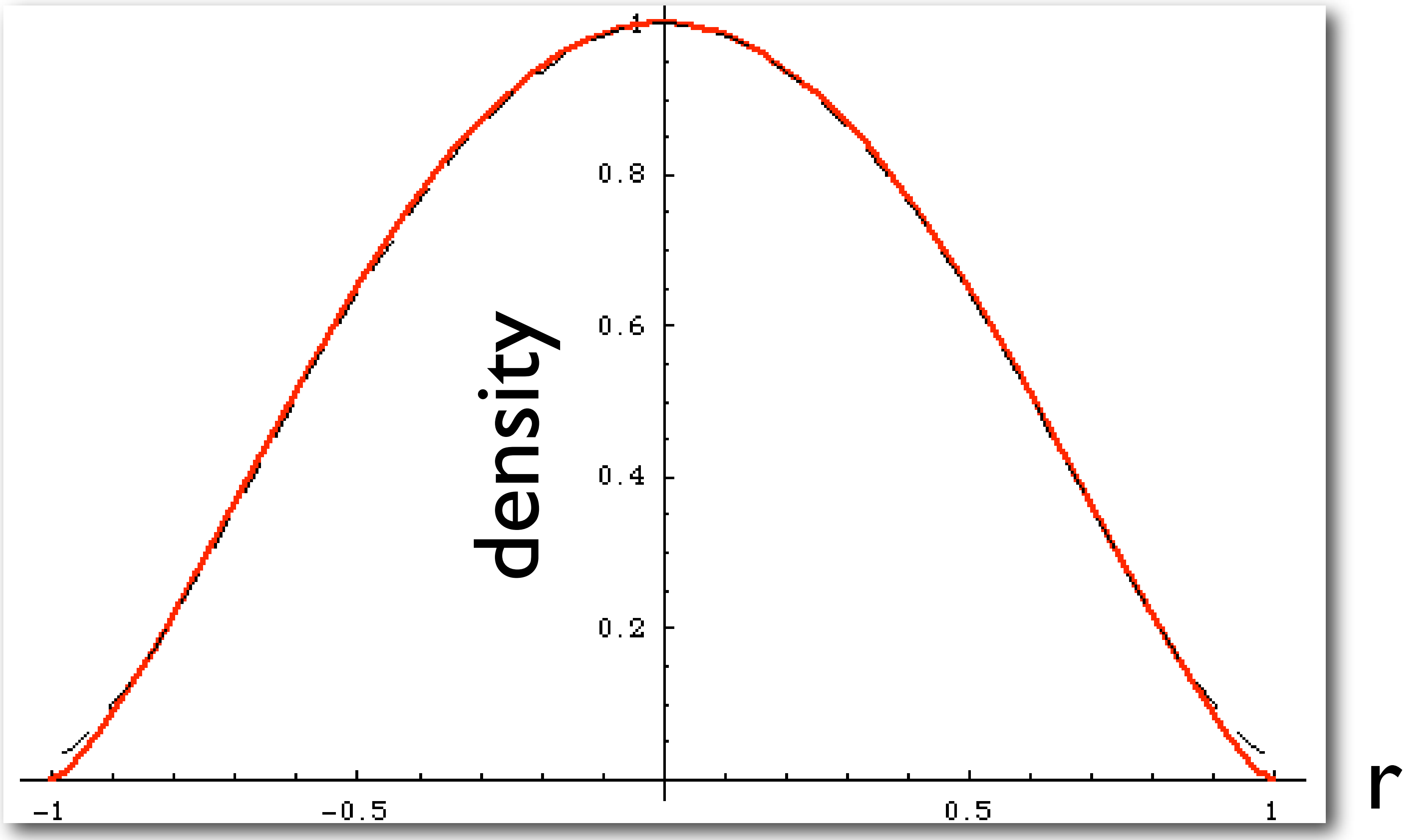


Theromdynamic function
Fugacity z
$g_{3/2}$
z
$f_{3/2}$
0.0
0.5
1.0
1.5
2.0
2.5
0.0
0.2
0.4
0.6
0.8
1.0

In-trap distribution at finite T (fermions)



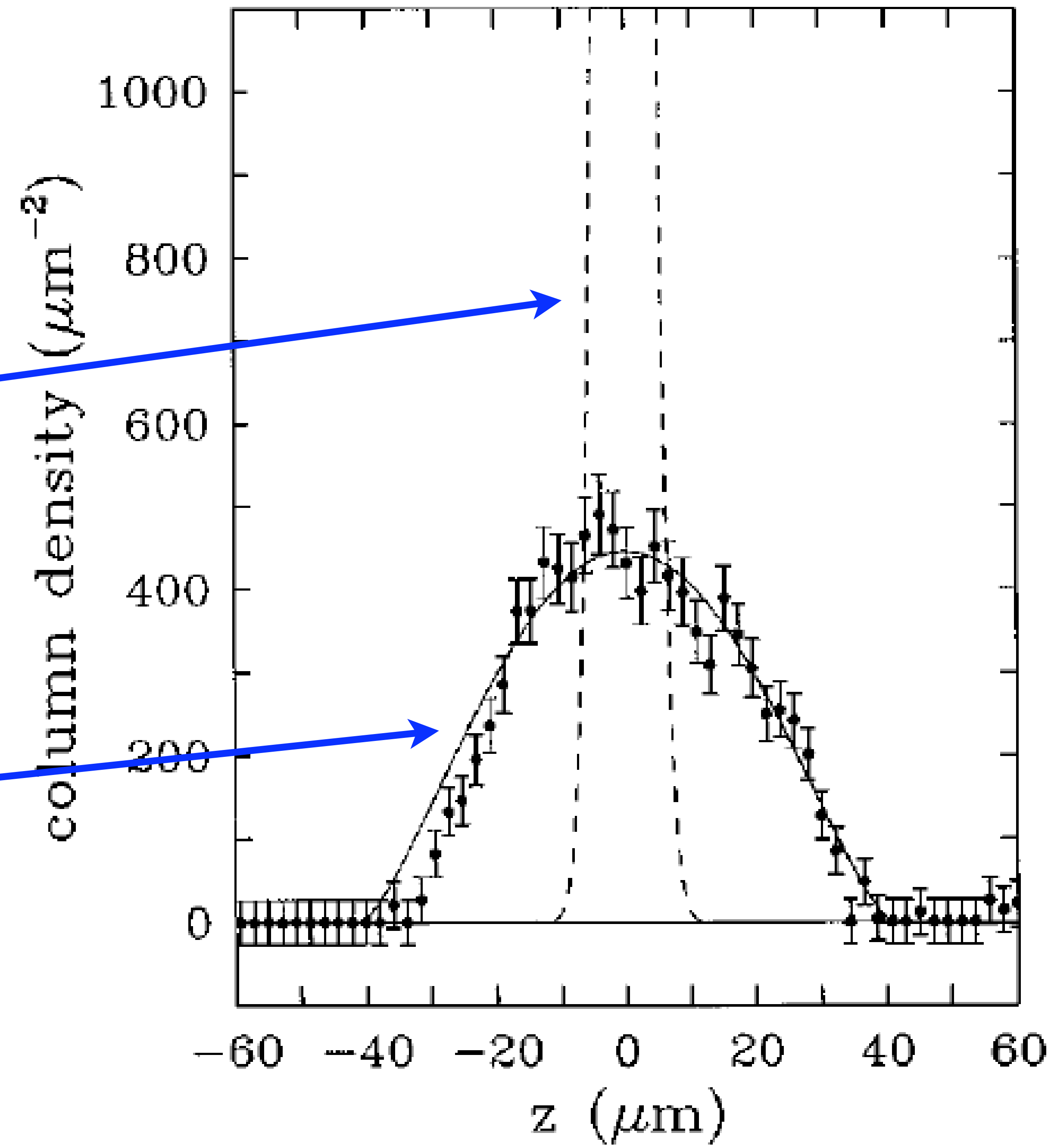
T=0 (Thomas Fermi) profile for fermions



Observation: shape of BEC

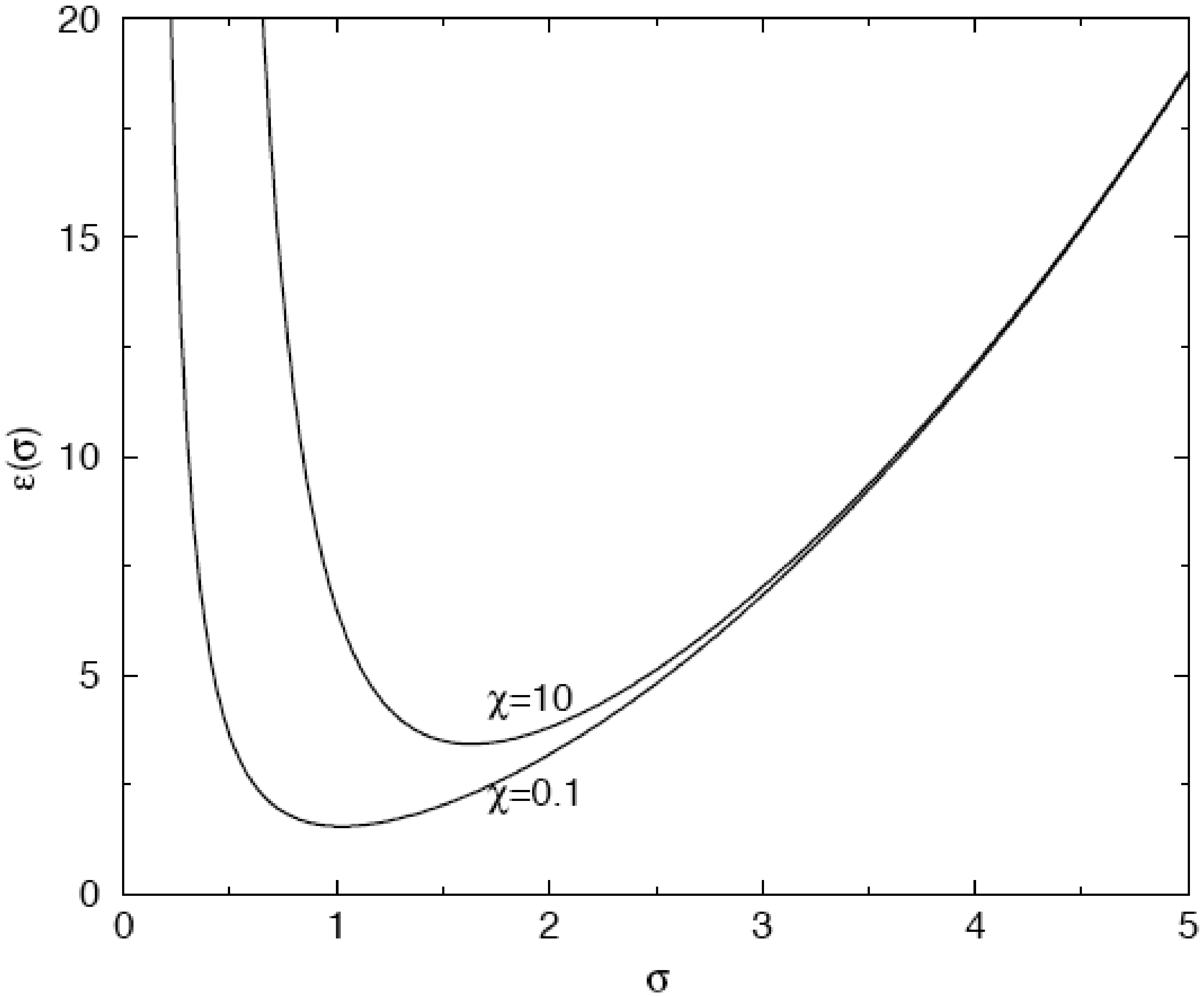
non-interacting theory

observed.



Hau (1998)

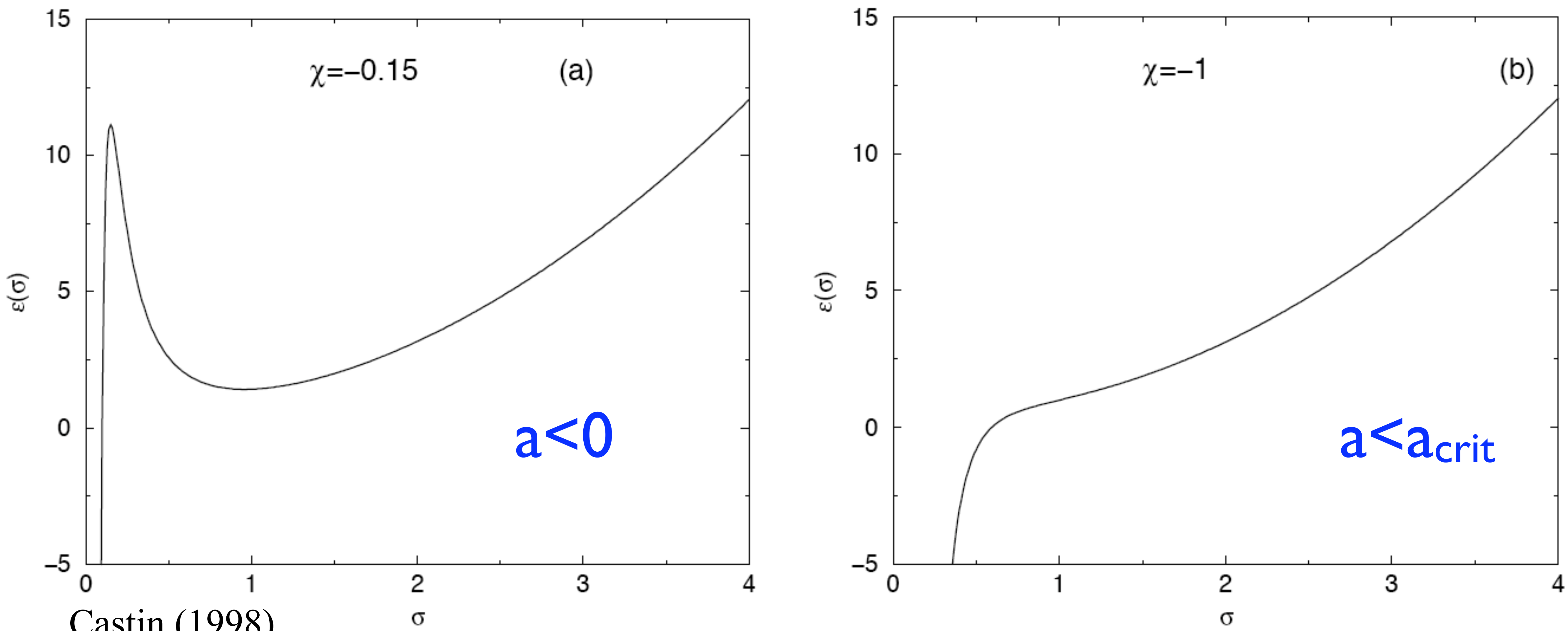
Gaussian ansatz: energy vs. size



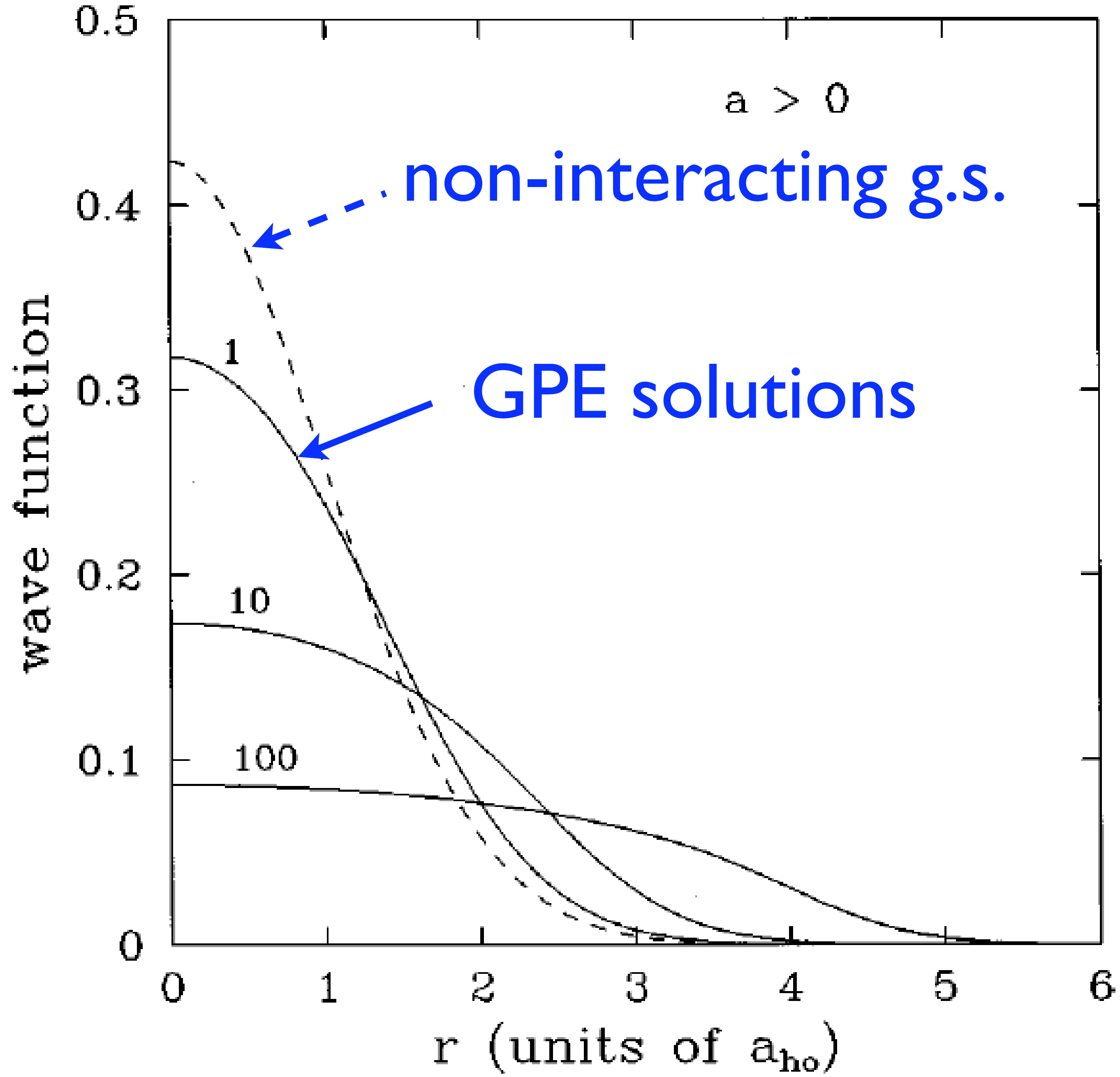
a>0
(repulsive)

Castin (1998)

attractive (a<0) case



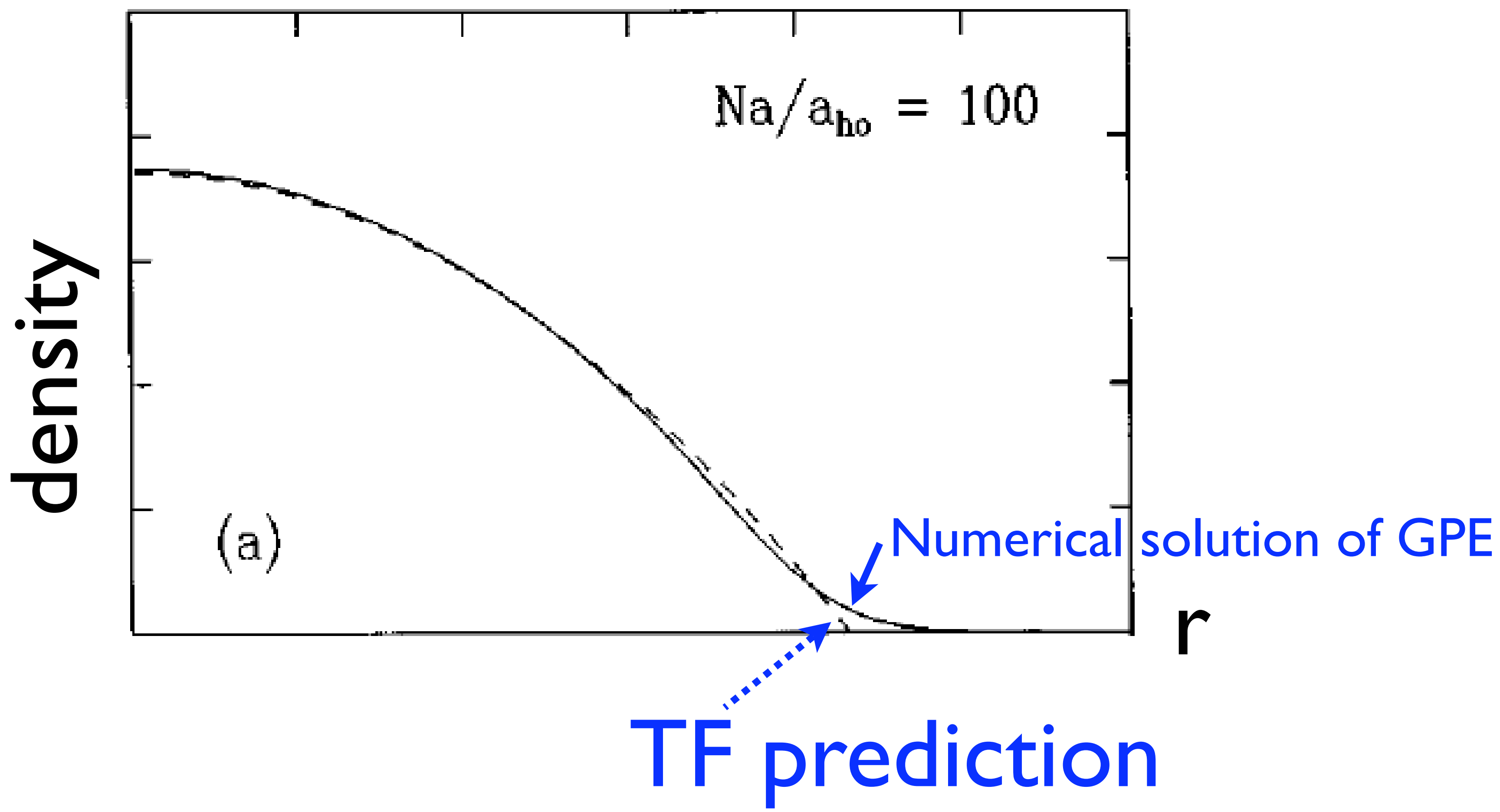
Castin (1998)



$N|a|/\sigma_{\text{ho}}$

Broadening of the BEC

Edge effects



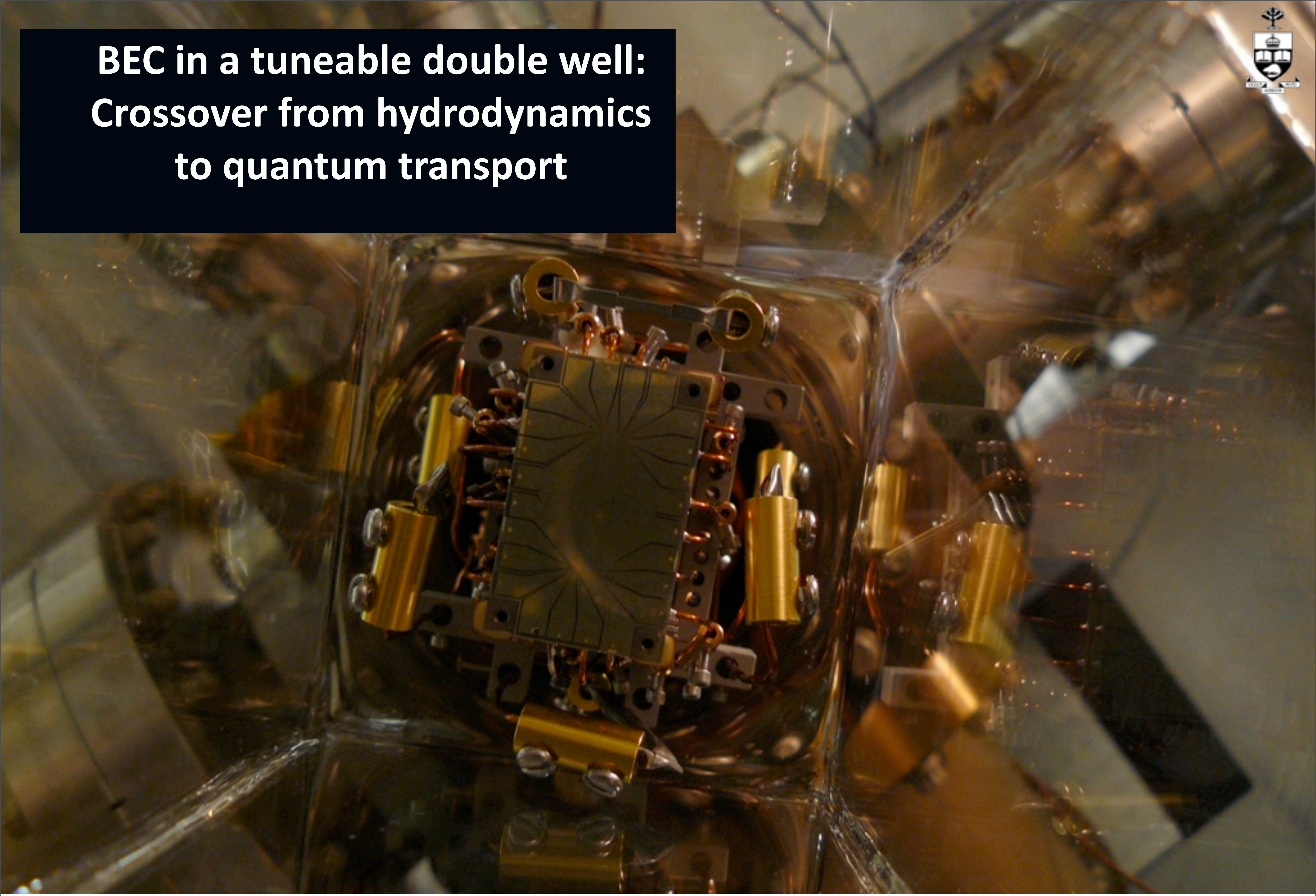
Is a superfluid a perfect fluid?

..or is there more to it?

Equations for a perfect fluid

$$
\begin{aligned}
\frac{\partial}{\partial t}\rho &= -\vec{\nabla}\cdot(\rho\,\vec{v}_s) \\
m\left(\frac{\partial}{\partial t}+\vec{v}_s\cdot\vec{\nabla}\right)\vec{v}_s &= -\vec{\nabla}\left[U+g\rho\right], \\
\vec{\nabla}\times\vec{v}_s &= 0
\end{aligned}
$$

BEC in a tuneable double well: Crossover from hydrodynamics to quantum transport



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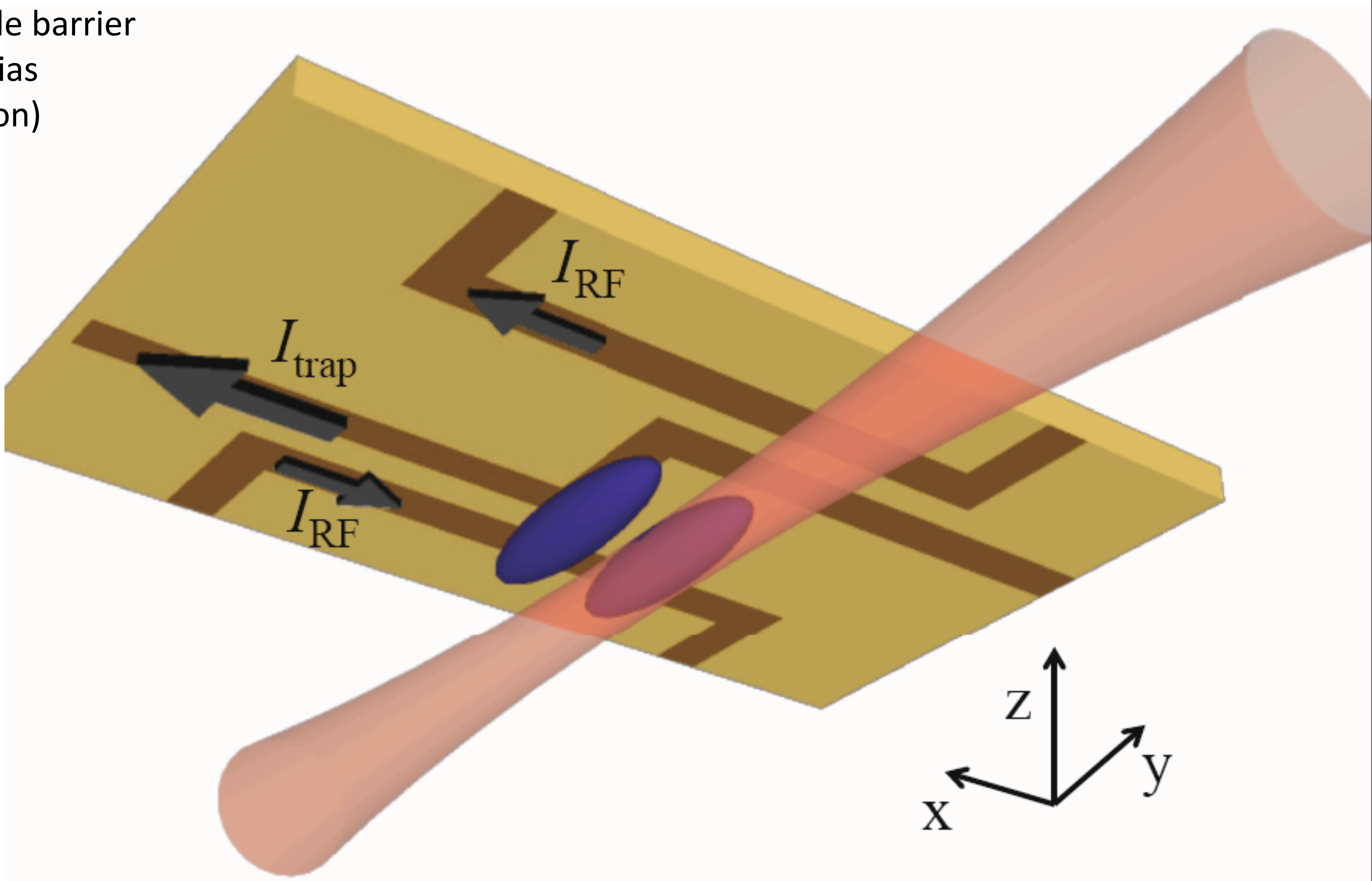
RF-dressed magnetic + optical trap

- Variable separation
- Adjustable barrier
- Optical bias

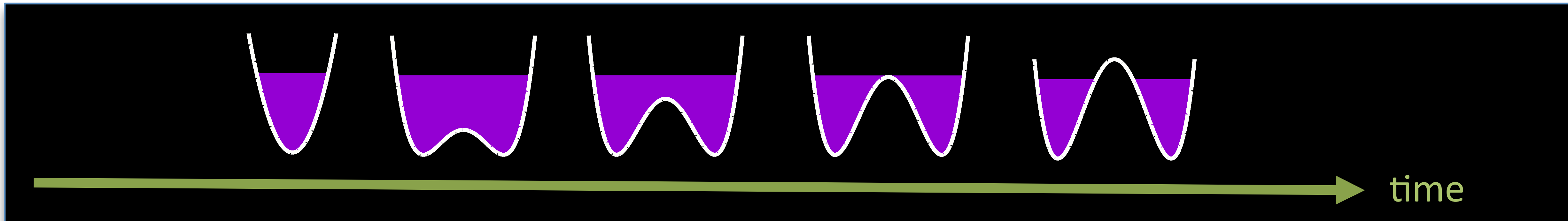
(+ levitation)

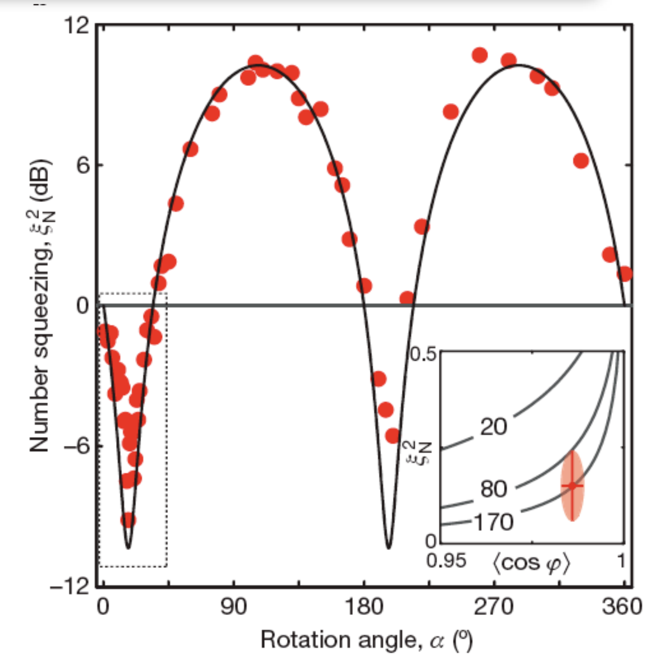


<u>RF-dressed traps:</u>

Schmiedmayer, Zobay, Perrin, DeMarco, Ketterle, van Druten, Phillips/Spielman/Porto, ...

Adiabaticity & generation of entangled states

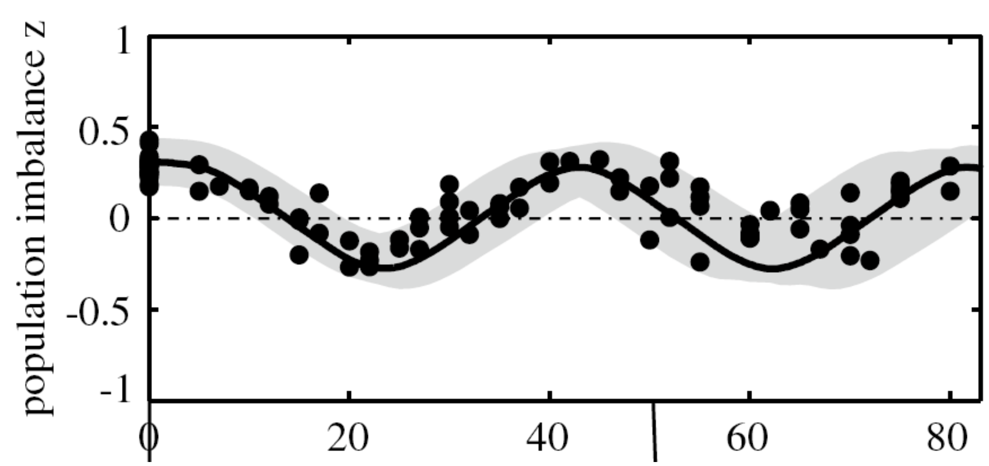




•Sweeping the barrier height used to create squeezed states, with application to clock improvement:
-Ketterle (2008,9)
-Oberthaler (2010)
-related: Vuletic (2010)

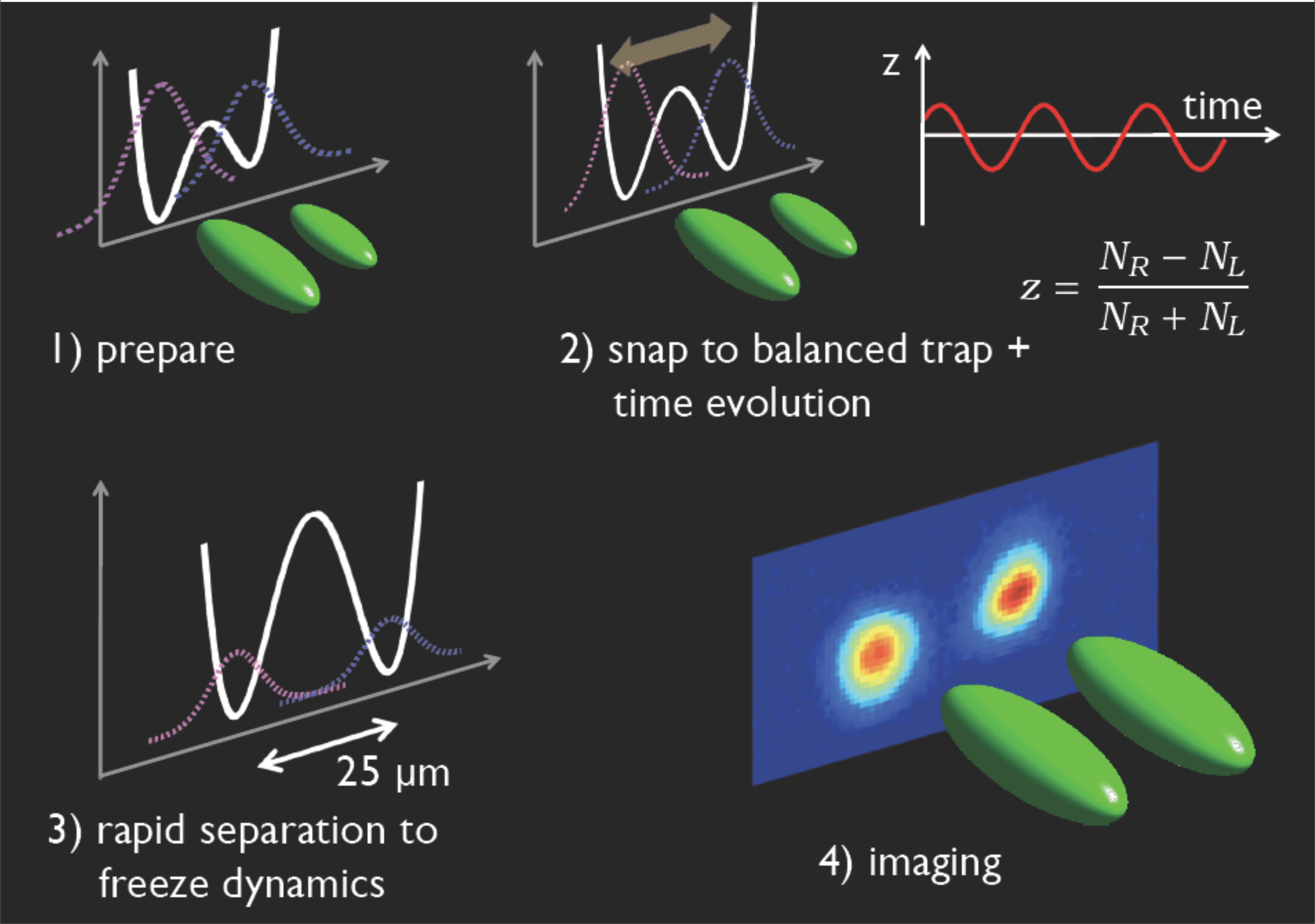
•Many-body adiabaticity necessary for these applications.
But do we understand dynamics throughout the single- to double-well crossover?

•Previous work observed Josephson-type plasma oscillations.
-Oberthaler (2005)
-Steinhauer (2007)

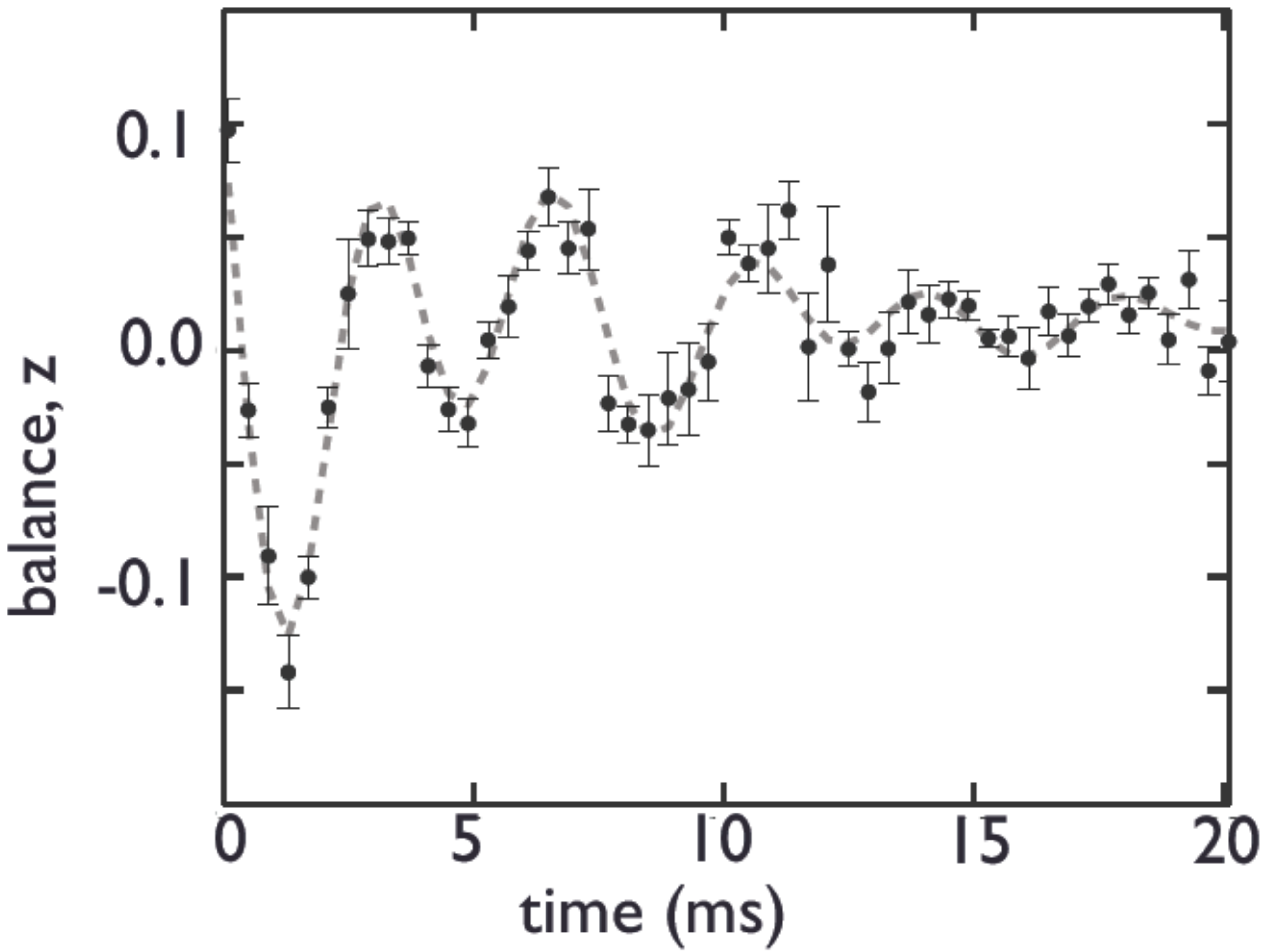


We observe multi-frequency dynamics.

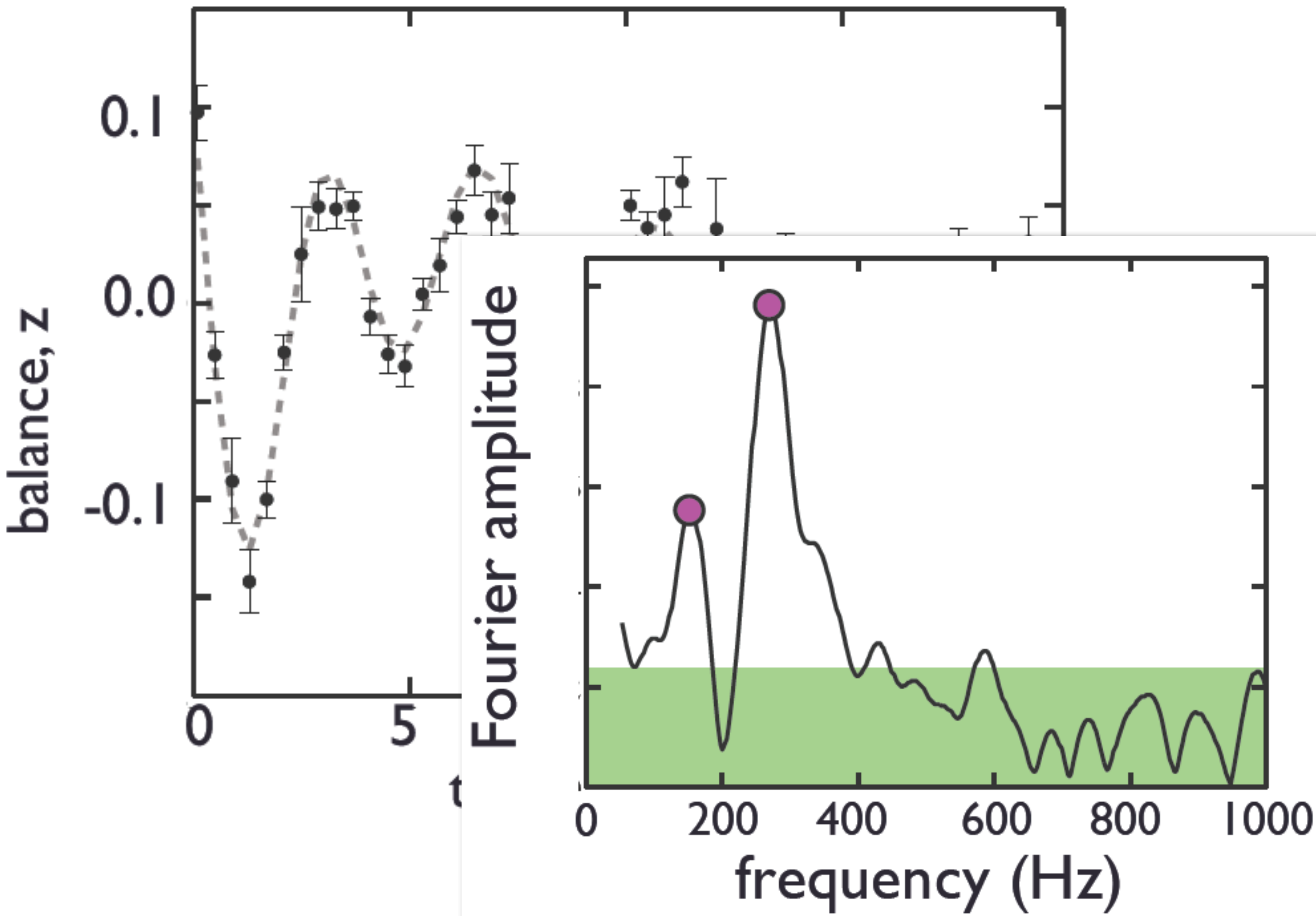
Measuring transport dynamics



Two-frequency dynamics



Two-frequency dynamics



Hydrodynamics

Writing the BEC wave function as $\Phi(r,t) = \sqrt{\rho(r,t)}e^{i\phi(r,t)}$

and defining the local velocity as $\vec{v}_s(\vec{r},t) = \frac{\hbar}{m}\vec{\nabla}\phi(\vec{r},t)$

immediately the fluid must be irrotational, and will follow perfect fluid equations

$$\begin{aligned} \frac{\partial}{\partial t}\rho &= -\vec{\nabla}\cdot(\rho\,\vec{v}_s) \\ m\left(\frac{\partial}{\partial t} + \vec{v}_s\cdot\vec{\nabla}\right)\vec{v}_s &= -\vec{\nabla}\left[U + g\rho\right] \end{aligned}$$

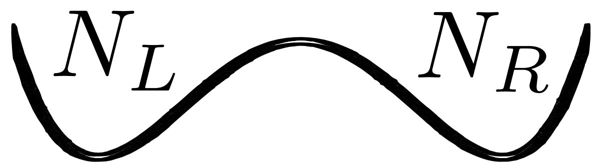
if we neglect a ``quantum pressure'' term. This is equivalent to a dynamic Thomas Fermi or local density approximation. The criterion for validity is that the **healing length must be much smaller than the system size.**

Two-mode model

Two-mode hamiltonian, zero T:

$$H = E_c \frac{n^2}{2} - E_J \sqrt{1 - \frac{4n^2}{N^2}} \cos\phi$$

interaction   tunnelling



relative number:

$$n \equiv (N_L - N_R)/2$$
$$z \equiv (N_L - N_R)/N$$

non-dimensional form:

$$H = \frac{\Lambda}{2} z^2 - \sqrt{1 - z^2} \cos\phi$$

interaction parameter:

$$\Lambda = N^2 E_c / 4E_J$$

small oscillations:

$$\omega_P = \frac{1}{\hbar} \sqrt{E_J \left(E_c + \frac{4E_J}{N^2} \right)}$$

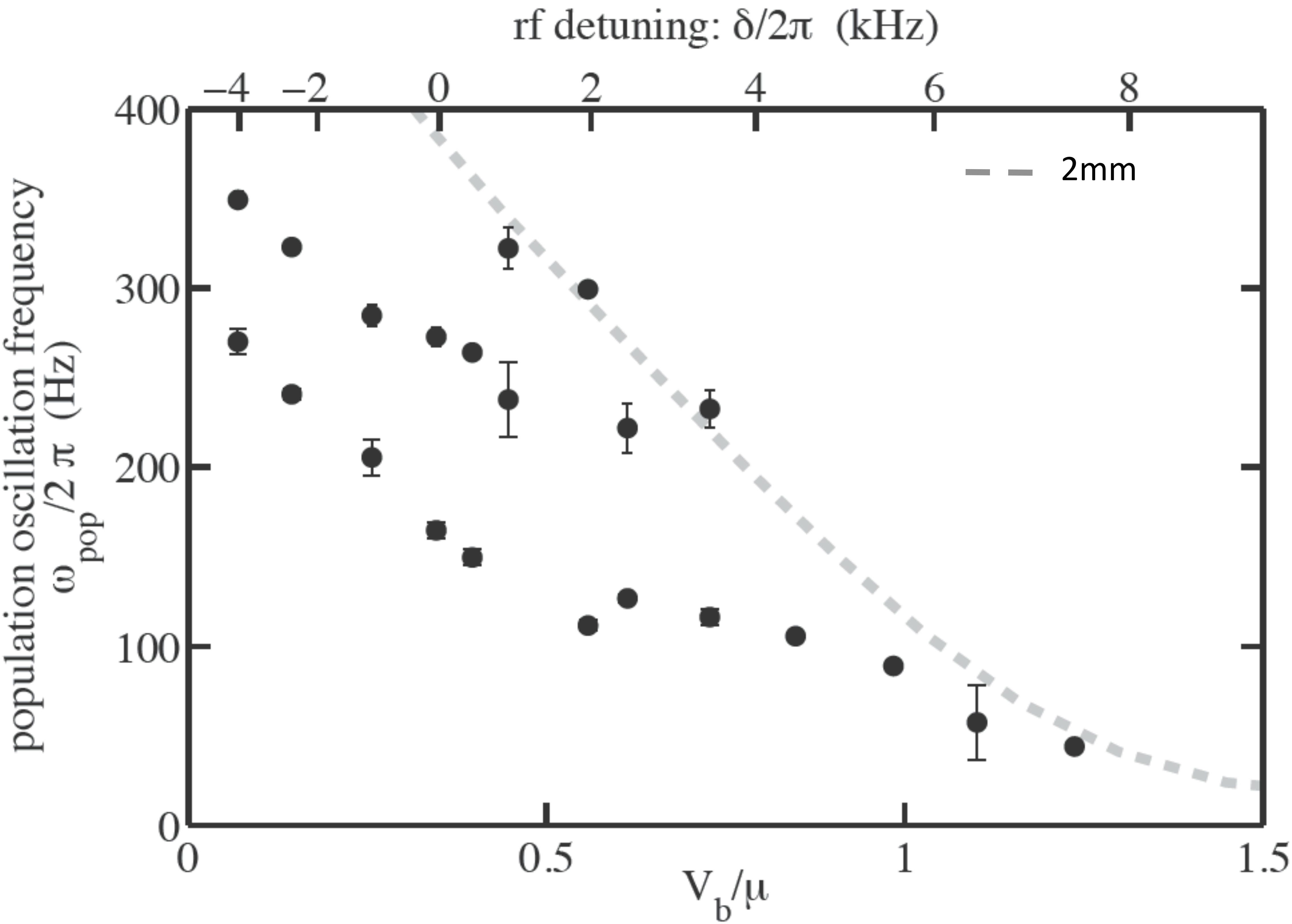
$$\xrightarrow{\Lambda \ll 1} 2E_J/N\hbar \equiv \omega_R \quad \text{(Rabi regime)}$$

$$\xrightarrow[1 \ll \Lambda \ll N^2]{} \sqrt{E_J E_c}/\hbar \quad \text{(Josephson regime)}$$

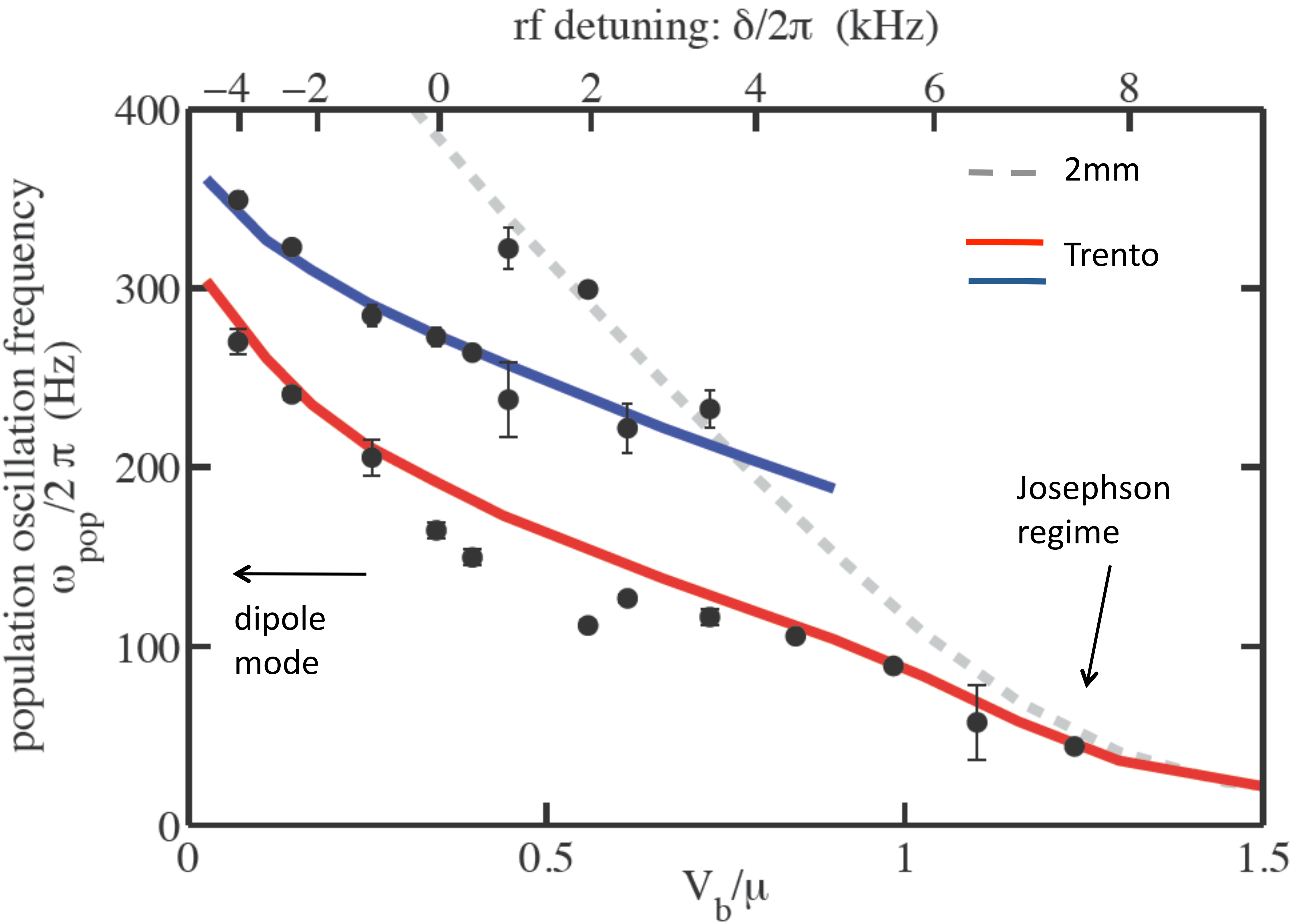
"Plasma frequency"
or "Josephson frequency"

Refs:
Gati and Oberthaler, J Phys B (2007), and references therein.
(Anderson, Leggett, Javanainen, Sipe, Smerzi, Sols, Walls, ...)

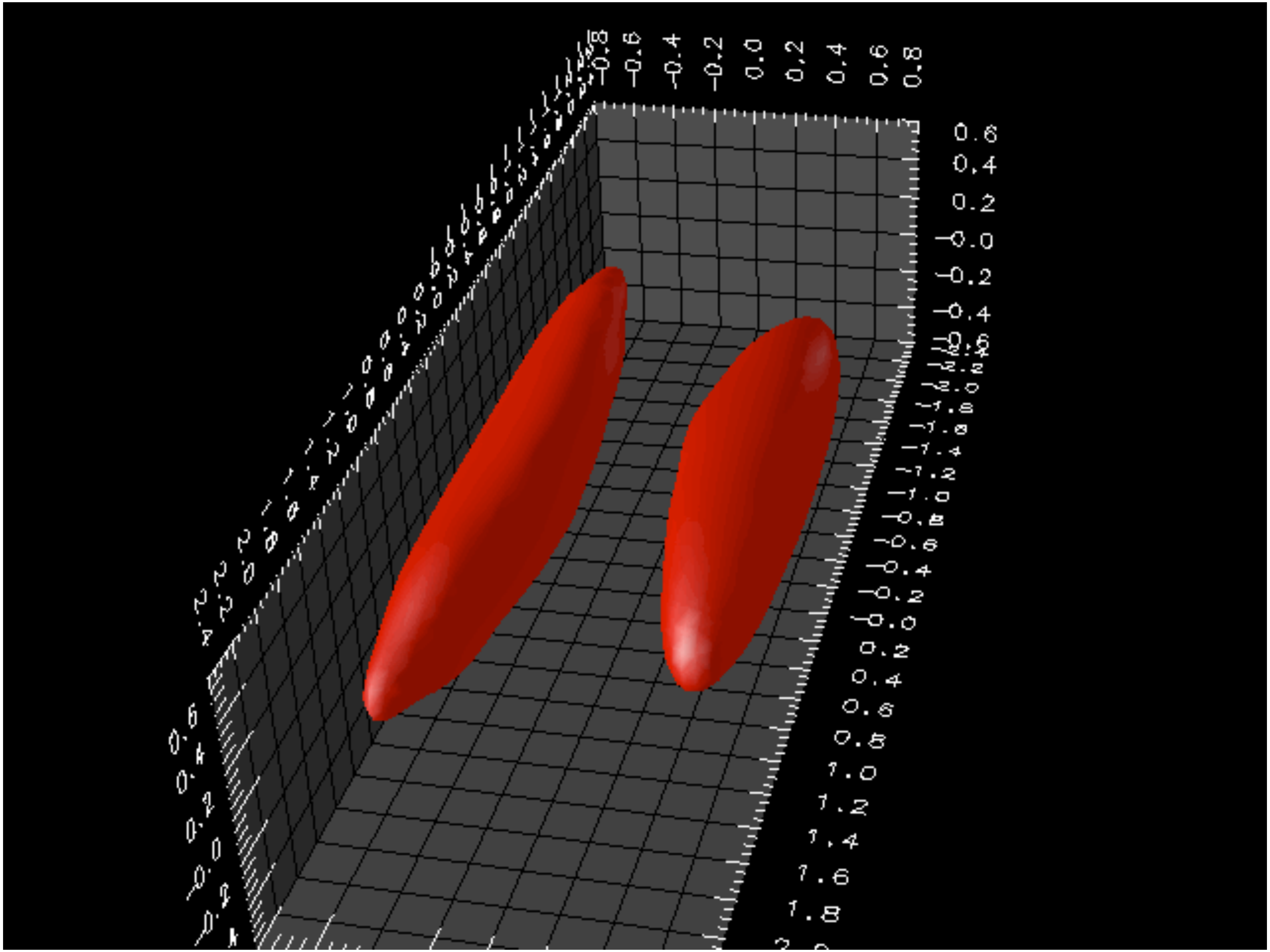
Frequency vs. barrier height



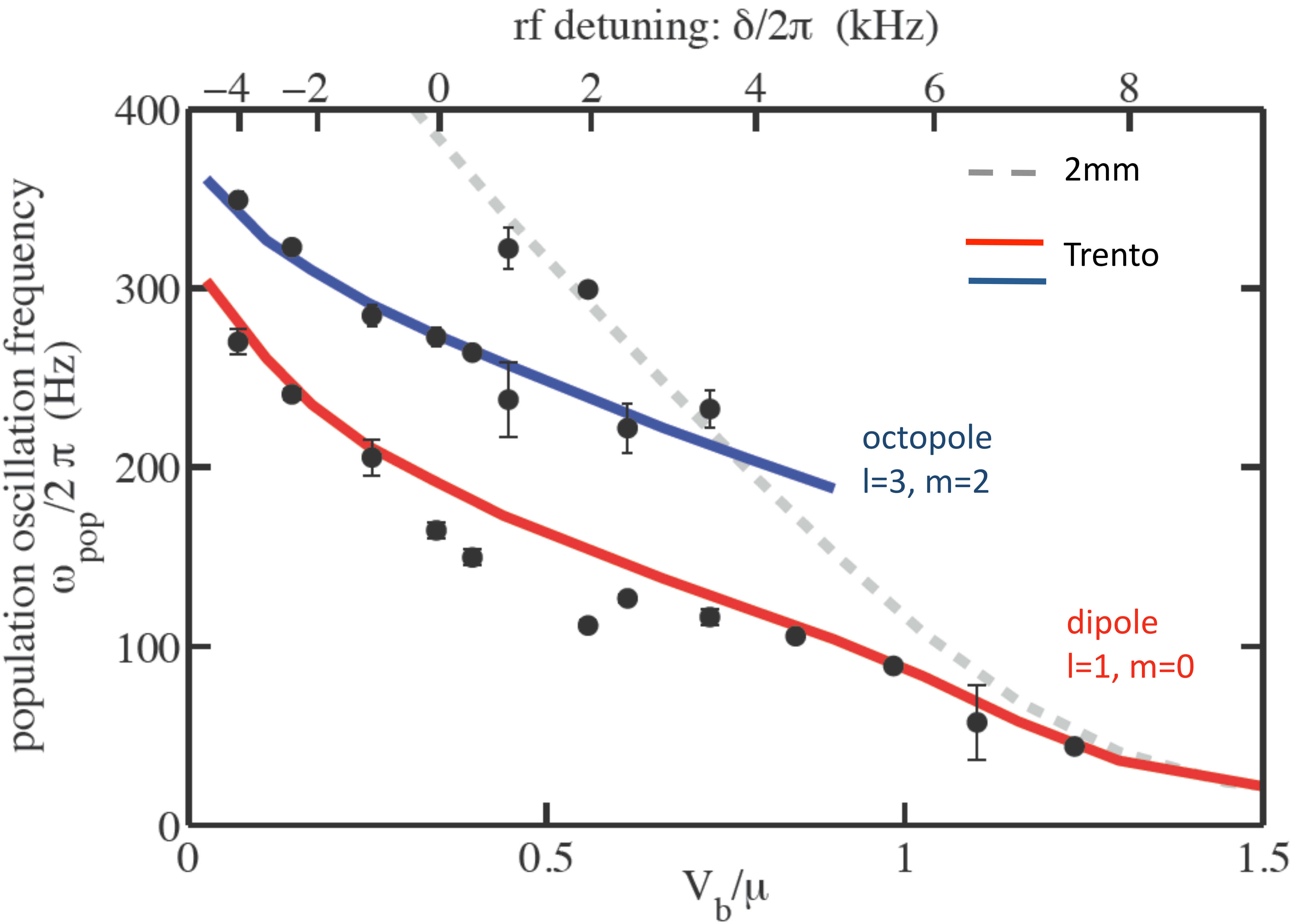
Frequency vs. barrier height



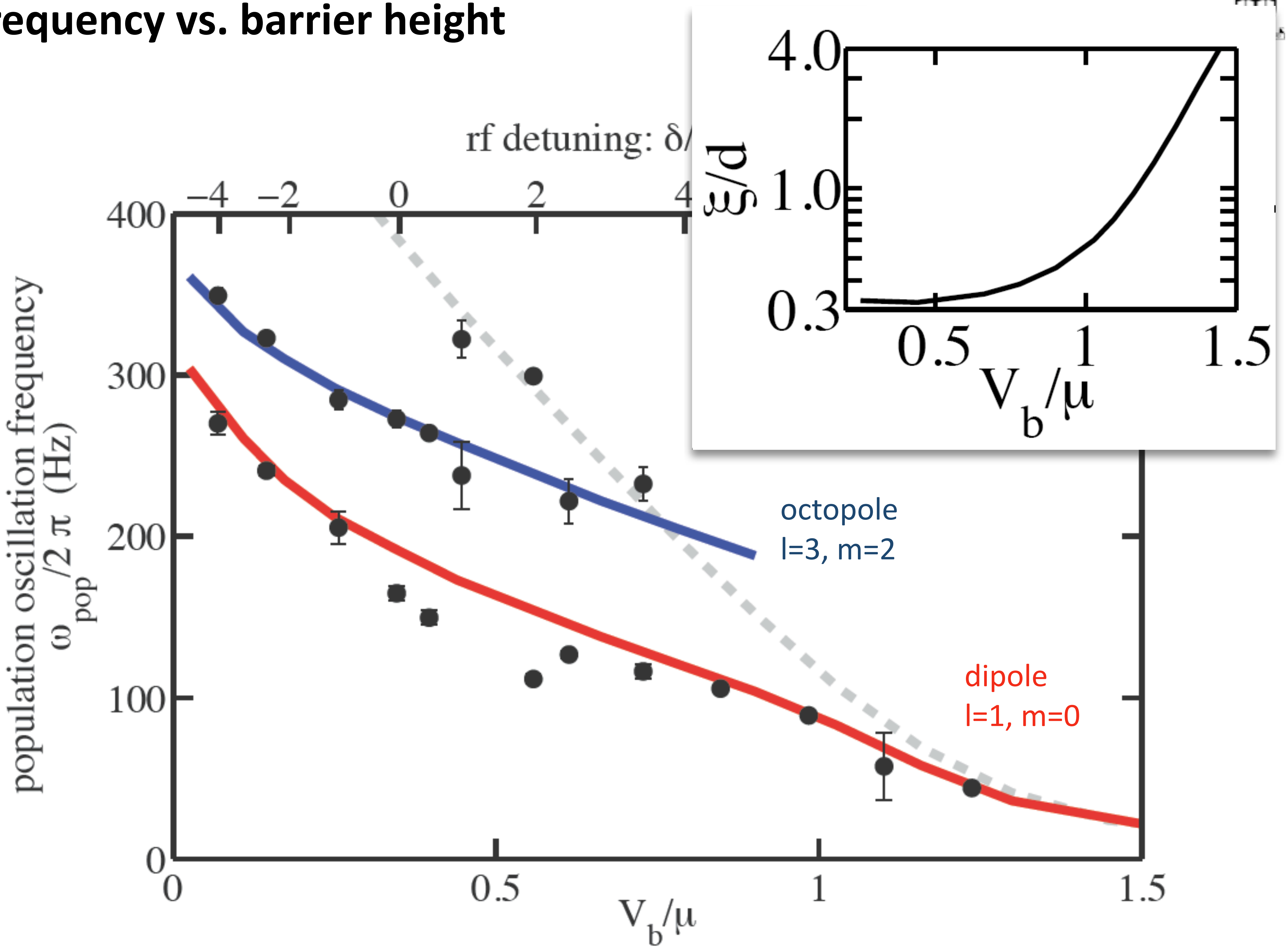
Nature of higher mode?



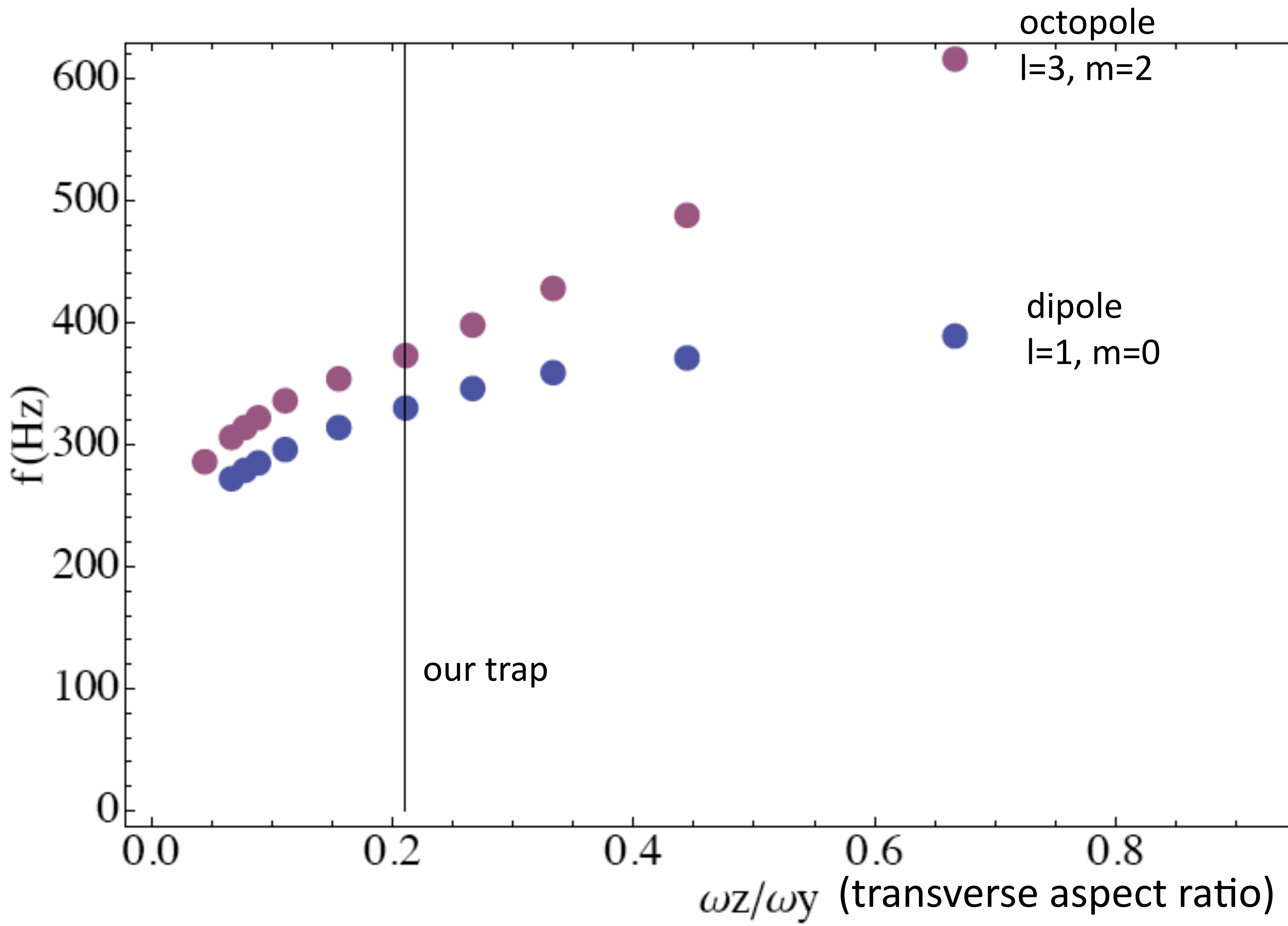
Frequency vs. barrier height



Frequency vs. barrier height



Who will rid me of this turbulent mode?



->important in traps without axial symmetry

Conclusions & Questions

•Observed quantum transport in a hybrid magnetic trap
-Explore transition from the Josephson regime to Hydrodynamic regime
-High quality dynamics reveals a surprising richness in structure

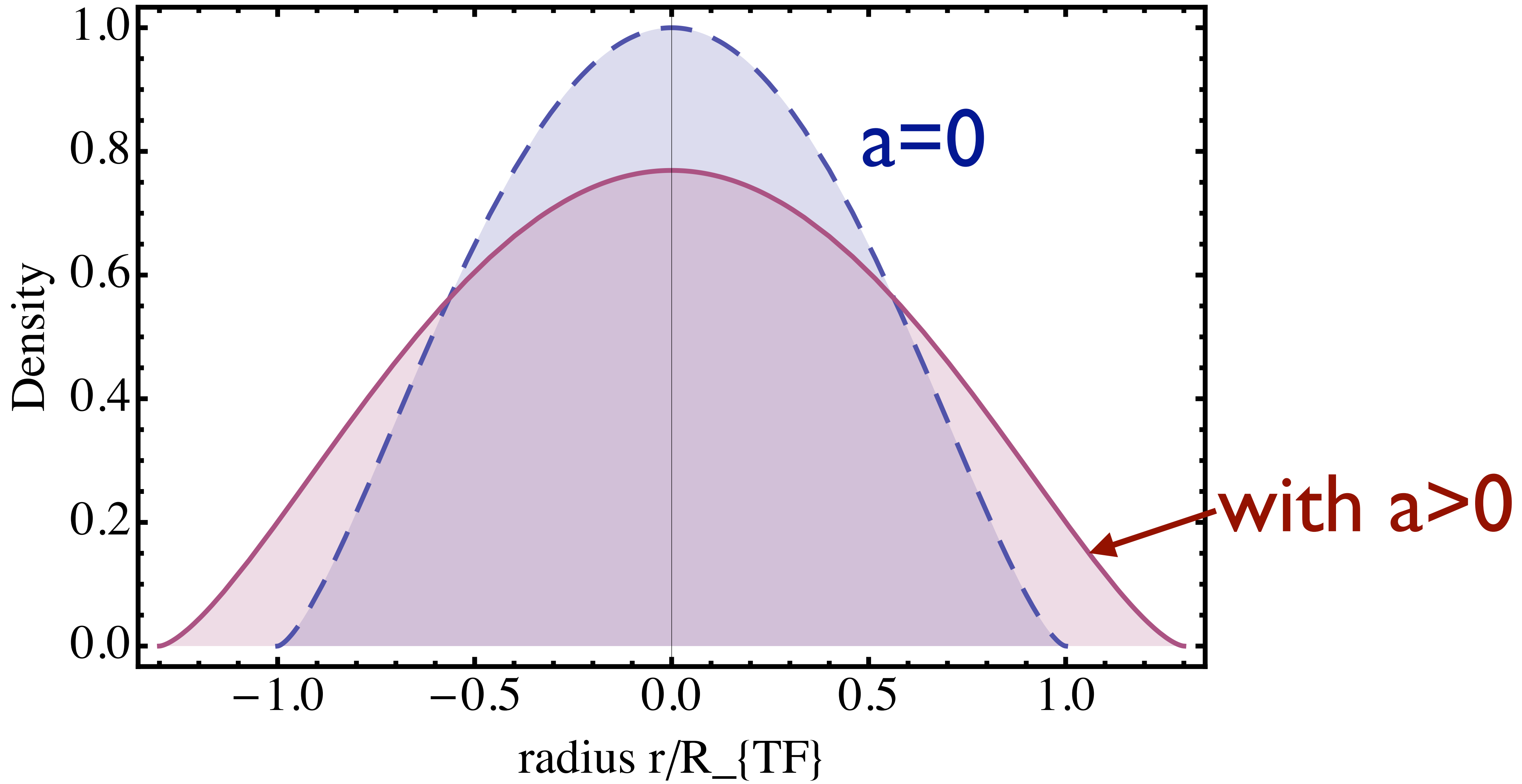
•Mode structure includes not only lowest dipole/Josephson mode, but also a low-lying octopole mode.
-2-mode-model fails at Vb ~ 1.2 μ
-new modes appear at Vb ~ 0.9 μ

•Open questions:
-Damping of Josephson mode
-Decay of Self-trapped state (not shown here)

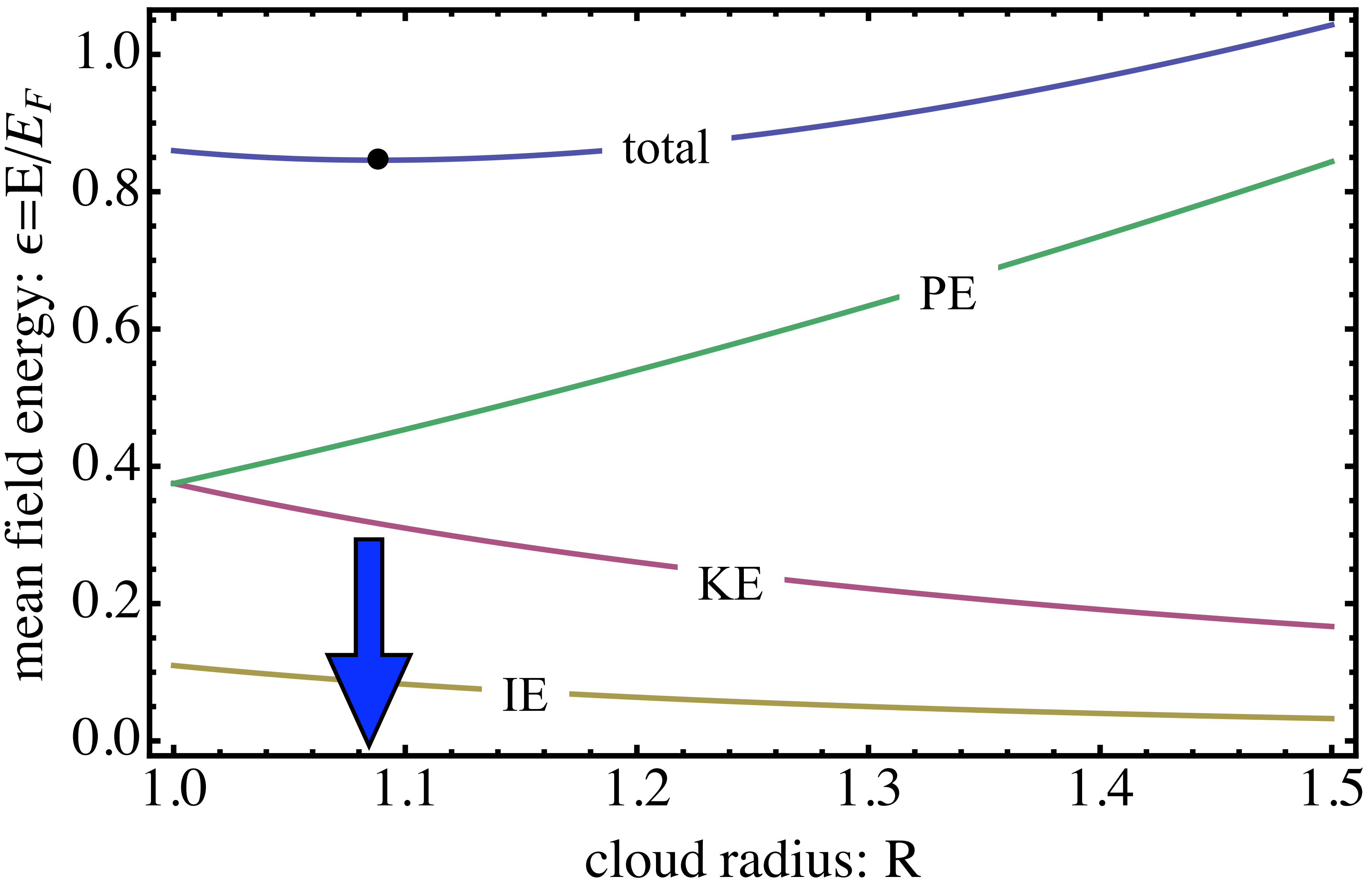
Interacting Fermi gases

3.1 Mean field: variational solution

TF ansatz:

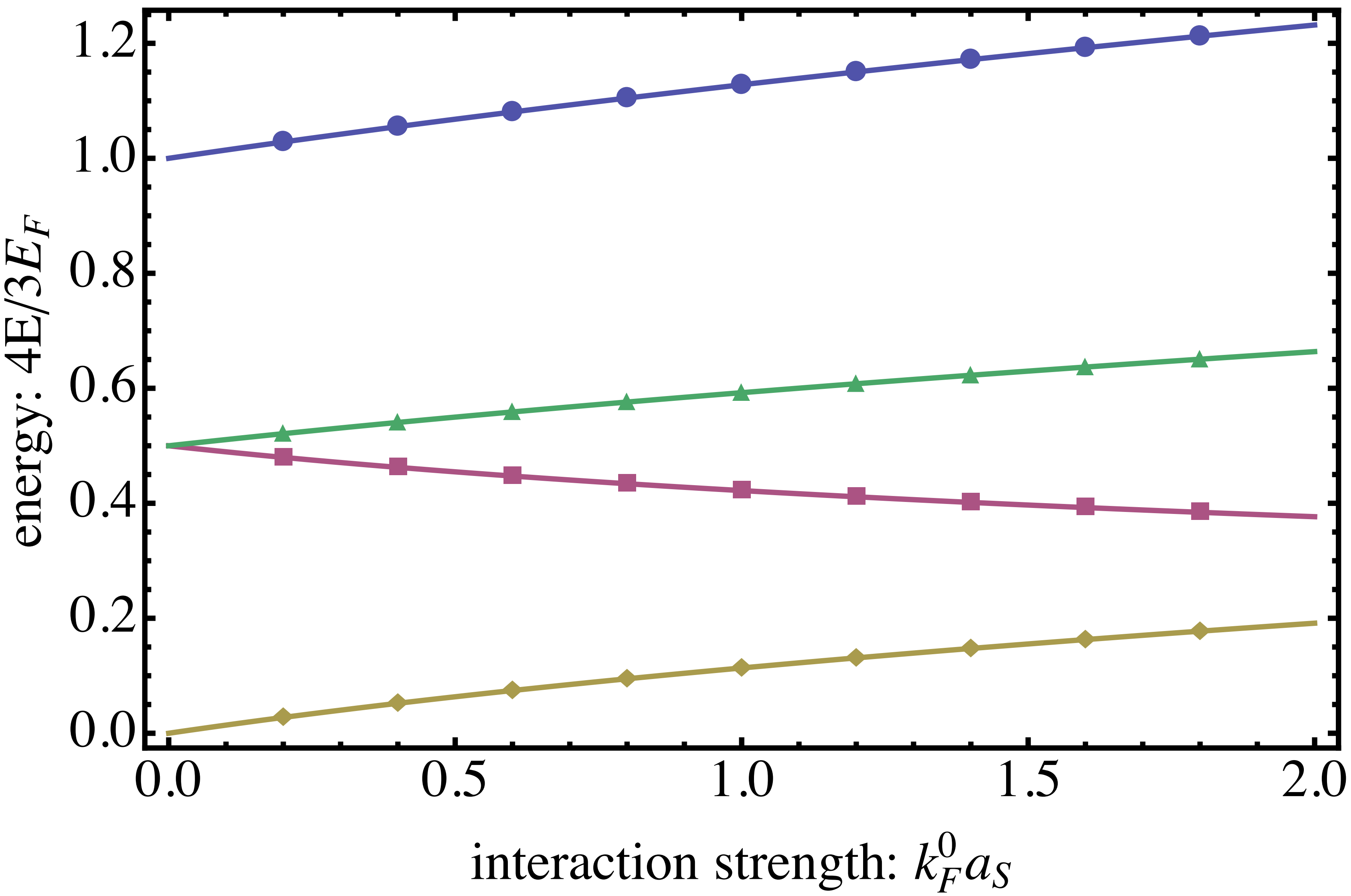


Variational solution: example of minimization

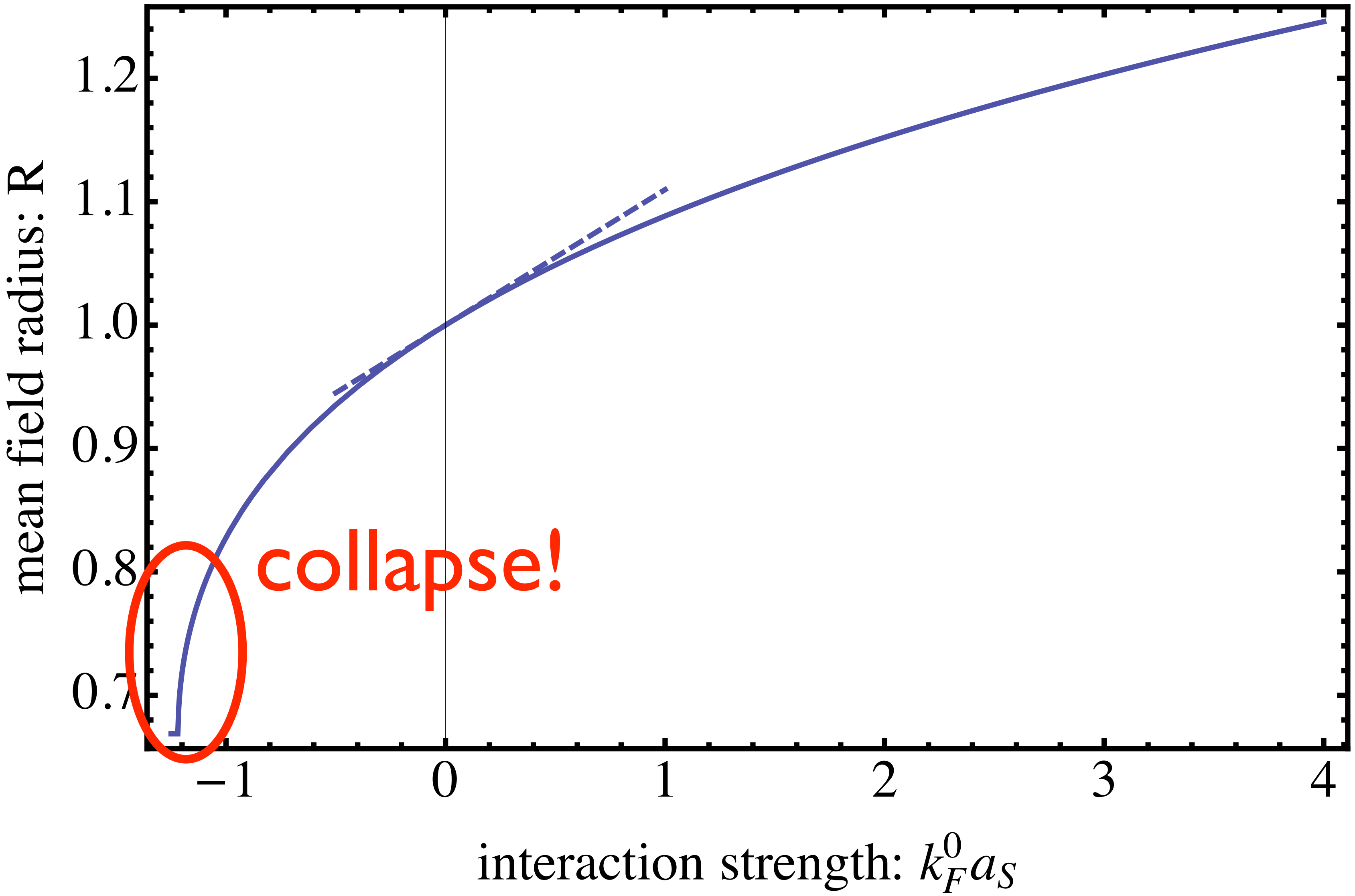


@$k_F a$=1

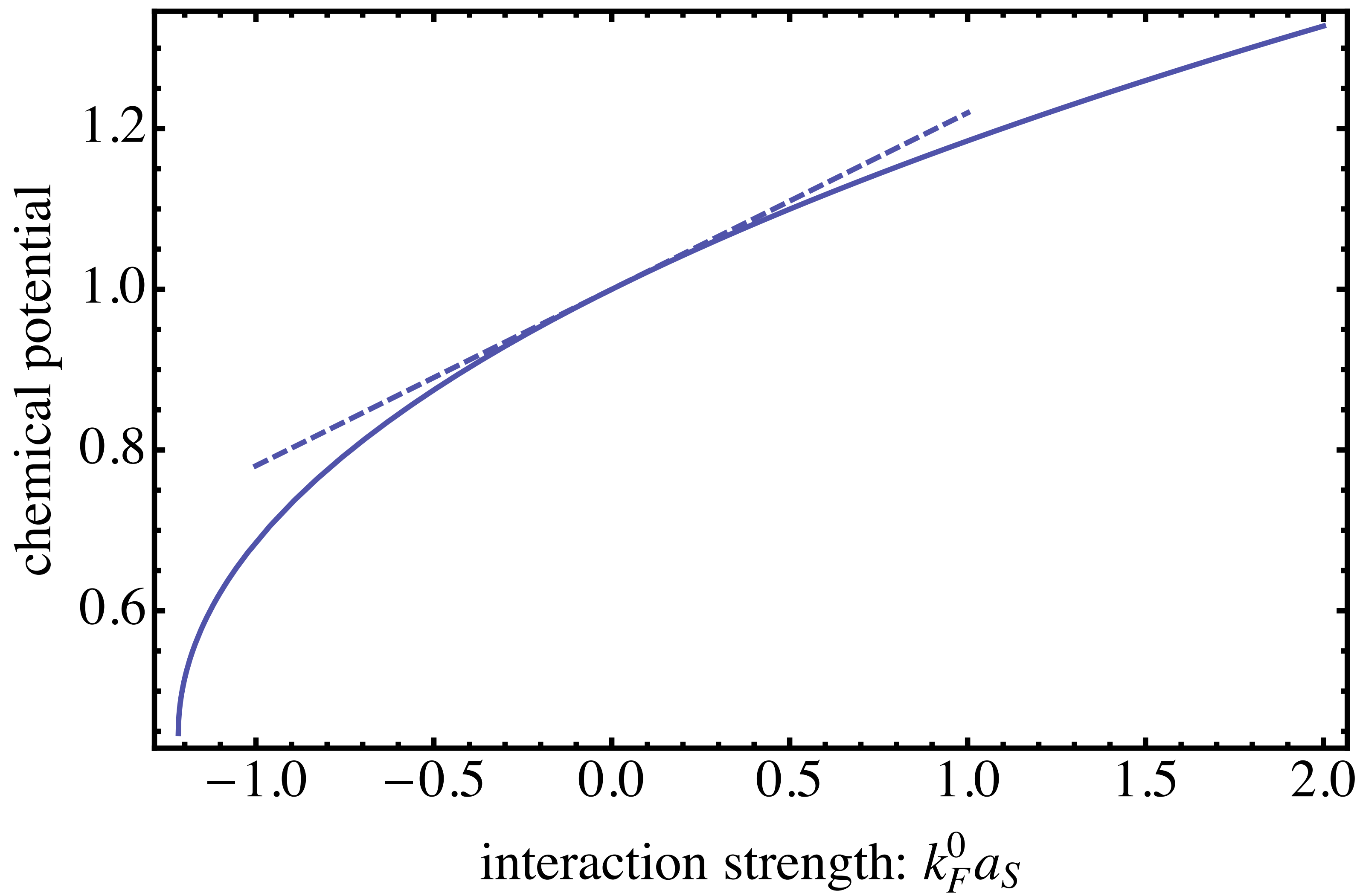
Comparison of TF ansatz energies to energy functional minimization



Radius of TF ansatz soln

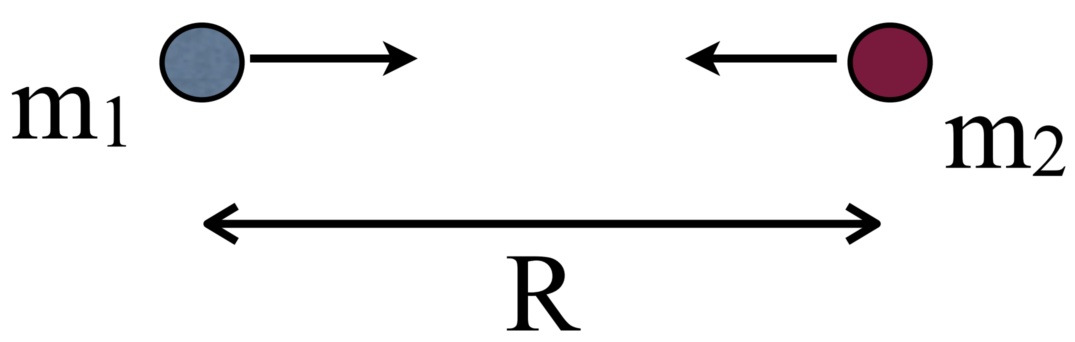


mean field chemical potential from TF ansatz



3.2 Scattering theory

Two-body collision

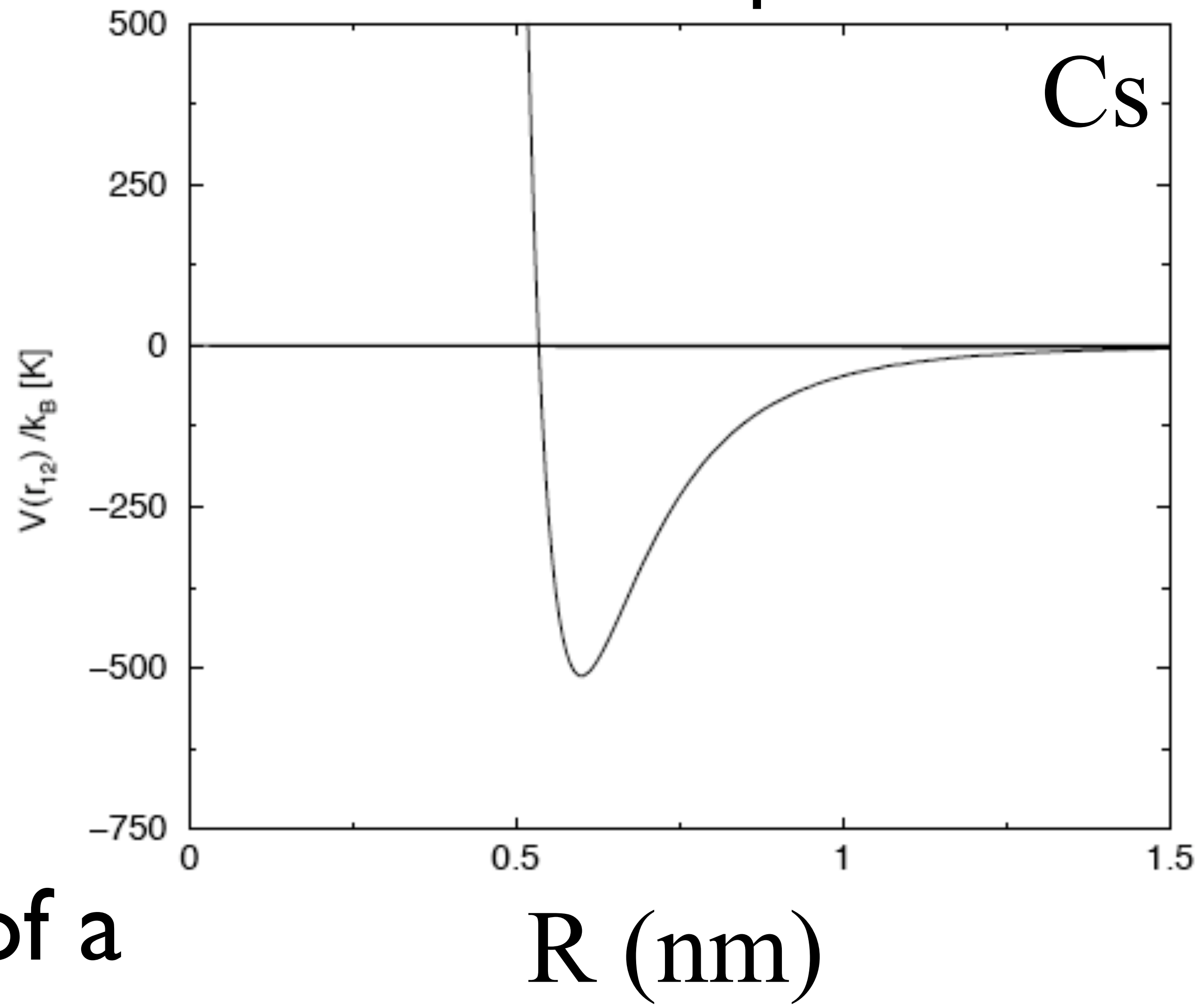


use an effective mass

$$\mu = m_1 m_2/(m_1 + m_2)$$

and write equations in terms of a single spatial degree of freedom, R.

inter-atomic potential



Setting up the problem

Assuming a potential with spherical symmetry, we can treat each (spatial) angular momentum separately:

$$\psi_\ell(R) = \phi_\ell(R)/R \qquad \ell = 0, 1, 2, \ldots \text{ for } s-, p-, d-, \ldots$$

Task: solve the Schrödinger equation

$$-\frac{\hbar^2}{2\mu}\frac{d^2\phi_\ell(R)}{dR^2} + V_\ell(R)\phi_\ell(R) = E\phi_\ell(R)$$

Start with an incoming plane wave with $E = \hbar^2 k^2/(2\mu)$

After collision, end up with phase-shifted plane wave:

$$\phi_\ell(R, E) \rightarrow c\frac{\sin(kR - \pi\ell/2 + \eta_\ell(E))}{\sqrt{k}} e^{i\eta_\ell(E)}$$

Centrifugal barrier

The potential in center-of-mass coordinates includes a centrifugal barrier:

$$V_\ell(R) = V(R) + \hbar^2 \ell(\ell+1)/(2\mu R^2)$$

This barrier is repulsive for l>0 but vanishes for l=0. Since its height is ~ 0.1mK, practically restricts ultracold atoms to s-wave collisions!

For *s-wave collisions*, our outgoing wave function has a phase shift defined by

$$k \cot \eta_0(E) = -\frac{1}{a} + \frac{1}{2} r_0 k^2$$

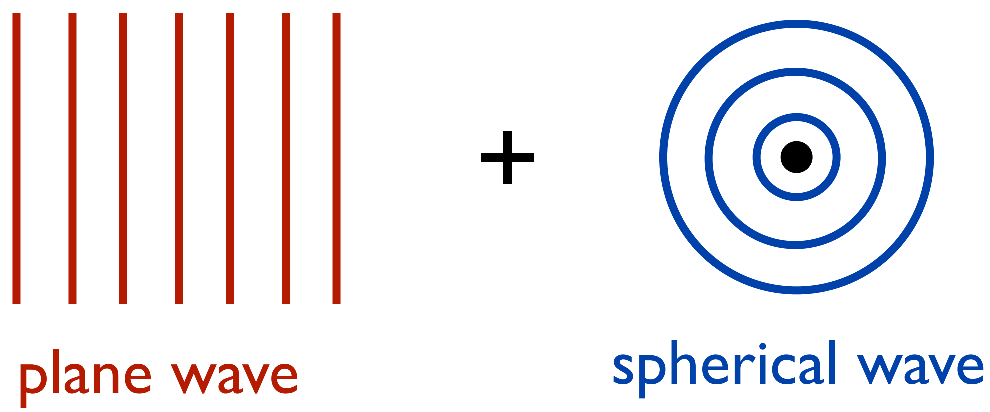
a = *scattering length*

r_0 = *effective range* (neglect at small k)

Scattering amplitude

Another way to write the scattered wave function is

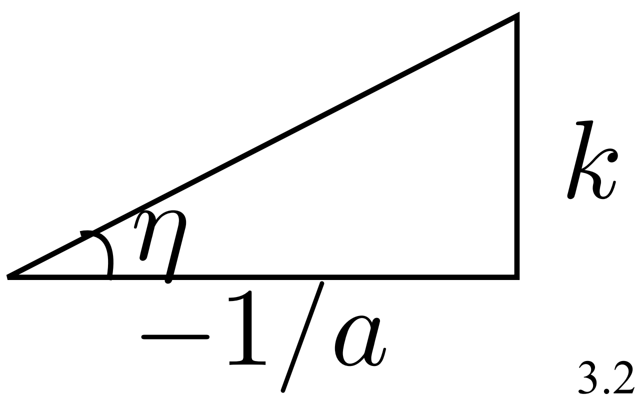
$$\psi_{\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} - \frac{a}{1+ika}\frac{e^{ikr}}{r}$$



The scattering term has an amplitude

$$f_k = -[1/a + ik]^{-1}$$ "scattering amplitude"

The phase shift of the scattered wave is its complex argument:



Results of low-energy scattering theory

The total cross section for scattering is given by

$$\sigma = 4\pi |f_{\vec{k}}(\vec{n})|^2 = \frac{4\pi a^2}{1 + k^2 a^2},$$

Results of low-energy scattering theory

The total cross section for scattering is given by

$$\sigma = 4\pi |f_{\vec{k}}(\vec{n})|^2 = \frac{4\pi a^2}{1 + k^2 a^2},$$

1. This is written for *non-identical particles*, like two different spin states colliding. For *identical bosons*, there is an additional factor of 2; for *identical fermions*, $\sigma = 0$.

2. In the low-k limit, get the well-known $\sigma = 4\pi a^2$, valid for weakly interacting degenerate atoms.

3. Note that the cross-section does not depend on the sign of the scattering length. Additional measurements are necessary to distinguish attractive (a<0) from repulsive (a>0).

What have we left out?

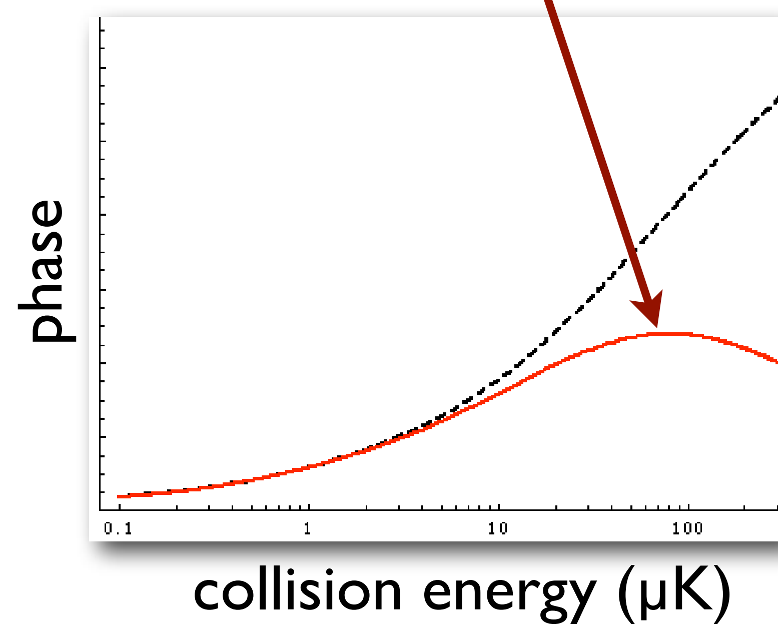
For higher relative momentum (but still below the p-wave threshold), need to know more than the scattering length.

$$f^{\ell=0} = -\left[1/a + ik + \frac{1}{2}k^2 r_e + \ldots\right]^{-1}$$

real theory

"Naive" theory, giving

$$\sigma = \frac{4\pi a^2}{1 + a^2 k^2}$$



effective range term reduces phase for a<0

Beyond the scattering length

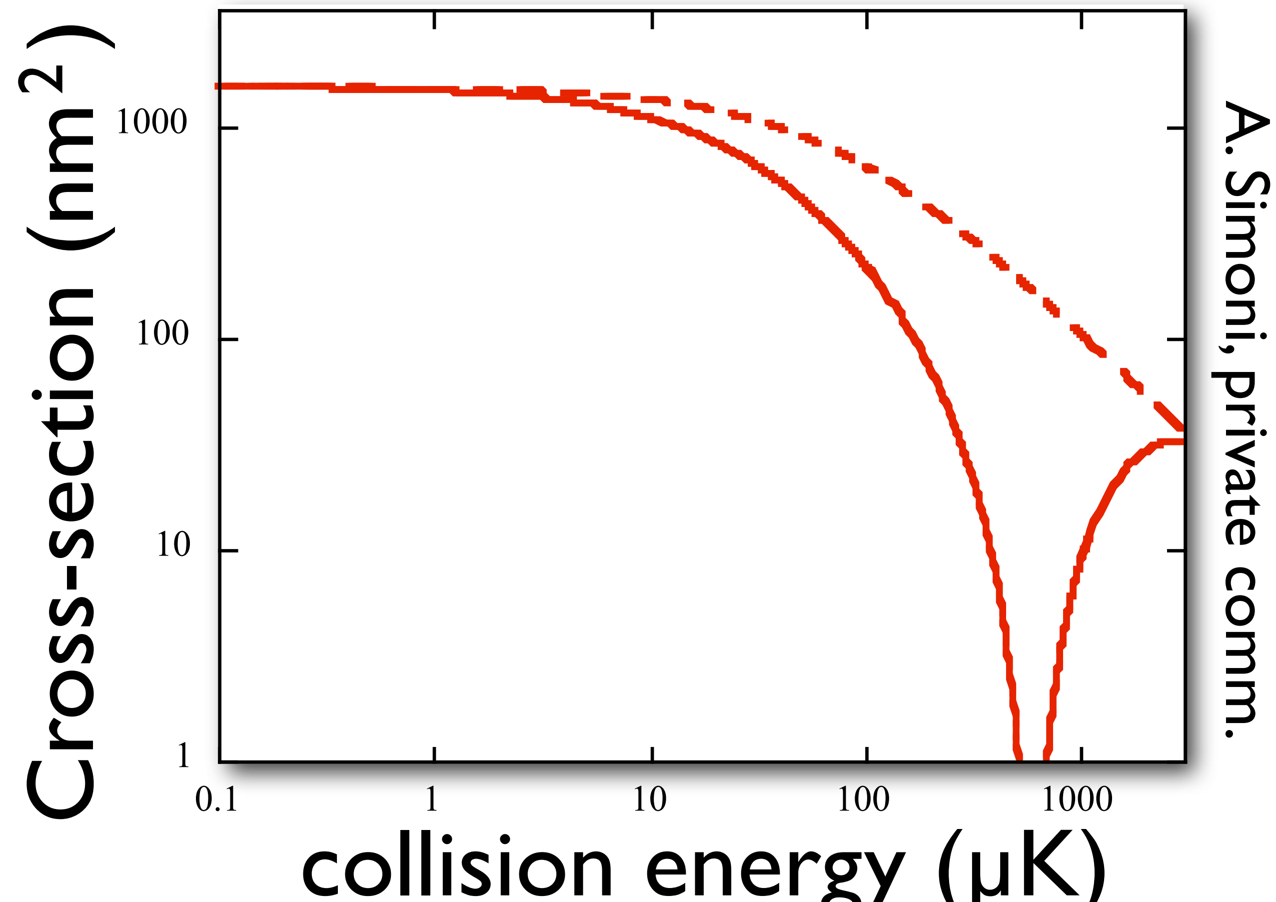
A reduction in phase will mean a reduction in cross-section:

$$\sigma_0 = \frac{4\pi}{k^2}\sin^2\eta_0$$

Ramsauer-Townsend effect

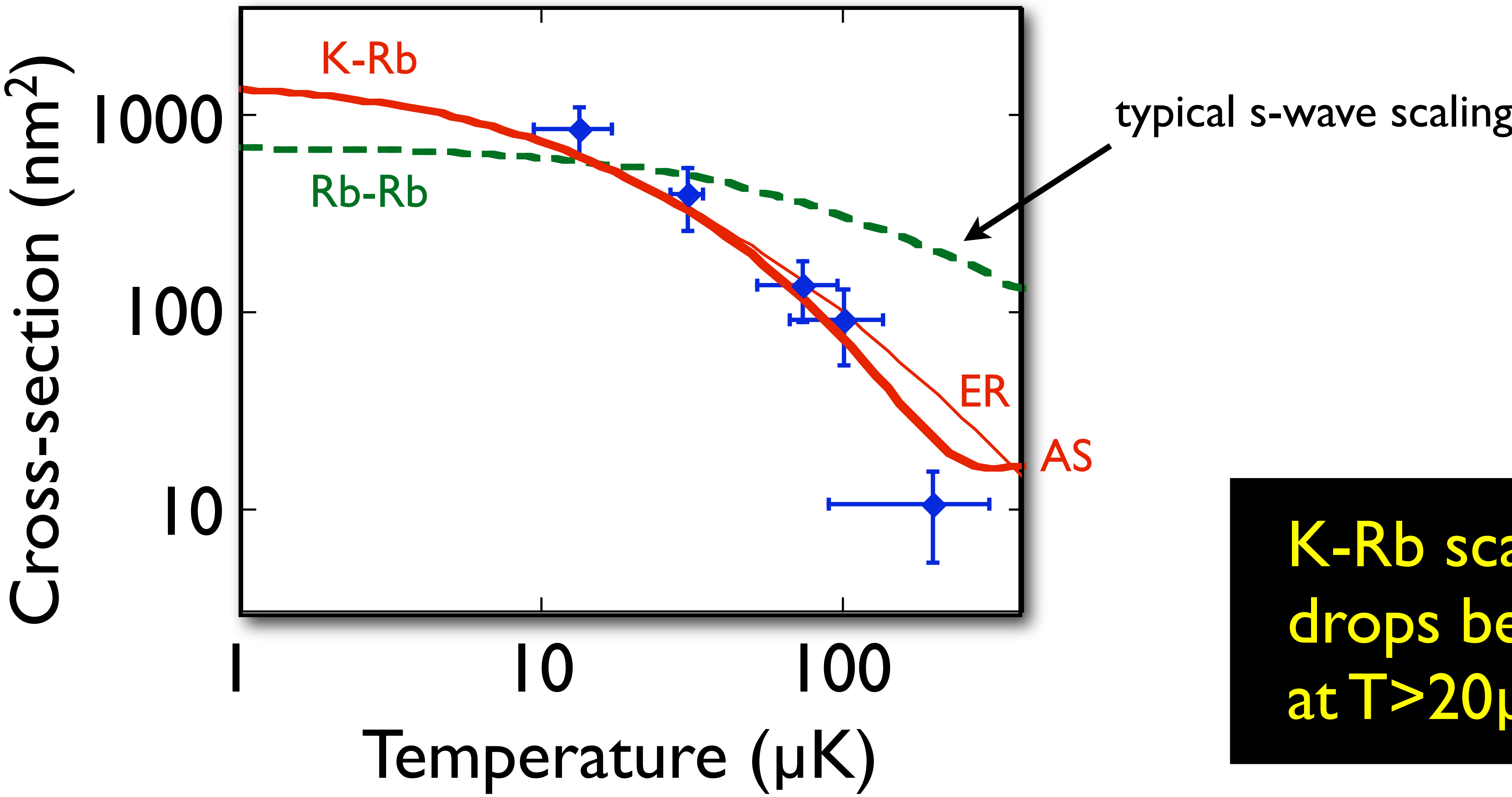
If $\eta_0 \to 0$ then cross-section can vanish!

For K-Rb, zero **@595μK** (collision energy)



(Effective range reduction is a precursor to Ramsauer-Townsend effect in ^{40}K-^{87}Rb scattering.)

Ramsauer-Townsend effect: observation



K-Rb scattering drops below Rb-Rb at T>20μK.

Physics: scattering phase drops to zero

Consequence: Slows down sympathetic cooling of 40K by 87Rb.

S. Aubin et al., *Nature Physics* **2**, 384 (2006)

See also Dalibard; Salomon; Wieman

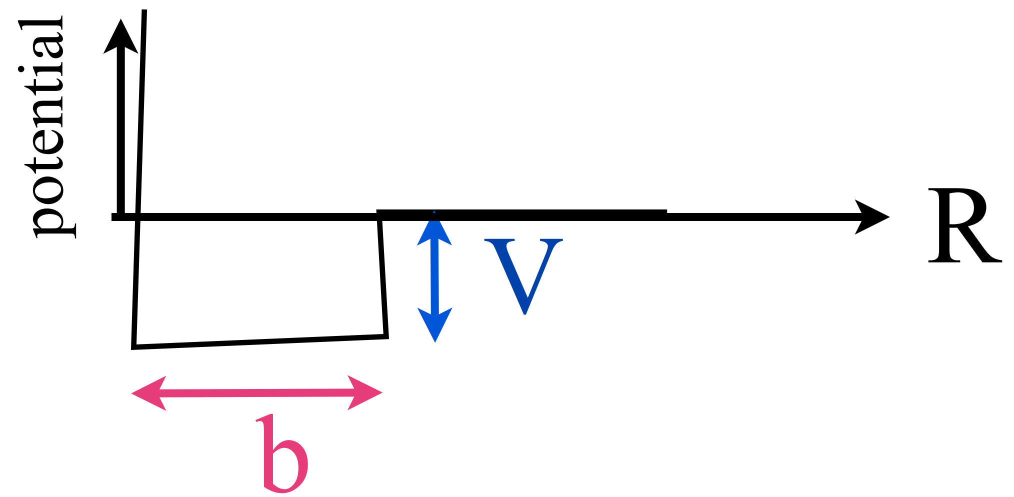
Equivalence of scattering potentials

Since the s-wave scattering results in only a single phase shift, parameterized by **a**, it doesn't matter what the full form of the scattering potential was!

Equivalence of scattering potentials

Since the s-wave scattering results in only a single phase shift, parameterized by **a**, it doesn't matter what the full form of the scattering potential was!

Let's then consider a square well potential:

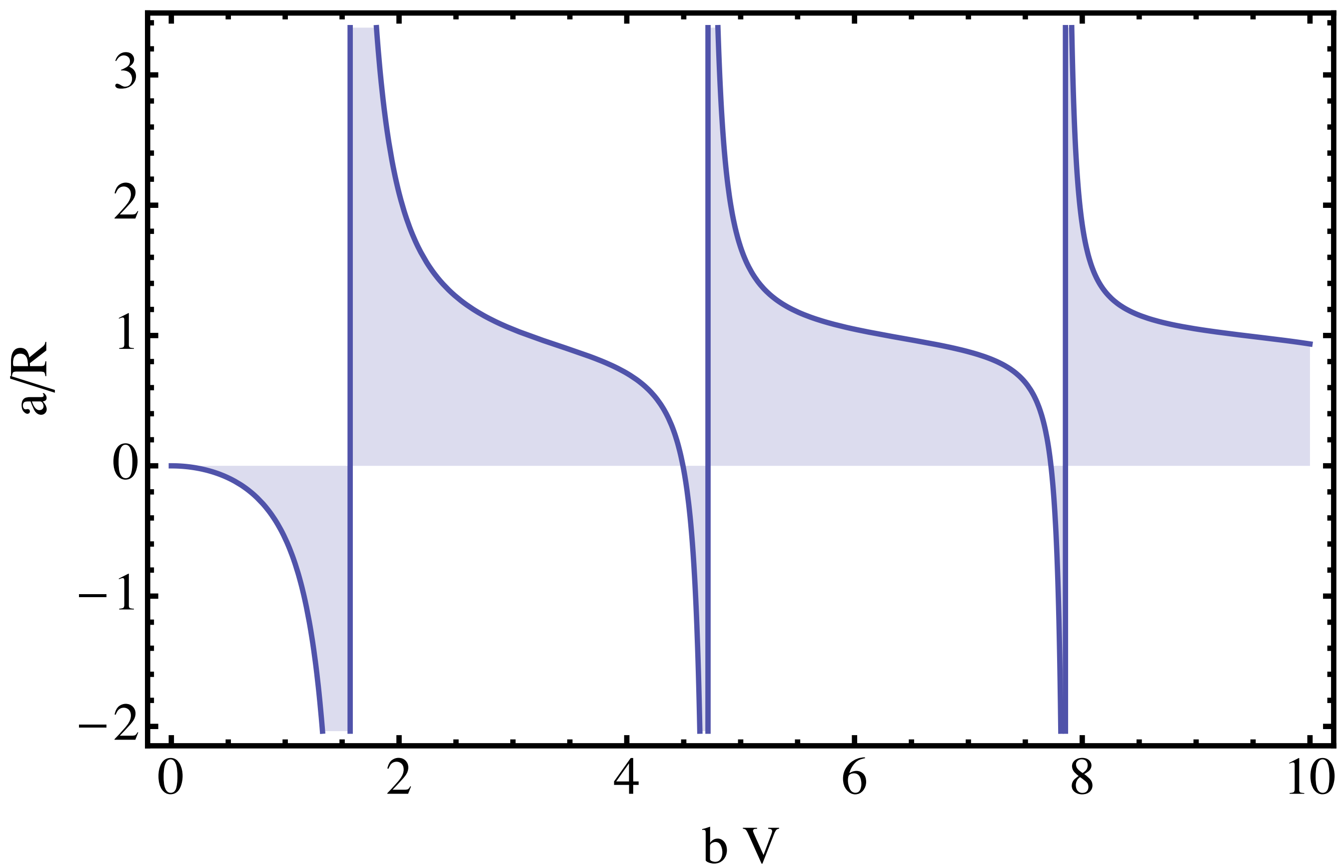


We find:

1. Resonances at

$$bV = (n + 1/2)\pi$$

when each new bound state appears.

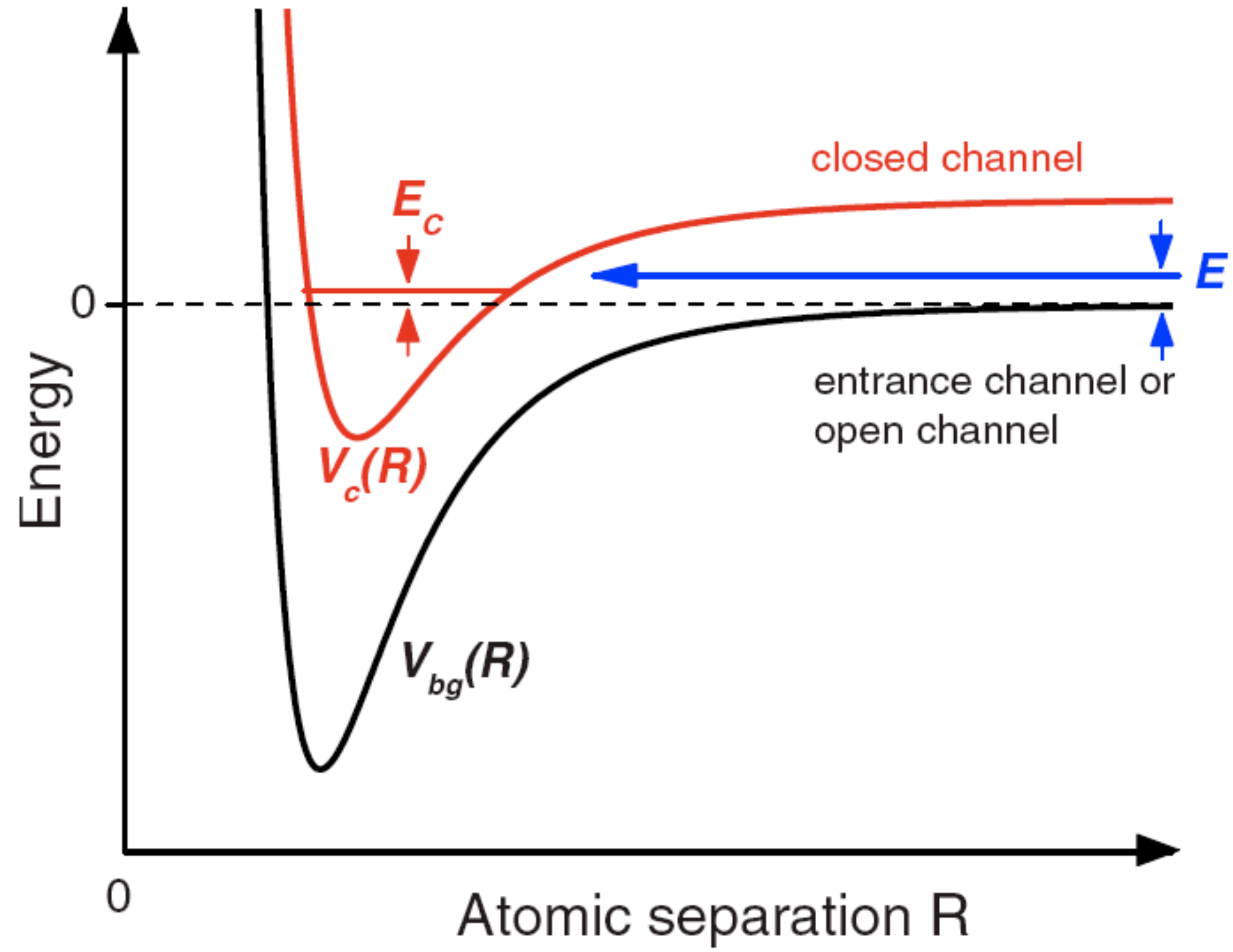


2. Mostly a>0. Near a resonance when a<0 (eg, Li.)

Feshbach resonances

*How can we tune the scattering length **a**?*

Yes! We can tune a molecular bound state into resonance with the free atoms, and affect net phase acquired during the collision.

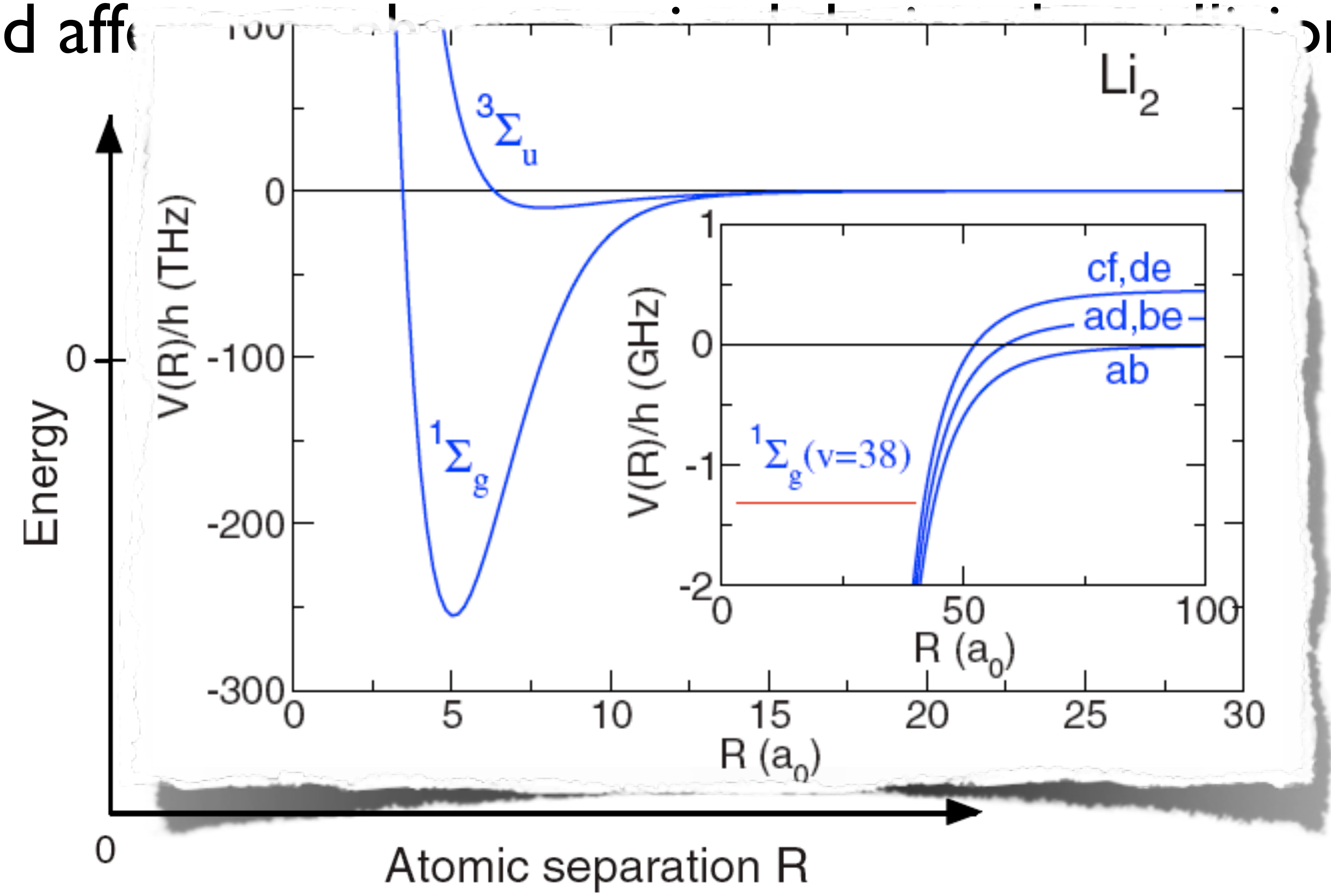


Result is indistinguishable from tuning the single-channel square well: it’s only the phase that matters.

Feshbach resonances

*How can we tune the scattering length **a**?*

Yes! We can tune a molecular bound state into resonance with the free atoms, and aff… …on.



Result is indistinguishable from tuning the single-channel square well: it's only the phase that matters.

Feshbach resonances

Near resonance the scattering length can be described as

$$a(B) = a_{\mathrm{bg}} \left(1 - \frac{\Delta}{B - B_0}\right)$$

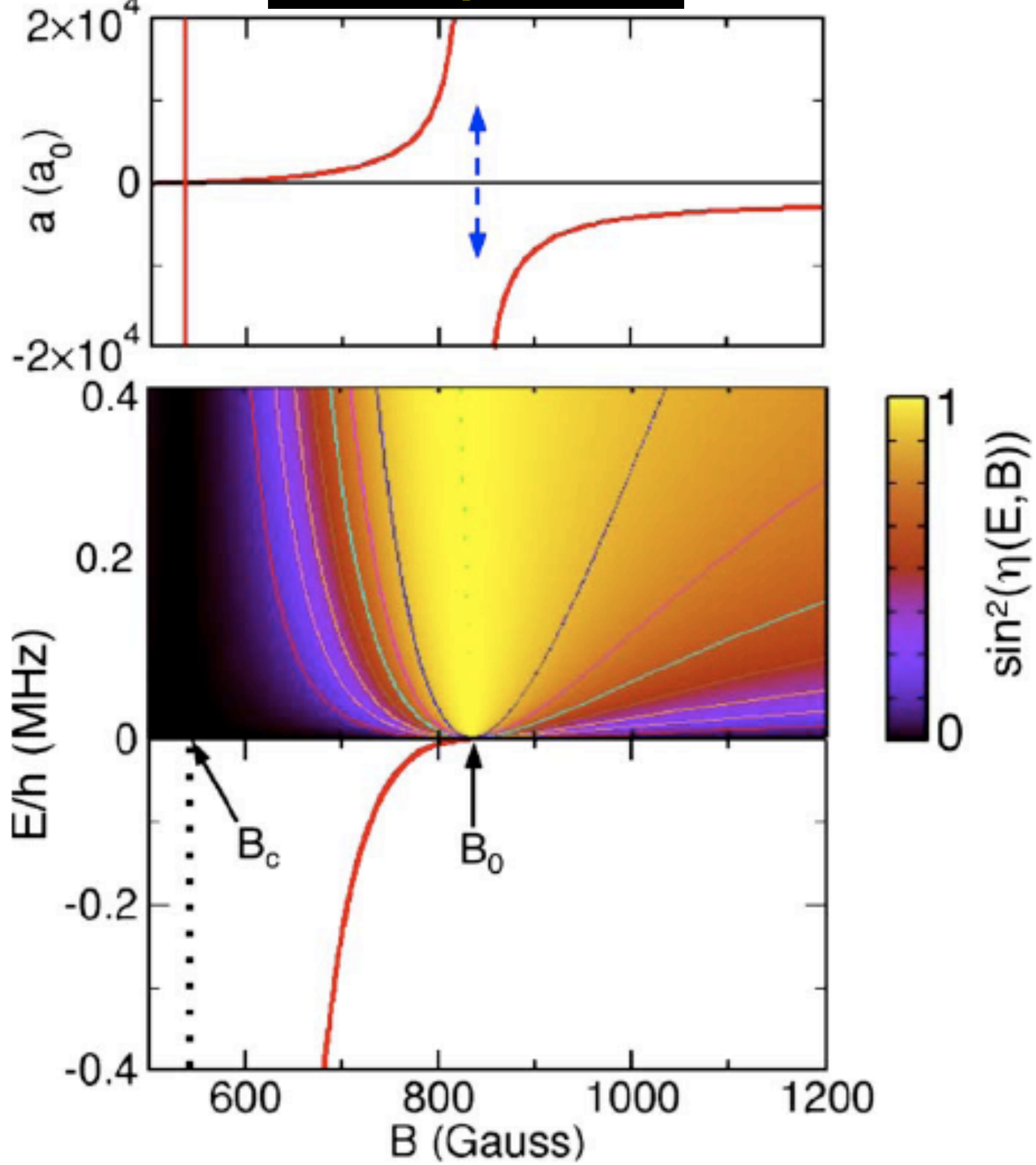
s-wave cross section is

$$\sigma_0 = \frac{4\pi}{k^2} \sin^2 \eta_0$$

For a>0, a bound state exists with binding energy

$$E_b = \frac{\hbar^2}{2\mu a^2}$$

Example: ^{6}Li



Unitarity

Near a Feshbach resonance, |a| diverges. The scattering cross section departs from its low-ka form:

$$\sigma = \frac{4\pi a^2}{1 + k^2 a^2} \to \frac{4\pi}{k^2}$$

This is just a manifestation of the optical theorem, which says that complete reflection corresponds to a finite scattering length. In terms of the de Broglie wavelength,

$$\sigma_{\mathrm{res}} = \lambda_{dB}^2 / \pi$$

You may be more familiar with the resonant atom-photon cross section (which has different constants because it is a vector instead of scalar field):

$$\sigma_{\mathrm{res}} = \frac{3}{2\pi} \lambda_L^2$$

Unitarity

For a many-body system, resonant interactions also saturate but are less easy to quantify. Certainly it is the case that a divergent **a** can no longer be a relevant physical quantity to the problem.

For fermions, the only remaining length scale is k_F^{-1}.

This means that interaction energies must scale with the Fermi E. In particular, for resonant attractive interactions,

$$\mu_{\mathrm{Local}} = (1+\beta)\epsilon_F$$

where $\beta \approx -0.58$ has been measured in various experiments. Using the LDA to integrate over the profile, we find

$$\begin{aligned}\mu_U &= \sqrt{1+\beta}E_{\mathrm{F}} \\ &\approx 0.65E_{\mathrm{F}}\end{aligned}$$

for $a \to -\infty$

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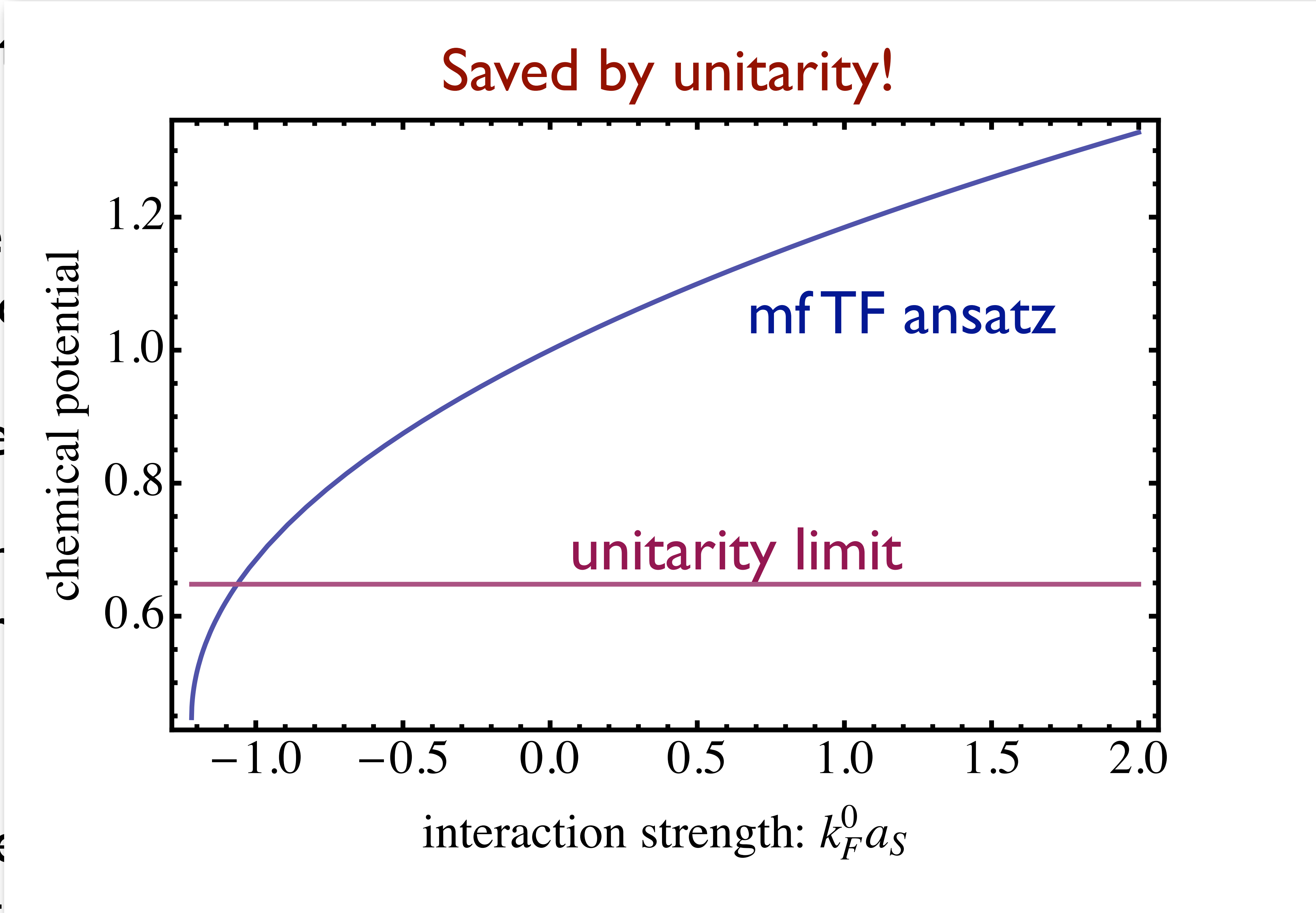

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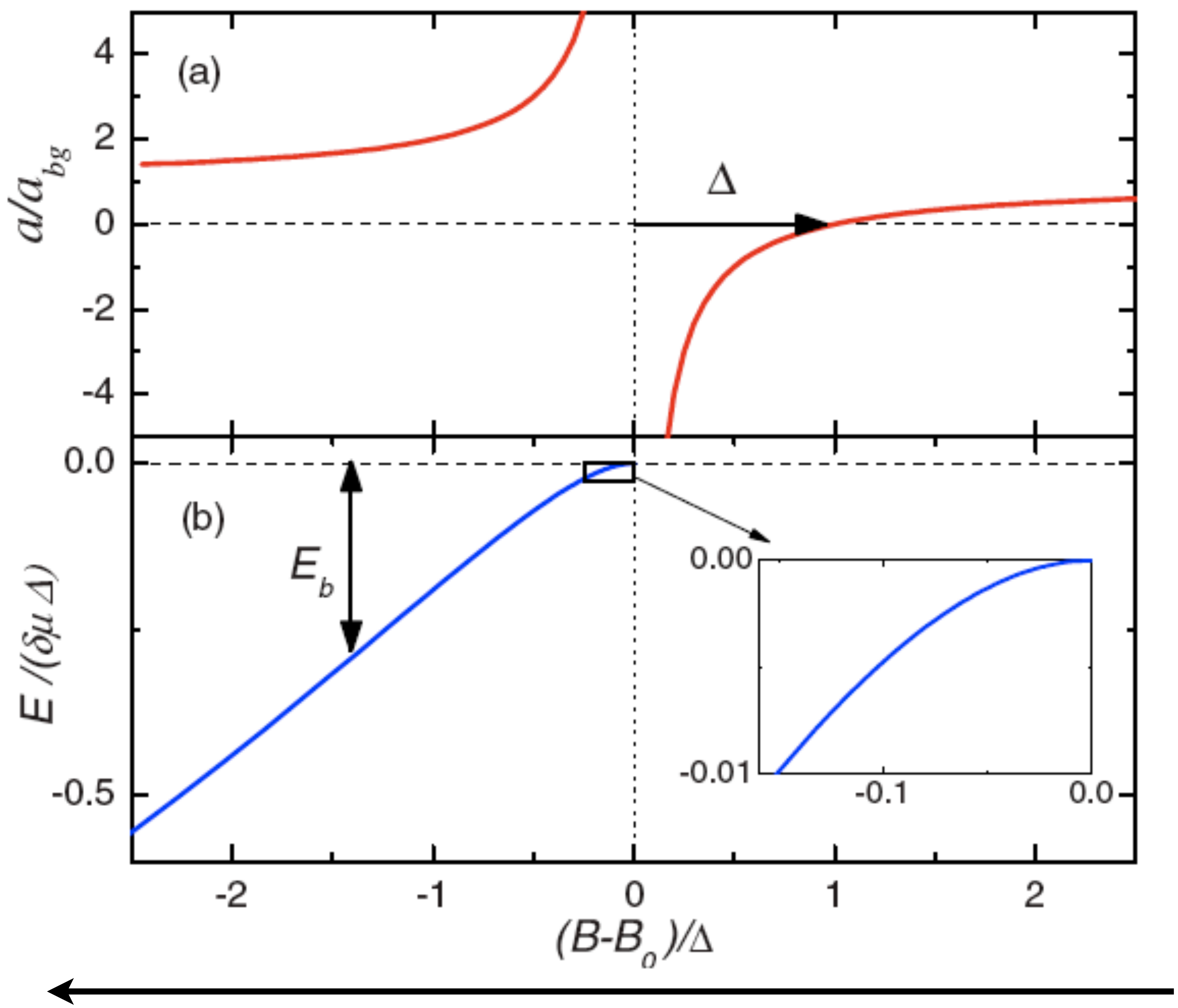
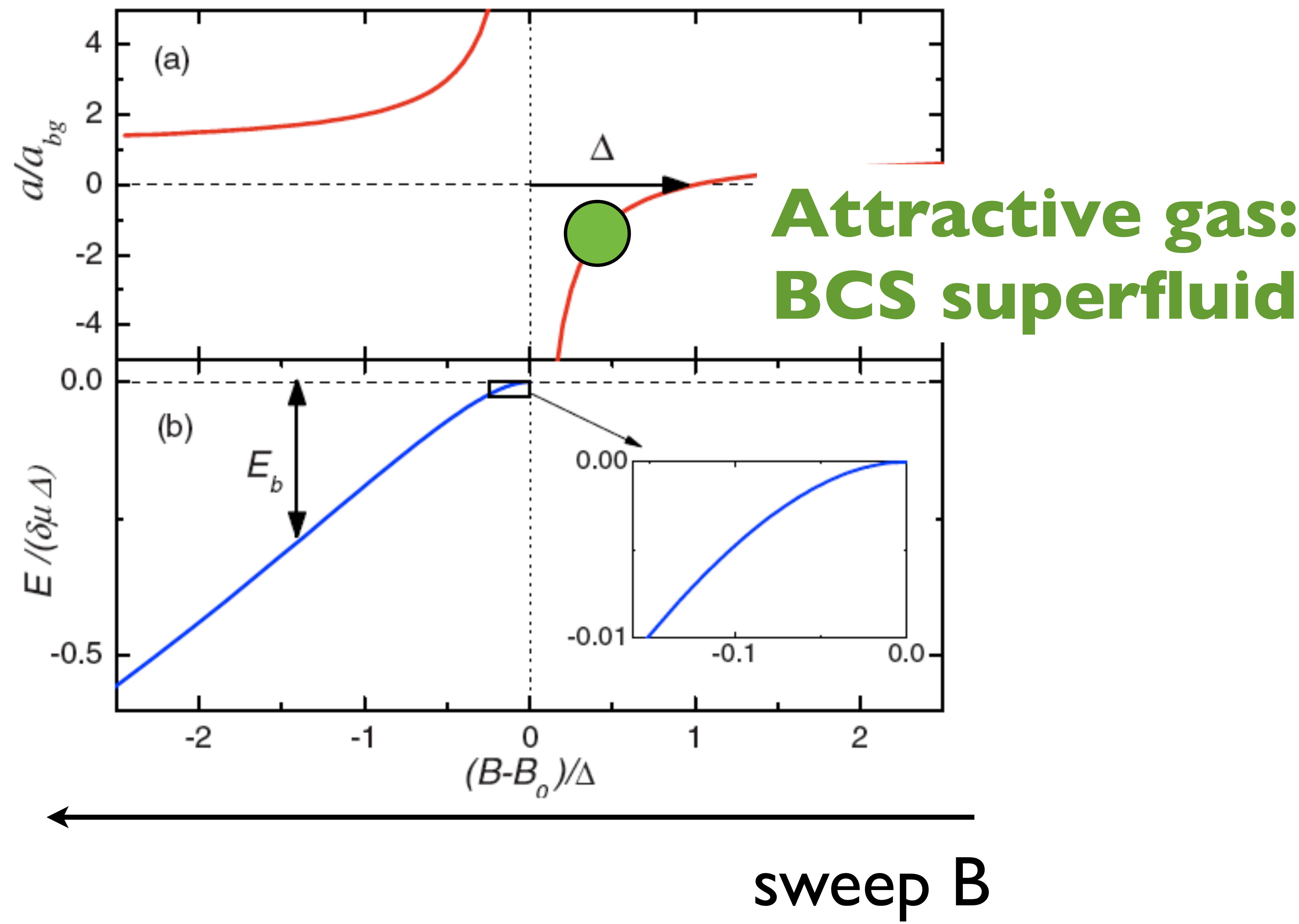
BEC-BCS crossover



sweep B

BEC-BCS crossover



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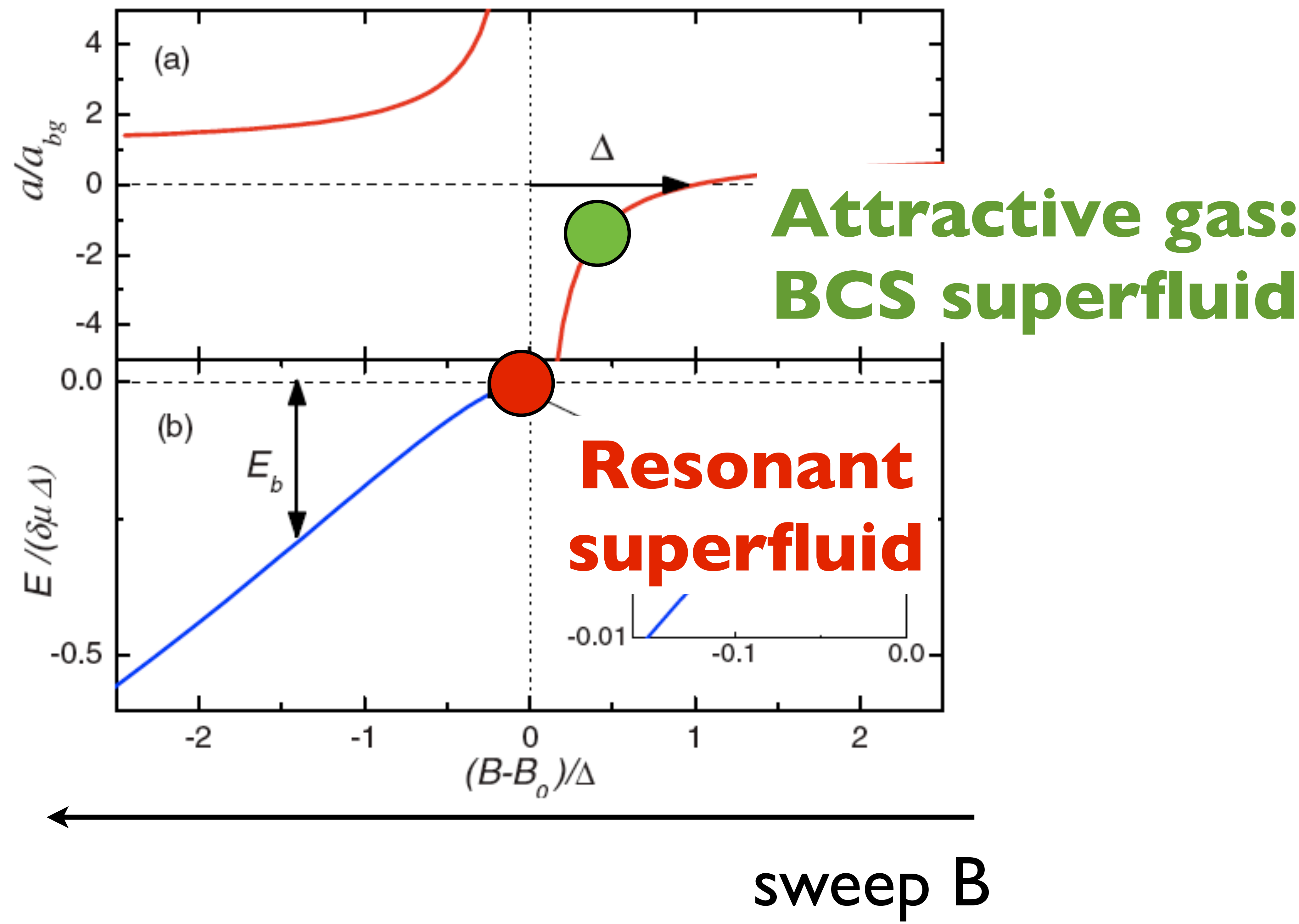
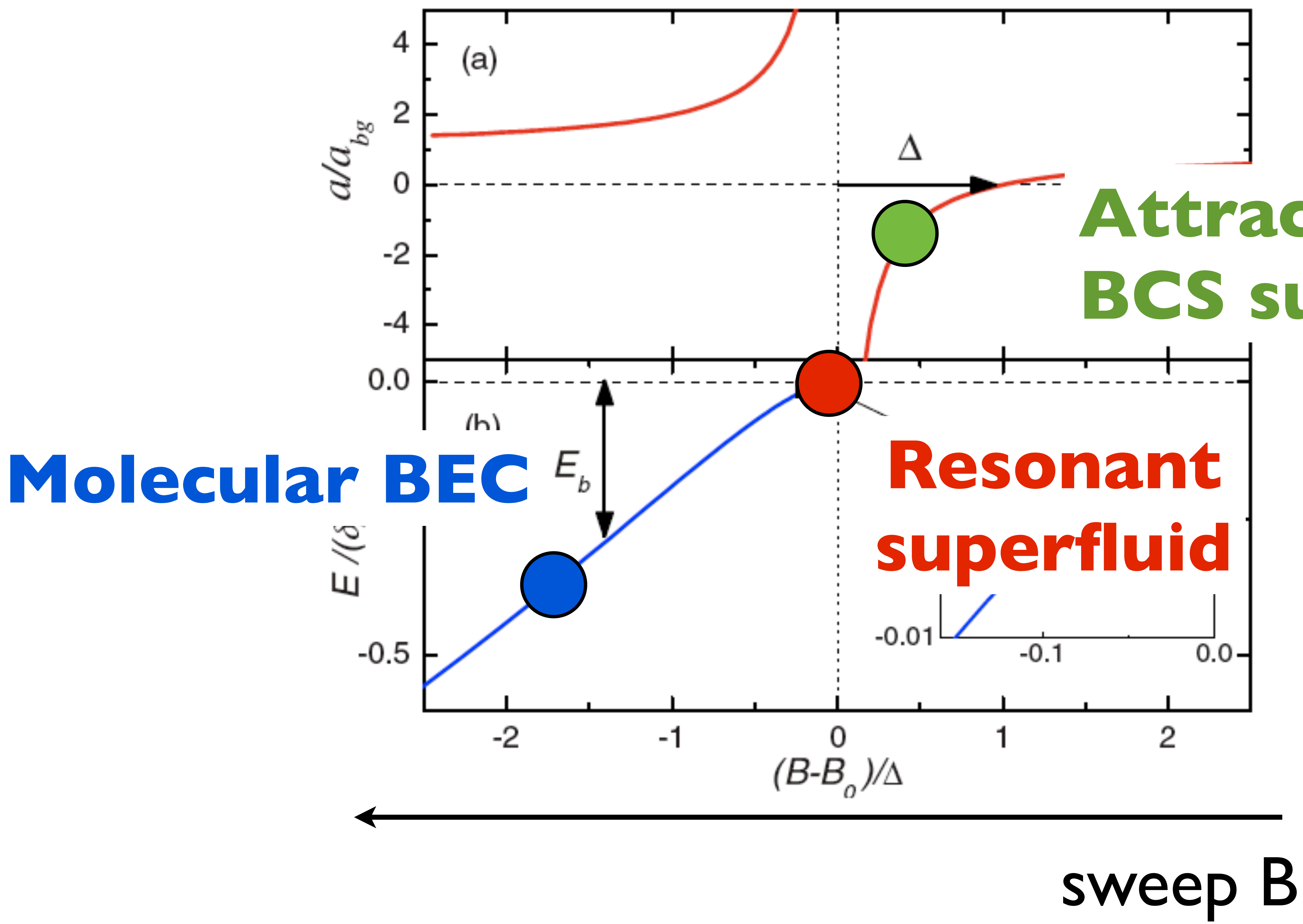
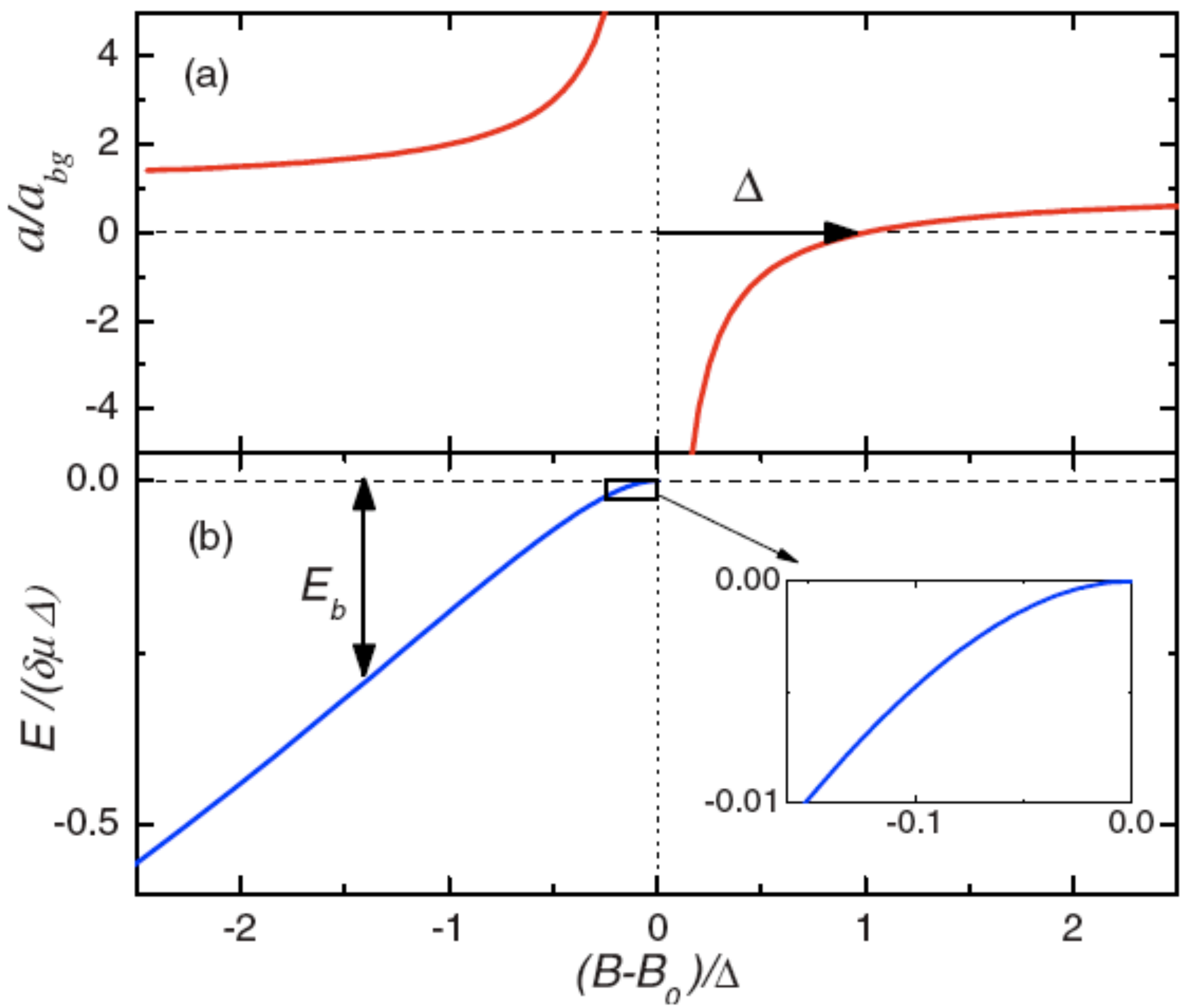


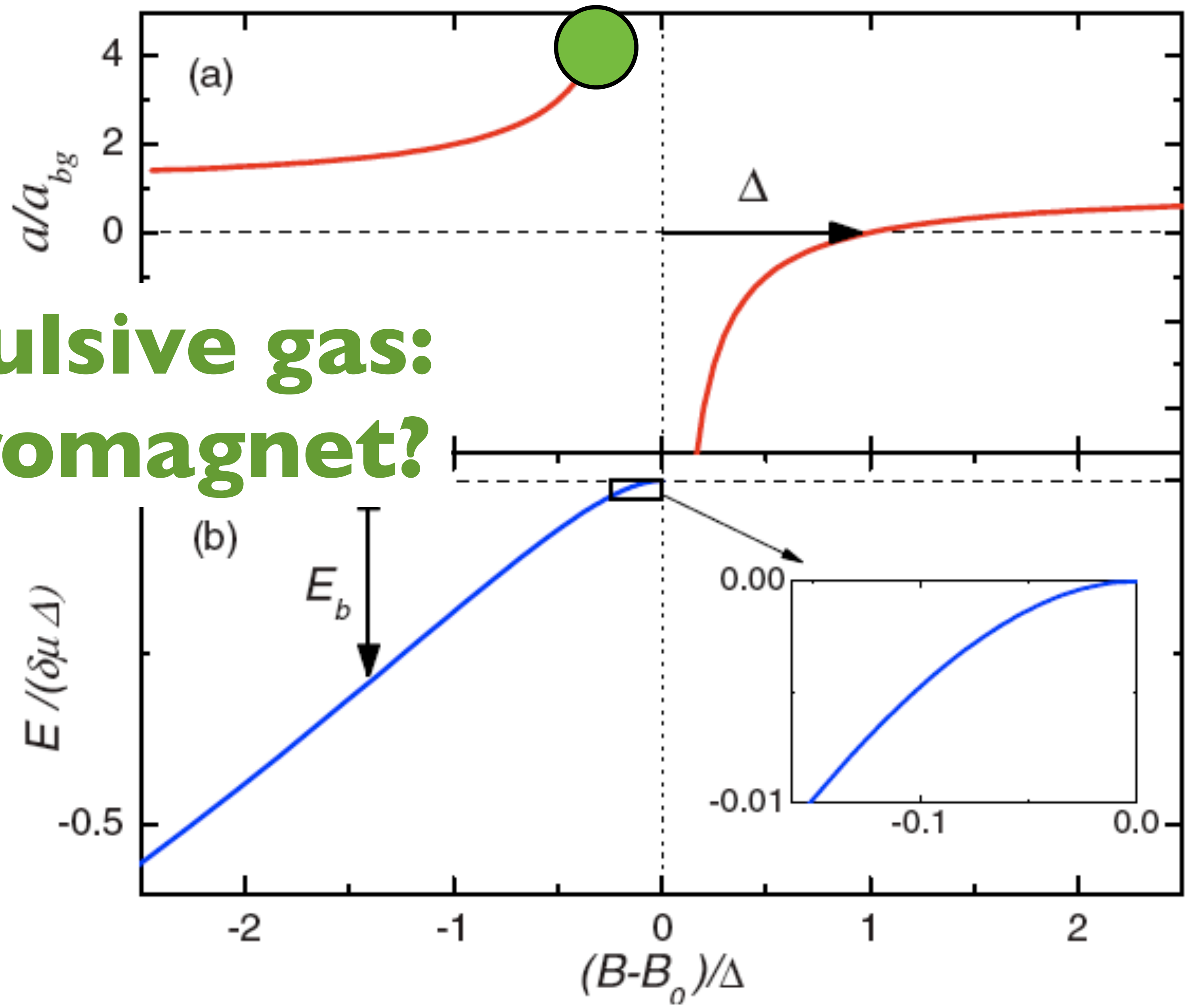
BEC-BCS crossover



Strongly repulsive Fermi gas



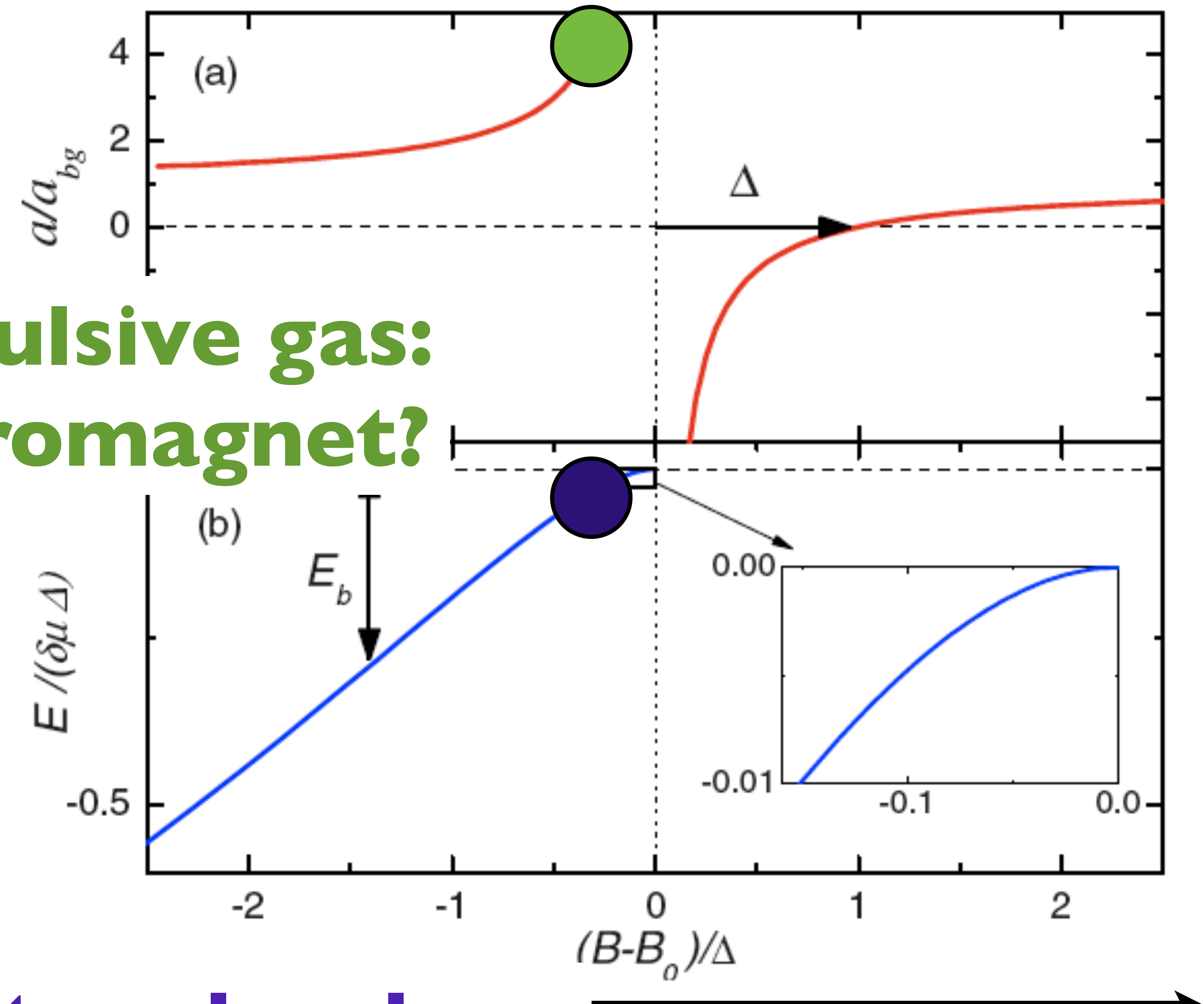
Strongly repulsive Fermi gas



Repulsive gas: Ferromagnet?

sweep B

Strongly repulsive Fermi gas



Repulsive gas: Ferromagnet?

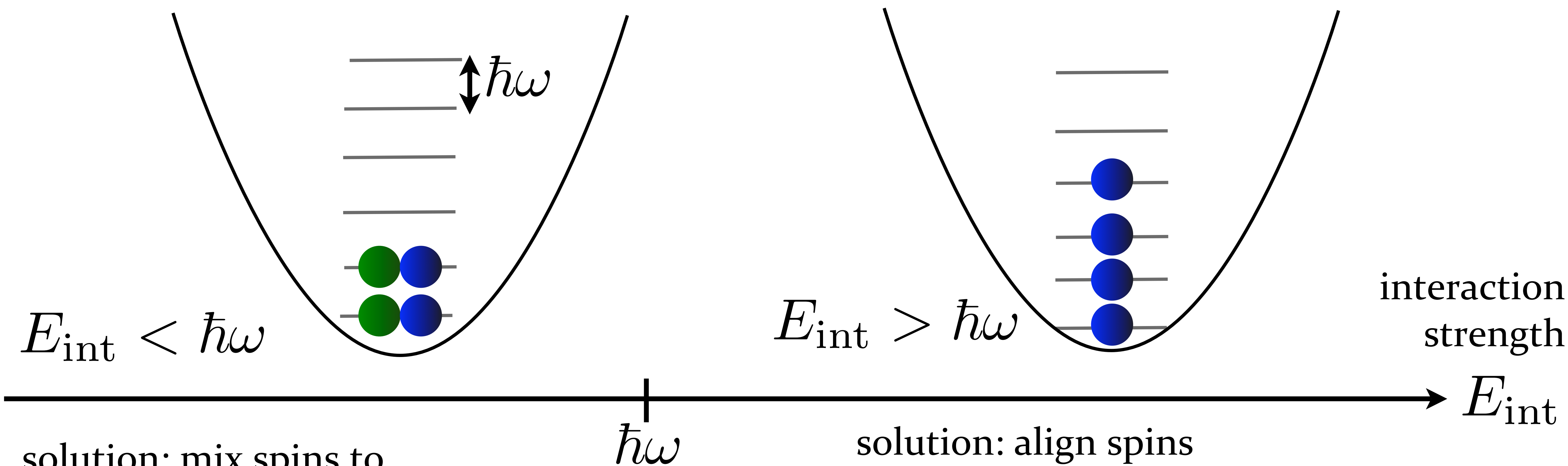
or just molecular decay?

Basic physics of ferromagnetism

Total energy = single-particle energy + interaction energy

$$E_{\text{tot}} = \hbar\omega \sum n_i + E_{\text{int}} N_{\uparrow} N_{\downarrow}$$

For example, what configuration minimizes energy for 4 particles ?



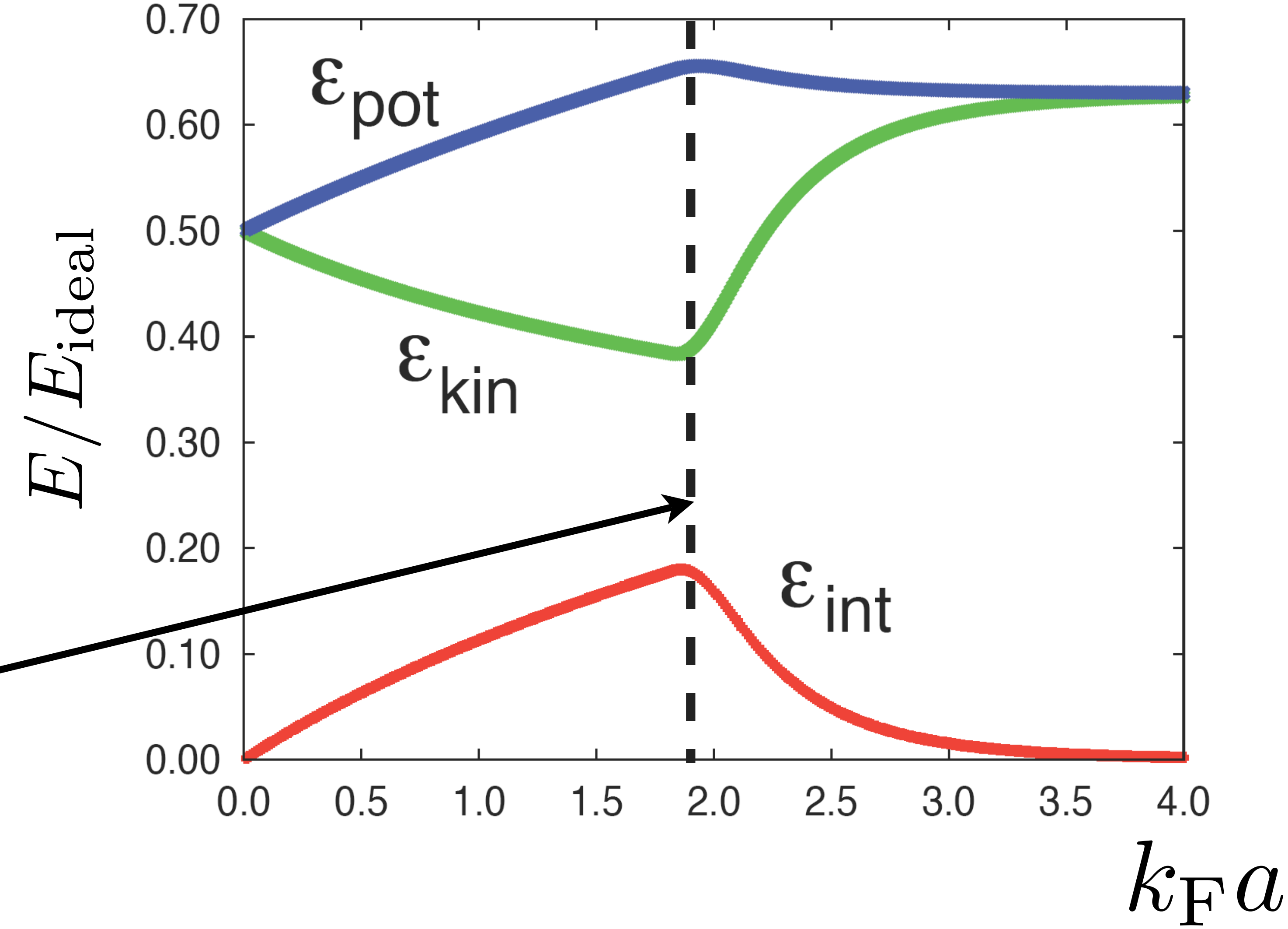
solution: mix spins to minimize maximum occupied level.

solution: align spins (polarize) to minimize interaction energy.

Ferromagnetic configuration is *strongly interacting*: Interaction energy must be higher than single-particle energy.

Mean field calculation of energies

- using calculated density profiles, find kinetic, potential, and interaction energies.
- compare expansion energy with and without tuning to a = 0 regime before release.
- “kink” in energy vs. interaction strength indicates a crossover to ferromagnetic regime



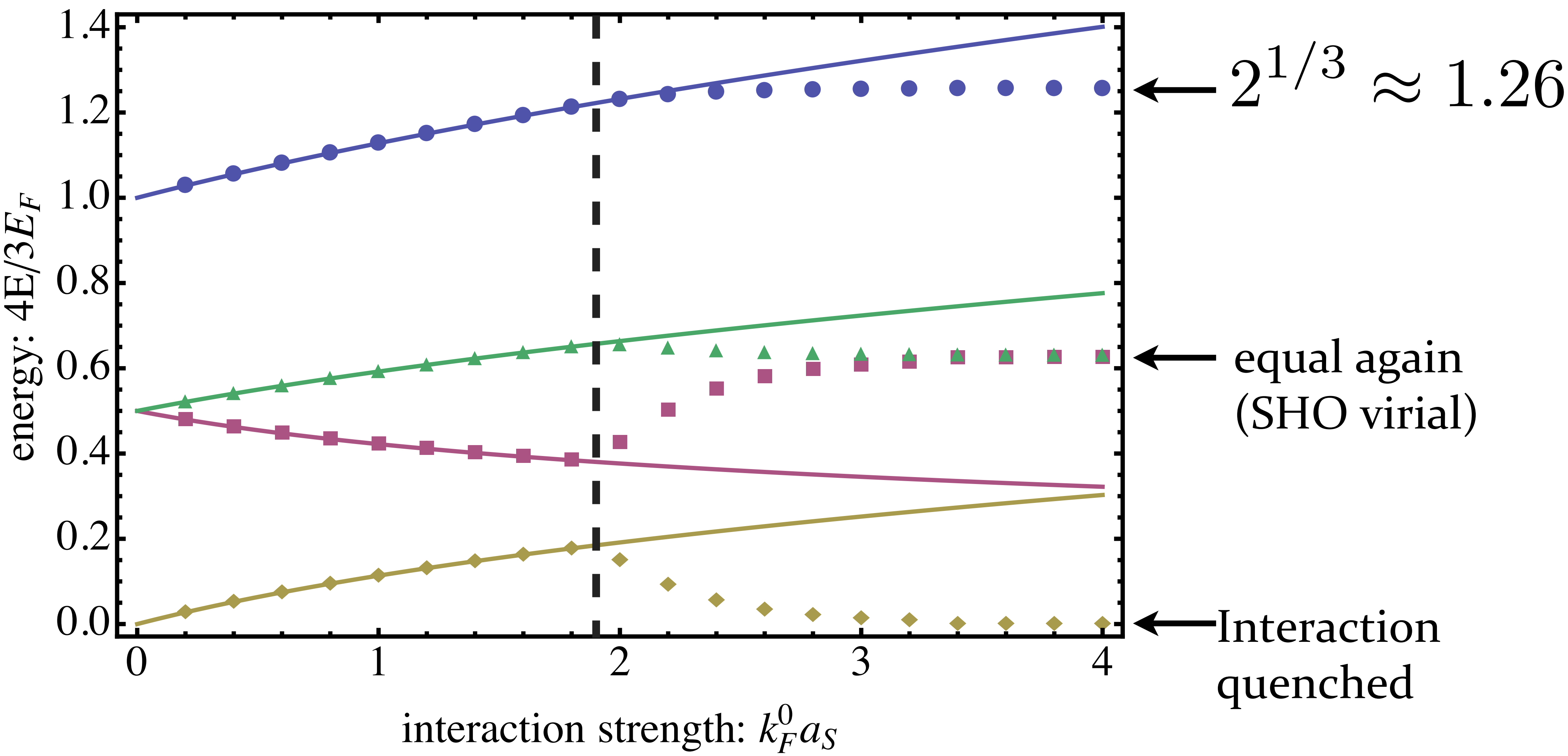
$$E[\{\rho_\sigma(\mathbf{r})\}] = \int d^3\mathbf{r} \left[\frac{3}{5} \sum_\sigma \frac{\hbar^2 (6\pi^2 \rho_\sigma)^{2/3}}{2m} \rho_\sigma(\mathbf{r}) + V(\mathbf{r}) \sum_\sigma \rho_\sigma(\mathbf{r}) + g \rho_\uparrow(\mathbf{r}) \rho_\downarrow(\mathbf{r}) \right]$$

kinetic energy, like $\frac{\hbar^2 k_F^2(\mathbf{r})}{2m}$

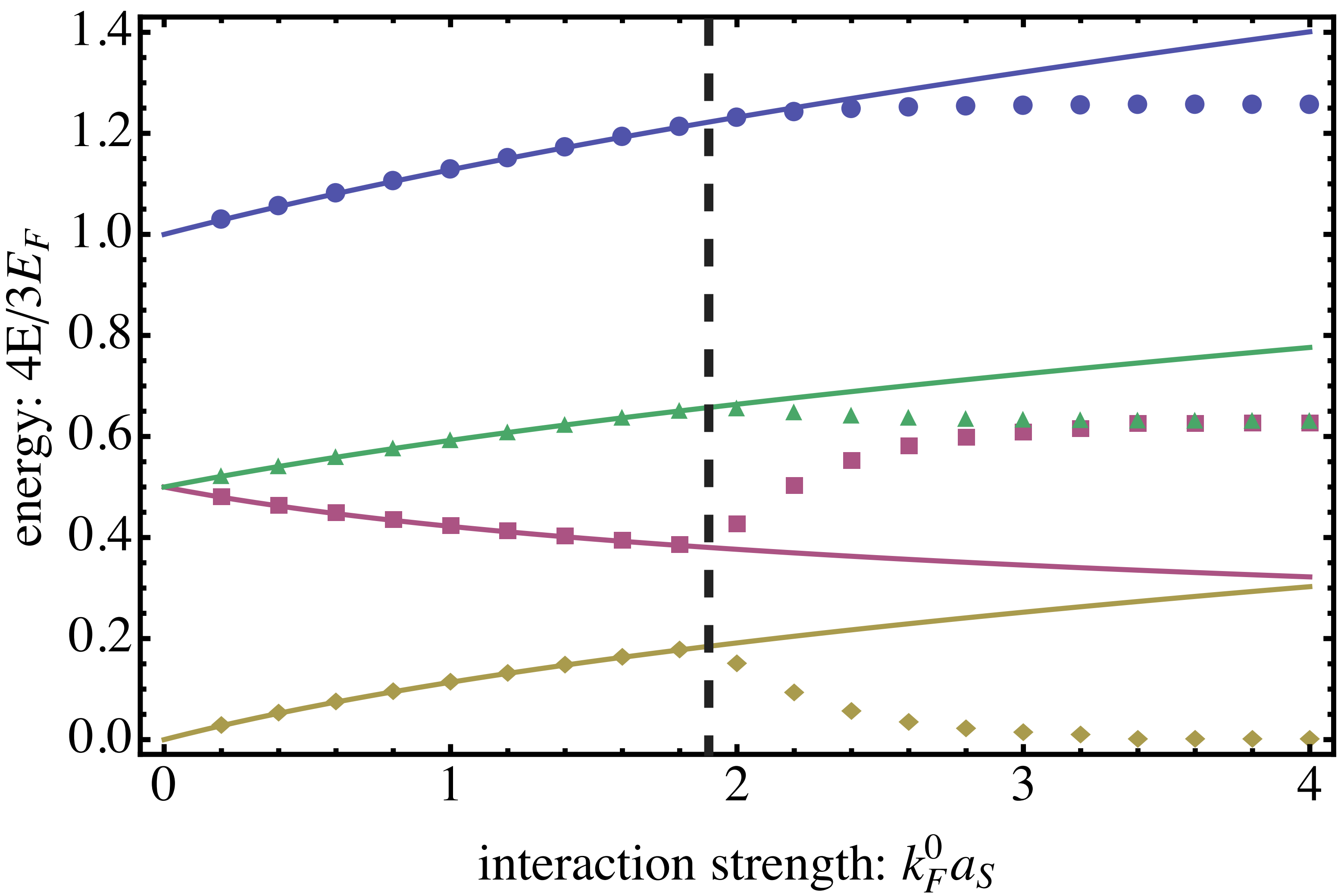
potential energy

interaction energy $g = \frac{4\pi a \hbar^2}{m}$

Comparison to TF ansatz energies



Comparison to TF ansatz energies



$\leftarrow 2^{1/3} \approx 1.26$

← equal again (SHO virial)

← Interaction quenched

Broken symmetry! e.g.

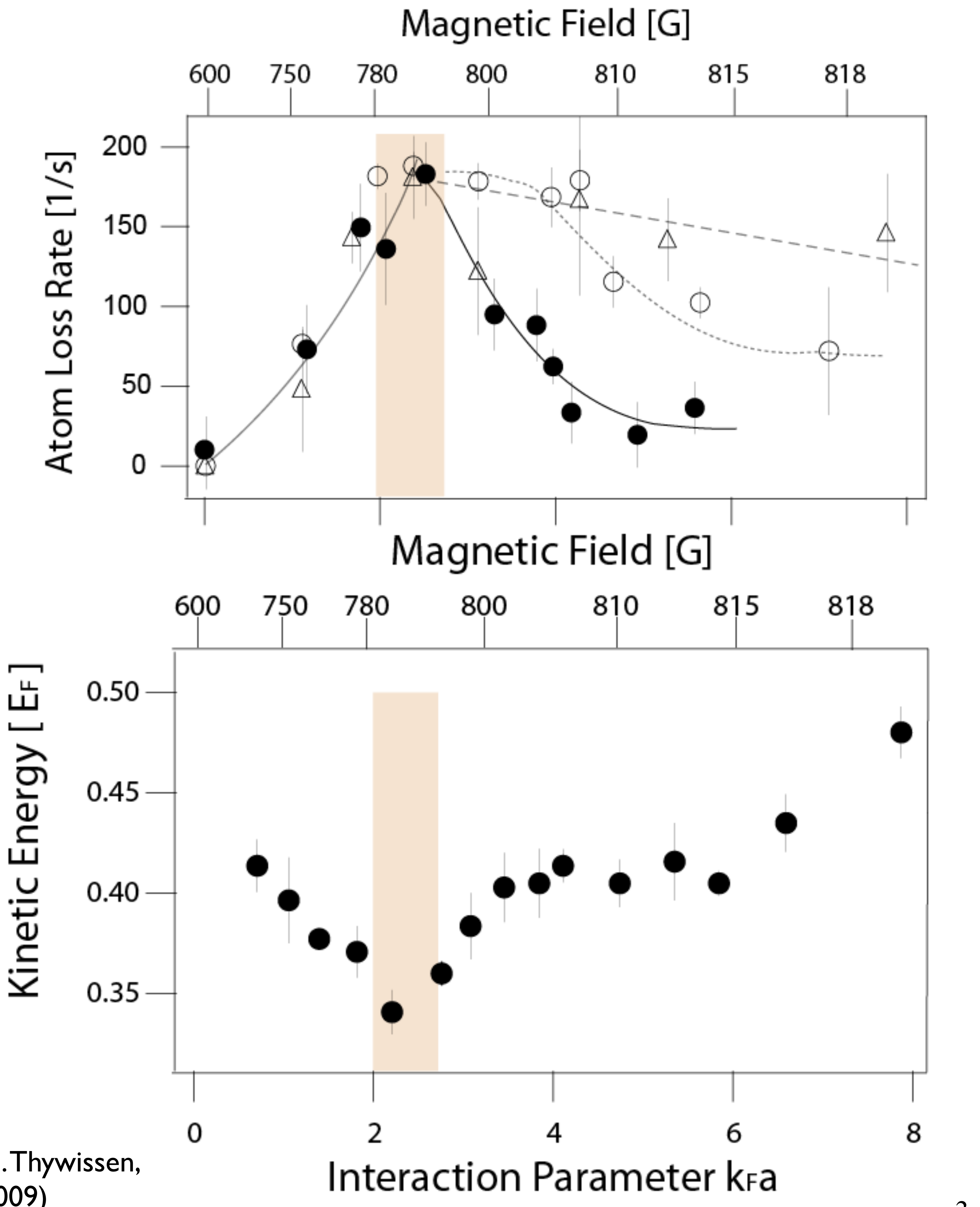
$$N_\downarrow : \quad N/2 \rightarrow 0$$

$$N_\uparrow : \quad N/2 \rightarrow N$$

recall

$$E_\mathrm{F} = \hbar\omega(6N_\sigma)^{1/3}$$

Experimental Observations



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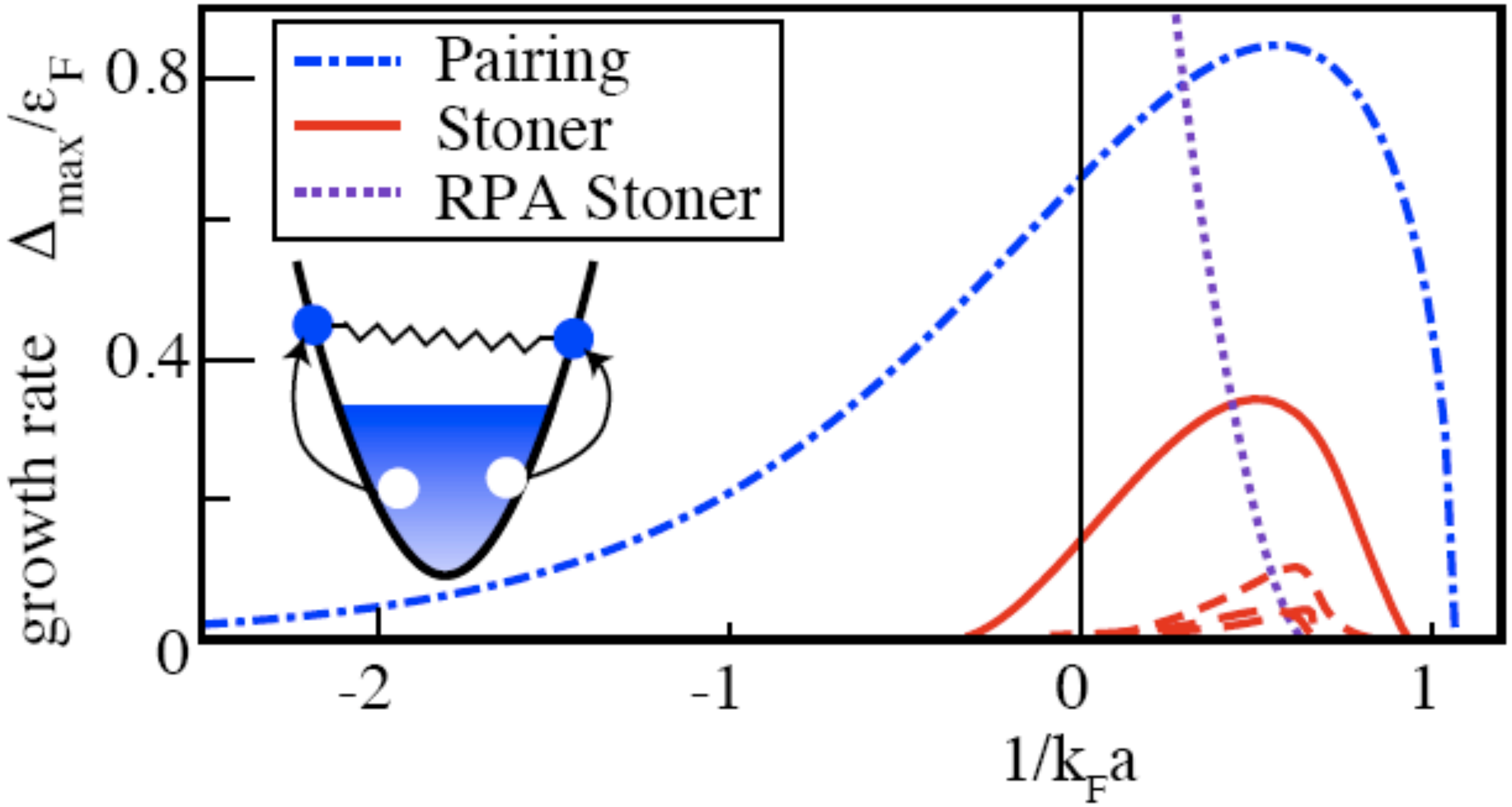
Competitive instabilities

Pekker...Demler: "Competition between pairing and ferromagnetic instabilities in ultracold Fermi gases near Feshbach resonances". arXiv:1005.2366



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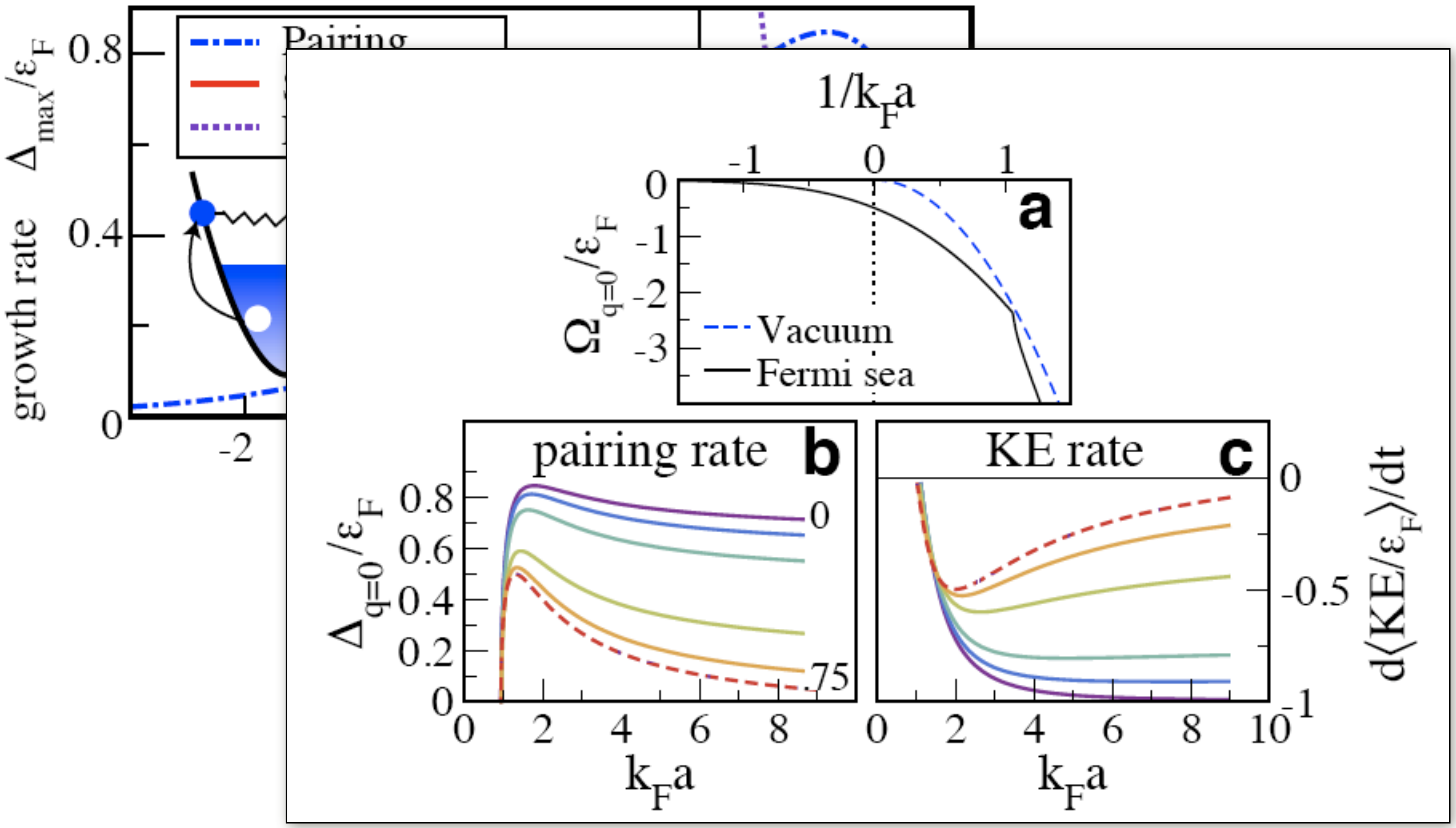
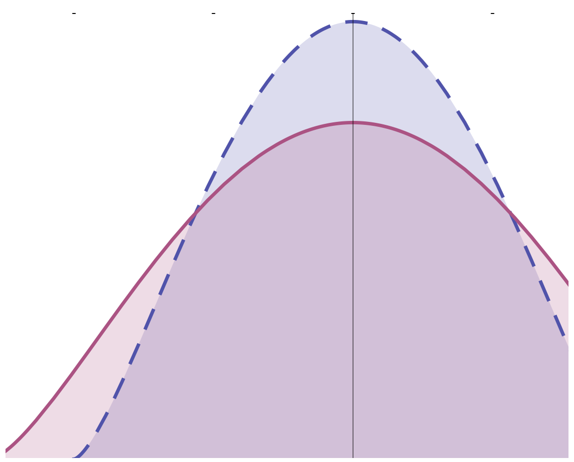


Summary & Conclusion

Overview of trapped cold atoms



1. Non-interacting quantum gases

Thermal DFG; T=0 limit; Bose gases

2. Interacting bosons

GPE; Gaussian Ansatz; TF solution; LDA idea & validity; Hydrodynamics

3. Interacting fermions

Spin degrees of freedom; Mean field variational solution; Scattering theory & unitarity; Feshbach resonances; BEC-BCS crossover; Repulsive gases: Ferromagnetic?

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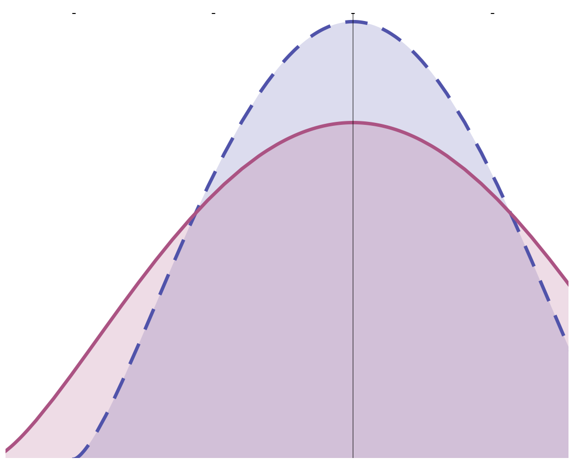

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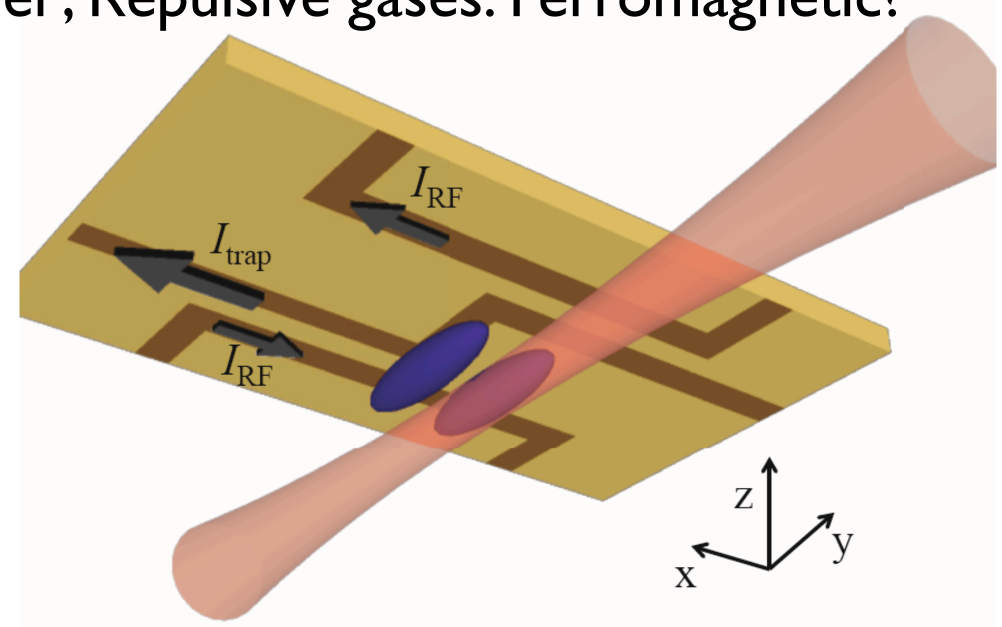
Quantum transport in a double well



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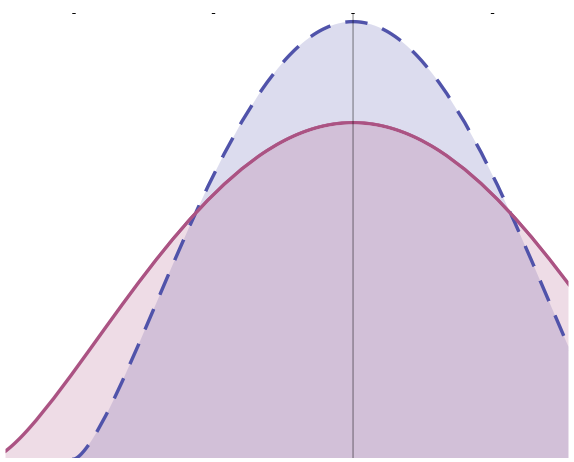

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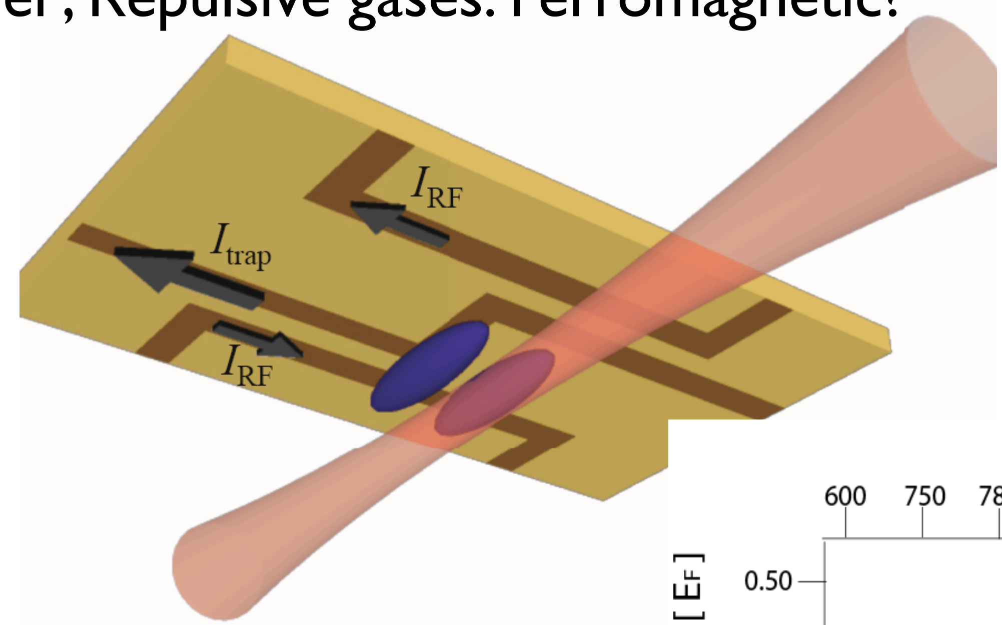
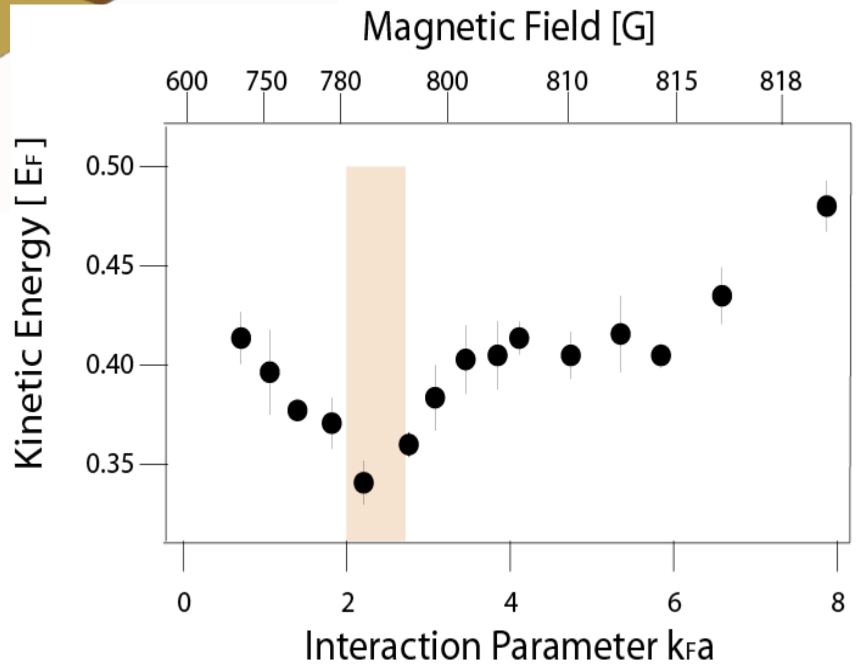


Non-monotonic energetics of a strongly interacting DFG: FM??



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Alex Piggot
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Alan Stummer
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Marcius Extavour (OSA congressional fellow)
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Stefan Myrskog (Morgan Solar)
Karl Pilch
Thorsten Schumm (TU Vienna)
Michael Sprague (Oxford)

Postdoc positions available!

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David Pritchard
Wolfgang Ketterle

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