BATS-R-US: a Multi-Physics and Multi-Application Code

Bart van der Holst, Gábor Tóth

Center for Space Environment Modeling
Center for RAdiative Shock Hydrodynamics
University Of Michigan

http://csem.engin.umich.edu
Space Weather Modeling Framework

BATS-R-US

MHD equations with anisotropic proton pressure
  Magnetosphere application

Electron physics
  Solar wind application

Summary
Space Weather Modeling Framework

SWMF Control & Infrastructure

- Eruption Generator
- Solar Corona
- Inner Heliosphere
- 3D Outer Heliosphere
- Lower Corona
- Synoptic Magnetograms
- Upstream Monitors
- Radiation Belts
- Polar Wind
- Plasmasphere
- Thermosphere & Ionosphere
- Lower Atmosphere
- Ionospheric Electrodynamics
- Radars Magnetometers In-situ
- Couplers
- F10.7 Flux Gravity Waves
- Flare/CME Observations

domains modeled by BATSRUS

SWMF is freely available at http://csem.engin.umich.edu and via CCMC
BATS-R-US
Block Adaptive Tree Solar-wind Roe Upwind Scheme

**Physics**
- Classical, semi-relativistic and Hall MHD
- Multi-species, multi-fluid, anisotropic ion pressure
- (Anisotropic) heat conduction, Alfvén wave turbulence
- Radiation hydrodynamics with grey/multigroup diffusion
- Multi-material, non-ideal equation of state

**Numerics**
- Conservative finite-volume discretization
- Parallel block-adaptive grid
- Cartesian and generalized coordinates
- Splitting the magnetic field into $B_0 + B_1$
- Divergence B control: 8-wave, CT, projection, parabolic/hyperbolic cleaning
- Shock-capturing TVD schemes: Rusanov, HLLE, AW, Roe, HLLD
- Explicit, point-implicit, semi-implicit, fully implicit time stepping

**Applications**
- Sun, heliosphere, magnetospheres, unmagnetized planets, moons, comets…

**100,000+ lines of Fortran 90 code with MPI parallelization**

http://csem.engin.umich.edu
Anisotropic MHD

- Different pressures parallel and perpendicular to the magnetic field
- Space physics applications
  - Reconnection
  - Magnetosphere
  - Coupling with inner magnetosphere models
  - Solar wind heating
- Difficulties
  - What is a proper conservative form?
  - Physical instabilities: fire-hose, mirror, proton cyclotron
- Combinations with more physics
  - Separate electron pressure
  - Hall MHD, semi-relativistic, multi-ion
Resistive MHD with electrons and anisotropic ion pressure

Mass conservation: \( \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \)

Momentum: \( \frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{uu} + P \mathbf{I}) + \mathbf{I} I_e \times \mathbf{B} \mathbf{J} \times \mathbf{B} = \) \( \mathbf{J} \times \mathbf{B} \)

Pressure: \( P = (p_\perp + p_e) I + (p_\parallel - p_\perp) \mathbf{b} \mathbf{b} \)

\( p = \frac{2p_\perp + p_\parallel}{3} \)

Induction: \( \frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0 \)

Pressure: \( \frac{\partial p_\perp}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = \frac{1}{3\tau} (p_\parallel - p_\perp) + \frac{2}{3\tau} (p_e - p) - p_\perp \nabla \cdot \mathbf{u} + p_\perp \mathbf{b} \cdot (\nabla \mathbf{u}) \cdot \mathbf{b} \)

\( \frac{\partial p_\parallel}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = \frac{2}{3\tau} (p_\perp - p_\parallel) + \frac{1}{\tau_{ie}} (p_e - p) - 2p_\parallel \mathbf{b} \cdot (\nabla \mathbf{u}) \cdot \mathbf{b} \)

Electron pressure: \( \frac{\partial p_e}{\partial t} + \nabla \cdot (p_e \mathbf{u}) = (\gamma - 1) \left[ - p_e \nabla \cdot \mathbf{u} + \eta J^2 + \nabla \cdot (\kappa \mathbf{b} \mathbf{b} \cdot \nabla T_e) \right] + \frac{2}{\tau_{ie}} (p - p_e) \)

Electric field: \( \mathbf{E} = -\mathbf{u} \times \mathbf{B} + \eta \mathbf{J} \)

Current: \( \mathbf{J} = \nabla \times \mathbf{B} \)
Shock capturing schemes require conservation laws to get proper jump conditions

- Double adiabatic invariants are not the right conservative variables
- Energy conservation only replaces one of the two pressure equations
- The anisotropy behind a shock is determined by instabilities

We use the energy equation and instability criteria to get proper jump conditions.
### Limiting the Anisotropy

#### Instabilities

- **Fire-hose:**
  \[
  \frac{p_\|}{p_\perp} > 1 + \frac{B^2}{p_\perp}
  \]  
  (destabilized Alfvén wave)

- **Mirror:**
  \[
  \frac{p_\perp}{p_\|} > 1 + \frac{B^2}{2p_\perp}
  \]

- **Proton cyclotron:**
  \[
  \frac{p_\perp}{p_\|} > 1 + 0.847 \left( \frac{B^2}{2p_\|} \right)^{0.48}
  \]

- In unstable regions we reduce anisotropy so it becomes stable

#### Ion-ion, ion-electron and/or wave-ion interactions:

- Push ion pressure towards isotropic distribution with time rate \( \tau \)
Idealized Steady Magnetosphere Run (Xing Meng)

- dipole axis aligned with Z
- Steady solar wind: \( n = 5 \) /cc, \( v = 400 \) km/s, \( B_Z = -5 \) nT
- Solve for energy and parallel pressure near bow shock
- Enforce stability conditions
- Relaxation rate towards isotropy: \( \tau = 20 \) s
Idealized Steady Magnetosphere Run (Xing Meng)

B and log Density

log P

Isotropic MHD
Electron thermal heat conduction

\[ C_T \frac{\partial T_e}{\partial t} = -\nabla \cdot \left[ \kappa_e T_e^{5/2} \frac{BB}{B^2} \cdot \nabla T_e \right] \]

- At resolution changes: interpolation of \( T_e \) at fine AMR block has to be third order to make the scheme second order. Use finite difference approach.

- At block faces:

\[
T_{-1/2, j, k} = \frac{5T_{-1/2, j-3/2, k-3/2} + 30T_{-1/2, j+1/2, k+1/2} - 3T_{-1/2, j+5/2, k+5/2}}{32}
\]

\[
T_{0, j, k} = \frac{8T_{-1/2, j, k} + 10T_{1, j, k} - 3T_{2, j, k}}{15}
\]

- Similar interpolation schemes are used for the block edges.
Collisional electron heat conduction from inner coronal boundary to approximately $5R_{\text{sun}}$, smoothly diminishes between $5R_{\text{sun}}$ and $10R_{\text{sun}}$.

Heating of protons by Alfvén wave dissipation.

Heating of electrons by collisional coupling with protons.

\[
\frac{\partial p_i}{\partial t} + \nabla \cdot (p_i \mathbf{u}) + (\gamma - 1) p_i \nabla \cdot \mathbf{u} = (\gamma - 1) \left[ Q_i + \lambda_{ei}(T_e - T_i) \right],
\]
\[
\frac{\partial p_e}{\partial t} + \nabla \cdot (p_e \mathbf{u}) + (\gamma - 1) p_e \nabla \cdot \mathbf{u} = (\gamma - 1) \left[ -\nabla \cdot \mathbf{q}_e + \lambda_{ei}(T_i - T_e) \right],
\]
\[
\mathbf{q}_e = -\kappa_e T_e^{5/2} \frac{BB}{B^2} \cdot \nabla T_e
\]
Wind acceleration: work done by wave pressure force

Coronal heating: formulation of the Kolmogorov dissipation by Hollweg (1986)

\[
\frac{\partial E^+_w}{\partial t} + \nabla \cdot \left[ E^+_w (u + u_A) \right] + p^+_w \nabla \cdot u = -Q^+,
\]

\[
\frac{\partial E^-_w}{\partial t} + \nabla \cdot \left[ E^-_w (u - u_A) \right] + p^-_w \nabla \cdot u = -Q^-,
\]

Ion heating \( Q_i = Q^+ + Q^- = \frac{E^+_{w 3/2}}{L \sqrt{\rho}} + \frac{E^-_{w 3/2}}{L \sqrt{\rho}} \), \( L = C/\sqrt{B} \).

Free parameter C in heating scale height L
Selected field lines showing streamer belt

Meridional slice showing bimodal wind due to Alfvén waves
High electron temperature above streamer due to heat conduction, cool electrons in fast wind due to adiabatic expansion

Protons mostly heated in coronal hole due to Alfvén waves
Non-alignment of magnetic- and rotation-axis gives appearance of fast and slow streams

Result: compression of plasma seen as spiral arms

Comparison with ACE satellite at L1 point
Anisotropic proton pressures in BATSRUS
- Enforcing stability limits
- Optional (ad hoc) relaxation term
- Comparison of magnetosphere simulations with observations in progress

Electron Physics in BATSRUS
- Separate electron pressure and electron thermal heat conduction
- Comparison of solar wind at 1AU with observations is rather good

Plans
- Combined electron pressure and anisotropic proton pressure in both the solar wind and magnetosphere
- Include counter-propagating Alfvén waves (partial reflection due to inhomogeneities in background)